Using Hadron resonance gas model to extract freezeout parameters at LHC energies

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Abstract. Hadron resonance gas (HRG) model gives a statistical description of hadrons in the grand canonical ensemble picture. It can be used to obtain the thermodynamic variables like Pressure, Entropy and Energy of the system. We have used one variant of the model called the excluded volume HRG in which the different hadrons are considered as hard spheres that follow the quantum statistics of bosons or fermions. We have used our HRG model to calculate the bulk thermodynamics of a gas of hadrons and compared those calculated from an ideal hadron gas (hadrons are considered as point particles) model and Lattice QCD data. Then we have used the number density of various hadrons calculated using the ideal HRG model, compared it to corresponding measured yields of hadrons in ALICE for Pb-Pb collisions at 2.76 TeV, to obtain the freeze-out volume and temperature.

Keywords: Hadron resonance gas (HRG) model, Freezeout parameters, LHC experiments.

1. THE HADRON RESONANCE GAS MODEL

The basic quantity required to compute the thermodynamical quantities is the partition function $Z(T, V)$ [1]. In the grand canonical (GC) ensemble, the partition function for a particle species $i$ in the limit of large volume takes the following form ($k = \hbar = c = 1$):

$$
\ln Z_i^{id.gas} = \frac{g_i V}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))
$$

(1)

Where, $g_i$ is the degeneracy factor, $E_i = \sqrt{p^2 + m_i^2}$ is the total energy of a particle with mass $m_i$ and $\mu_i$ is the chemical potential of the $i^{th}$ species.

This integral can be solved analytically to obtain $\ln Z_i^{id.gas}$ as an infinite sum of bessel functions of second kind.

$$
T \ln Z_i^{id.gas} = \frac{g_i m_i^2 V T^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{(\pm 1)^{n-1}}{n^2} K_2 \left( \frac{nm_i}{T} \right) \exp \left( \frac{n\mu_i}{T} \right)
$$

(2)

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Pressure ($P$), number density ($n_i$), entropy density ($s_i$) and energy density ($\epsilon_i$) obtained as:

$$ n_{id.gas}^{i}(T,\mu_i) = \frac{T}{V} \left( \frac{\partial \ln Z_{id.gas}^{i}}{\partial \mu_i} \right)_{V,T} \tag{3} $$

$$ P_{id.gas}^{i}(T,\mu_i) = \frac{T}{V} \ln \left( Z_{id.gas}^{i} \right) \tag{4} $$

$$ \epsilon_{id.gas}^{i}(T,\mu_i) = -\frac{1}{V} \left( \frac{\partial \ln Z_{id.gas}^{i}}{\partial (1/T)} \right)_{\mu/T} \tag{5} $$

$$ s_{id.gas}^{i}(T,\mu_i) = \frac{1}{V} \left( \frac{\partial \ln Z_{id.gas}^{i}}{\partial (T)} \right)_{V,\mu} \tag{6} $$

2. INTERACTING HADRON RESONANCE GAS MODEL

The preceding section describes hadrons that do not interact with each other. That is often not the case. Therefore, we also explore the cases where the hadrons interact. We start with adding a repulsive interaction by giving the hadrons dimensions (Excluded volume HRG).

2.1 Excluded volume HRG

For deriving the expressions for the thermodynamic variables in the excluded volume HRG, we need to solve the following transcendental equations [2][3]:

$$ p^{ex}(T,\mu_1,\mu_2,...,\mu_n) = p(T,\hat{\mu}_1,\hat{\mu}_2,...,\hat{\mu}_n) \tag{7} $$

$$ \hat{\mu}_i = \mu_i - v_0 p^{ex}(T,\mu_1,\mu_2,...,\mu_n) \tag{8} $$

Where, $v_0 = (16\pi/3)R_i^3$, $R_i$ being the radius of the $i^{th}$ hadron.

The other thermodynamic variables can be calculated as,

$$ n_i^{ex} = \left( \frac{\partial p^{ex}}{\partial \mu_i} \right)_T = \frac{n_i(T,\hat{\mu}_i)}{1 + \sum_k v_0 n_k(T,\hat{\mu}_k)} \tag{9} $$

$$ s_i^{ex} = \left( \frac{\partial s^{ex}}{\partial T} \right)_{(\mu_i)} = \frac{s_i(T,\hat{\mu}_i)}{1 + \sum_k v_0 n_k(T,\hat{\mu}_k)} \tag{10} $$

$$ \epsilon_i^{ex} = T s - P + \sum_k \mu_k n_k = \frac{\epsilon_i(T,\hat{\mu}_i)}{1 + \sum_k v_0 n_k(T,\hat{\mu}_k)} \tag{11} $$

One then can numerically calculate and compare the thermodynamic variables $P, s$ and $\epsilon$ drawn from Ideal and Excluded volume HRG and lattice QCD calculations [4].
Figure 1. Pressure (top), Entropy density (middle) and Energy density (bottom) with ideal and excluded volume HRG, and comparison with lattice QCD[4].

From Fig. 1, it is apparent that the excluded volume consideration suppresses the pressure,
entropy density and energy density compared to the ideal case. This suppression becomes larger with temperature, as the mean distance between the hadrons keeps getting smaller compared to twice the radius considered in excluded volume HRG. Hadrons up to 2.25 GeV in mass have been considered, which includes 63 mesons and 59 baryons.

3. RESONANCE DECAY AND ESTIMATION OF YIELD

Until now, we were taking all the hadrons to be stable, and not considering their decays. But that is not the case. Most of the hadrons taken are unstable resonances that decay into lower mass hadrons. Considering resonance decay is imperative to estimating the yields of different particles at freezeout[5].

Therefore, we calculate the final multiplicity of a hadron species $h$ as,

$$\langle N_h \rangle = V n_h + V \sum_R \langle n_h \rangle_R n_R$$

(12)

Where, $V$ is the volume of the fireball, $V n_h$ is the primary yield of hadron $h$, $n_R$ is the primary yield of resonance $R$ and $\langle n_h \rangle_R$ is the average number of particles of species $h$ from a decay of resonance $R$ (also called the branching ratio). In this work, we have compared the yields obtained from the Ideal- HRG model, considering resonance decay to the yields of $\pi^\pm, K^\pm, K_0, p, \bar{p}, \Lambda, \Xi^\pm, \Omega^\pm$ corresponding measured yields in ALICE Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV[6]. The branching ratios are obtained from the PDG[7].

As for LHC energies, the chemical potential tends to zero, we have set $\mu_B, \mu_S$ and $\mu_Q$ to zero. The free parameters are the freezeout temperature ($T$) and volume ($V$). We have used the $\chi^2$ minimization method to obtain $T$ and $V$, where, $\chi^2$ is defined as,

$$\frac{\chi^2}{N_{dof}} = \frac{1}{N_{dof}} \sum_{h=1}^{N} \frac{(\langle N_{h}^{\text{exp}} \rangle - \langle N_{h} \rangle)^2}{\sigma_h^2}$$

(13)

Where, $\langle N_{h}^{\text{exp}} \rangle$ and $\langle N_{h} \rangle$ are the experimental and theoretical hadron yields respectively, $\sigma_h$ is the error in experimental yields and $N_{dof}$ is the number of degrees of freedom calculated as the difference between the number of particles considered and the number of free parameters.

We calculated the $\chi^2$ for a range of temperature and volume and then found the $T$ and $V$ for which the value of $\chi^2$ is the lowest (shown in Fig.2).

For our considerations, we get a minimum $\chi^2/N_{dof}$ value of 8.08, at a Temperature of $(163^{+9}_{-8})$ MeV and a fireball volume of $V = (3960^{+1240}_{-566})$ fm$^3$. 

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A comparison of the calculated and experimental yields have also been done in both graphically (Fig. 3) and tabular manner (Table 1).

From table 1, we observe, that the light hadron yields are better determined than their heavier counterparts. This is because the limited number of resonance decays taken into account in our calculations. The heavier hadrons have more significant contributions from the heavy resonance decays than the lighter ones.
Figure 3. A comparison of experimental data[6] and model calculations for different particles.

Table 1. Experimental and Model Yields.

<table>
<thead>
<tr>
<th>Particle</th>
<th>$\langle N_{h}^{\text{exp}} \rangle$</th>
<th>$\sigma_{h}$ (Experimental error)</th>
<th>$\langle N_{h} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{+}$</td>
<td>733</td>
<td>54</td>
<td>693.46</td>
</tr>
<tr>
<td>$\pi^{-}$</td>
<td>732</td>
<td>52</td>
<td>695.88</td>
</tr>
<tr>
<td>$K^{+}$</td>
<td>109</td>
<td>9</td>
<td>143.06</td>
</tr>
<tr>
<td>$K^{-}$</td>
<td>109</td>
<td>9</td>
<td>143.097</td>
</tr>
<tr>
<td>$K_{0}$</td>
<td>110</td>
<td>10</td>
<td>138.055</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>34</td>
<td>3</td>
<td>42.17</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>33</td>
<td>3</td>
<td>42.17</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>26</td>
<td>3</td>
<td>22.94</td>
</tr>
<tr>
<td>$\Xi^{+}$</td>
<td>3.28</td>
<td>0.247</td>
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</tr>
<tr>
<td>$\Xi^{-}$</td>
<td>3.34</td>
<td>0.238</td>
<td>2.53</td>
</tr>
<tr>
<td>$\Omega^{+}$</td>
<td>0.6</td>
<td>0.103</td>
<td>0.78</td>
</tr>
<tr>
<td>$\Omega^{-}$</td>
<td>0.58</td>
<td>0.098</td>
<td>0.78</td>
</tr>
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</table>
4. CONCLUSION

Through $\chi^2$ minimization, we obtain the freezeout temperature and volume as $(163_{-8}^{+9})$MeV and $(3960_{-560}^{+1240})$ fm$^3$ respectively for heavy-ion collisions at LHC energies.

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References