Rainbow and Acid Rain

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Abstract: This experimental study investigates the possibility of measurement of rainbow angle and using it to get an estimate of the acidity or pH of rainwater falling at a distant place. We studied the rainbow angle in the laboratory using a pendent drop of a liquid, and measured the total angle of deviation for the first and the second order rainbow for various values of pH of the acidic solution. We used sulfuric acid and nitric acid which make up most of the acid rain. In the case of sulfuric acid, the variation was found to be 1.202° and 2.074° for angle of deviation corresponding to first and second order rainbow, respectively. In case of nitric acid, the variation in angle of deviation corresponding to first order and second order rainbow was found to be 0.376° and 1.507°, respectively. The experimental results show that there is small change in the rainbow angle with variation of pH of the acidic solution of the drop.

Keywords: Rainbow, acid rain, refractive index, acidity (pH) measurement

1. INTRODUCTION

Rainbow is one of the most beautiful phenomena observed in nature. On a rainy day, a rainbow is produced when white light from the sun falls and gets deviated to the eyes of an observer on the earth due to a large number of raindrops in the sky (Figure 1).

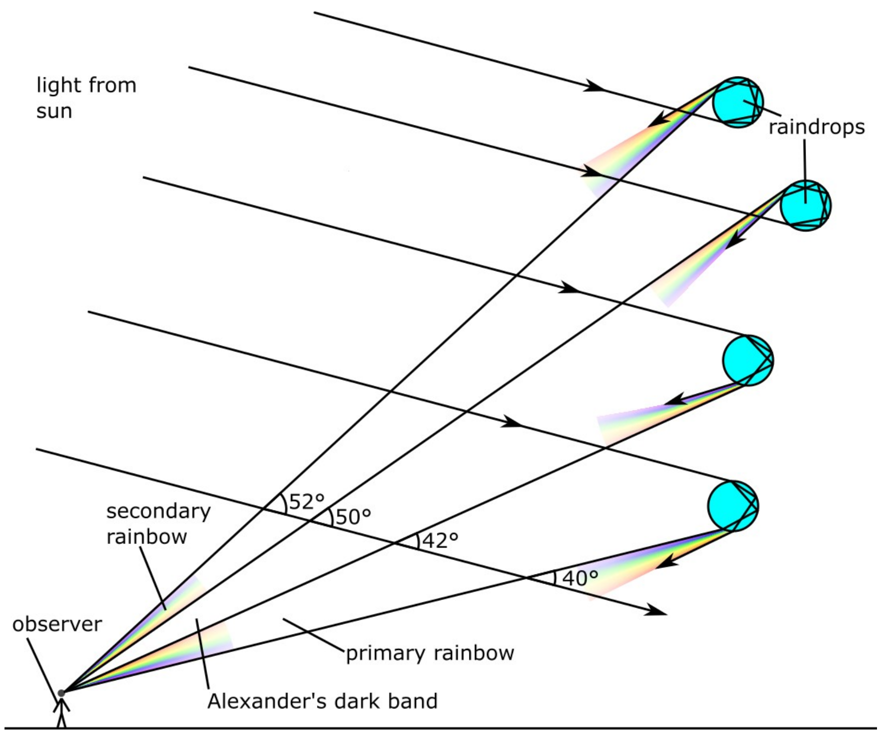


Figure 1: Formation of primary (first order) and secondary (second order) rainbow

The several colors in a rainbow are due to the fact that sunlight is made up of a range of colors, and the light of different colors is refracted at different angles, when it passes from one medium into another. Thus 'dispersion' occurs as sunlight propagates through the drop. From the surface of the earth, we can easily observe primary (first order) and secondary (second order) rainbows and the Alexander's dark band between the two rainbows.

Acid rain results when sulfur dioxide (SO_2) and nitrogen oxides (NO_x) present in the atmosphere get absorbed in the precipitating rainwater [1]. The acids (having different refractive indices) and their interaction with water molecules together contribute to the change in the refractive index of rainwater, which affect the angle (with respect to the incident sunlight) at which the rainbow is observed.

The relation between the angle of deviation for different order rainbows and refractive index of the liquid is well defined in geometrical optics [2]. There exists almost a linear relation between refractive index and low concentrations of acid in solutions [3], [4], and these concentrations can be determined using pH in the range of 0 to 7, hence it is possible to show the variation of angle of deviation of different order rainbow with the pH. This provides a possibility that the measurement of rainbow angle can be used to estimate the acidity of rainwater falling at a distant place. We conducted a systematic study of this possibility using a single droplet in a laboratory. We studied the variation of the angle at which rainbow is observed by using a drop of acidic solution with varying pH of the drop.

In this laboratory investigation on formation of rainbow with acidic solutions, we use a red He-Ne laser as a source of parallel beam of monochromatic light. Here, we do not observe various colors as in the case of a rainbow in the sky formed due to sunlight falling on the water drops. We still use the term 'rainbow' that here refers to the first bright fringe (from a set of fringes) emerging from a pendent drop of a liquid which is observed through the telescope of a prism spectrometer.

2. THEORY

In view of geometrical optics, a rainbow formation takes place because of a ray getting reflected and refracted in a droplet of water with refractive index μ. Consider Figure 2 for the first order rainbow (k = 1) formed because of a spherical droplet with a single ray. If we follow along the path of the ray, we find that the ray undergoes a refraction then an internal reflection and finally a refraction emerging at C. If i and r be the angle of incidence and the angle of refraction respectively, and if φ_1 is the total angle of deviation of the emerging ray from the ray incident on the drop then,

$$\varphi_1 = 180° + 2i - 4r$$

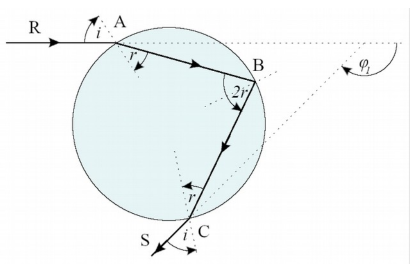


Figure 2: Ray diagram for the first order rainbow formed from a spherical drop

For the second order rainbow instead of one internal reflection there are two internal reflections as shown in Figure 3 and if φ_2 is the total angle of deviation of the emerging ray from the ray incident on the drop then,

$\varphi_2 = 360° + 2i - 6r$

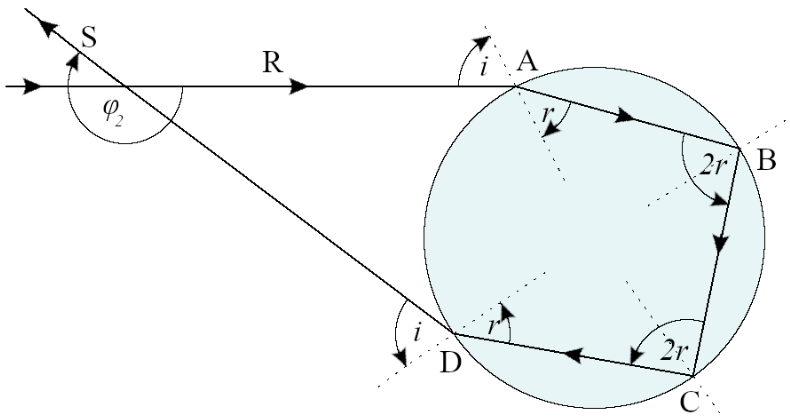


Figure 3: Ray diagram for the second order rainbow formed from a spherical drop

In the same manner, for k[th] order rainbow, as derived in [5], if φ_k is the total angle of deviation of the emerging ray from the ray incident on the drop then,

$\varphi_k = 180k + 2i - 2(k+1)r$

We minimize φ_k with respect to angle i (and use Snell's law), for finding the minimum angle of deviation at which rainbow formation takes place. We find the minimum φ_k to be:

$$\varphi_k = 180k + 2\cos^{-1}\sqrt{\frac{\mu^2-1}{k(k+2)}} - 2(k+1)\sin^{-1}\left(\frac{\sin\cos^{-1}\sqrt{\frac{\mu^2-1}{k(k+2)}}}{\mu}\right) \quad \text{….(Eq. 1)}$$

We plotted using Desmos [6], a theoretically expected graph of variation of total angle of deviation or rainbow angle φ_k as given by equation (1) with refractive index μ for various orders of rainbow i.e., k = 1, 2, 3, 4 and 5 as shown in Figure 4.

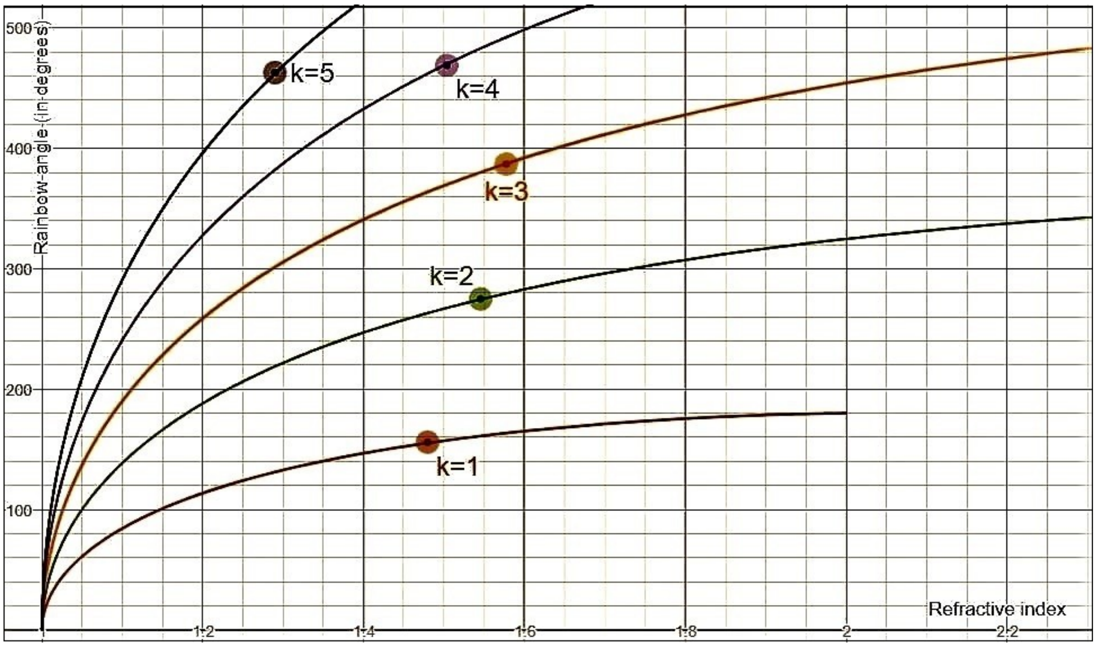


Figure 4: Plot of variation of rainbow angle φ_k with refractive index μ for various values of k

3. EXPERIMENTAL SETUP

The complete experimental arrangement is as shown in Figure 5 and Figure 6. The experimental setup used in this study consisted of the following main components:

(a) A He-Ne red laser source (wavelength 632.8 nm with 5 mW of power) was used as a source of monochromatic and parallel beam of light. The laser source was mounted on a precision laboratory jack using a laser mount as shown in Figure 6.

(b) A prism spectrometer (9 inches diameter scale and having least count of 0.0028°, Make: Optiregion [7]) was used for measuring the angle of deviation for the first and second order rainbows. A specially designed mount with a syringe holder was used for holding the syringe on the prism table of the spectrometer. At the front (where light enters) of the collimator an aluminum sheet with a hole of 2 mm diameter, was stuck using foam tape. Lens in the collimator was replaced with a pin hole of diameter 2 mm. This was done to ensure that the incident laser beam passes through the center of the spectrometer symmetrically. The objective of the telescope was also removed.

(c) A pair of polarizer and analyzer (Make: Melles Griot) were used for controlling the intensity of the laser beam as we were observing the rainbow and the direct beam through the telescope for measuring the angles.

(d) An Abbe's Refractometer with a least count of 0.001 (Make: Optiregion [7]) as shown in Figure 7 (a) was used to measure the refractive index μ of the acidic solution. This refractometer measures the refractive index of the liquid placed between two glass prisms. Light source is focused at the bottom of the illuminating prism. The refractometer is first calibrated using plain water. After calibration, the water was wiped using soft, splinter-free tissue paper. Then the acidic solution was placed and the refractive index measured by turning the scale knob to get a clear interface between dark and bright regions. The crosswire is then moved to the interface and the value of the refractive index is noted.

(e) A pH meter (Make: Equiptronics, Model: EQ-614A [8]) as shown in Figure 7 (b), was used along with its probe to measure the value of pH of the solution used to study the rainbow angle for each case.

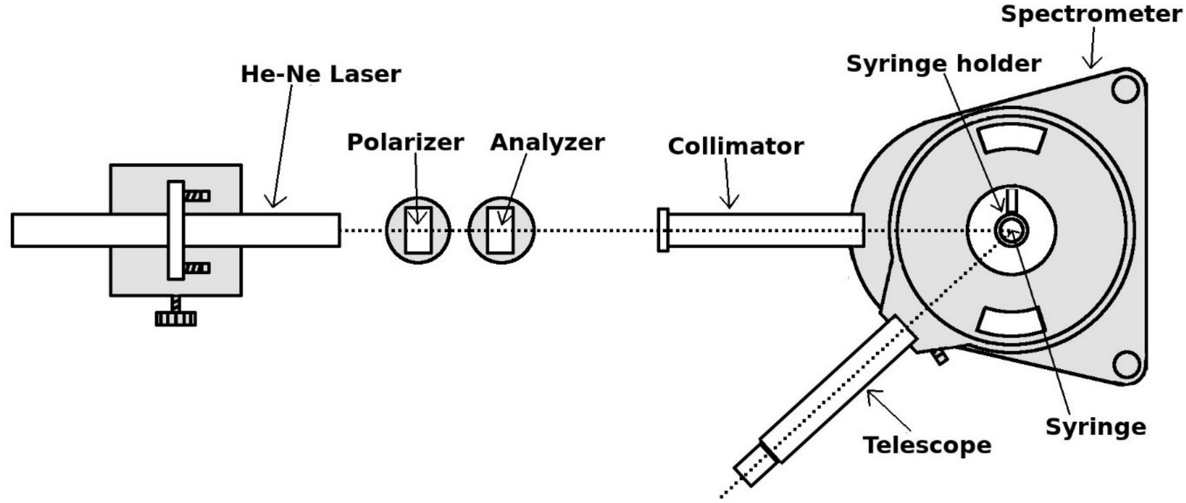


Figure 5: Schematic diagram (top view) of the experimental setup

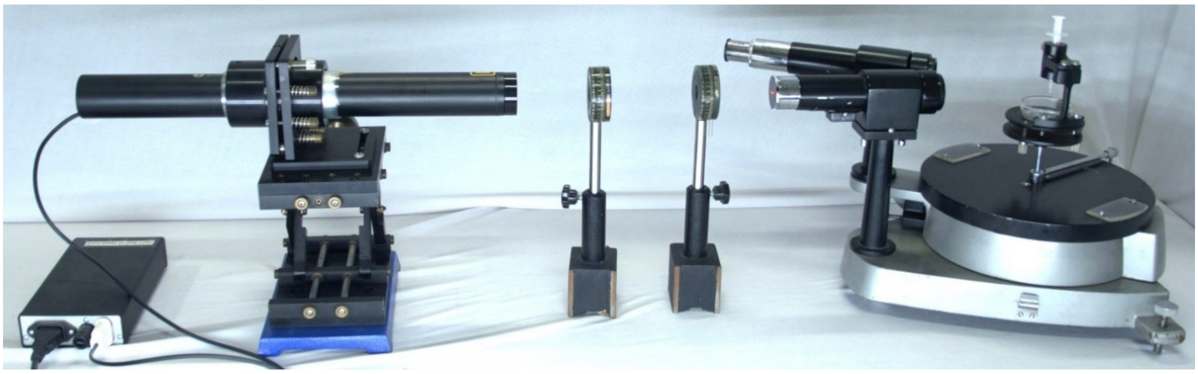
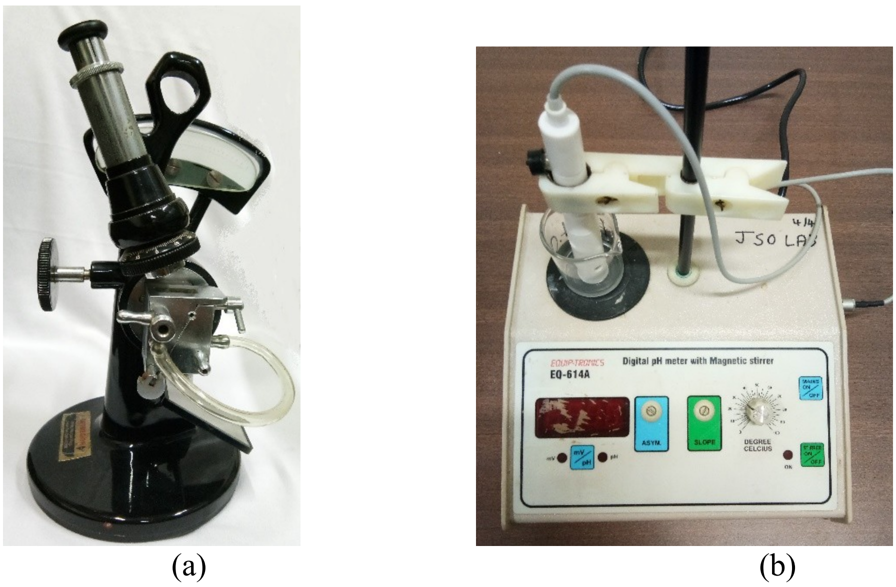


Figure 6: Photograph (oblique view) of the experimental setup



(a) (b)

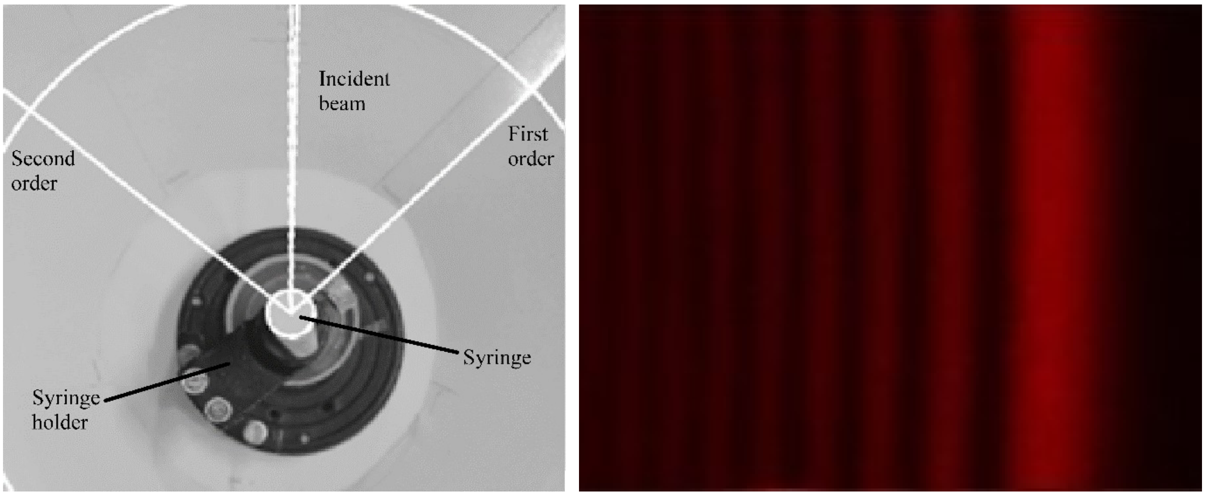
Figure 7: Photograph of the Abbe's refractometer (a) and the pH meter (b) used in this study

4. OBSERVATIONS AND MEASUREMENTS

The following process was used to collect the required experimental data. First the distilled water was taken in a beaker. Using buffer solutions of pH 4.0, 7.0 and 9.2, the measurement probe of the pH meter was calibrated. Then the probe was washed and placed in the beaker. Using micropipette, a few drops of sulfuric acid (H_2SO_4) (whose concentrated solution was 98% wt./wt.) were added in the beaker and the solution was stirred. Then, a 2 mL syringe was rinsed with the solution in the beaker. A sample of the solution was taken in the syringe and the syringe was mounted on the syringe holder. The laser was switched ON and the intensity of light was reduced using a polarizer and analyzer and then direct beam angle was noted. Now, a pendent drop was formed at the center of the line of the laser by carefully pressing the piston of the syringe. The telescope of the prism spectrometer was moved to the positions where rainbows were forming. The intensity of the laser beam was varied using an analyzer such that the width of the rainbow was reduced and the angle of the first bright fringe was noted for both the orders. We thus measured the total angle of deviation φ_1 and φ_2 for the acidic solution made using sulfuric acid.

Now syringe was taken out and a few drops from it were placed in the Abbe's refractometer and the refractive index μ of the solution was measured. Each time the prism surface inside the refractometer was cleaned using isopropanol and allowed to dry. Also, the value of pH of the solution in the beaker was measured using a pH meter. This entire process was repeated for several solutions with pH varying from 7.0 to 0.0. Further, the entire process was repeated for nitric acid (HNO_3) whose concentrated solution was 70% (wt./wt.).

Figure 8 (a) gives the angular positions of the first (on right) and second (on left) order rainbows as seen from top of the syringe mount, and (b) gives the first bright fringe (which is used for the measurement of the angle of deviation) and interference fringes as observed through the telescope for both the orders. These interference fringes are observed because of the path difference introduced between the emerging rays due to the finite width of the laser beam which is incident on the drop around a particular angle of incidence [2] for each order.



(a) (b)

Figure 8: (a) Angular positions of the first and second order rainbow from top of the syringe, and (b) the interference fringes as observed through the telescope

5. DATA AND RESULTS

Table 1 and Table 2 shows the data collected for the first and second order rainbows for the acidic solutions made using H_2SO_4 and HNO_3. Here, we give the data for measured values pH, refractive index μ and the values of total angle of deviation φ_1 and φ_2 measured using a spectrometer, for 10 measurements performed. Further, using Equation (1) and the experimentally measured value of refractive index μ, we have calculated the theoretically expected values of deviation φ_1 and φ_2. These values are also given in the tables. Note that the telescope focuses at the center of the spectrometer table, since the final rays come out from the surface of the drop which is slightly off-center of the spectrometer table, hence the measured angle of deviation was corrected for this error.

Table 1: Data collected for first and second order rainbow for solutions made with H_2SO_4

Sr. No.	pH	Refrac-tive index (μ)	For first order ϕ_1 /(^{o})		For second order ϕ_2 /(^{o})	
			Calculated	Measured	Calculated	Measured
1	7.00	1.332	137.776	137.711	230.628	230.849
2	5.36	1.332	137.776	137.757	230.628	230.850
3	4.86	1.331	137.630	137.762	230.365	230.780
4	3.15	1.332	137.776	137.854	230.628	230.790
5	2.61	1.332	137.776	137.917	230.628	230.706
6	2.08	1.332	137.776	137.854	230.628	230.823
7	1.48	1.333	137.922	137.806	230.891	230.901
8	1.03	1.333	137.922	137.972	230.891	231.083
9	0.52	1.335	138.212	138.260	231.414	231.599
10	0.00	1.340	138.929	138.913	232.709	232.923

Table 2: Data collected for first and second order rainbow for solutions made with HNO_3

Sr. No.	pH	Refractive index (μ)	For first order ϕ_1 / (°)		For second order ϕ_2 /(°)	
			Calculated	Mea-sured	Calculated	Measured
1	7.00	1.332	137.776	137.734	230.628	230.734
2	6.10	1.332	137.776	137.884	230.628	230.417
3	4.05	1.332	137.776	137.628	230.628	230.857
4	3.63	1.332	137.776	137.756	230.628	230.769
5	3.02	1.332	137.776	137.642	230.628	230.817
6	2.03	1.332	137.776	137.862	230.628	230.894
7	1.36	1.333	137.922	137.744	230.891	230.927
8	0.91	1.333	137.922	137.908	230.891	231.260
9	0.58	1.333	137.922	138.040	230.891	231.273
10	0.05	1.337	138.500	138.110	231.934	232.241

We plotted a graph of angle of deviation φ_1 and φ_2 versus pH for both the acids as shown in Figure 9 and 10.

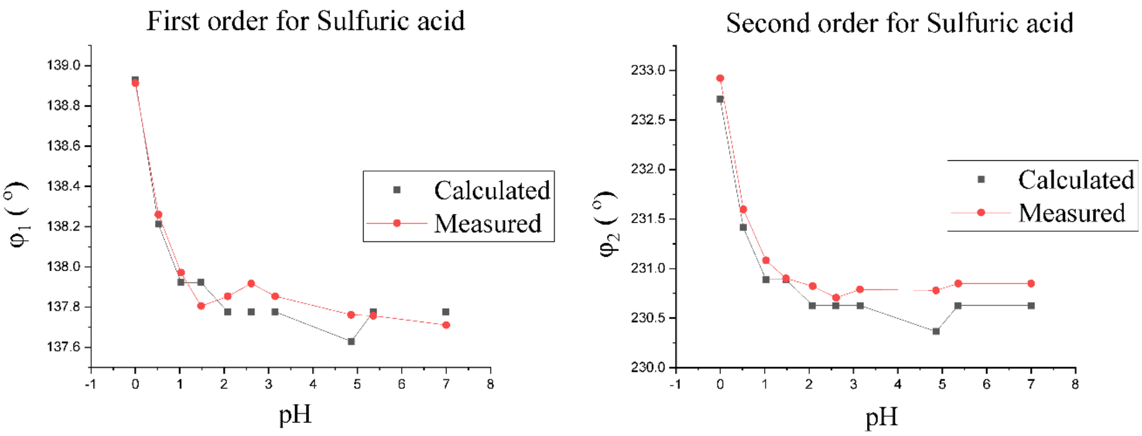


Figure 9: Plots of angle of deviation φ_1 and φ_2 versus pH for the sulfuric acid (H_2SO_4)

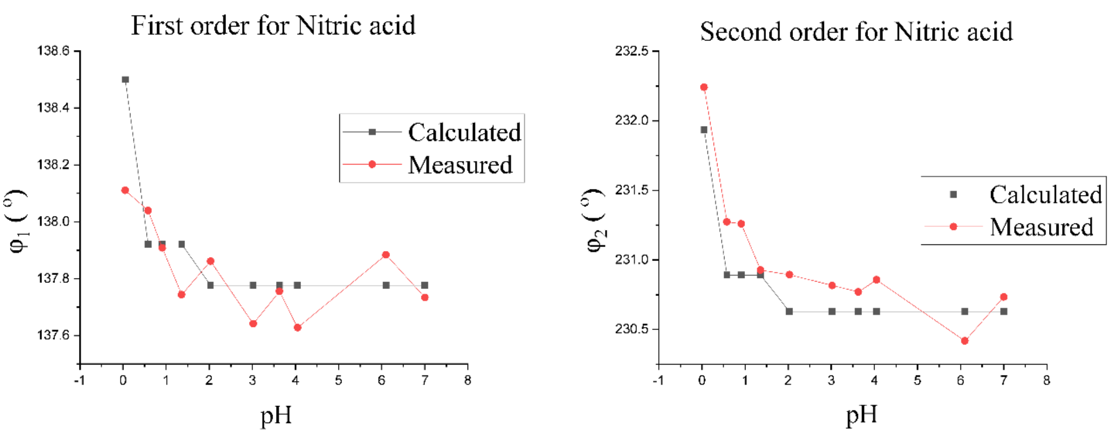


Figure 10: Plots of angle of deviation φ_1 and φ_2 versus pH for the nitric acid (HNO_3)

From the data shown in Table 1 and 2, we calculated the difference in the measured angle of deviation and the angle of deviation calculated from the measured value of the refractive index and Equation (1). Table 3 shows this data for the acidic solutions made with both the acids.

Table 3: Difference in the angle of deviation

For acidic solution made with	Calculated difference for		Measured difference for	
	first order	second order	first order	second order
H_2SO_4 (pH 7.00 to 0.00)	1.153°	2.081°	1.202°	2.074°
HNO_3 (pH 7.00 to 0.05)	0.724°	1.306°	0.376°	1.507°

6. CONCLUSION

We studied the rainbow formation in a single pendent drop of acidic solution in a laboratory set-up and measured the variation of the angle of deviation of the rainbow with the value of pH of the solution of the drop. It was observed that for sulfuric acid, as the pH varied from 7.0 to 0.0, the angle for the first order rainbow varied from 137.711° to 138.913° and for the second order from 230.849° to 232.923. For nitric acid, the angle for first order rainbow varied from 137.734° to 138.110° and for the second order from 230.734° to 232.241, for pH varying from 7.0 to 0.05. Our results show that there is relatively small change in the rainbow angle with variation of pH of the acidic solution of the drop in the typical range for the acid rain (pH of about 4.0 to 5.5). However, strong acid rains can be identified by making simple observations of rainbow angle in the sky. Further, even within the normal range of acidity, with accurate measurements of rainbow angle, one may be able to detect acid rain and get an estimate of the acidity or pH remotely.

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