Large scale anisotropy in Cosmic microwave background radiation

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January 21, 2015

Outline

- **Introduction**
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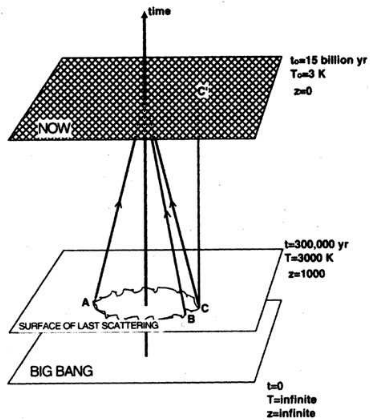
Introduction

Cosmological Principle

- Modern Cosmology based on Cosmological Principle which states that Universe is homogeneous and isotropic on large scales ($\geq$ 100Mpc).
- Supported by High degree of isotropy seen in the CMB blackbody spectrum at a mean temperature of 2.725 K with a peak in the microwave regime.

Continued...

Brief review of CMB



Last scattering surface

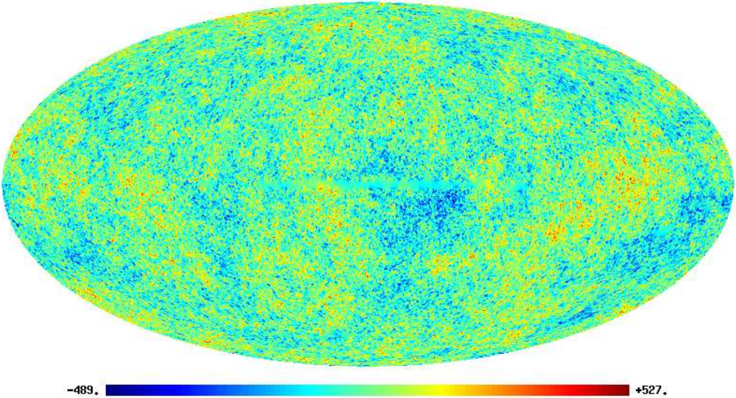
- The observed CMB support the Hot Big Bang cosmological model.
- Photons were in thermal equilibrium with the inhomogeneous baryon density field upto the redshift $z = 1100$
- Expansion of universe leads to formation of bound states ("Recombination").
- After this epoch the photon mean free path increased to greater than the present Hubble radius so that they could free stream to us.

Continued...

The COBE spacecraft detected the anisotropy in CMB which carries an imprint of the primordial perturbations via small temperature fluctuation of the order of $10^{(-5)}$ on angular scales $> 7^o$.

Continued...

Universe at Large Distance Scale



CMB seen by Planck (380,000 years old universe), fluctuate 1 part in 100,000.

Continued...

CMB Anomalies

- Alignment of $l = 2, 3$ multipoles of the cosmic microwave background radiation with significance $3 - 4\sigma$.
  Costa et al. 2004, Ralston & Jain 2004
- Hemispherical power asymmetry or Dipole modulation in temperature.
  Eriksen et al. 2004, Hoftuft et al. 2009. Bennett et al. 2011, Ade et al. 2014
- Parity asymmetry in the CMB data.
  Kim et al. 2010, Aluri & Jain 2012a
- Cold spot in CMB of radius 10^o located at $l = 207^o$ and $b = -56^o$ in the Southern hemisphere.
  Vielva et al. 2004, Cruz et al. 2005

Continued...

- Dipole anisotropy in radio polarizations.
  Birch 1982, Jain & Ralston 1999
- Large scale alignment of optical polarizations.
  Hutsemekers 1998
- Dipole anisotropy in distribution of radio galaxies
  Singal 2011, Tiwari et al. 2015 and cluster peculiar velocities
  Kashlinsky et al. 2010.

Continued...

- Remarkably several of these indicate a preferred direction towards the Virgo cluster.
- **The indications for a preferred direction in the data have motivated many theoretical studies of inflationary models which violate statistical isotropy and homogeneity, giving rise to a direction dependent power spectrum.**

continued...

Theoretical models...

- At very early times the Universe may be anisotropic and inhomogeneous.
- Bianchi models are for homogeneous, but anisotropic universes.
- For times $t < t_{iso}$ $(= \sqrt{\frac{3}{\Lambda}})$, the universe is anisotropic and becomes isotropic later on.
- Modes generated during this anisotropic phase leave the horizon at early times. *Aluri & Jain 2012b*.
- They re-enter the horizon later during radiation and dark matter dominated phases and leads to structure formation and CMB anisotropies.

Continued...

Our Work...

We revisit, the two of the anomalies observed in the CMBR temperature data which is provided by WMAP and recently by PLANCK team.

- Alignment of the quadrupole ($l = 2$) and octopole($l = 3$).
- Hemispherical power asymmetry.

Continued...

Alignment of quadrupole and octopole...

We show that the anisotropic modes arising during the very early phase of inflation can consistently explain the alignment of CMB quadrupole and octopole within the framework of Big Bang cosmology.

Continued...

Anisotropic metric...

We consider three different anisotropic metrices,

$$ds^2 = dt^2 - 2\sqrt{\sigma}dzdt - a^2(t)(dx^2 + dy^2 + dz^2) \quad \text{Model I}$$

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2) - b^2(t)dz^2) \quad \text{Model I}$$

$$ds^2 = dt^2 - a_1^2(t)(dx^2 + dy^2) - a_2^2(t)dz^2 \quad \text{Model III}$$

where $a_2(t) = a_1(t) + \sigma$, σ being a constant, independent of time.

Continued...

Quantization of field

- The action may be written as,

$$S_\phi = \int d^4x \, \mathcal{L}$$

and the Lagrangian density,

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \, .$$

The Quantized scalar field $\phi(x,t)$ in fourier space,

$$\phi_I(x,t) = \int \frac{d^3k}{(2\pi)^3}\left(e^{i\vec{k}\cdot\vec{x}}\phi_k(t)a_k + e^{-i\vec{k}\cdot\vec{x}}\phi_k^*(t)a_k^\dagger\right) .$$

with $[a_k, a_{k'}^+] = (2\pi)^3\delta(k-k')$.

- Euler-Lagrange equation of motion,

$$\frac{1}{\sqrt{-g}}\partial_\nu(g^{\mu\nu}\sqrt{-g}\partial_\mu\phi) = 0 \, .$$

Continued...

Direction dependent power spectrum

Model I

- The Hamiltonian in interaction-picture,

$$H_I = \int d^3x \sqrt{\sigma} \left(\frac{d\phi_I(x,\eta)}{d\eta} \frac{d\phi_I(x,\eta)}{dz} \right) .$$

- Two point correlations to first order,

$$\begin{aligned} \langle \phi(x_1,t)\phi(x_2,t) \rangle &\equiv \langle \phi_I(x_1,t)\phi_I(x_2,t) \rangle + \\ & i\int_0^t dt' \langle [H_I(t'), \phi_I(x_1,t)\phi_I(x_2,t)] \rangle . \end{aligned}$$

- To the leading order of the perturbation the anisotropic part of the two point correlation vanishes, hence gives no correction to the primordial power spectrum.

Continued...

Model II

- The power spectrum obtained in *Ackerman et al. 2007*,

$$P'(k) = P_{iso}(k)\left[1 + g(k)(\hat{k}\cdot\hat{n})^2\right] \quad \text{Model II}$$

- The anisotropic function, $g(k)$, at the end of inflation (time t_*) is,

$$g(k) = \frac{9}{2}\epsilon_H \log\left[\frac{q(t_*)}{\bar{H}}\right],$$

  with, $\bar{H} = \frac{1}{3}(2H_a + H_b)$, $\epsilon_H = \frac{2}{3}\left(\frac{H_b - H_a}{\bar{H}}\right)$ and the physical wavelength $q(t_*) = \frac{k}{a(t_*)}$.

- Here the perturbative correction to the isotropic power spectrum is neglected.

Model III

Similarly for model III, the modified power spectrum is given by,

$$P'(k) = P'_{iso}(k)\left[1 + (\hat{k}\cdot z)^2 g(k)\right],$$

where,

$$g(k) = -\frac{\sigma}{a_I k}\left[k\cos\left(\frac{2k}{a_I\bar{H}}\right) - \frac{5}{2}a_I\bar{H}\sin\left(\frac{2k}{a_I\bar{H}}\right) + 2a_I\bar{H}\mathrm{Si}\left(\frac{2k}{a_I\bar{H}}\right)\right]$$

with

$$\mathrm{Si}\,(x) \equiv \int_0^x \mathrm{d}x'\,\frac{\sin x'}{x'}\,.$$

Here $P'_{iso}(k)$, includes a perturbative correction to $P_{iso}(k)$ as
$P'_{iso}(k) = P_{iso}(k)\left[1 - \frac{g(k)}{3}\right]$.

Continued..

Effect of the power spectrum on the CMB

- Temperature fluctuation,

$$\delta T(\hat{n}) = T_0 \int dk \sum_l \frac{2l+1}{4\pi} (-i)^l P_l(\hat{k} \cdot \hat{n}) \delta(k) \Theta_l(k)$$

- For the low-l multipoles, only the Sachs-Wolfe effect contributes effectively to the transfer function, $\Theta_l(k) = \frac{3}{10} j_l(k\eta_0)$.

- Spherical harmonic coefficients a_{lm},

$$a_{lm} = \int d\Omega Y^*_{lm}(\hat{n}) \delta T(\hat{n}).$$

- Two point correlation function of a_{lm}'s implies,

$$\langle a_{lm} a^*_{l'm'} \rangle = \langle a_{lm} a^*_{l'm'} \rangle_{iso} + \langle a_{lm} a^*_{l'm'} \rangle_{aniso}.$$

Continued...

- Isotropic part,

$$\langle a_{lm} a^*_{l'm'} \rangle_{iso} = \delta_{ll'} \delta_{mm'} \int_0^\infty k^2 dk P_{iso}(k) \Theta_l^2(k),$$

following the statistical isotropy, the two point correlation implies,

$$\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l.$$

Continued...

- Anisotropic part,

$$\langle a_{lm} a^*_{l'm'} \rangle_{aniso} = (-i)^{l-l'} \xi_{lm;l'm'} \int_0^\infty k^2 dk P_{iso}(k) g(k) \Theta_l(k) \Theta_{l'}(k),$$

with,

$$\xi_{lm;l'm'} = \sqrt{\frac{4\pi}{3}} \int d\Omega_k (Y_{lm}(\hat{k}))^* Y_{l'm'}(\hat{k}) \times \left(n_+ Y_1^1(\hat{k}) + n_- Y_1^{-1}(\hat{k}) + n_0 Y_1^0(\hat{k}) \right),$$

where we have used the spherical components of the unit vector n as,

$$n_+ = -\left(\frac{n_x - i n_y}{\sqrt{2}} \right), n_- = \left(\frac{n_x + i n_y}{\sqrt{2}} \right), n_0 = n_z.$$

- This leads to,

$$\langle a_{lm} a^*_{l'm'} \rangle = C_{ll'mm'}$$

Continued...

Statistics

- Defined a second rank real symmetric Power tensor matrix as,

$$A_{ij}(l) = \frac{1}{l(l+1)} \sum_{mm'm''} \langle lm|J_i|lm'\rangle \langle lm''|J_j|lm\rangle a_{lm'} a^*_{lm''}$$

  where J_i are the angular momentum operators in spin$-l$ representation.

- The eigenvectors of the power tensor defines an invariant frame for each multipole and the corresponding eigenvalues yield the power along each of these vectors.

- For a given multipole, the eigenvector associated with the largest eigenvalue of the power tensor is known as the principal eigenvector (PEV) which defines a preferred direction for that multipole. (*Ralston 2004, Samal 2008*).

Continued...

- We measure for dispersion in the power entropy using the normalized eigenvalue $\tilde{\lambda}$ of the power tensor. The Power entropy is defined as,

$$S_P(l) = -\sum_a \tilde{\lambda}_a \log \tilde{\lambda}_a.$$

- $S_P(l)$ can take values in the range 0 to $\log(3)$, with 0 being the value for the maximal dispersion , and hence anisotropy.
- We attribute the difference $\Delta S_P(l) = S_P(l, data) - S_P(l, mean)$ to the contribution due to the anisotropic term, where $S_P(l, data)$ and $S_P(l, mean)$ are the power entropy of the observed CMB data and the mean of random samples.
- The theoretical estimate of $\Delta S_{P,th} = S_{P,th} - \log(3)$.

Continued...

Result

- In model II, the metric is anisotropic during the entire inflationary period, hence we expect that all the modes, independent of the k values, would violate the isotropy and lead to the alignment among CMB multipoles of arbitrary l values.
- For each l, we expect two equal eigenvalues out of three and the third eigenvalue is found to be larger than these two eigenvalues for negative values of the anisotropy parameter ϵ_H.
- Hence in this case, the principal eigenvector would align with the preferred $z-$axis.

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Continued...

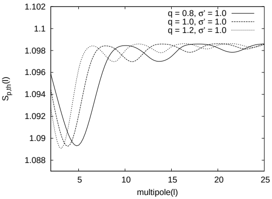
Model II

- For WMAP-9 year data, $\Delta S_P(2) = -0.0056$ and $\Delta S_P(3) = -0.064$.
- The observed value is very close to the isoptropic case for $l = 2$ and for $l = 3$, the deviation is larger but still less than 1σ.
  $\Longrightarrow$ not statistically significant.
- The best fit value of the anisotropy paramater ϵ_H is found to be -0.0054 by minimizing the square error.
- The value of $\Delta S_{P,th}$ is found to be independent of l since the anisotropy is equally effective for all the multipoles.

Continued...

Model III

- In model III, $\Delta S_{P,th}$ oscillates, with amplitude decreasing with l.
- We find that the best fit is driven to relatively large values of anisotropic parameter σ', where the peturbation theory becomes unreliable.
- The largest perturbative contribution is obtained for the isotropic part of the power spectrum. Hence here we only give results by fixing the parameter $\sigma' = 1.0$.
- We also find $\Delta S_{P,th}$ approaches to log(3) for large l as expected.



Continued...

conclusion

- In models I and III, the Universe is anisotropic during the early stages of inflation and quickly evolves into a de Sitter space-time.
- The perturbative contributon decays rapidly for higher l.
- In model II, the anisotropy is present throughout the period of inflationary expansion.
- In these models, we choose the preferred direction same as the principal axis of the CMBR $l = 2$ mode which points roughly in the direction of the virgo supercluster.
- However each multipole is affected such that its principal axis is aligned with the preferred axis of the model, hence leads to the alignment of the principle axis corresponding to $l = 2, 3$ multipoles.

Rath et al. JCAP04(2013)007

Hemispherical Power asymmetry in CMBR

- The CMBR data also shows a hemispherical power asymmetry with excess power in the southern ecliptic hemisphere compared to northern ecliptic hemisphere.
- This power asymmetry can be described in the low-l regime by a phenomenological dipole modulation model,

$$\Delta T(\hat{n}) = g(\hat{n})\left(1 + A\hat{\lambda}\cdot\hat{n}\right).$$

- The dipole amplitude A and direction for $l \leq 64$ are:
  - WMAP 5yr data: $A = 0.072 \pm 0.022$, $(l, b) = (224^o, -22^o) \pm 24^o$. (*Hoftuft et al. 2009*)
  - Planck SMICA data: $A = 0.073 \pm 0.010$, $(l, b) = (217^o, -20^o) \pm 15^o$. (*Ade et al. 2014*)
- This anisotropy dies off at $l \sim 500$. (*Donoghue et al. 2005, Hanson et al. 2009*)

continued...

Tests of the dipole modulation model

- We suggest several tests of the dipole modulation model, both in real and multipole space.
- In real space, we squared the observed temperature fluctuation field and extract the dipole.
  - If the field is statistically isotropic, none of the multipoles would be significantly different from those corresponding to a random realization.
  - If the dipole modulation is present, then we should detect a significant dipole.
- In multipole space, we compute the two point correlation of the spherical harmonic coefficients a_{lm}.
  - The dipole modulation model predicts a correlation between a_{lm} and $a_{l+1,m}$ and it does not give any contribution to the power $\langle a_{lm} a^*_{lm} \rangle$.

continued...

Testing in Real space

- In real space, the observed squared temperature fluctuation field,

$$f(\theta,\phi) = (\Delta T(\theta,\phi))^2 \, .$$

- If the dipole modulation model, provides an accurate description of the real data, we can write,

$$\langle f(\theta,\phi)\rangle \approx \sum_l \frac{2l+1}{4\pi} C_l \left(1 + 2A\hat{\lambda}\cdot\hat{n}\right)$$

- The dipole amplitude of the temperature squared field, essentially sums over $(2l+1)C_l$ and hence puts lower weight on higher multipoles.
- We remove $l = 0 - 4$ and $l = 0 - 8$ multipoles from the temperature field for the sensitive probe of the model.
- The dipole amplitude and the direction extracted from the squared field is in well agreement with those obtained from the hemispherical analysis.

continued...

Test in multipole space

- In the presence of the dipole modulation, SI violated,

$$\langle a_{lm} a^*_{l'm'} \rangle = \langle a_{lm} a^*_{l'm'} \rangle_{iso} + \langle a_{lm} a^*_{l'm'} \rangle_{dm} .$$

Taking the preferred axis along $z - axis$,

$$\langle a_{lm} a^*_{l'm'} \rangle_{dm} = A(C_{l'} + C_l)\xi^0_{lm;l'm'}$$

where,

$$\begin{aligned} \xi^0_{lm;l'm'} &= \int d\Omega Y^{*m}_l(\hat{n}) Y^{m'}_{l'}(\hat{n}) \cos\theta \\ &= \delta_{m',m}\left[\sqrt{\frac{(l-m+1)(l+m+1)}{(2l+1)(2l+3)}}\delta_{l',l+1} \right. \\ &\quad \left. + \sqrt{\frac{(l-m)(l+m)}{(2l+1)(2l-1)}}\delta_{l',l-1}\right]. \end{aligned}$$

Continued...

statistics

We test for the correlation in real data by defining,

$$C_{l,l+1} = \frac{l(l+1)}{2l+1} \sum_{m=-l}^{l} a_{lm} a^*_{l+1,m}$$

The sum of $C_{l,l+1}$ over a chosen range of multipoles defines our statistic, $S_H(L)$,

$$S_H(L) = \sum_{l=2}^{L} C_{l,l+1}$$

Following the range of observed hemispherical power asymmetry, we consider the multipole range $l = 2 \leq l \leq 64$.

	$S_H^{data}(L)$ (mK^2)	$S_H^{data}(L)$ (mK^2) (bias corrected)	(l, b)	P-value
WMAP-ILC	0.023 ± 0.007	0.025 ± 0.008	$(227^o, -14^o)$	0.55%
SMICA	0.021 ± 0.005	0.023 ± 0.006	$(229^o, -16^o)$	2.6%
SMICA (in-painted)	0.027 ± 0.007	–	$(232^o, -12^o)$	0.38%

Continued...

conclusion

- We find a significant signal of the dipole modulation model both in WMAP and PLANCK data.
- In real space, we find a significant signal of anisotropy by eliminating a few low l multipoles and the dipole amplitude and direction are found to be in agreement, within errors.
- In multipole space, the test gives correlation between l and $l+1$ multipoles.
- We extract the maximum value of the defined statistics $S_H(L)$ after searching over all possible directions. The extracted value agrees well with this prediction.

Pranati K.Rath, Pankaj Jain, JCAP12(2013)014

Inhomogeneous Power spectrum

We propose an inhomogeneous primordial Power spectrum model and show that it leads to a correlation among different multipoles corresponding to $m' = m$ and $l' = l + 1$, as in the case of the dipole modulation.

Continued...

Two point correlation...

- In real space the two point correlation,

$$F(\vec{\Delta},\vec{X}) = \langle \tilde{\delta}(\vec{x})\tilde{\delta}(\vec{x}')\rangle \,,$$

where, $\vec{\Delta} = \vec{x} - \vec{x}'$ and $\vec{X} = (\vec{x} + \vec{x}')/2$.

- Fourier transform implies,

$$\langle \delta(\vec{k})\delta^*(\vec{k}')\rangle = \int \frac{d^3X}{(2\pi)^3}\frac{d^3\Delta}{(2\pi)^3} e^{i(\vec{k}+\vec{k}')\cdot\vec{\Delta}/2} e^{i(\vec{k}-\vec{k}')\cdot\vec{X}} F(\vec{\Delta},\vec{X})\,.$$

- If we assume that $F(\vec{\Delta},\vec{X})$ depends only on the magnitude $\Delta \equiv |\vec{\Delta}|$, we obtain, $\langle \delta(\vec{k})\delta^*(\vec{k}')\rangle = P(k)\delta^3(\vec{k} - \vec{k}')$, where, $k \equiv |\vec{k}|$.

continued...

- We assume a simple inhomogeneous model,

$$F(\vec{\Delta},\vec{X}) = f_1(\Delta) + \hat{\lambda}\cdot\vec{X}\left(\frac{1}{\eta_0}f_2(\Delta)\right),$$

where $f_1(\Delta)$ and $f_2(\Delta)$ depend only on the magnitude, Δ. The correlation in Fourier space,

$$\langle\delta(\vec{k})\delta^*(\vec{k}')\rangle = \tilde{F}_0(\vec{k},\vec{k}') + \Delta\tilde{F}(\vec{k},\vec{k}'),$$

the second term is given by,

$$\Delta\tilde{F}(\vec{k},\vec{k}') = -i\hat{\lambda}_i\left[\frac{\partial}{\partial k_{-i}}\delta^3(\vec{k}_-)\right]\; g(\vec{k}_+),$$

where,

$$g(k_+) = \frac{1}{\eta_0}\int\frac{d^3\Delta}{(2\pi)^3}e^{i\vec{k}_+\cdot\vec{\Delta}}f_2(\Delta).$$

and $\vec{k}_+ = (\vec{k}+\vec{k}')/2$.

continued...

Aplication in CMB

- Two point correlation function of the spherical harmonic coefficients, a_{lm},

$$\langle a_{lm} a^*_{l'm'} \rangle = (4\pi T_0)^2 \int d^3k d^3k' (-i)^{l-l'} \Theta_l(k) \Theta_{l'}(k') \times Y^*_{lm}(\hat{k}) Y_{l'm'}(\hat{k}') \langle \delta(\vec{k}) \delta^*(\vec{k}') \rangle \,.$$

- Violation of statistical isotropy implies,

$$\langle a_{lm} a^*_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'} + A(l, l') \,,$$

where the anisotropic part,

$$A(l, l') = T_0^2 \int d^3k d^3k' (-i)^{l-l'+1} \Theta_l(k) \Theta_{l'}(k') \times Y^*_{lm}(\hat{k}) Y_{l'm'}(\hat{k}') g(k_+) \left(\frac{\partial}{\partial k_{-z}} \delta^3(\vec{k}_-) \right) \,.$$

Continued...

Result...

- The homogeneous power, $P(k) = k^{n-4}A_\phi/(4\pi)$, with $n = 1$ and $A_\phi = 1.16 \times 10^{-9}$ (*Gorbunov et al. 2011*).
- For the inhomogeneous term, we assume the form,

$$g(k) = g_0 P(k) \frac{(k\eta_0)^{-\alpha}}{\eta_0} .$$

- Setting $\alpha = 0$, the best fit value for $l = 2 - 64$ are $g_0 = 0.048 \pm 0.011$ for WMAP-ILC 9 year and $g_0 = 0.044 \pm 0.012$ for SMICA.
- The 1σ limit is $-0.34 < \alpha < 0.4$ and $-0.24 < \alpha < 0.39$ for WMAP-ILC 9 year and SMICA respectively.
- Over the three multipole bins, $l = 2 - 22, 23 - 43$ and $l = 44 - 64$, $g_0 = 0.047 \pm 0.008$ with $\chi^2 = 0.90$ and $g_0 = 0.043 \pm 0.007$ with $\chi^2 = 0.84$ for WMAP-ILC 9 year and SMICA respectively.

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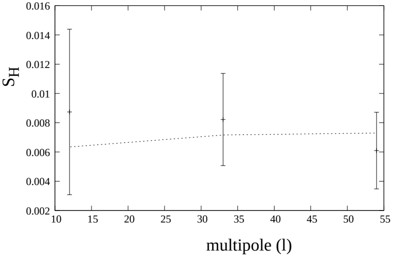
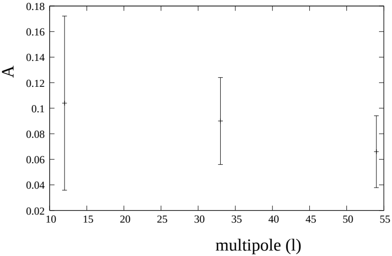


S_H^{data}, as a function of the multipole l. The dotted line represents the theoretical fit corresponding to $\alpha = 0, g_0 = 0.047 \pm 0.008$.



The dipole modulation parameter, A, as a function of the multipole (l), after fixing the direction parameters, for the three chosen multipole bins, $2-22,\ 23-43,\ 44-64$.

Continued...

Conclusion

- We find that the effective dipole modulation parameter A slowly decreases with the multipole, l.
- The spectral index α of the inhomogeneous power spectrum is consistent with zero.

  Pranati K. Rath, Pankaj Jain, arXiv:1403.2567[(astro-ph.CO)], accepted in Phys. Rev. D.

Anisotropy Power spectrum model

- We argue that an anisotropic dipolar imaginary primordial power spectrum is possible within the framework of noncommutative space-times.
- It provides a good description of the observed dipole modulation in CMBR data.

Continued...

- In a classical framework,

$$\langle\tilde{\delta}(\vec{x})\tilde{\delta}(\vec{x}')\rangle = \langle\tilde{\delta}(\vec{x}')\tilde{\delta}(\vec{x})\rangle,$$

  this implies, it can only be an even function of $\vec{\Delta}$.

- The simplest anisotropic function is, therefore,

$$F(\vec{\Delta},\vec{X}) = f_1(\Delta) + B_{ij}\Delta_i\Delta_j f_2(\Delta)$$

  where $B_{i,j}$, $i,j = 1,2,3$ are parameters. Such a model cannot give rise to a dipole modulation, which requires a term linear in Δ_i.

- In a noncommutative space-time, a term linear in Δ_i is permissible.
- Applicable when quantum gravity effects were not negligible.

- The noncommutativity of space-time may be expressed as,

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$$

where, $\theta_{\mu,\nu}$ are parameters and the coordinate functions, $\hat{x}_\mu(x)$, depends on the choice of the coordinate system. In a particular coordinate system, we may set $\hat{x}_\mu(x) = x_\mu$.

- We propose the anisotropic power spectrum in real space as,

$$F(\vec{\Delta}) = f_1(\Delta) + \hat{\lambda} \cdot \vec{\Delta} f_2(\Delta).$$

- For this case,

$$\varphi(\mathbf{x}, t)\varphi(\mathbf{x}', t') \neq \varphi(\mathbf{x}', t')\varphi(\mathbf{x}, t)$$

even for space like separations.

Continued...

- The two point correlation leads to

$$\langle\delta(\vec{k})\delta^*(\vec{k}')\rangle = \delta^3(\vec{k}-\vec{k}')P(k)[1+i(\hat{k}\cdot\hat{\lambda})g(k)],$$

- where the anisotropic component of the power spectrum, $g(k)$,

$$g(k) = -\frac{1}{P(k)}\,\frac{d\tilde{f}_2(k)}{dk}$$

Continued...

- In the small k limit, we find,

$$\hat{\lambda} = -\frac{\vec{\theta}^0}{|\vec{\theta}^0|}$$

$$g(k) = kH|\vec{\theta}^0|$$

- Hence we find that in this limit the anisotropic term increases linearly with k.
- The precise form of the correlation predicted within the framework of non- commutative geometry is model dependent.
- Here we don′t confine ourselves to a particular model and instead extract the anisotropic power directly from data. For this purpose, we assume the following parametrization of $g(k)$,

$$g(k) = g_0(k\eta_0)^{-\alpha}.$$

- The noncommutative model, implies $\alpha = 1$.

Continued...

- We set $\alpha = 0$, the best fit value for the entire multipole range $l = 2 - 64$ is found to be $g_0 = 0.28 \pm 0.09$ for WMAP-ILC 9 year data.
- Making a fit over the three multipole bins, the best fit leads to $g_0 = 0.24 \pm 0.06$ with $\chi^2 = 0.41$.
- The one σ limit on α is $-0.28 < \alpha < 0.28$.
- The preliminary results obtained using CAMB software for $\alpha = 0$ is $g_0 = 0.17$ with $\chi^2 = 1.46$.

Continued...

conclusion

- Data suggests that the anisotropic power, $g(k)$, is independent of k for small k and decays for the larger values of k.
- The data indicates a value of α close to zero for the range of multipole $l = 2 - 64$.
- Observed hemispherical anisotropy might represent the first observational signature of noncommutative geometry and hence of quantum gravity.

P. Jain, Pranati K. Rath, arXiv:1407.1714 [astro-ph.CO].

Publications

Direction dependence of the power spectrum and its effect on the Cosmic Microwave Background Radiation
Pranati K. Rath, Tanmay Mudholkar, Pankaj Jain, Pavan K. Aluri, Sukanta Panda
JCAP04(2013)007.

Testing the Dipole Modulation Model in CMBR
Pranati K.Rath, Pankaj Jain
JCAP12(2013)014.

Relating the inhomogeneous power spectrum to the CMB hemispherical anisotropy
Pranati K. Rath, Pankaj Jain
arXiv:1403.2567[(astro-ph.CO)](Accepted in Phys. Rev. D).

Noncommutative Geometry and the Primordial Dipolar Imaginary Power Spectrum
P. Jain, Pranati K. Rath
arXiv:1407.1714 [astro-ph.CO](in review).

Phenomenological dipole modulation model
R. Kothari, S. Ghosh, P. K. Rath, G. Kashyap, P. Jain(In preparation).

Cosmological Power spectrum in Non-commutative space time

P. K .Rath, R. Kothari, A. Kumar, P.Jain(In preparation).

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Thank You

continued...

	$\mathcal{C}_1(4)\ (mK^4)$	(θ, ϕ)	P-value
NILC	$4.07 \times 10^{-7}\ (2.91 \times 10^{-7})$	$(135^o, 226^o)$	6.05%
SMICA	$4.15 \times 10^{-7}\ (3.01 \times 10^{-7})$	$(135^o, 224^o)$	5.85%
SEVEM	$3.93 \times 10^{-7}\ (2.80 \times 10^{-7})$	$(136^o, 229^o)$	6.82%
WMAP-ILC	$4.06 \times 10^{-7}\ (2.28 \times 10^{-7})$	$(131^o, 216^o)$	6.65%

Table : The extracted dipole power of the temperature squared field and the direction parameters removing $l = 2 - 4$ multipoles.

	$\mathcal{C}_1(8)\ (mK^4)$	(θ, ϕ)	P-value
NILC	$2.04 \times 10^{-7}\ (1.15 \times 10^{-7})$	$(130^o, 245^o)$	0.92%
SMICA	$2.06 \times 10^{-7}\ (1.16 \times 10^{-7})$	$(130^o, 244^o)$	0.89%
SEVEM	$2.05 \times 10^{-7}\ (1.14 \times 10^{-7})$	$(130^o, 246^o)$	0.92%
WMAP-ILC	$1.8 \times 10^{-7}\ (1.15 \times 10^{-7})$	$(126^o, 242^o)$	1.45%

Table : Dipole power of the temperature squared field and the direction parameters removing $l = 2 - 8$ multipoles.

Continued...

- The relevant quantity is the deformed quantum field. defined in *Akofor et al. 2008*,

$$\varphi_\theta = \varphi_0 \, e^{\frac{1}{2} \overleftarrow{\partial} \wedge P}$$

where

$$\overleftarrow{\partial} \wedge P \equiv \overleftarrow{\partial}_\mu \theta^{\mu\nu} P_\nu .$$

- For a deformed field,

$$\varphi_\theta(\mathbf{x}, t)\varphi_\theta(\mathbf{x}', t') \neq \varphi_\theta(\mathbf{x}', t')\varphi_\theta(\mathbf{x}, t)$$

even for space like separations.

Continued...

- The two point correlation leads to

$$\langle\delta(\vec{k})\delta^*(\vec{k}')\rangle = \delta^3(\vec{k}-\vec{k}')P(k)[1+i(\hat{k}\cdot\hat{\lambda})g(k)],$$

- where the anisotropic component of the power spectrum, $g(k)$,

$$g(k) = -\frac{1}{P(k)}\frac{d\tilde{f}_2(k)}{dk}$$

Continued...

Power spectrum..

The inflaton field $\varphi(\vec{x}, t)$ can be expanded around the background field as,

$$\varphi(\vec{x}, t) = \varphi^{(0)}(t) + \delta\varphi(\vec{x}, t)$$

$$\Phi(\vec{k}, t) = \int d^3x \delta\varphi(\vec{x}, t) e^{-i\vec{k}\cdot\vec{x}} \ .$$

We define, $\tilde{\Phi}(\vec{k}, \eta) = a(\eta)\Phi(\vec{k}, \eta)$. In terms of the creation and annihilation operatorsas,

$$\tilde{\Phi}(\vec{k}, \eta) = u(\vec{k}, \eta)\hat{a}(\vec{k}) + u^*(-\vec{k}, \eta)a^\dagger(-\vec{k})$$

where $u(\vec{k}, \eta)$ is the standard mode function,

$$u(\vec{k}, \eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) .$$

Continued...

- The power spectrum odd in $\vec{k}$ must by imaginary has already been recognized in *Koivisto et al. 2011.*
- They suggested the correlator of the anti-self-adjoint part of the operator as,

$$\langle ... \rangle \longrightarrow \alpha \langle ... \rangle_M + i(1-\alpha)\langle ... \rangle_A .$$

- It correctly points out the basic problem that the operator whose expectation value is being computed is not Hermitian.
- However it is not clear how this suggestion can be implemented at a fundamental level. The action of any operator on a state is well defined. Hence it is not clear how one can have different definitions of expectation values for different parts of an operator.
- So the problem is not really solved and we need to look for alternatives.

Continued...

New Real twist element...

- Define a new Real twist element,

$$\mathcal{F}_{\theta_R} = \exp\left(-\frac{1}{2}\theta^{\alpha\beta} P_\alpha \otimes P_\beta\right)$$

  with $P_\alpha = -i\partial_\alpha$.

- Leads to define a new product of two functions, $f(x)$ and $g(x)$,

$$(f \diamond g)(x) = \left[e^{-\frac{1}{2}\theta^{\mu\nu} P_\mu \otimes P_\nu} f \otimes g\right](x)$$

- This additional product rule does not conflict with any rule of noncommutative geometry or quantum mechanics.
- We emphasize that it is not necessary that only the star product must be taken while computing the power.
- our use of diamond product is a phenomenological prescription which is consistent with all rules of quantum field theory and noncommutative geometry.

Continued...

- Under this real twist element

$$(\phi_{\theta_R}(x_1)\phi_{\theta_R}(x_2))^\dagger = \phi_{\theta_R}(x_1)\phi_{\theta_R}(x_2).$$

is hermitian since the fields commute for space-like separations.

- Power spectrum for the expanding Universe,

$$\langle 0|\Phi_{\theta_R}(\vec{k},\eta)\Phi_{\theta_R}(\vec{k}',\eta)|0\rangle \approx (2\pi)^3 P(k)\left(1 - iH\vec{\theta}^0\cdot\vec{k}\right)\delta(\vec{k}+\vec{k}').$$

Where we used,

$$\eta^{\pm}(\vec{k}) = \eta\left(t \pm \frac{i\vec{\theta}^0\cdot\vec{k}}{2}\right) = \eta(t)e^{\mp\frac{i}{2}H\vec{\theta}^0\cdot\vec{k}}$$