"Black holes & Exotic branes in string theory"

Masaki Shigemori (YITP, Kyoto U)

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Refs:

- Peet, hep-th/0008241
- de Boer + MS, 1209.6056
- Bena, El-Showk, Vercnocke "BHs in string theory" (avaliable on internet)

0. Intro

- String thy. has various extended objects. e.g. D-branes
  ↳ played crucial role in understanding nonpert. phys of string thy.
  e.g. BH, AdS/CFT...

- String thy. contains low-codim branes such as D7, D8, D9
  (# of transverse directions.)

  their role hasn't been appreciated.

  ↳ They must also be important for nonpert. phys. of string thy.

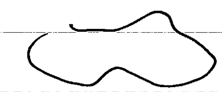
  e.g. BH ?

  Interesting on their own right.

I String theory & branes

I.1 What's s.t.?

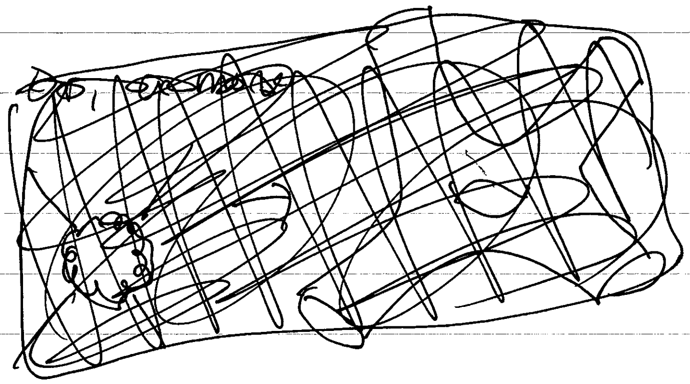
Theory of string, extending in one spatial dir.

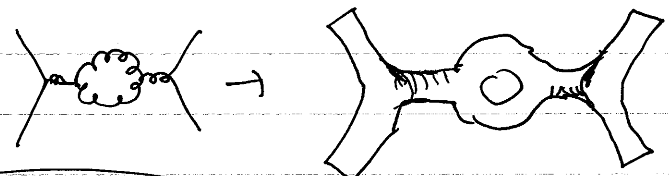


$$S^{F1}_{NG} = -\frac{1}{2\pi l_s^2}\int \sqrt{-\det G}$$

l_s : str. lth $\sim 10^{-35}$ m?

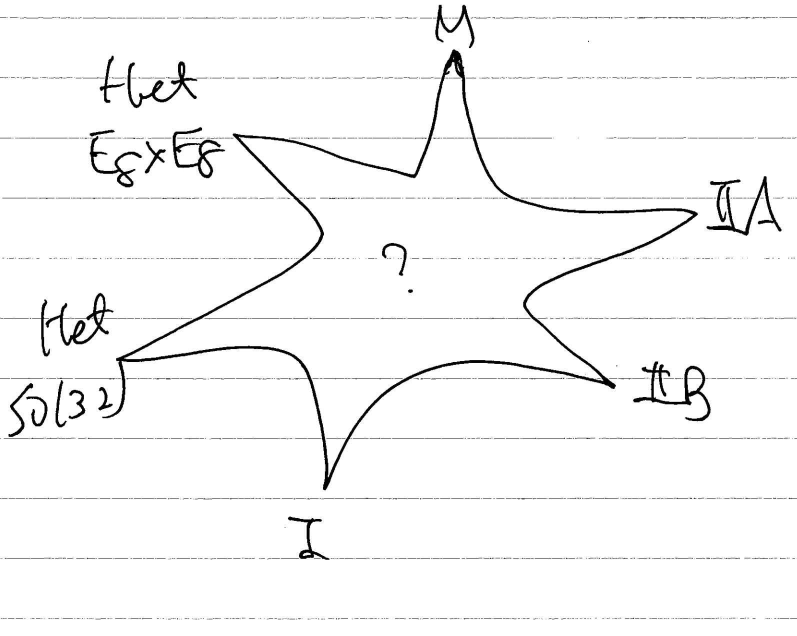
$\alpha' = l_s^2$





A more modern view: ← obtained by NP understanding

A "tentative" name for the huge monster thy. which has strings as fund. excitations in certain lims.



We'll focus on Type II and M-theory.

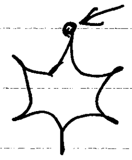
- Reduces in low E to 10D Type II sugra and 11D sugra.

- Non-perturbatively, includes extended obj ("branes") in addition to fund. string.

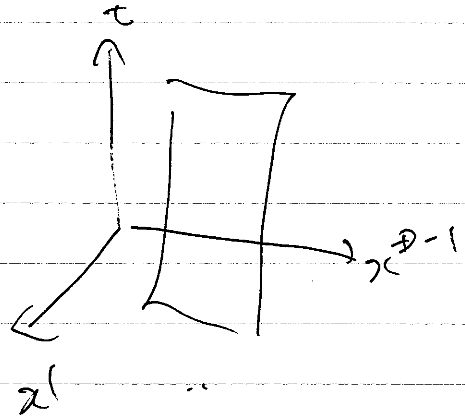
II : D-branes, NS5, KKM

M: M2, M5

- Note the meaning of "M" in these lectures (I only mean quantum membrane thy that reduces to 11D sugra in low E). i.e. tip on top



I.2 p-branes →



extends in p spatial directions, (worldvolume is (p+1)-dim'l)

- Couples electrically to a (p+1)-form A_{p+1}

$$A_{p+1} = \frac{1}{(p+1)!} A_{\mu_1 \cdots \mu_{p+1}} dx^{\mu_1} \wedge \cdots dx^{\mu_p}$$

$$S_{min} = Q \int A_{p+1}$$

- A_{p+1} is a gauge field.
  - gauge trfm
  $$\delta A_{p+1} = \delta \lambda_p$$
  - field str (p+2)-form
  $$F_{p+2} = dA_{p+1} \quad \text{(gauge inv)}$$

- Action

$$S_{gauge} = -\frac{1}{2}\int d^D x \sqrt{-g}\, \underbrace{|F_{p+2}|^2}_{\frac{1}{(p+2)!} F_{\mu_1 \cdots \mu_{p+2}} F^{\mu_1 \cdots \mu_{p+2}}}$$

diff. forms (convention)

p-form

$$\omega_p = \frac{1}{p!}\omega_{\mu_1 \cdots \mu_p} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}$$

Hodge dual in D-dims

$$(*\omega)_{\mu_1 \cdots \mu_{D-p}} = \frac{1}{p!}\epsilon_{\mu_1 \cdots \mu_{D-p}}{}^{\nu_1 \cdots \nu_p}\omega_{\nu_1 \cdots \nu_p}$$

$$*(dx^{\nu_1} \wedge \cdots dx^{\nu_p}) = \frac{1}{(D-p)!} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-p}} \epsilon_{\mu_1 \cdots \mu_{D-p}}{}^{\nu_1 \cdots \nu_p}$$

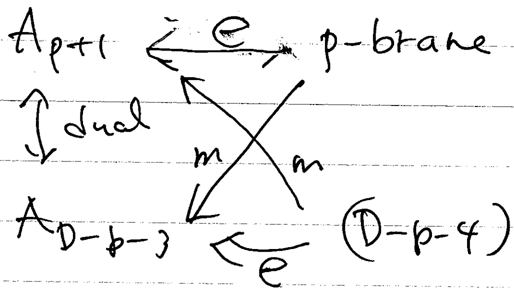
Define e/m dual field

$$F_{D-p-2} \equiv \pm * F_{p+2}$$

$\pm$ depends on conv., purpose

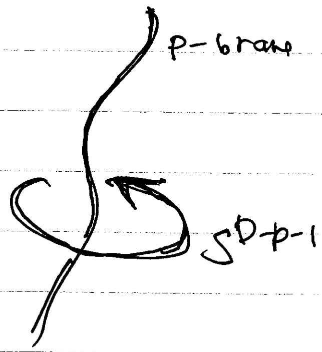
dual gauge pot:

$$F_{D-p-2} = dA_{D-p-3}$$ → elect'ly couples to $(D-p-4)$-brane



p-brane chg: measured with dual field.

$$Q_{\text{p-brane}} = \int_{S^{D-p-2}} F_{D-p-2}$$



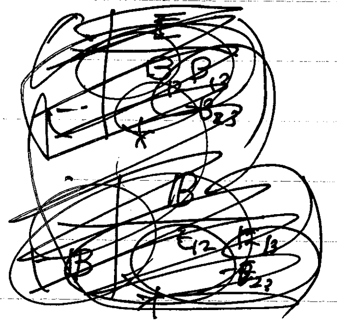
Cf. elec. chg in 4D
= 0-brane

$$S_{min} = Q_e \int A_1 \qquad A: \text{ Maxwell gauge field.}$$

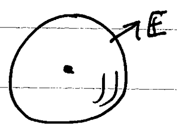
$$\underbrace{F_2}_{\text{field str.}} = dA_1, \quad \underbrace{\tilde{F}_2}_{\text{dual field str.}} = *F_2$$

$F_{0i} = E_{0i}$
$F_{ij} = \epsilon_{ijk} B_k$

$\tilde{F}_{0i} = B_i$
$\tilde{F}_{ij} = \epsilon_{ijk} E_k$



$$Q_e \sim \int_{S^2} \tilde{F}_2 = \int_{S^2} E \cdot dS$$



I.3. Branes in s.t.

- Spacetime fields in s.t. (know from pert. s.t.)

	IIA				IIB		
NS⊗NS →	$g_{\mu\nu}$	(2)	35		$g_{\mu\nu}$	(2)	35
	B_2	[2]	28		B_2	[2]	28
	Φ	[0]	1		Φ	[0]	1
R⊗R →	C_1	[1]	8		C_0	[0]	1
	C_3	[3]	56		C_2	[2]	28
					C_4	[4]	35
NS⊗R, R⊗NS →	$\Psi^{I=1,2}_{\mu\alpha}$		56+56'		$\Psi^{I}_{\mu\alpha}$		56+56
	λ^{I}_{α}		8+8'		λ^{I}_{α}		8+8'
			128+128				128+128.

MW: $16_R \xrightarrow{\text{EOM}} 8_R$

$\underset{8}{\mu}\ \underset{8}{\alpha} \to 8 \cdot 8 = 64 \xrightarrow[\gamma\text{-trace}]{} 56$

- M-thy (known from 11D sugra)

g_{MN} 44

A_3 84

Ψ_{MA} 128

Maj: $32 \xrightarrow[\text{EOM}]{} 16$

$16 \times (\underset{\text{transv.}}{9} - \underset{\gamma\text{-trace}}{1}) = 128.$

▶ Branes in IIA/B, M

IIA

D0 C_1 e/m
D2 C_3
D4 C_5
D6 C_7

F1 B_2
NS5 B_6

IIB

D(-1) C_0
D1 C_2
D3 C_4
D5 C_6
D7 C_8

M

M2
M5

Dp branes: $\underbrace{\text{realized}}_{\text{in pert. string}}$ as an object on which strings can end with D. b.c.

$$S_{Dp} = -\frac{1}{(2\pi)^p g_s l_s^{p+1}} \int d^{p+1}\xi \sqrt{-\det(G)}$$

$$+ \frac{1}{(2\pi)^p l_s^{p+1}} \int C_{p+1} \quad \left(\begin{array}{c}\text{ignored coupling} \\ \text{to } B_2, \\ F_2\end{array}\right)$$

Dp tension: $T_{Dp} = \dfrac{1}{(2\pi)^p g_s l_s^{p+1}}$

NP Non-pert.

Mass $M = T_{Dp} V_p$

Volume that Dp is wrapping

$$T_{F1} = \frac{1}{(2\pi) l_s^2}$$

$$T_{NS5} = \frac{1}{(2\pi)^5 g_s^2 l_s^6}$$

$$T_{M2} = \frac{1}{(2\pi)^2 l_{11}^3}$$

$$T_{M5} = \frac{1}{(2\pi)^5 l_{11}^6}$$

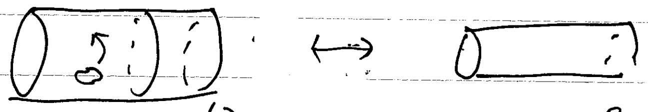
I.4

• Duality in string theory

Discrete sym. of string theory

Relates different descriptions of the same sys. (gauge sym)

T-duality (IIA $\leftrightarrow$ IIB)

S^1_y  $S^1_{\tilde{y}}$ $\qquad \tilde{g}_s = \frac{l_s}{R_y} g_s$

$\overleftrightarrow{R_y}$ $\qquad \tilde{R}_y = \frac{l_s^2}{R_y}$

mom. $M = \frac{n}{R_y}$ $\leftrightarrow$ wind $M = \frac{m\tilde{R}_y}{l_s^2}$ $\qquad P \overset{T}{\leftrightarrow} F1$

wind $M = \frac{w R_y}{l_s^2}$ $\leftrightarrow$ mom $M = \frac{w}{\tilde{R}_y}$

$Dp\,(y \ldots) \underset{T_y}{\longleftrightarrow} D(p-1)\,(\ldots)$ $\qquad$ Dirichlet $\leftrightarrow$ Neumann.

$Dp\,(\ldots) \underset{T_y}{\longleftrightarrow} D(p+1)\,(y \ldots)$

e.g. $D1(y)$ $\quad M = T_{D1} \cdot 2\pi R_y = \frac{R_y}{g_s l_s^2}$

$$\tilde{M} = \frac{l_s^2/\tilde{R}_y}{\left(\frac{l_s}{\tilde{R}_y}\tilde{g}_s\right) l_s^2} = \frac{1}{g_s l_s} =: T_{D0} \qquad D0.$$

$NS5\,(y1234) \underset{T_y}{\longleftrightarrow} NS5\,(y1234)$

$NS5\,(12345) \underset{T_y}{\longleftrightarrow} KKM\,(12345, y)$ $\quad$ special circle

S-duality (IIB)

Non-pert. Weak ↔ strong.

$$g_s \to \frac{1}{g_s} \qquad l_s \to g_s^{1/2} l_s$$

$F1 \overset{S}{\leftrightarrow} D1$

$D3 \circlearrowleft S \qquad KKM \circlearrowleft S$

$NS5 \overset{S}{\leftrightarrow} D5$

$$F1(1): \quad M_{F1} = \frac{R_1}{l_s^2} \xrightarrow{S} \frac{R_1}{(g_s^{1/2} l_s)^2} = \frac{R_1}{g_s l_s^2} = M_{D1}.$$

$\mathrm{IIA} \underset{\text{red.}}{\overset{\text{lift}}{\rightleftarrows}} M$

10D	11D	$S^1_\natural$
IIA	M	

$D0 \longleftrightarrow P$

$D2(..) \longleftrightarrow M2(..)$

$D4(...) \longleftrightarrow M5(\natural ....)$

$D6(....) \leftrightarrow KKM(...., \natural)$

$F1(.) \leftrightarrow M2(\natural .)$

$NS5(.....) \leftrightarrow M5(.....)$

$$\begin{cases} l_s = l_{11}^{3/2} R_\natural^{-1/2} \\ g_s = l_{11}^{-3/2} R_\natural^{3/2} \end{cases}$$

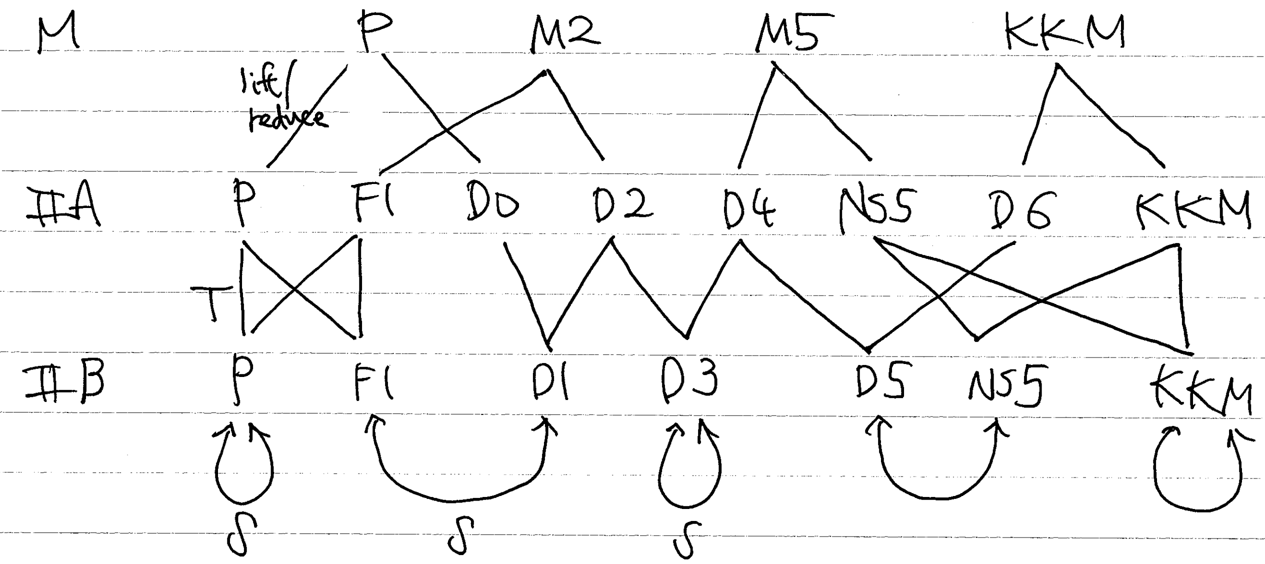
l_{11}: 11D Planck lch.

$C_3 \longleftrightarrow A_3$, $B_2 \longleftrightarrow A_3$

$C_1 \leftrightarrow g_{\natural\mu}$

Famous

Duality web



U-duality

$$S\text{-duality} + T\text{-duality} = \text{"}U\text{-duality"}$$

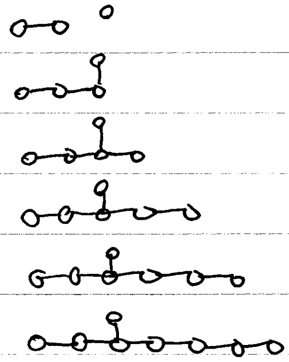
Also, shift potentials e.g. $B_2 = b\, dx_1 \wedge dx_2$ $\qquad b \to b + \frac{l_s^2}{R_1 R_2}\mathbb{Z}$ is allowed.

$$e^{\frac{i}{2\pi l_s^2}\int B_2} = e^{\frac{i 2\pi R_1 R_2}{l_s^2} b}$$

$$\frac{2\pi R_1 R_2}{l_s^2}\Delta b = 2\pi \mathbb{Z} \Rightarrow \Delta b = \frac{l_s^2}{R_1 R_2}\mathbb{Z}$$

All other potentials can be shifted.

d	$G(\mathbb{Z})$
10A	$\mathbb{1}$
10B	$SL(2,\mathbb{Z})$
9	$SL(2,\mathbb{Z})\times\mathbb{Z}_2$
8	$SL(3,\mathbb{Z})\times SL(2,\mathbb{Z})$
7	$SL(5,\mathbb{Z})$
6	$SO(5,5,\mathbb{Z})$
5	$E_{6(6)}(\mathbb{Z})$
4	$E_{7(7)}(\mathbb{Z})$
3	$E_{8(8)}(\mathbb{Z})$



I.5

Brane sol'ns in sugra

String/M reduces to supergravity (SUGRA) in low E lim.

IIA/B string (10D) $\rightarrow$ 10D Type IIA/B sugra ($\mathcal{N}=2$)

M-th'y $\rightarrow$ 11D sugra ($\mathcal{N}=1$)

- Deviation from SUGRA enters as higher der. terms

$$\int \sqrt{-g}\,(R + \#\alpha'^3 R^4 + \cdots)$$

Suppressed by powers of $\frac{1}{M_{str}}$.

There are sol'ns corresponding to Branes (actually how they were discovered first Horowitz-Strominger 1991)

Action (bosonic) for completeness

$$2\kappa_{10}^2 S_{IIA} = \int d^{10}x \sqrt{-g}\left\{e^{-2\Phi}\left[R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2\right] - \frac{1}{2}|G_2|^2 - \frac{1}{2}|G_4|^2\right\}$$
$$- \frac{1}{2}\int B_2 \wedge dC_3 \wedge dC_3$$

$$\left\{\begin{array}{ll} 2\kappa_{10}^2 = (2\pi)^7 l_s^8, & H_3 = dB_2 \\ G_2 = dC_1 & G_4 = dC_3 - H_3 \wedge C_1 \\ e^{\Phi(\infty)} = g_s & \end{array}\right.$$

IIB: Similar (modulo self-duality of G_5)

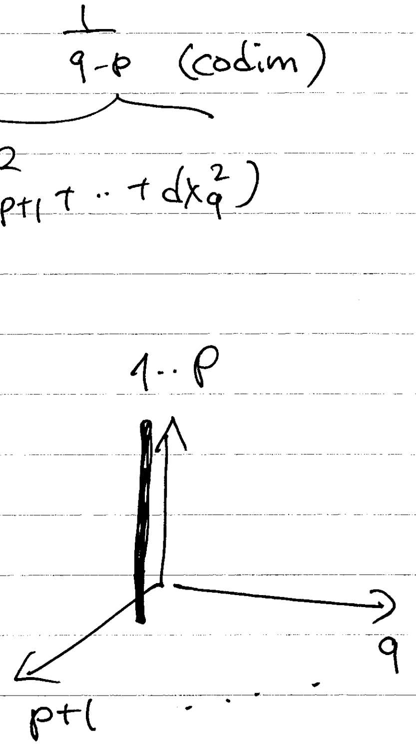
$$2\kappa_{11}^2 S_M = \int d^{11}x \sqrt{-g}\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{3!}\int A_3 \wedge F_4 \wedge F_4$$

$$2\kappa_{11}^2 = (2\pi)^8 l_{11}^9, \quad F_4 = dA_3$$

$$dG_{p+2} - H_3 \wedge G_p = 0$$
$$G_{p+2} \equiv dC_{p+1} - H_3 \wedge C_{p-1}$$

Dp (12.. p)

$$\begin{cases} ds_{10}^2 = H_{Dp}^{-\frac{1}{2}}\underbrace{(-dt^2 + dx_1^2 \cdots + dx_p^2)}_{\substack{\| \\ p+1}} + H_{Dp}^{\frac{1}{2}}\underbrace{(dx_{p+1}^2 + \cdots + dx_9^2)}_{\substack{\| \\ 9-p\ (\text{codim})}} \\ e^{\Phi} = g_s H_{Dp}^{\frac{3-p}{4}} \\ C_{p+1} = g_s^{-1}(1 - H_{Dp}^{-1})\, dx_0 \wedge \cdots \wedge dx_p \end{cases}$$



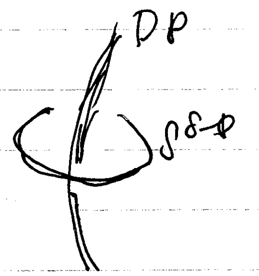
$$H_{Dp} = \underset{\uparrow\ \text{asympt flat}}{1} + \frac{c_p g_s l_s^{7-p} N}{r^{7-p}}$$

N = # of Dp-branes (= 1, 2, 3 ...)

$$r^2 = x_{p+1}^2 + \cdots + x_9^2$$

$$c_p = \frac{(2\pi)^{7-p}}{(7-p)\,\Omega_{8-p}} = (2\sqrt{\pi})^{5-p}\,\Gamma\!\left(\frac{7-p}{2}\right)$$

p	0	1	2	3	4	5	6	7
codim	9	8	7	6	5	4	3	2
c_p	$60\pi^3$	$32\pi^2$	$6\pi^2$	4π	π	1	$1/2$	∞



Charge quantization :

$$N = -\frac{1}{(2\pi l_s)^{7-p}} \int_{S^{8-p}} {*G_{p+2}}$$

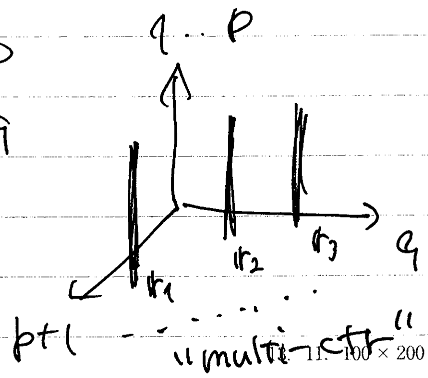
coeff here is det'd by some other way e.g.

Superposition

Actually, H_{Dp} can be any harmonic func satisfying

$$\partial_i^2 H_{Dp} = 0$$

$$i = p+1 \cdots 9$$



SUSY ($\frac{1}{2}$ BPS)
$\Leftrightarrow$ grav. attr | cancel
Coulomb repl. |
$M = |Q|$

$$H = 1 + \frac{Q_1}{|r - r_1|^{7-p}} + \frac{Q_2}{|r - r_2|^{7-p}} + \cdots$$

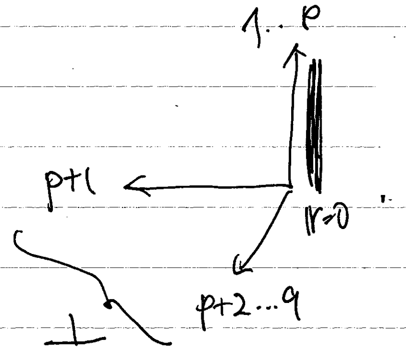
$$r = (x_{p+1}, \ldots, x_9)$$

Day 2

Smeaning

Important technique to obtain approx. sol'n in sugra for multi- brane configs

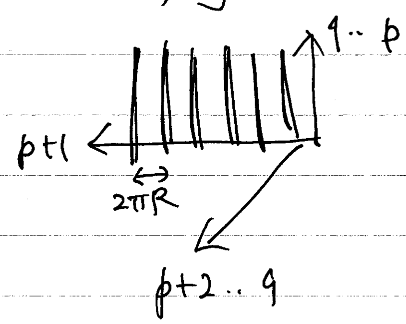
$$H_{Dp} = 1 + \frac{g_s l_s^{7-p} c_p N}{r^{7-p}}$$



Compactify one of the ⊥ directions, say x^{p+1}

↓

Same as arraying $\underbrace{}_{\infty}$ branes with periodicity $2\pi R$



$$H = 1 + \sum_{n=-\infty}^{\infty} \frac{g_s l_s^{7-p} c_p N}{\left[(x^{p+1} - 2\pi R n)^2 + \hat{r}^2\right]^{\frac{7-p}{2}}}$$

$$\hat{r}^2 \equiv x_{p+2}^2 + \cdots + x_9^2$$

If R is very small (e.g. $R \sim l_s$), can approximate in sugra as

$$\simeq 1 + \frac{g_s l_s^{7-p} c_p N}{2\pi R} \underbrace{\int_{-\infty}^{\infty} \frac{dz}{(z^2 + \hat{r}^2)^{\frac{7-p}{2}}}}_{\frac{\sqrt{\pi}}{\hat{r}^{6-p}} \frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}}$$

$$c_p = (2\sqrt{\pi})^{5-p}\,\Gamma\left(\frac{7-p}{2}\right)$$

$$= 1 + \frac{g_s l_s^{7-(p+1)} C_{p+1} N}{\hat{r}^{7-(p+1)}} \left(\frac{l_s}{R} \right)$$

$\Downarrow$

Same as $(p+1)$-brane up to $\left(\frac{l_s}{R}\right)$.

We've made it into a $(p+1)$-brane

If smear along q dirs,

$$H = 1 + \frac{g_s l_s^{7-(p+q)} C_{p+q} N}{\hat{r}^{7-(p+q)}} \frac{l_s^q}{R_1 \cdots R_q}.$$

$\rightarrow$ same as $(p+q)$-brane

<u>sugra duality at work</u>

T-Duality : $Dp(1 \cdots p) \xrightarrow[T_{p+1}]{} Dp(1 \cdots p+1)$?

Rule :

$$\begin{cases} ds_{10}^2 = e^{2\sigma}(dy + v_1)^2 + g_{\mu\nu} dx^\mu dx^\nu \\ B_2 = b_2 + b_1 \wedge (dy + v_1) \end{cases} \qquad \Phi = \varphi + \frac{\sigma}{2}$$

$$C^{IIA} = c_{odd} + c_{even} \wedge (dy + v_1)$$
$$C^{IIB} = \tilde{c}_{even} + \tilde{c}_{odd} \wedge (dy + v_1') \qquad G_{p+2} = dC_{p+1} - H_3 \wedge C_{p-1}$$

$\Downarrow$

$$\tilde{\sigma} = -\sigma, \quad \tilde{\varphi} = \varphi, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu}$$
$$\tilde{v}_1 = -b_1 \quad \tilde{b}_1 = -v_1 \quad \tilde{b}_2 = b_2 + b_1 \wedge v_1 ,$$
$$\tilde{c}_{odd} = c_{odd} \quad \tilde{c}_{even} = c_{even}$$

$$\text{or} \quad e^{\tilde{\Phi}} = \frac{e^{\Phi}}{e^{\sigma}}$$

This maps a sol'n in IIA/B into another in IIB/A.

Present case:

Dp(1.. p), smeared along $x_{p+1} = y$

$$\begin{cases} ds_{10}^2 = H^{-\frac{1}{2}}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + H^{\frac{1}{2}}(\underbrace{dx_{p+1}^2} + \cdots + dx_9^2) \\ B_2 = 0 \end{cases}$$

$$e^{2\sigma} = H^{\frac{1}{2}}, \qquad v_1 = b_1 = b_2 = 0$$

$$e^{\Phi} = g_s H^{\frac{3-p}{4}}$$

$$C_{p+1} = g_s^{-1}(1 - H^{-1})\, dx_0 \wedge \cdots \wedge dx_p$$

$\downarrow$ T-dual

$$e^{2\tilde{\sigma}} = e^{-2\sigma} = H^{-\frac{1}{2}}, \qquad \tilde{v}_1 = \tilde{b}_1 = \tilde{b}_2 = 0$$

$$\begin{cases} d\tilde{s}_{10}^2 = H^{-\frac{1}{2}}(-dt^2 + dx_1^2 + \cdots + dx_{p+1}^2) + H^{\frac{1}{2}}(dx_{p+2}^2 + \cdots + dx_9^2) \\ \tilde{B}_2 = 0 \end{cases}$$

$$e^{\tilde{\Phi}} = \frac{e^{\Phi}}{e^{\sigma}} = g_s H^{\frac{3-(p+1)}{4}}$$

$$\tilde{C}_p = g_s^{-1}(1 - H^{-1})\, dx_0 \wedge \cdots \wedge dx_p \wedge dx_{p+1}$$

$\rightarrow$ exactly D(p+1) (1.. p+1).

Can check other duality relations too: $\rightarrow$ next pg.

S-duality

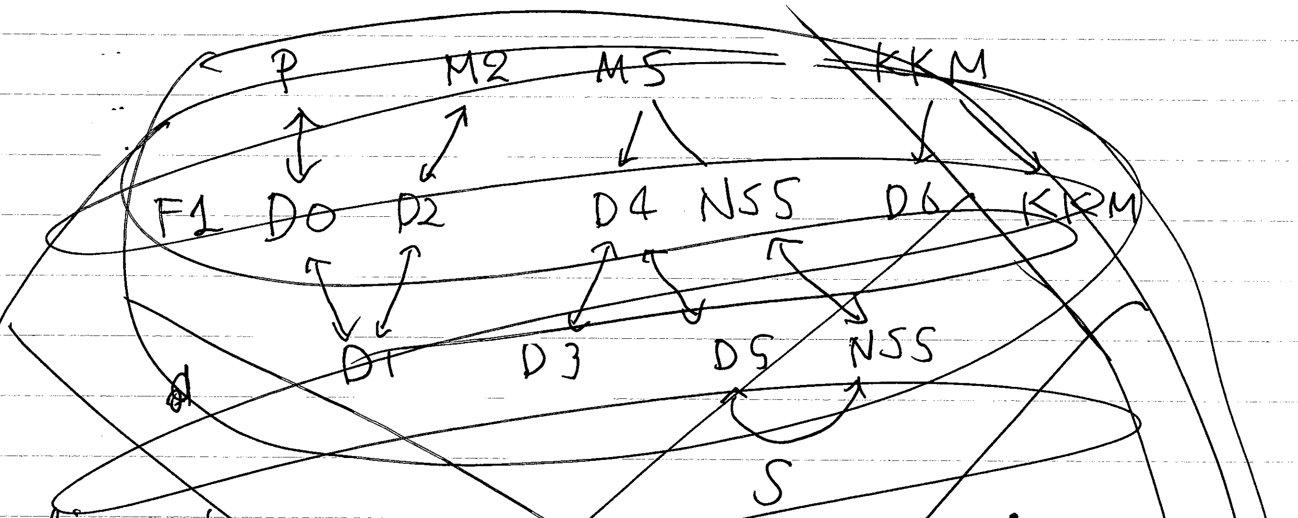
$$\tilde{G}_{\mu\nu} = G_{\mu\nu} e^{-\Phi}, \quad \tilde{B}_2 = +C_2, \quad \tilde{C}_2 = -B_2$$

$$\tilde{G}_5 = g_s^{\frac{1}{2}} G_5, \quad \tilde{l}_s = l_s$$

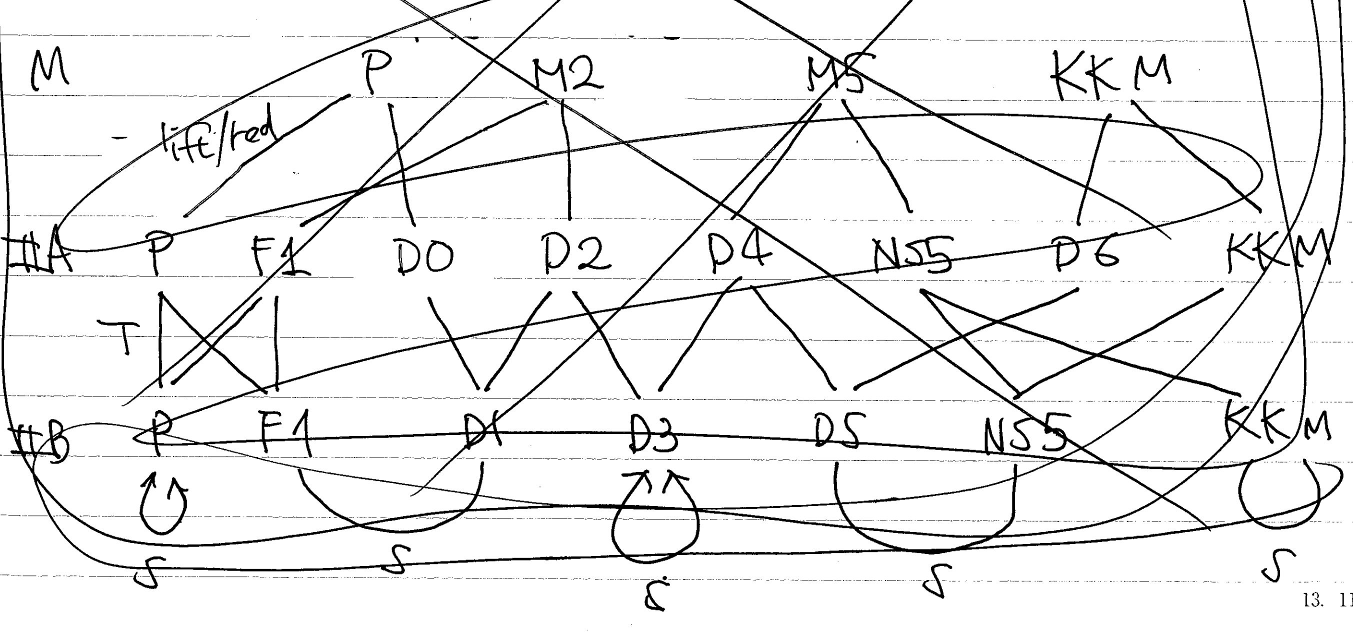
IIA/M

$$ds_{11}^2 = e^{-\frac{2}{3}\Phi} ds_{10}^2 + e^{\frac{4}{3}\Phi}(dx_{11} + C_1)^2$$

$$C_{\mu\nu\rho} = A_{\mu\nu\rho}, \quad B_{\mu\nu} = A_{11\mu\nu}$$



Duality web



#15

skip

o In sugra, quantization of U-duality grp isn't visible

related to chg quant'n

e.g. IIB string has $SL(2,\mathbb{Z})$

$$C_0 + ie^{-\Phi} \equiv \tau \quad \in \text{upper half plane}$$

$$\rightarrow \quad \tilde{\tau} = \frac{a\tau + b}{c\tau + d}, \qquad ad - bc = 1, \quad a,b,c,d \in \mathbb{Z} \qquad (\text{def of } SL(2,\mathbb{Z}))$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} & -1 \\ 1 & \end{pmatrix} \rightarrow \tilde{\tau} = -\frac{1}{\tau}$$

(If $C_0 = 0$, $\tilde{\Phi} = -\Phi$)

S-duality

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow \tilde{C}_0 = C_0 + 1 \quad \text{shift sym.}$$

D(-1) action $2\pi \int C_0$

IIB sugra:

$$S_{sugra} \supset \int \frac{|d\tau|^2}{\tau_2^2}$$

: inv under $a,b,c,d \in \mathbb{R}$

$\rightarrow SL(2,\mathbb{R})$.

I 6

BH sol'n

Dp(1⋯p) $ds_{10}^2 = H^{-\frac{1}{2}} dx_{\parallel}^2 + H^{\frac{1}{2}} dx_{\perp}^2$

$e^{\Phi} = g_s H^{\frac{3-p}{4}}$

$C_{p+1} = g_s^{-1}(1-H^{-1})\, dx_0 \wedge \cdots \wedge dx_p$

M2(12) $ds_{11}^2 = H^{-\frac{2}{3}} dx_{\parallel}^2 + H^{\frac{1}{3}} dx_{\perp}^2$

$A_3 = (1-H^{-1})\, dx_0 \wedge dx_1 \wedge dx_2$

- Harmonic rule (empirical)

The sol'n for multiple stacks of branes that are mutually BPS can be constructed by multiplying harmonic funcs.

- D1(5)−D5(56789) sys

$$\begin{array}{c|cccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline D1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 & \sim & \sim & \sim & \sim \\ D5 & 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 & 0 \end{array}$$

$\overbrace{6\,7\,8\,9}$ compact T^4

$$ds_{10}^2(D1(5)) = H_1^{-\frac{1}{2}}(-dt^2 + dx_5^2) + H_1^{\frac{1}{2}}(dx_{12346789}^2), \quad e^{\Phi} = g_s H_1^{\frac{1}{2}}$$

$$ds_{10}^2(D5(56789)) = H_5^{-\frac{1}{2}}(-dt^2 + dx_{56789}^2) + H_5^{\frac{1}{2}}(dx_{1234}^2), \quad e^{\Phi} = g_s H_5^{-\frac{1}{2}}$$

$\downarrow$

$$ds_{10}^2(D1\text{-}D5) = H_1^{-\frac{1}{2}} H_5^{-\frac{1}{2}}(-dt^2 + dx_5^2) + H_1^{\frac{1}{2}} H_5^{\frac{1}{2}} \underbrace{dx_{1234}^2}_{\parallel} + H_1^{\frac{1}{2}} H_5^{-\frac{1}{2}} dx_{6789}^2$$

$$\left(dr^2 + r^2 d\Omega_3^2, \quad r^2 = x_1^2 + \cdots + x_4^2 \right.$$

$$e^{\Phi} = g_s H_1^{\frac{1}{2}} H_5^{-\frac{1}{2}}$$

smeared

$$H_1 = 1 + \frac{g_s l_s^6 / v}{r^2} N_1 \qquad H_5 = 1 + \frac{g_s l_s^2}{r^2} N_5$$

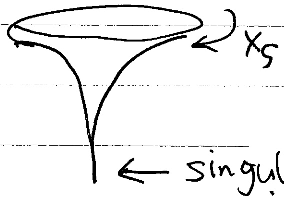
Note: $c_s = 1$, volume of T^4 : $(2\pi)^4 v$

"Decoupling lim" $r \to 0$

$$ds_{10}^2 \to \underbrace{\frac{r^2}{R^2}(-dt^2+dx_5^2) + \frac{R^2}{r^2}dr^2}_{AdS_3} + \underbrace{R^2 d\Omega_3^2}_{S^3} + \underbrace{c\, dx_{6789}^2}_{T^4}$$

$$e^{\Phi} \Rightarrow g_s c, \quad R = \left(\frac{g_s l_s^8}{v} N_1 N_5\right)^{\frac{1}{4}}, \quad c = \sqrt{\frac{l_s^4 N_1}{v N_5}}$$

$\to$ AdS_3/CFT_2 duality.



If we compactify x_5, $\to$ singular

$\leftarrow$ singularity at $r=0$

But CFT_2 cpt'fied is well-def'd and has $S \simeq 2\sqrt{2}\pi\sqrt{N_1 N_5}$. $\to$ the bulk singularity must somehow be resolved.

- 3-chg BH in 5D

	0	1‥4	5 6	7 8	9 11
$M2_1$	0	‥‥	0 0	~ ~	~ ~
$M2_2$	0		~ ~	0 0	~ ~
$M2_3$	0		~ ~	~ ~	0 0

$$dr^2 + r^2 d\Omega_3^2$$

$$ds_{11}^2 = -(123)^{-\frac{2}{3}}dt^2 + (123)^{\frac{1}{3}}dx_{1234}^2 + \left(\frac{23}{1^2}\right)^{\frac{1}{3}}dx_{56}^2 + \left(\frac{31}{2^2}\right)^{\frac{1}{3}}dx_{78}^2 + \left(\frac{12}{3^2}\right)^{\frac{1}{3}}dx_{9\natural}^2$$

$$H_1 = 1 + \frac{l_{11}^2}{r^2} N_1 \quad \cdots \qquad r^2 = x_1^2 + \cdots + x_4^2$$

This is a BH! at $r=0$, $g_{tt}=0$, $g_{rr}=\infty$

Set the radii of circles to l_{11}.

At $r=0$,

$$ds_{11}^2 = (dt, dr) + \underbrace{l_{11}^2(N_1N_2N_3)^{\frac{1}{3}}}_{\text{"}R^2} d\Omega_3^2 + \left(\frac{N_2N_3}{N_1^2}\right)^{\frac{1}{3}} dx_{56}^2 + \cdots$$

The area:

$$A = R^3 \cdot \Omega_3 \cdot (2\pi l_{11})^6_{\sqrt{\cdot}} = \frac{1}{2}(2\pi)^8 l_{11}^9 \sqrt{N_1N_2N_3}$$

$$G_{11} = \frac{1}{8}(2\pi)^7 l_{11}^9$$

$$\therefore S = \frac{A}{4G} = 2\pi\sqrt{N_1N_2N_3}. \quad \to$$

can be reproduced by CFT by dualizing it to D1-D5-P sys (cf. Strominger–Vafa)

- 4-chg BH in 4D (MSW)

	0	1 2 3	45	6 7	89
$D4_1$	O		~ ~	O O	O O
$D4_2$	O		O O	~ ~	O O
$D4_3$	O		O O	O O	~ ~
$D0_4$	O		~ ~	~ ~	~ ~

$$ds^2 = -\frac{dt^2}{\sqrt{1234}} + \sqrt{1234}\, dx_{123}^2 + \sqrt{\frac{14}{23}}\, dx_{45}^2 + \sqrt{\frac{24}{13}}\, dx_{67}^2 + \sqrt{\frac{34}{12}}\, dx_{89}^2$$

$$Z_i = 1 + \frac{g_s l_s N_i}{2r}, \quad i = 1234. \qquad \text{set radii. to } l_s.$$

$$S = 2\pi\sqrt{N_1 N_2 N_3 N_4}$$

$\to$ can be reproduced by CFT (lift to M.).

$\downarrow$

MSW CFT

[Maldacena-Strominger-Witten]

Day 3

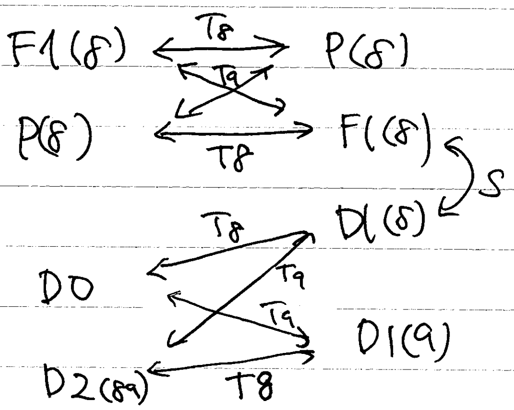
II. Exotic branes

II.1. Exotic branes

By duality, one can relate various objs

e.g. particle mult. in 8D

IIA/T^2_{89} $\quad$ IIB/T^2_{89}



IIA	P	2	U-duality multiplet
	F1	2	
	D0	1	
	D2	1	
IIB	P	2	
	F1	2	
	D1	2	
M	P	3	
	M2	3	

$G = SL(2)\times SL(3)$

Can identify the obj by the mass.

$$T_y : R_y \to \frac{l_s^2}{R_y} \qquad g_s \to \frac{l_s}{R_y} g_s$$

$$S : g_s \to \frac{1}{g_s} \qquad l_s \to g_s^{\frac{1}{2}} l_s$$

$$IIB \quad F1(8) \quad M = \frac{R_8}{l_s^2} \xrightarrow{T_8} M \Rightarrow \frac{l_s^2/R_8}{l_s^2} = \frac{1}{R_8} \ : P(8)$$

$$\xrightarrow{S} M \to \frac{R_8}{(g_s^{\frac{1}{2}} l_s)^2} = \frac{R_8}{l_s^2 g_s} \ : D1(8).$$

All members of mult. has higher-D (10D) interpretation as std branes wrapped on it'l cycles.

Same down to 4D.

• "Exotic branes" (~ 97–98)

But things change in 3D. IIA/B on $T^7_{345..9}$

IIB D7 (3456789) $M = \dfrac{R_3 \cdots R_9}{g_s l_s^8}$ a.k.a. "(0,1) 7-br"

$\downarrow S$ $\uparrow$

$M = \dfrac{R_3 \cdots R_9}{(1/g_s)(g_s^{1/2} l_s)^8} = \dfrac{R_3 \cdots R_9}{g_s^3 l_s^8}$ "7_3"

KKM (56789, 4) $M = \dfrac{R_5 \cdots R_9 R_4^2}{g_s^2 l_s^8}$

$\downarrow T_3$

$M = \dfrac{R_5 \cdots R_9 R_4^2}{(g_s l_s/R_3)^2 l_s^8} = \dfrac{R_5 \cdots R_9 (R_3 R_4)^2}{g_s^2 l_s^{10}}$ "5^2_2"

No interpretation as std branes

"Exotic states"

Notation: $b^c_n : M = \dfrac{R^b (R^c)^2}{g_s^n}$

$b^{(d,c)}_n : M = \dfrac{R^b (R^c)^2 (R^d)^3}{g_s^n}$

M-thy: no subscr n

Keep doing this

• IIA/T^7 p (1) F1 (7) D0 (1) D2 (21)

D4 (35) D6 (7) NS5 (21)

KKM (42)

and 5^2_2 (21).

6^1_3 (7) 4^3_3 (35) 2^5_3 (21) 0^7_3 (1)

1^6_4 (7) $0^{(1,6)}_4$ (7)

240 objects.

related to non-zero roots of $e_{8(8)}$

Easy to derive rules for T, S dualities & M-lift using trfm law for mass

e.g. IIB $5^2_2(56789;34)$

$$M = \frac{R_5 \cdots R_9 (R_3R_4)^2}{g_s^2 l_s^{10}} \xrightarrow{T_5} 5^2_2(5..9;34)$$

$\downarrow S$

$$M = \frac{R_5 \cdots R_9 (R_3R_4)^2}{g_s^3 l_s^{10}} \quad : 5^2_3(5..9;34)$$

$\swarrow T5$ $\qquad \searrow T4$

IIA $4^3_3(6789;345)$ $\qquad 6^1_3(456789;3)$

$\downarrow \mathrm{lift}_{\natural}$ $\qquad \downarrow \mathrm{lift}_{\natural}$

M: $5^3(6789\natural;345)$ $\qquad$ KKM $(456789;3)$

The 8D duality grp. is $SL(3)\times SL(2)$

(related to non-zero roots of sl_3)

(related to non-zero roots of sl_2)

Not restricted to 3D.
It happens for codim 2

e.g. II/T^2_{89}. take 5-branes in d=8 thy.

IIA: D6 (2), KKM (2), 6^1_3 (2)
IIB: D7 (1) D5 (1) NS5 (1)
7_3 (1) 5^2_2 (1) 5^2_3 (1)

M: KKM (6)

$T^2_{89\natural}$. KKM6

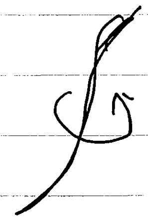
NS5 (1) 5^2_2 (1)
KKM (2)

M5 (1) 5^3 (1)

Summary
(Codim-2 multiplets include exotic branes
not std ones.
includes $M \sim \frac{1}{g_s^3}, \frac{1}{g_s^4}$

"Nonstd" branes

Codim-2 : defect branes (cf. D7)
(exo. branes)



Codim-1 : Domain walls
(cf. D8) $\to C_9 \leftrightarrow F_{10}$: space-filling flux.
$\updownarrow$
Class'n of flux cpt'n
(1206.5697)
Bergshoeff + Kleinschmidt
+ Riccioni)

Codim-0 : space-filling
(cf. D9) $\to$ IIB $\to$ Type I.
Diff'nt s.t.
New, unexplored s.t. ?

10/11D origin

Back to $d=3$ (II/T^7 or M/T^8)

- 3D sugra
- Duality grp: $G = E_{8(8)}(\mathbb{Z})$
- Many scalars coming from internal compns. of various 10/11D field (128)

e.g. IIB

packaged in matrix $\leftarrow$

$V \in E_{8(8)}(\mathbb{Z}) \backslash E_{8(8)}(\mathbb{R}) / SO(16)$

G_{ij}, B_{ij}, Φ

$C^{(0)}$ $C^{(2)}_{ij}$ $C^{(4)}_{ijkl}$

(Also vectors in 3D can be dualized to scalars)

$G_{i\mu}, B_{i\mu}, C_{i\mu}, C_{ijk\mu}$

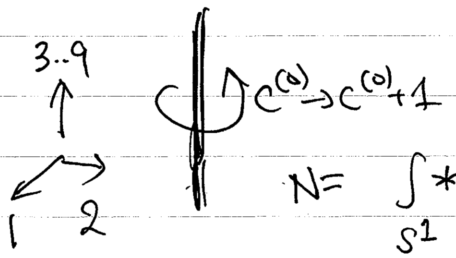
Take D7

elec'ly coupled to $C^{(8)}$
mag'ly $C^{(0)}$

$G^{(9)} = dC^{(8)}$

$G^{(1)} = dC^{(1)}$

$G^{(1)} = *G^{(9)}$



$C^{(0)} \to C^{(0)} + 1$

$N = \int_{S^1} *G_9 = \int_{S^1} dC^{(0)} = \Delta C^{(0)}$

So, $C^{(0)}$ = (scalar in 3D) is "twisted" by an $SL(2,\mathbb{Z})$ duality.

$\tau = C^{(0)} + ie^{-\Phi}$ $\quad \tau \to \frac{a\tau+b}{c\tau+d}$

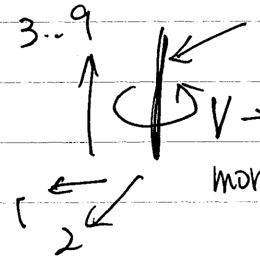
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Now, recall

- τ is just a part of 128 scalars of 3D theory
- $SL(2,\mathbb{Z})$ is just a part of $E_{8(8)}(\mathbb{Z})$

↓

U-duality means: That we can consider a more general obj with general $E_{8(8)}$ twisting



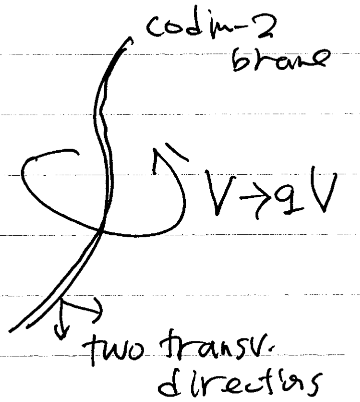
codim-2 obj

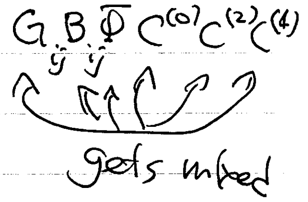
$V \to gV$, $g \in E_{8(8)}(\mathbb{Z})$

monodromy

- V is not single-valued (has monodromy)
- V contains G, B, Φ, C.

  $V \to gV$ mixes all components!





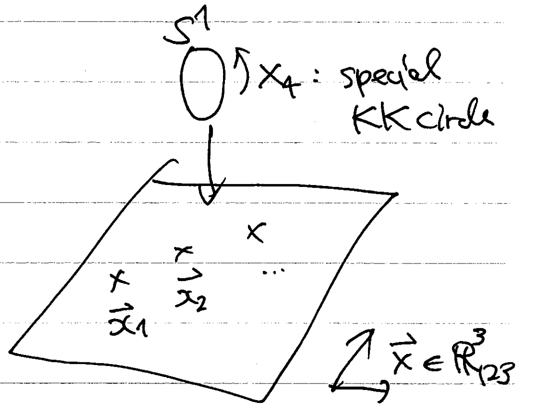
- Non-geometric "U-fold" (T-, S-..)
- Being U-dual to std. branes, represents a dyn. obj (can wiggle etc)
- Higher-D metric is multivalued but 3D Einst. metric is not (just scalars have monodromy)

2.3 Sugra description

Example: 5_2^2

$$\mathrm{KKM}(56789;4) \xrightarrow[T_3]{} 5_2^2(56789;34)$$

Need to cptfy x_3 → smear KKM
(radius $\tilde{R}_3$) (array)



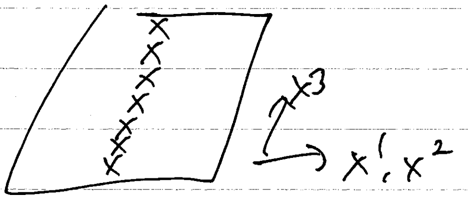
KKM(56789; 4) metric (a.k.a. Multi TN)

$$ds^2 = -dt^2 + H d\vec{x}^2 + H^{-1}(dx_4+\omega) + dx^2_{56789}$$

$$e^{2\Phi} = 1 \qquad B_2 = 0, \qquad d\omega = *_3 dH$$

See 7.1 for explanation of smoothness

$$H = 1 + \sum_p H_p, \qquad H_p = \frac{R_4}{2|\vec{x}-\vec{x}_p|}$$



$$H(r) = \sum_p \frac{R_4}{2|\vec{x}-\vec{x}_p|} = \sum_{n=-\infty}^{\infty} \frac{R_4}{2\sqrt{r^2+(x_3-2\pi\tilde{R}_3 n)^2}}$$

$$= \sigma \int \frac{d\xi}{2\sqrt{r^2+(x_3-\xi)^2}}, \quad \sigma = \frac{R_4}{2\pi\tilde{R}_3}$$

$$\sim h + \sigma \log\left(\frac{\mu}{r}\right)$$

$$\omega = -\sigma\theta\, dx^3$$

Note: the integral diverges.
this is an approx near the KKM core
but not too far away. ($\mu \sim$ IR cutoff distance)

Hopf Fibration & KKM

Write $\mathbb{R}^4$ as

$$X_1 + iX_2 = \sqrt{2R_4 r}\cos\frac{\theta}{2}\, e^{i\frac{x_4}{R_4} + i\phi}$$

$$X_3 + iX_4 = \sqrt{2R_4 r}\sin\frac{\theta}{2}\, e^{i\frac{x_4}{R_4}},$$

$$0 \le r < \infty, \quad 0 \le \theta \le \pi,$$

$$x_4 \simeq x_4 + 2\pi R_4, \quad \phi \simeq \phi + 2\pi$$

$$ds_4^2 = dX_1^2 + \cdots + dX_4^2$$

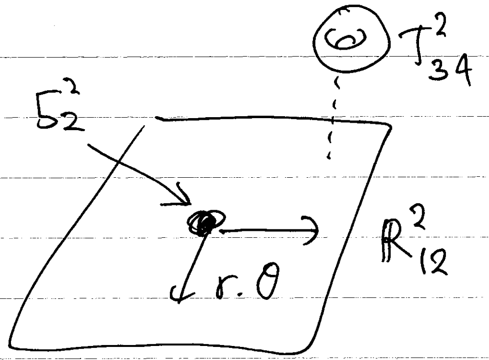
$$= H(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)) + H^{-1}\left(dx_4 + \frac{R_4}{2}(1+\cos\theta)d\phi\right)^2$$

$$H = \frac{R_4}{2r}$$

$\Downarrow$ T-dualize along x_3

$5^2_2(56789;34)$ metric

$$\begin{cases} ds^2_{10,\,\text{str}} = -dt^2 + H(dr^2 + r^2 d\theta^2) + HK^{-1} dx^2_{34} + dx^2_{56789} \\ B^{(2)} = -K^{-1}\theta\sigma\, dx_3 \wedge dx_4 \qquad e^{2\Phi} = HK^{-1} \\ K \equiv H^2 + \sigma^2\theta^2, \\ H \equiv h + \sigma \log\frac{\mu}{r}, \qquad \sigma = \frac{R_3 R_4}{2\pi l_s^2} \end{cases}$$



~~(θ-dep.: nongeom. T-fold structure appears)~~

- Non-geom. T-fold structure

$\theta = 0$: $g_{33} = g_{44} = H^{-1}$

$\theta = 2\pi$: $\dfrac{H}{H^2 + (2\pi\sigma)^2}$.

T^2_{34} doesn't come back to itself

~~(Now want to insert the monodromy discussion using $\Omega = \begin{pmatrix} 1 & 0 \\ 2\pi\sigma & 1 \end{pmatrix}$)~~

Day 4

Comments

- Easy to get sugra metric for other exotic branes.

  = Monodromy appear

  = $T \sim \frac{1}{g_s^3}, \frac{1}{g_s^4}$ → validity? (cf. 5^2_2 has $T \sim \frac{1}{g_s^2}$)

- Codim 2 obj <u>not</u> well-def'd as stand-alone objects.

  — Log div.

$$H = h + \sigma \ln \frac{\mu}{r}$$

$$> 0 \text{ for } r \ll \mu$$

$$< 0 \text{ for } r \gg \mu$$

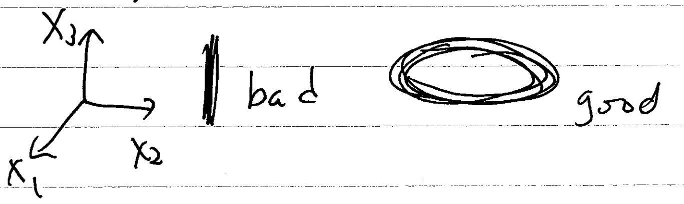
$$ds^2 = -dt^2 + \underbrace{H}_{<0 \text{ is bad}} dx^2 + \cdots$$

Resolutions (i) superposition

cf. F-theory 7 branes

24 branes
↓
transv space becomes S^2.

(ii) Configs which larger codim at large distance



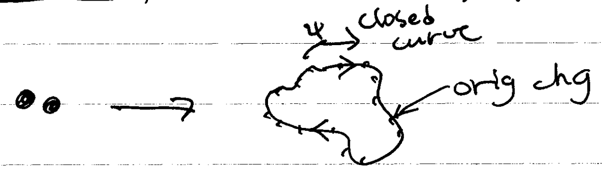
III. Supertube transitions & exotic microstates

III.1 Supertube transition

Combine a particular pair of chgs in S.T. (mutually $\frac{1}{2}$ BPS)

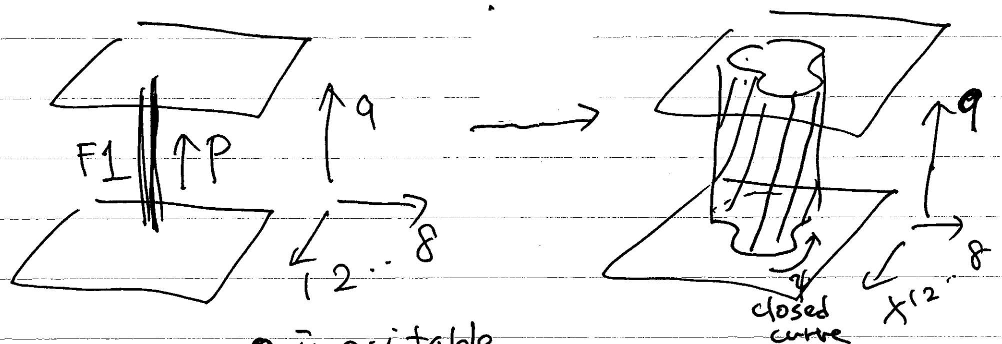
$\Downarrow$

Spontaneously polarize (puff up) into
a new dipole charge along
an arbitrary curve (supertube)



- IIA/B on S^1_9

F1(9) + P(9) ($\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ BPS sys. "2-chg sys")



"barber's pole"

$X^i = f^i(t - x^9)$

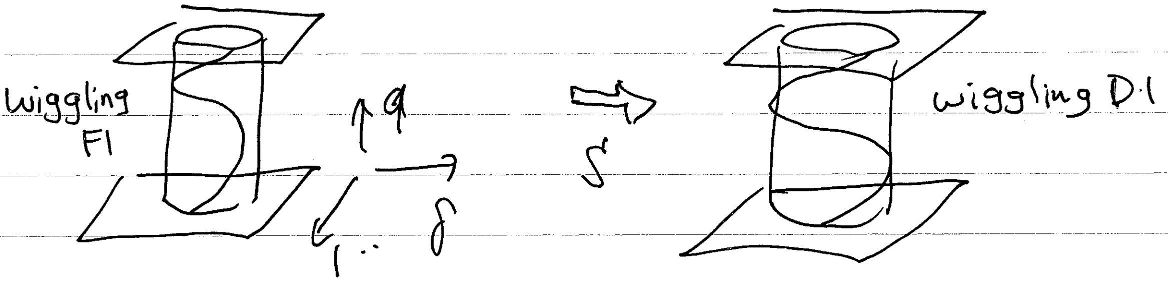
↑ arbitrary func.

- inevitable
- gain size in transv ($x^1 .. x^8$) directions
- genuine bound state of F1 & P
- carries dipole chg f1(ψ), p(ψ) ← lowercase to repr. dipole
- Arbitrary curve f^i → accounts for large entropy $S \sim \sqrt{nw}$ of 2-chg sys.

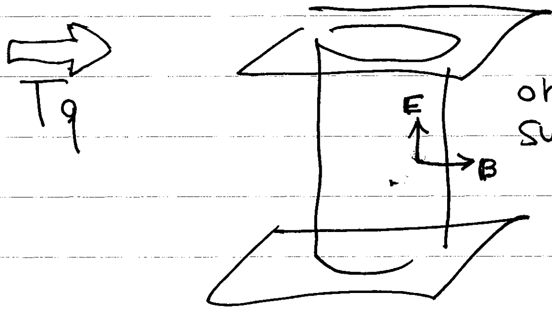
	0	1	2	3	4	5	6	7	8	9
F1	O	.	.	.	.	.	.	.	.	O
P	O	.	.	.	.	.	.	.	.	O
f1	O	←		4	→					~
p	O	←		4	→					~

• Dual picts. of spont. polar'n phenomenon

$P(9)+F1(9) \to f1(\psi)$ $\qquad\qquad$ $P(9)+D1(9) \to d1(\psi)$



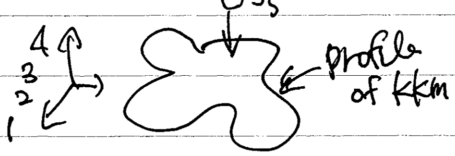
$F1(9) + D0 \quad \to d2(\psi 9)$



original
superfube [Mateos + Townsend 2001]
F1, D∅ dissolved into w.v. flux in d2

$\xrightarrow{T_{678}} \xrightarrow{S} \xrightarrow{T_{59}}$ $\underbrace{D1(5)+D5(56789)}$ $\to$ kkm$(\psi 6789, 5)$

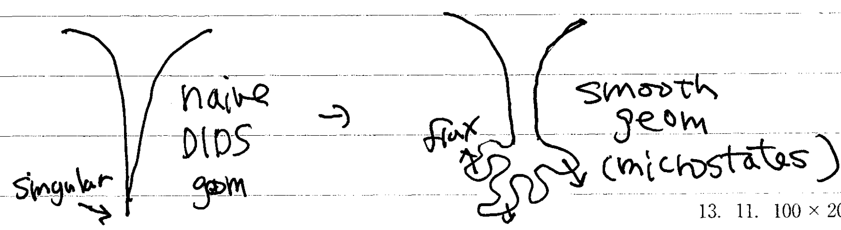
Famous D1-D5 system. we've seen.



• Wiggly kkm = smooth geometry
(Lunin-Mathur geom)
└ explicit metric known.

LM 0109154
LMM 0212210

• The orig D1, D5 chgs have turned into flux penetrating kkm



↓

can quantize these geoms.
to get expected entropy growth

$S_{geom} \propto \sqrt{N_1 N_5}$ ✓

$\sim S_{\substack{micro \\ CFT}} = 2\sqrt{2}\pi\sqrt{N_1 N_5}$

[Rychkov 0512053]

↓

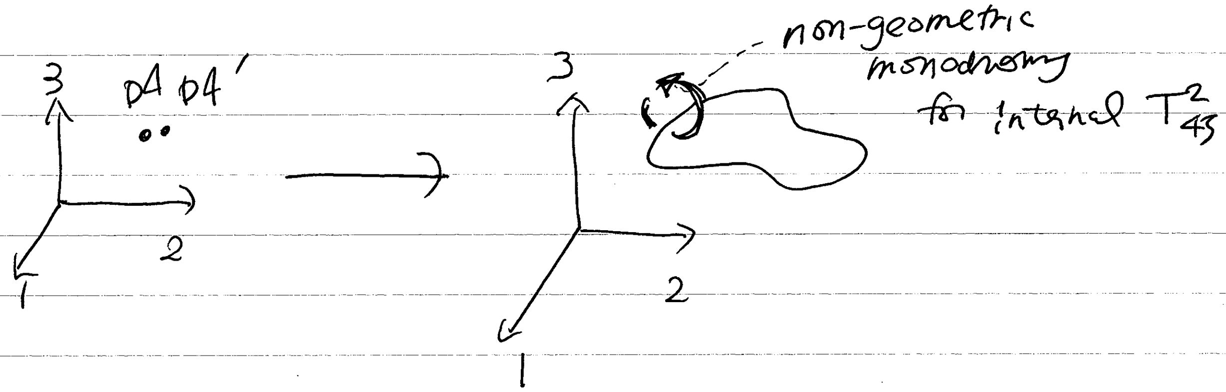
cf. fuzzball
[Mathur]

III 2. Exotic supertubes

$$D1(5) + D5(56789) \to \mathrm{kkm}(\psi 6789;5)$$

$$\downarrow T_{467}$$

$$D4(4567) + D4(4589) \to 5_2^2(\psi 6789;45)$$



More examples, e.g.

tension

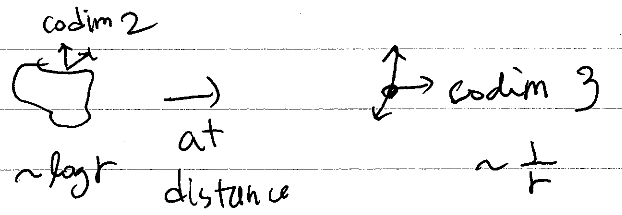
$$D3(589) + NS5(46789) \to 5_3^2(4567\psi;89) \quad T \sim \frac{1}{g_s^3}$$

$$NS5(46789) + KKM(46789;5) \to 1_4^6(\psi;456789) \quad T \sim \frac{1}{g_s^4}$$

$$g_s^{-a} + g_s^{-b} \to g_s^{-a-b}$$

$$R \sim g_s^{\frac{a+b}{2}}$$

- Std branes can polarize into exotic branes
- Only dipoles → larger codim at large distance
  no log problem.



- Explicit sol'n for $D4 + D4 \to 5^2_2$

$D4(6789) + D4(4589) \to 5^2_2(4567\psi; 89)$

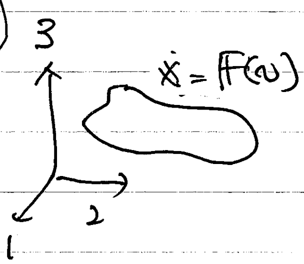
cf. D4-D4 "naive" metric

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}}dt^2 + \sqrt{f_1 f_2}\,dx_{123}^2 + \underbrace{\sqrt{\frac{f_1}{f_2}}dx_{45}^2 + \sqrt{\frac{f_2}{f_1}}dx_{67}^2 + \frac{1}{\sqrt{f_1 f_2}}dx_{89}^2}_{\text{internal}}$$

$$f_1 = 1 + \frac{Q_1}{r}, \quad f_2 = 1 + \frac{Q_2}{r}$$

singular but nothing fancy

The exotic supertube:



$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}}(dt - A)^2 + \quad \text{''} \quad + \frac{1}{\sqrt{f_1 f_2} + \frac{\gamma^2}{\sqrt{f_1 f_2}}}dx_{89}^2$$

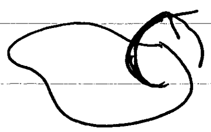
$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}, \quad f_2 = 1 + \frac{Q_1}{L}\int_0^L dv \frac{|\dot{\vec{F}}(v)|^2}{|\vec{x} - \vec{F}(v)|}$$

$$A = -\frac{Q_1}{L}\int_0^L dv \frac{\dot{\vec{F}}(v)\cdot d\vec{x}}{|\quad|}$$

$$d\beta_I = *_3 df_I, \quad d\gamma = *_3 dA$$

- Monodromy $\beta_I \to \beta_I - 2Q_I$, $\gamma \to \gamma + c$, $\left(c = \frac{4\pi n Q_1}{L}\right)$

As we go around,



$$G_{88} = G_{99} = \frac{1}{\sqrt{f_1 f_2}} \to \frac{1}{\sqrt{f_1 f_2} + \frac{c^2}{\sqrt{\;}}}$$

Same T-fold structure as we saw for straight 5^2_2.

- Asymptotically flat 4D (no long-distance log)

- Non-geom. microstates!

↳ cf. Ashoke's claim in 0908.3402

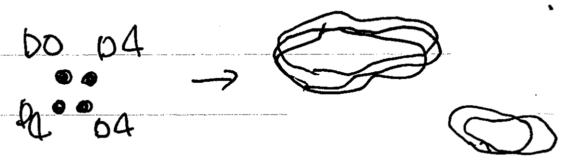
Generic brane sys : expected to undergo supertube trans.
& produce all kinds of chgs,
including exotic ones
(non-geom)

E.g. 4-chg BH

D0
D4(6789)
D4(4589)
D4(4567)

$\to 5^2_2(\psi 4567; 89)$
$\to 5^2_2(\psi 6789; 45)$
$\to 5^2_2(\psi 4589; 67)$

$\to$ NS5(6789ψ)
⋮



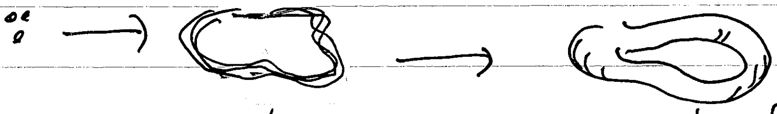
multi
std/exo tubes

3-chg BH

M2(56)
M2(78)
M2(9ψ)

$\to$ M5(5678ψ)
$\to$ M5$(789\psi\psi)$
$\to$ M5$(569\psi\psi)$

$\to 5^3(569\psi\phi, 78\psi)$
$\to 5^3(5678\phi, 9\psi\psi)$
$\to 5^3(789\psi\phi, 56\psi)$



arb. curve — arb. sfc? "superstratum"

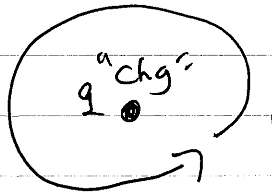
Conj:

Generic BH microstates involve non-geom. exotic branes produced by multiple supertube transitions (work in progress)

IV. Aspects of exotic branes

Charge as monodromy

How to define chge = monodromy?

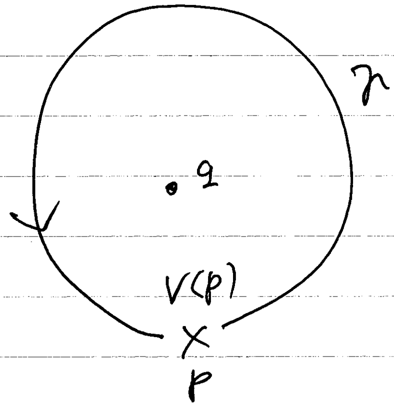


$V \to qV$

More precisely,

- Base pt p and value of moduli there, $V(p)$
- path γ
- Monod. $q \in G(\mathbb{Z})$

  $V \to qV$



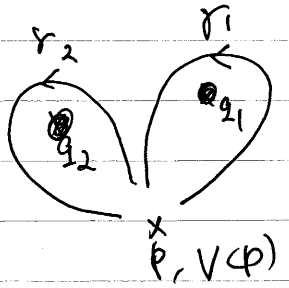
↓

Other moduli values

$\tilde{V} \equiv UV$

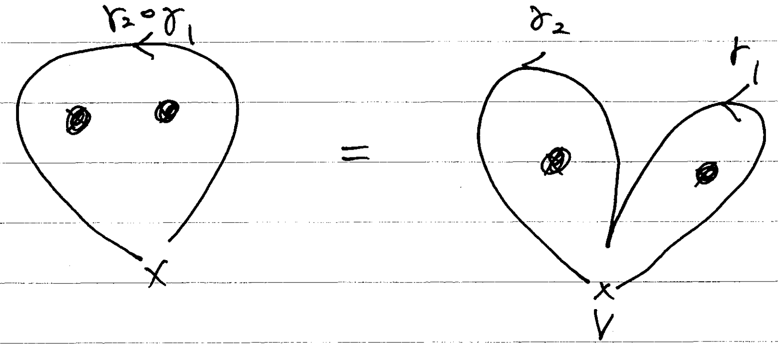
$\tilde{V} \to \tilde{q}\tilde{V}$, $\tilde{q} \equiv UqU^{-1}$

Fun w/ monodromies



$$V \xrightarrow{\gamma_1} g_1 V$$

$$V \xrightarrow[\gamma_2]{} g_2 V$$



$\gamma_1 : \; V \to g_1 V$

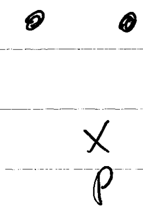
$\gamma_2 : \; g_1 V \equiv \tilde{V} \longrightarrow \tilde{g}_2 \tilde{V}$, $\tilde{g}_2 = g_1 g_2 g_1^{-1}$

$= g_1 g_2 g_1^{-1} \cdot g_1 V = g_1 g_2 V$

$\therefore \; \gamma_2 \circ \gamma_1 : \; V \to g_1 g_2 V$

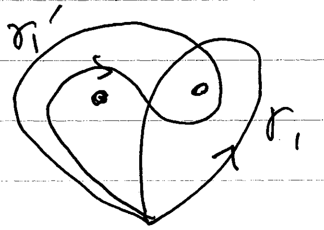
opposite

Charge & defining paths.



What's the chg of this?

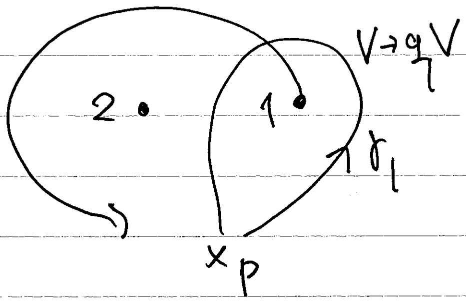
multiple ways

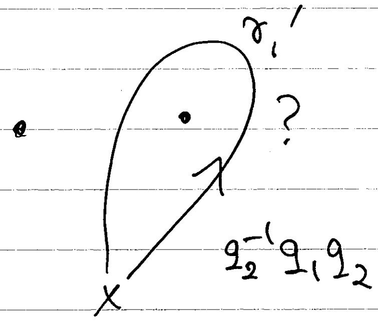


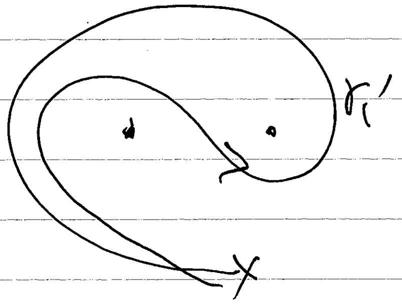
→ different def of chg.

- Need to fix path to define chg & stick to it
- Different defs related by U-duality

Moving chg around each other



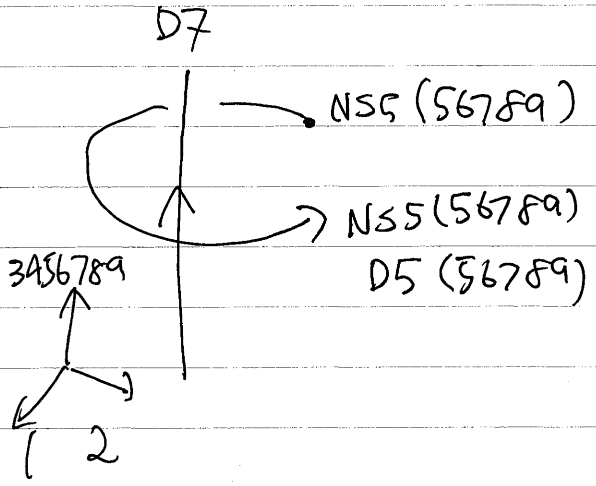




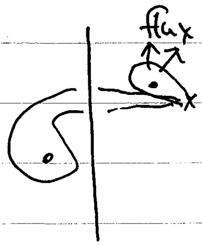
- looks as if chg has Changed from g_1 to $g_2^{-1} g_1 g_2$
- We are changing the def of chg.

Stick to one def → no prob.

Lower chg



• Stick to one def
→ No D5.
Always NS5.
chg consv'd.



(We use "Page charge"
See Marolf 0006117)
dB+MS 1209.6056 App. D

$\tau \to \tau + 1$

$SL(2,\mathbb{Z})$

$$\begin{pmatrix} Q'_{NS5} \\ Q'_{D5} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Q_{NS5} \\ Q_{D5} \end{pmatrix} = \begin{pmatrix} Q_{NS5} + Q_{D5} \\ Q_{D5} \end{pmatrix}$$

• Duality means
NS5 → NS5 + D5.
We change def.