

Talking to Neutrinos at the India-based Neutrino Observatory (INO)

Sanjib Kumar Agarwalla

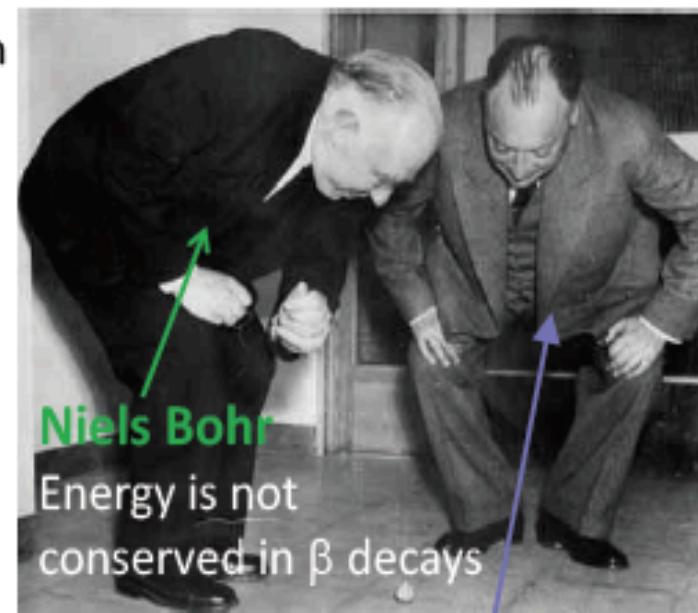
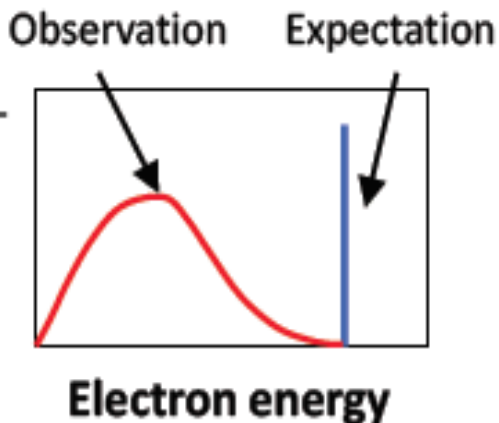
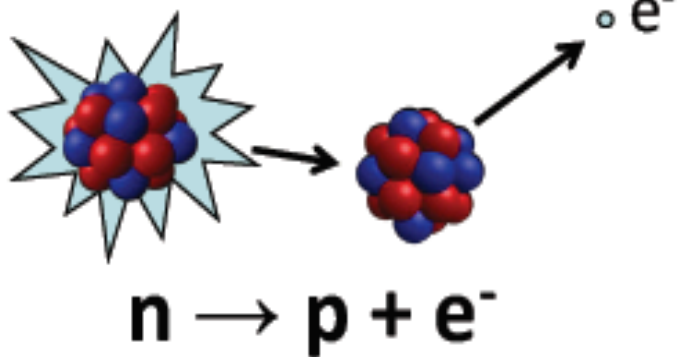
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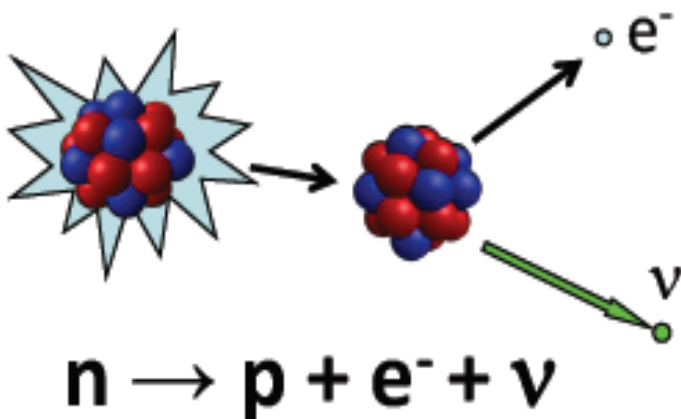


Mission Impossible: Detect Neutrinos

The problem (1914)



The desperate remedy (1930)



There is a neutral particle able to cross all detectors without leaving any trace and carrying all the missing energy

«I have done a terrible thing. I have postulated a particle that cannot be detected.» (1930)

Fortunately Pauli was wrong and neutrinos have been detected successfully


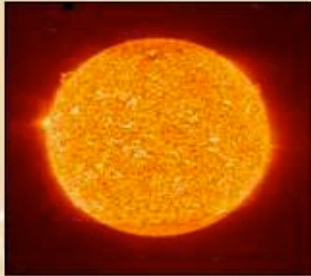





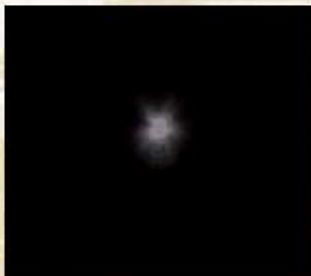
Few Unique Features of Neutrinos

- ⊙ **After photon, neutrino is the most abundant particle in the universe**
100 billion neutrinos pass through our thumbnail every second
- ⊙ **Nature's most elusive messenger, interacts very rarely, very hard to detect**
100 billion neutrinos + the whole Earth = only one interaction
- ⊙ **Arrives 'unscathed' from the farthest reaches of the Universe**
Carry information about its source

Few Unique Features of Neutrinos

- ⊙ **Known to undergo flavor change**
(neutrino mass: first clue of physics beyond the Standard Model)
- ⊙ **Masses are anomalously low**
(from CMB data $m_\nu < 0.2 \text{ eV}/c^2 = 0.0000004 m_e$)
- ⊙ **Only fundamental fermion that can be its own anti-particle**
(Majorana particle)
- ⊙ **May open window on the GUT Scale ($\Lambda_{\text{GUT}} \sim 10^{16} \text{ GeV}$)**
(via seesaw mechanism)
- ⊙ **Could explain the matter/anti-matter asymmetry of the Universe**
(leptogenesis)

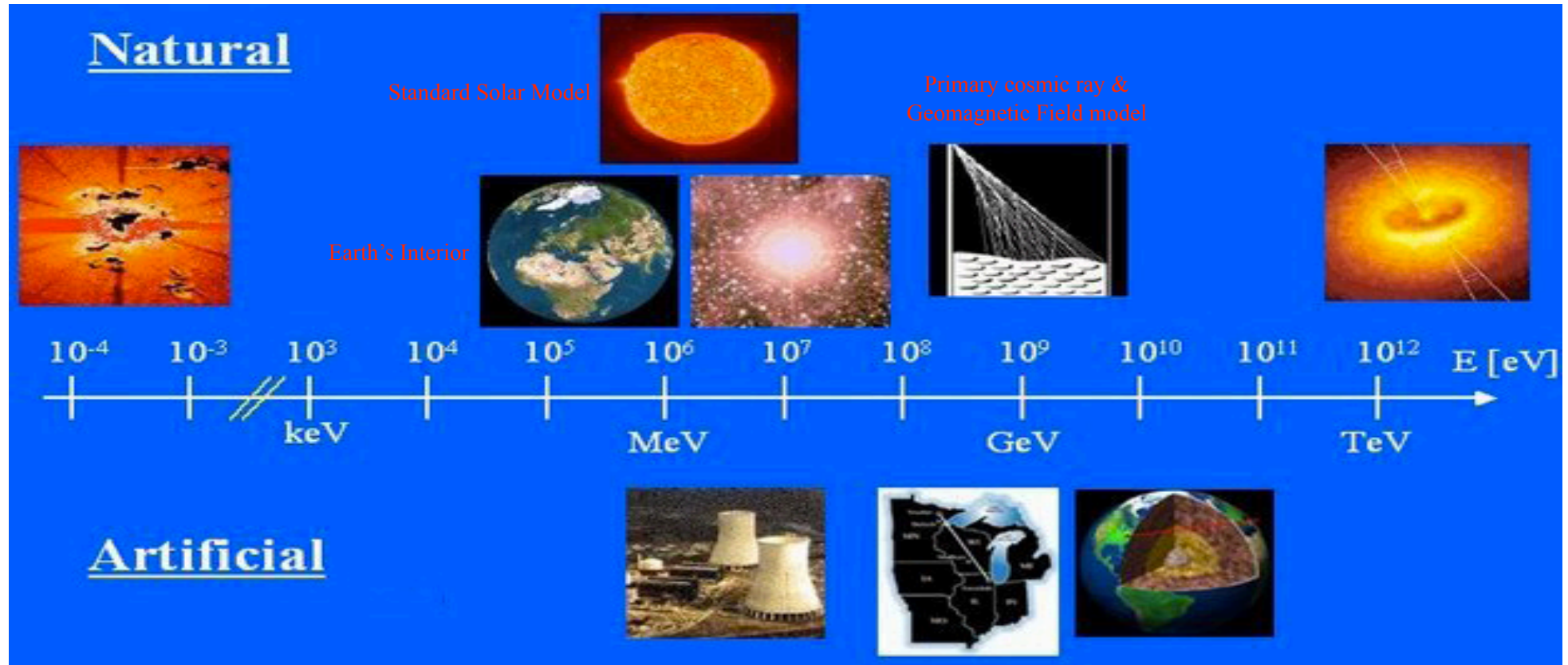
Neutrinos are omnipresent

Detected (1950s) ✓ Nuclear Reactors			Detected (1960s) Sun ✓
Created & Detected (1960s) ✓ Particle Accelerators			Detected (1980s) Supernovae (Stellar Collapse) SN 1987A ✓
Detected (1960s) ✓ Earth Atmosphere (Cosmic Rays)			Recently, IceCube reports the detection of 37 extremely HE neutrinos Astrophysical Accelerators Soon ?
Detected (2000s) ✓ Earth Crust (Natural Radioactivity)			Not even close Cosmic Big Bang (Today $330 \nu/\text{cm}^3$) Indirect Evidence

Extremely rich and diverse neutrino physics program

Neutrinos: Exceptional Probe for Environments

Neutrino Observation: Go Beyond optical and radio observation

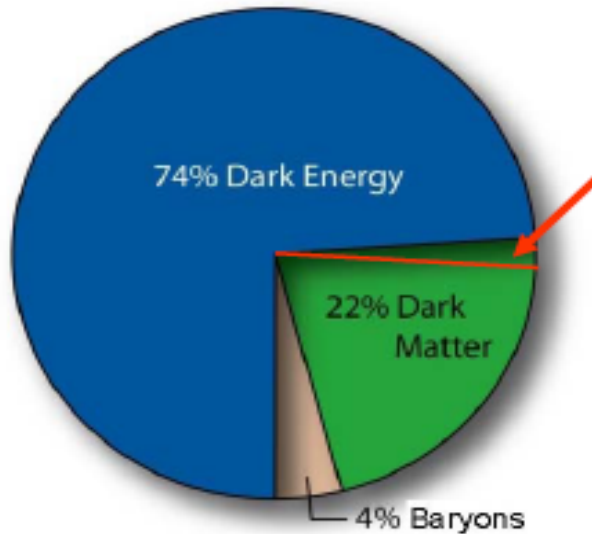


Detect neutrinos from the Sun, Supernovae, AGN, GRBs: Era of Neutrino Astronomy

ν detection involves several methods on surface, underground, under the sea, or in the ice

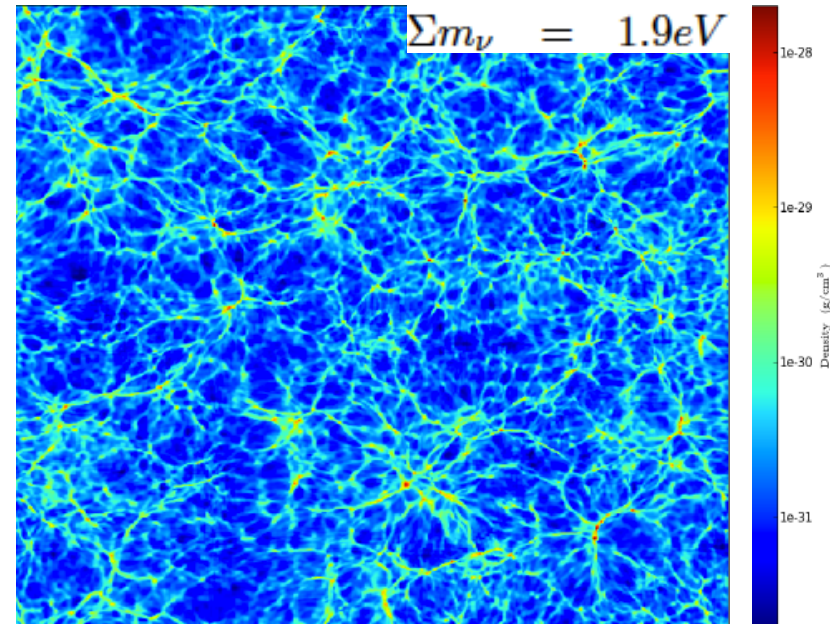
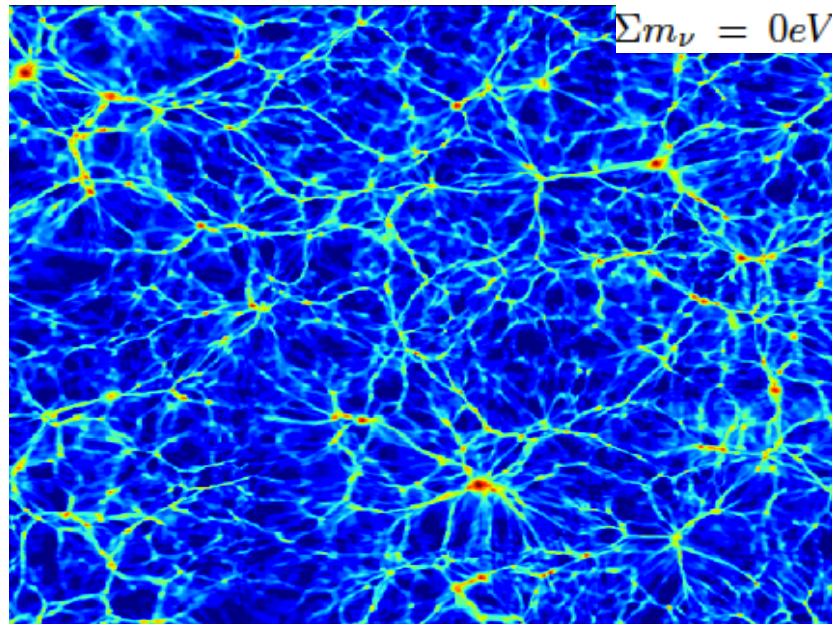
ν detector masses range from few kgs to megatons, with volumes from few m^3 to km^3

Neutrinos and Dark Matter



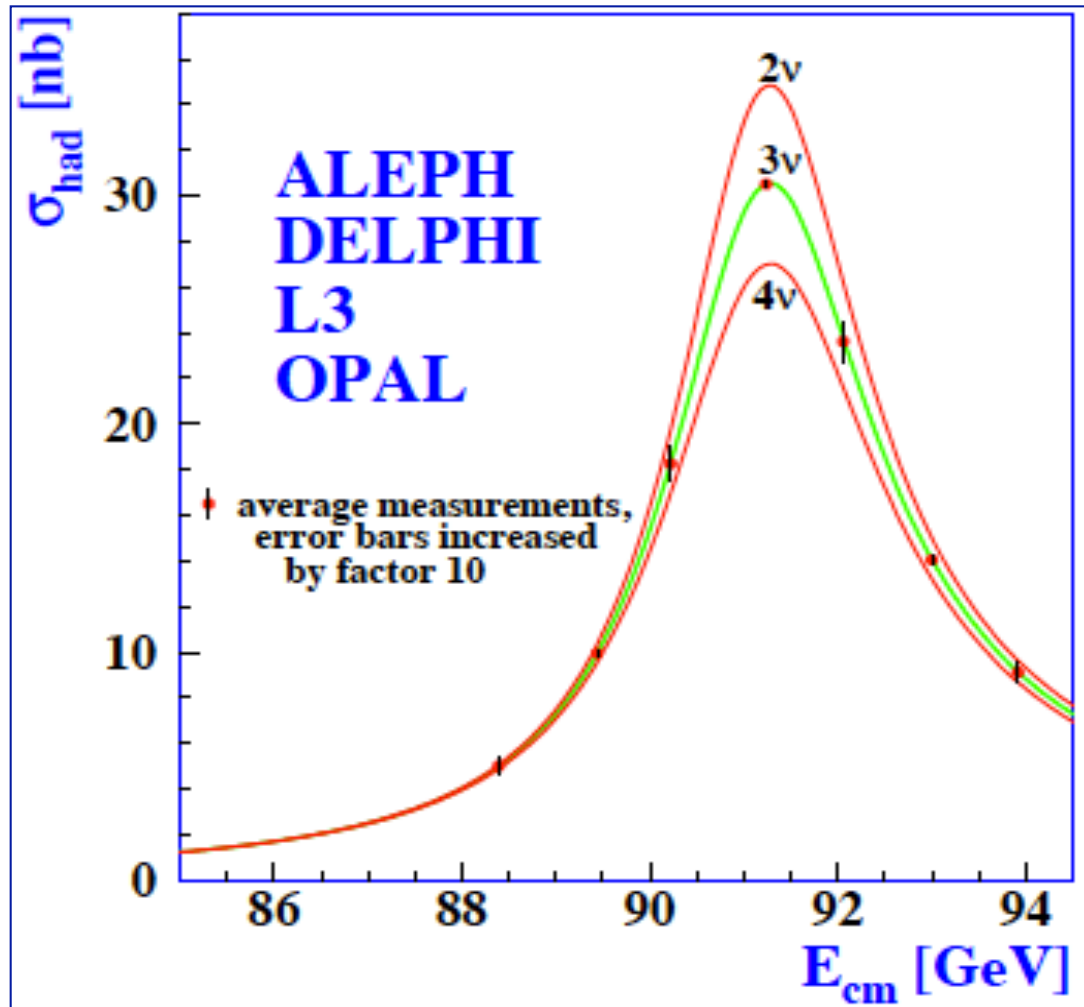
Neutrinos constitute a hot dark matter component and affect the formation of clusters of galaxies

Neutrinos are too hot for being trapped in the gravitational wells in the early Universe (**free streaming**) and move freely, smoothing out the structures (**galaxies**) at small (**cosmological**) distances



Agarwal, Feldman, arXiv:1006.0689 [astro-ph.CO]

Three Light Active Neutrinos



*Precision data of the Z-decay width
at the e^+e^- collider at LEP*

$$e^+e^- \rightarrow Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a$$

$$N_{\nu_{\text{active}}} = 2.9840 \pm 0.0082$$

[LEP, Phys. Rept. 427 (2006) 257, hep-ex/0509008]

3 light active flavor neutrinos

$$\nu_e \nu_\mu \nu_\tau$$

The Standard Model: Massless Neutrinos

The Standard Model is a gauge theory & it unifies strong, weak & electromagnetic forces!

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

$(1, 2)_{-\frac{1}{2}}$	$(3, 2)_{\frac{1}{6}}$	$(1, 1)_{-1}$	$(3, 1)_{\frac{2}{3}}$	$(3, 1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

3-fold repetition of the same representation!

- 3 active neutrinos: ν_e, ν_μ, ν_τ
- Neutral elementary particles of Spin $\frac{1}{2}$
- Only couple to weak force (& gravity)
- Only left handed neutrinos
- There are no right-handed neutrinos
- No Dirac Mass term: $m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$

Neutrinos are massless in the Basic SM

- ❑ Over the past decade, marvelous data from world class neutrino experiments firmly established that they change flavor after propagating a finite distance
- ❑ Neutrino flavor change (oscillation) demands non-zero mass and mixing

Non-zero ν mass: first experimental proof for physics beyond the Standard Model

!! An extension of the Standard Model is necessary !!

Neutrino Physics: An Exercise in Patience

Three most fundamental questions were being asked in the past century...

1. How tiny is the neutrino mass? (Pauli, Fermi, '30s)

Planck + BAO + WMAP polarization data: upper limit of **0.23 eV** for the sum of ν masses!

Planck Collaboration, arXiv:1303.5076 [astro-ph.CO]

2. Can a neutrino turn into its own antiparticle? (Majorana, '30s)

Hunt for ν -less Double- β decay ($Z, A \rightarrow Z+2, A$) is still on, demands **lepton number violation!**

Nice Review by Avignone, Elliott, Engel, Rev.Mod.Phys. 80 (2008) 481-516

3. Do different ν flavors 'oscillate' into one another? (Pontecorvo, Maki-Nakagawa-Sakata, '60s)

B. Pontecorvo, Sov. Phys. JETP 26, 984 (1968) [Zh. Eksp. Teor. Fiz. 53, 1717 (1967)]

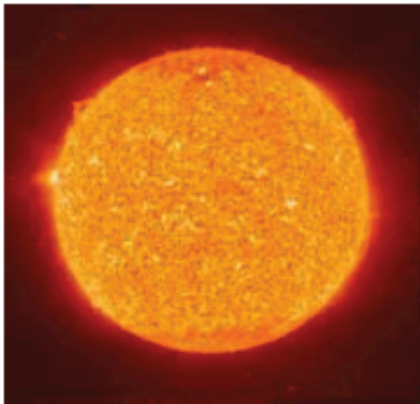
Last question positively answered only in recent years. Now an established fact that **neutrinos are massive** and leptonic flavors are not **symmetries of Nature!**

Recent measurement of θ_{13} , a clear first order picture of the 3-flavor lepton mixing matrix has emerged, signifies a major breakthrough in ν physics!

This year marks the 60th anniversary since ν detector of Reines & Cowan was turned on

Golden Age of Neutrino Physics (1998 – 2014 & Beyond)

sun



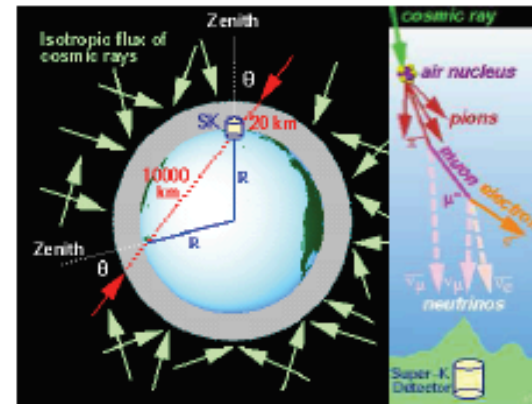
Homestake, SAGE, GALLEX
SuperK, SNO, Borexino

reactors



KamLAND, CHOOZ
Double Chooz, Daya Bay, RENO

atmosphere



SuperKamiokande
IceCube

accelerators



K2K, MINOS, T2K
NOvA

Over the last sixteen years or so, precious data from world-class experiments

- Solar neutrinos (ν_e)
- Atmospheric neutrinos ($\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$)
- Reactor anti-neutrinos ($\bar{\nu}_e$)
- Accelerator neutrinos ($\nu_\mu, \bar{\nu}_\mu$)

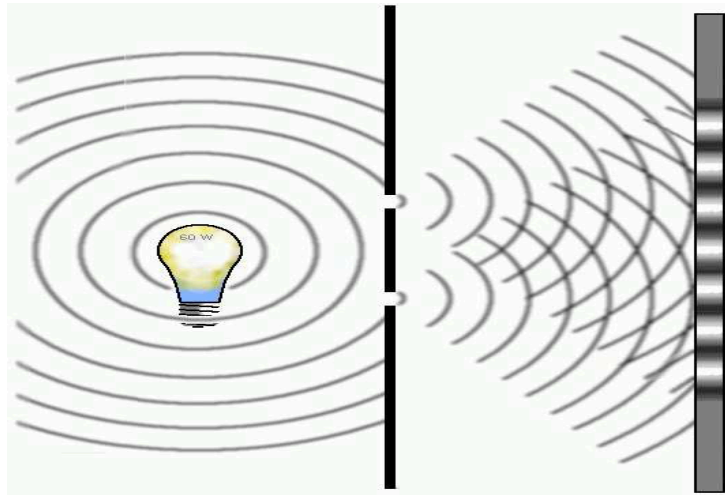


Data from various neutrino sources and vastly different energy and distance scales

We have just started our journey in the mysterious world of neutrinos

Neutrino Flavor Oscillations

1957: Bruno Pontecorvo proposed **Neutrino Oscillations** in analogy with $K^0 \leftrightarrow \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955)

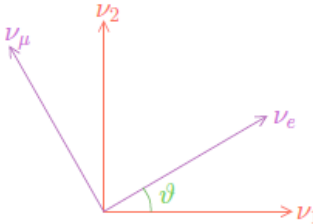


- **Neutrino oscillation:**
Quantum Mechanical interference phenomenon
- **Like electrons in the double slit experiment**
- **In Neutrino Oscillation:**
Neutrino changes flavor as it propagates
- **It happens if neutrinos have masses (non-degenerate) and there is mixing**

Neutrino Oscillations: 2 Flavors

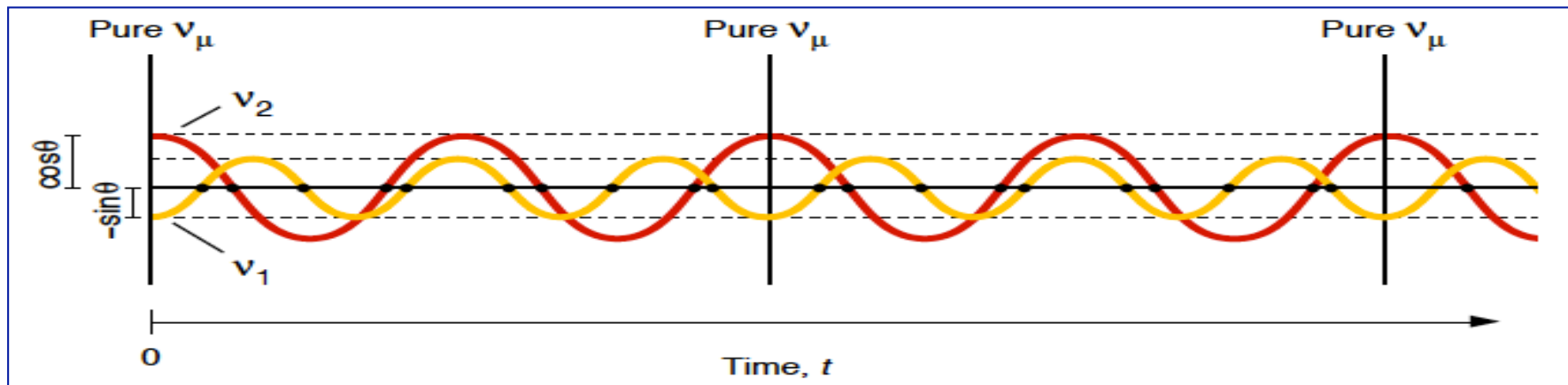
- **Flavor States :** ν_e and ν_μ (produced in Weak Interactions)
- **Mass Eigenstates :** ν_1 and ν_2 (propagate from Source to Detector)

A Flavor State is a linear superposition of Mass Eigenstates

$$|\nu_\alpha\rangle = \sum_{k=1}^2 U_{\alpha k} |\nu_k\rangle \quad (\alpha = e, \mu)$$


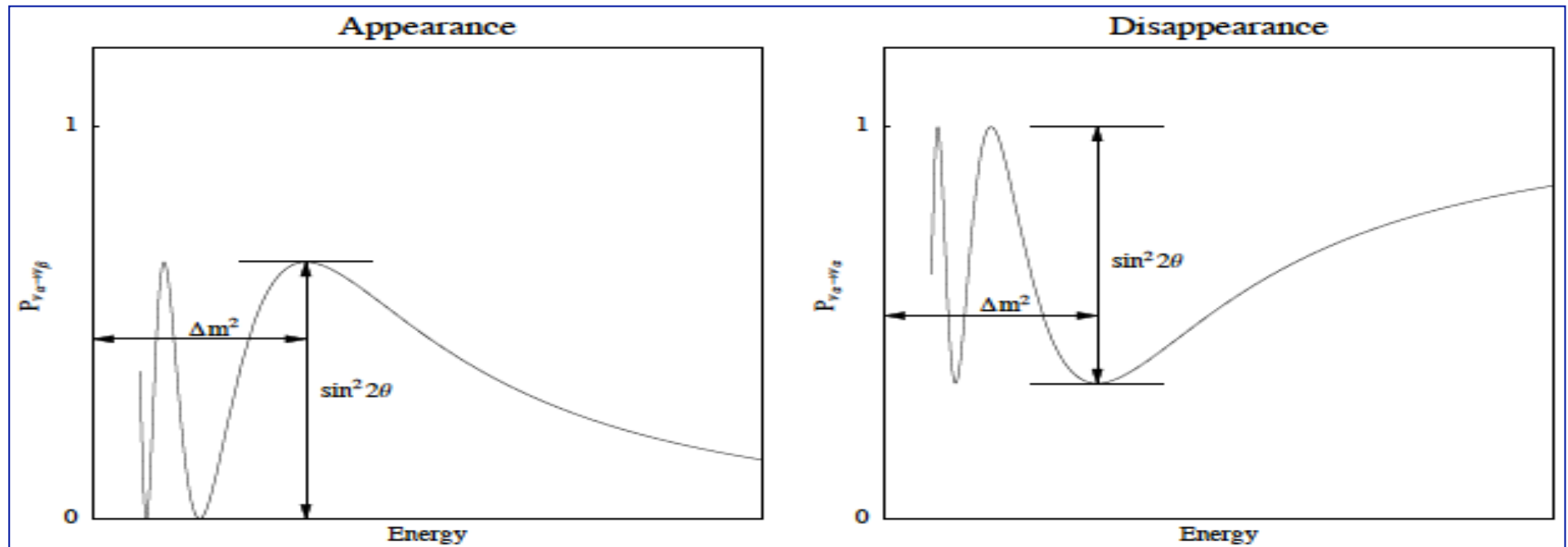
$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{aligned} |\nu_e\rangle &= \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle \end{aligned}$$



If the masses of these two states are different then they will take different times to reach the same point and there will be a phase difference and hence interference

Oscillation Probabilities in 2 Flavors



$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2 2\theta \sin^2\left(1.27\Delta m^2 \frac{L}{E}\right)$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2 2\theta \sin^2\left(1.27\Delta m^2 \frac{L}{E}\right)$$

Δm^2 is in eV^2 , L is in m (km) and E in MeV (GeV)

$$\lambda = 2.47\text{km} \left(\frac{E}{\text{GeV}}\right) \left(\frac{\text{eV}^2}{\Delta m^2}\right) \Rightarrow \text{oscillation length}$$

Neutrino Oscillations only sensitive to mass squared difference but not to the absolute Neutrino mass scale

Neutrino Oscillations in 3 Flavors

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$

θ_{23} : $P(\nu_\mu \rightarrow \nu_\mu)$ by Atoms, ν and ν beam

θ_{13} : $P(\nu_e \rightarrow \nu_e)$ by Reactor ν
 θ_{13} & δ : $P(\nu_\mu \rightarrow \nu_e)$ by ν beam

θ_{12} : $P(\nu_e \rightarrow \nu_e)$ by Reactor and solar ν

Three mixing angles: $\theta_{23}, \theta_{13}, \theta_{12}$ and one CP violating (Dirac) phase δ_{CP}

$$\tan^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2 \theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin \theta_{13} e^{-i\delta}$$

3 mixing angles simply related to flavor components of 3 mass eigenstates

Over a distance L , changes in the relative phases of the mass states may induce flavor change!

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}[U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*] \sin^2 \Delta_{ij} - 2 \sum_{i>j} \text{Im}[U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^*] \sin 2\Delta_{ij}$$

$$\Delta_{ij} = \Delta m_{ij}^2 L / 4E_\nu$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

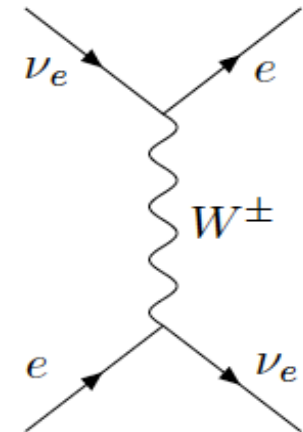
2 independent mass splittings Δm_{21}^2 and Δm_{32}^2 , for anti-neutrinos replace δ_{CP} by $-\delta_{CP}$

Neutrino Oscillations in Matter

Neutrino propagation through matter modify the oscillations significantly

Coherent forward elastic scattering of neutrinos with matter particles

Charged current interaction of ν_e with electrons creates an extra potential for ν_e



Wolfenstein matter term: $A = \pm 2\sqrt{2}G_F N_e E$ or $A(\text{eV}^2) = 0.76 \times 10^{-4} \rho (\text{g/cc}) E(\text{GeV})$

N_e = electron number density, + (-) for neutrinos (anti-neutrinos), ρ = matter density in Earth

Matter term changes sign when we switch from neutrino mode to anti-neutrino mode

$P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq 0 \implies$ even if $\delta_{CP} = 0$, causes fake CP asymmetry

Matter term modifies oscillation probability differently depending on the sign of Δm^2

$\Delta m^2 \simeq A \Leftrightarrow E_{\text{res}}^{\text{Earth}} = 6 - 8 \text{ GeV} \implies$ Resonant conversion – Matter effect

	ν	$\bar{\nu}$
$\Delta m^2 > 0$	MSW	-
$\Delta m^2 < 0$	-	MSW



Resonance occurs for neutrinos (anti-neutrinos) if Δm^2 is positive (negative)

Oscillation Parameters After Neutrino 2014

	bfp $\pm 1\sigma$	3σ range	Relative 1σ Precision
$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.270 \rightarrow 0.344$	4%
$\theta_{12}/^\circ$	$33.48^{+0.77}_{-0.74}$	$31.30 \rightarrow 35.90$	
$\sin^2 \theta_{23}$	$[0.451^{+0.001}_{-0.001}] \oplus 0.577^{+0.027}_{-0.035}$	$0.385 \rightarrow 0.644$	9.6%
$\theta_{23}/^\circ$	$[42.2^{+0.1}_{-0.1}] \oplus 49.4^{+1.6}_{-2.0}$	$38.4 \rightarrow 53.3$	
$\sin^2 \theta_{13}$	$0.0219^{+0.0010}_{-0.0011}$	$0.0188 \rightarrow 0.0251$	4.8%
$\theta_{13}/^\circ$	$8.52^{+0.20}_{-0.21}$	$7.87 \rightarrow 9.11$	
$\delta_{CP}/^\circ$	251^{+67}_{-59}	$0 \rightarrow 360$	(Not Known)
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \rightarrow 8.09$	2.4%
$\frac{\Delta m_{31}^2}{10^{-3} \text{ eV}^2}$ (N)	$[+2.458^{+0.002}_{-0.002}]$	$+2.325 \rightarrow +2.599$	1.9%
$\frac{\Delta m_{32}^2}{10^{-3} \text{ eV}^2}$ (I)	$-2.448^{+0.047}_{-0.047}$	$-2.590 \rightarrow -2.307$	

Non-maximal
 $> 1.4\sigma$

Non-zero
 $> 10\sigma$

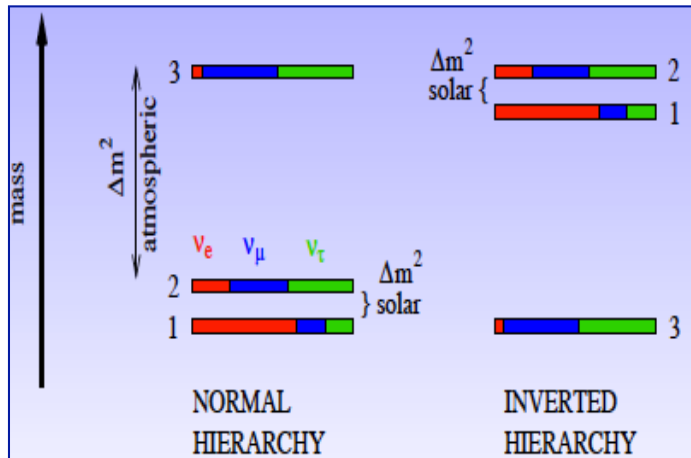
$\sin \delta_{CP} < 0$
at 90% C.L.

Based on the data available after Neutrino 2014 conference

Gonzalez-Garcia, Maltoni, Salvado, Schwetz, <http://www.nu-fit.org>

Fundamental Unknowns in Neutrino Oscillation

1. What is the hierarchy of the neutrino mass spectrum, normal or inverted?



- The sign of $\Delta m_{31}^2 = m_3^2 - m_1^2$ is not known!
- Currently do not know which neutrino is the heaviest?
- Only have a lower bound on the mass of the heaviest ν !

$$\sqrt{2.5 \cdot 10^{-3} \text{eV}^2} \sim 0.05 \text{ eV}$$

2. What is the octant of the 2-3 mixing angle, lower ($\theta_{23} < 45^\circ$) or higher ($\theta_{23} > 45^\circ$)?

Measure θ_{23} precisely, Establish deviation from maximality at higher C.L. Then look for Octant

2. Is there CP violation in the leptonic sector, as in the quark sector?

Mixing can cause CP violation in the leptonic sector (if δ_{CP} differs from 0° and 180°)

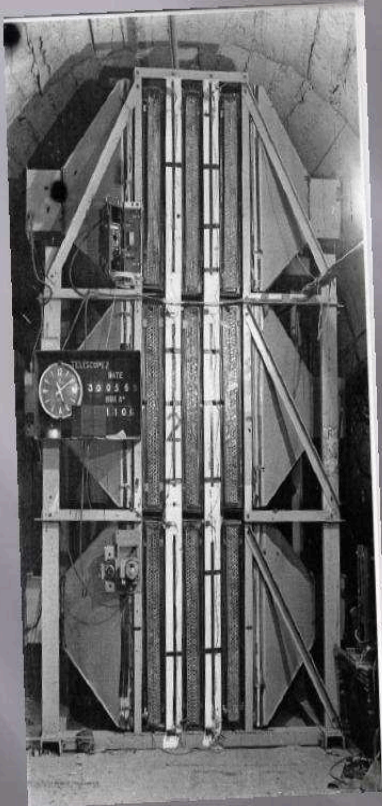
Need to measure the CP-odd asymmetries: $\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta; L) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; L)$ ($\alpha \neq \beta$)

With current knowledge of θ_{13} , resolving these unknowns fall within our reach
Sub-leading 3 flavor effects are extremely crucial in current & future oscillation expts

An Old Saga of Underground Laboratory in India

- *KGF: Deepest underground lab in world till 1992 > 6500 MWE*
- *In 1965, at KGF at a depth of 2.3km, first atmospheric neutrino was observed by the TIFR-Osaka-Durham group*
- *During early 80s dedicated detectors were setup at KGF by TIFR-Osaka collaboration to look for proton decay*

Atmospheric neutrino detection in 1965



DETECTION OF MUONS PRODUCED BY COSMIC RAY NEUTRINO
DEEP UNDERGROUND

C. V. ACHAR, M. G. K. MENON, V. S. NARASIMHAM, P. V. RAMANA MURTHY
and B. V. SREEKANTAN,
Tata Institute of Fundamental Research, Colaba, Bombay

K. HINOTANI and S. MIYAKE,
Osaka City University, Osaka, Japan

D. R. CREED, J. L. OSBORNE, J. B. M. PATTISON and A. W. WOLFENDALE
University of Durham, Durham, U.K.

Received 12 July 1965

Physics Letters 18, (1965) 196, dated 15th Aug 1965

EVIDENCE FOR HIGH-ENERGY COSMIC-RAY NEUTRINO INTERACTIONS*

F. Reines, M. F. Crouch, T. L. Jenkins, W. R. Kropp, H. S. Gurr, and G. R. Smith
Case Institute of Technology, Cleveland, Ohio

and

J. P. F. Sellschop and B. Meyer
University of the Witwatersrand, Johannesburg, Republic of South Africa
(Received 26 July 1965)

PRL 15, (1965), 429, dated 30th Aug. 1965

Atmospheric neutrino detector
at Kolar Gold Field -1965

Introducing INO Collaboration



Ahmadabad: Physical Research Laboratory
Aligarh: Aligarh Muslim University
Allahabad: HRI
Bhubaneswar: IoP, Utkal University
Calicut: University of Calicut
Chandigarh: Panjab University
Chennai: IIT-Madras, IMSc
Delhi: University of Delhi
Kalpakkam: IGCAR
Kolkata: SINP, VECC, University of Calcutta
Lucknow: Lucknow University
Madurai: American College
Mumbai: BARC, IIT-Bombay, TIFR, CMEMS
Mysore: University of Mysore
Srinagar: University of Kashmir
Varanasi: Banaras Hindu University

Nearly 100 scientists from 23 research institutes & universities all over India

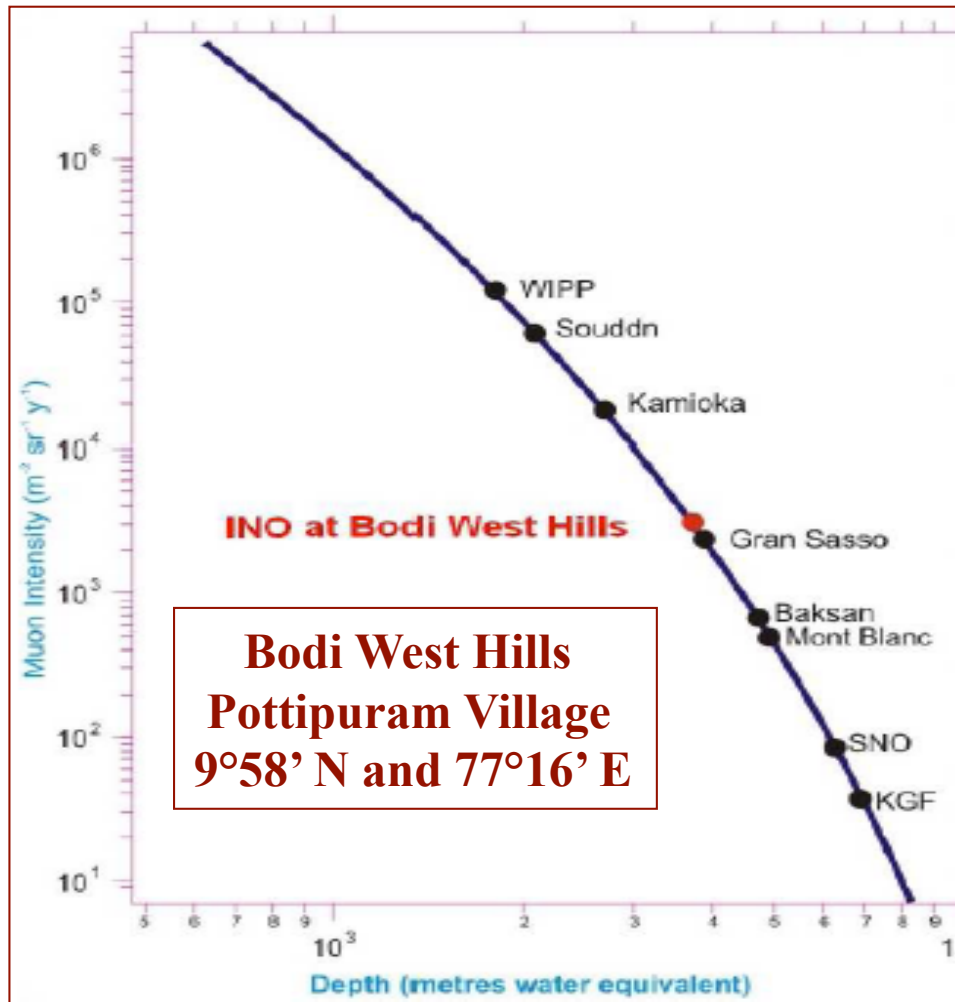
One of the largest basic science projects in India in terms of man power & cost as well

We are growing day by day

India-Based Neutrino Observatory

- *A multi-institutional attempt to build a world-class underground facility to study fundamental issues in science with special emphasis on neutrinos*
- *With ~1 km all-round rock cover accessed through a 2 km long tunnel.
A large and several smaller caverns to pursue many experimental programs*
- *Complementary to ongoing efforts worldwide to explore neutrino properties*
- *A mega-science project (~250 M\$) in India, jointly funded (50:50) by the Department of Atomic Energy and the Department of Science and Technology*
- *INO project was discussed and approved by the Atomic Energy Commission*
- *Regarding Final approval: Clearance from the Cabinet expected soon*
- *International Community is welcome to participate in ICAL@INO activity.
INO facility is also available to the entire community for setting up experiments like Neutrino-less Double Beta Decay, Direct Dark Matter searches*

Coordinates of INO

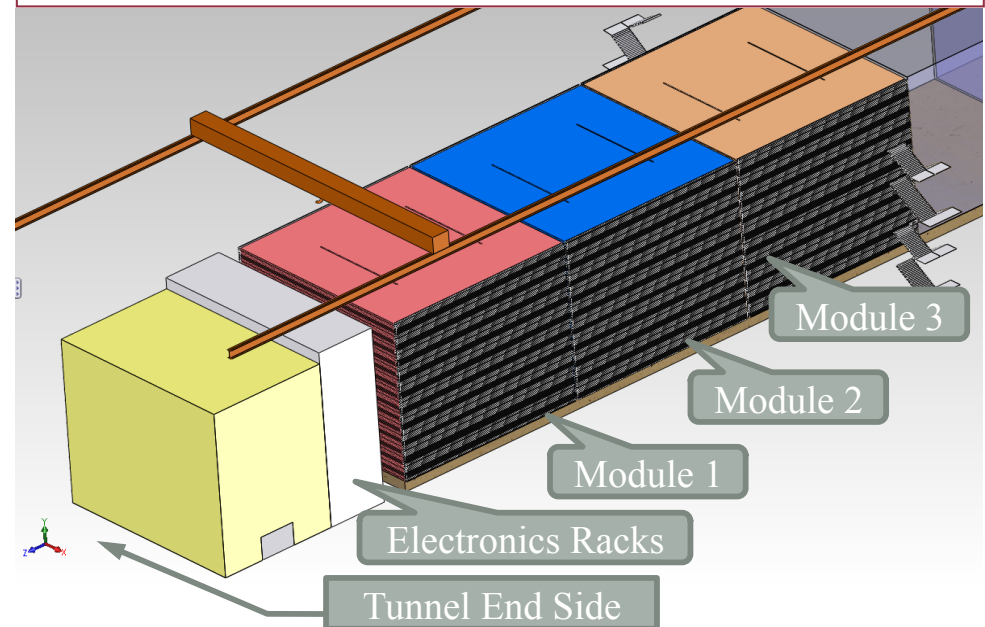
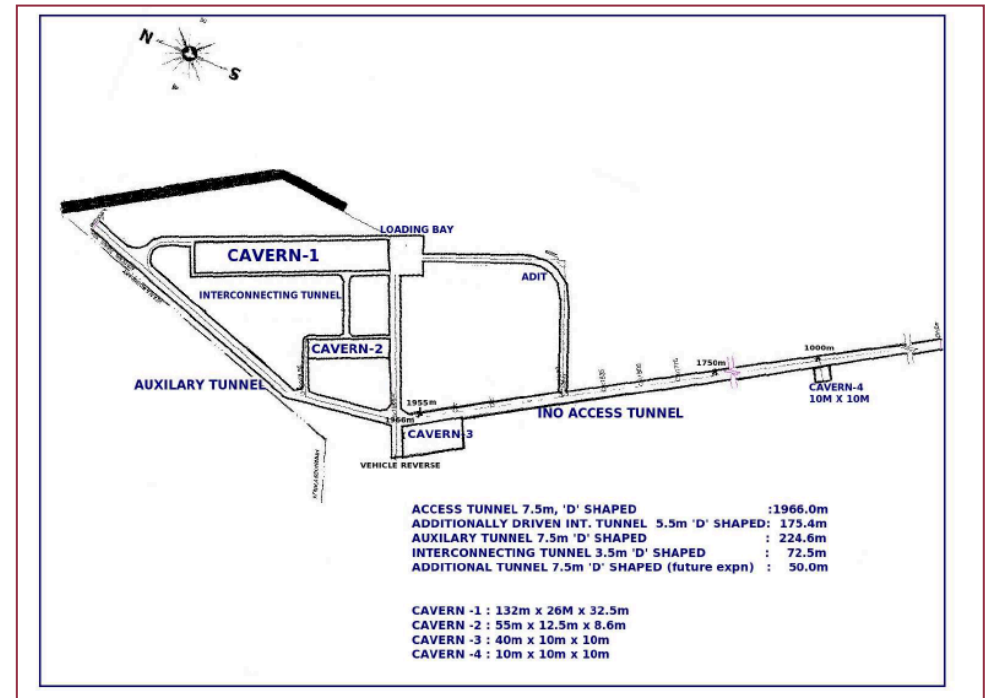


Located 115 km west of the Madurai city in the Theni district of Tamil Nadu

Madurai has an International Airport

Approved projects under INO

- Come up with an underground lab & surface facilities near Pottipuram village in Theni district of Tamil Nadu
- Build massive 50 kt magnetized Iron calorimeter (ICAL) detector to study properties of neutrinos
- Construction of INO centre at Madurai: Inter-Institutional Centre for High Energy Physics (IICHEP)
- Human Resource Development (INO Graduate Training Program)
- Completely in-house Detector R&D with substantial INO-Industry interface
- Time Frame for 1st module: 2019



Study Atmospheric neutrinos w/ a wide range of Baselines & Energies

Recent discovery of large θ_{13} : A good news for ICAL-INO

What do we want to achieve?

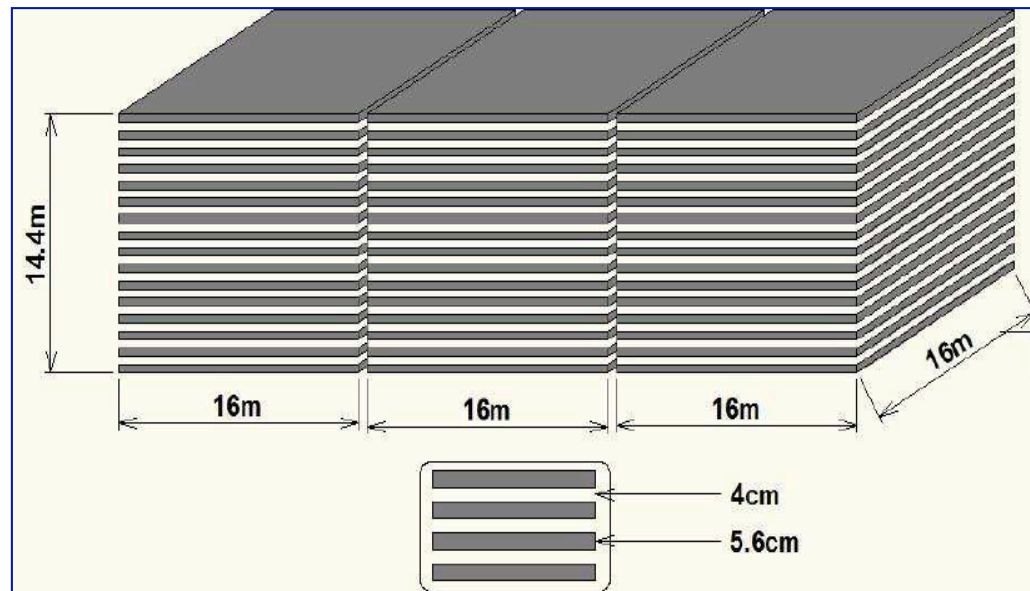
- ❖ *Reconfirm neutrino oscillations using neutrinos and anti-neutrinos separately*
- ❖ *Improved precision of atmospheric oscillation parameters*
- ❖ *Determine neutrino mass hierarchy using matter effects via charge discrimination*
- ❖ *Measure the deviation of 2-3 mixing angle from its maximal value and its octant*
- ❖ *Test bed for various new physics like NSI, CPT violation, long range forces*
- ❖ *Detect Ultra High Energy Neutrinos, Cosmic Muons, Indirect searches of DM*

Detector Characteristics

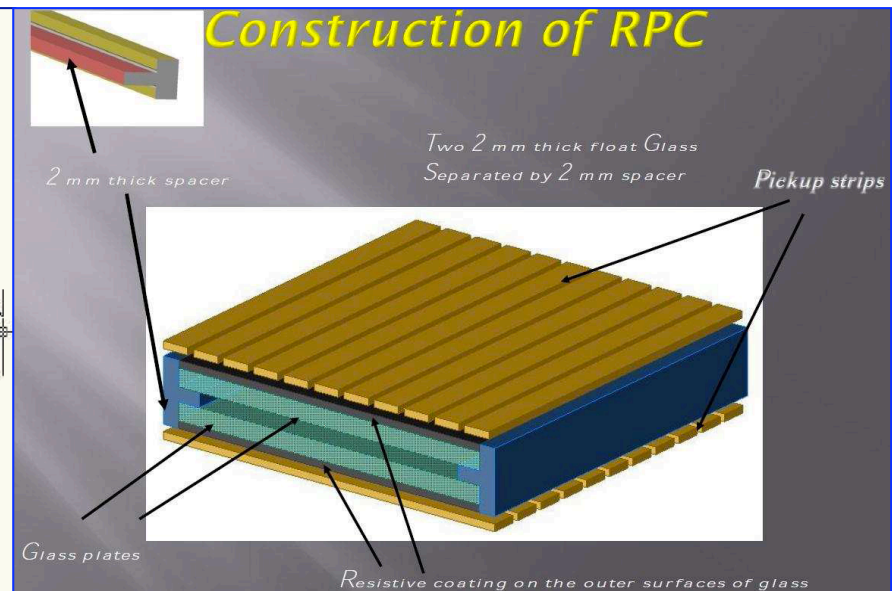
- *Should have large target mass (50 – 100 kt)*
- *Good tracking and Energy resolution (tracking calorimeter)*
- *Good directionality for up/down discrimination (nano-second time resolution)*
- *Charge identification (need to have uniform, homogeneous magnetic field)*
- *Ease of construction & Modularity*
- *Complementary to the other existing and proposed detectors*

Our choice

Magnetized iron (target mass): ICAL



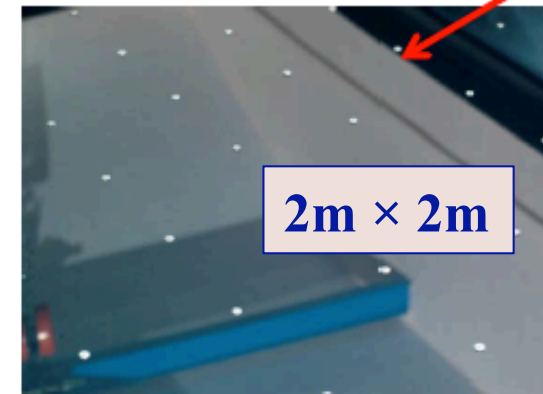
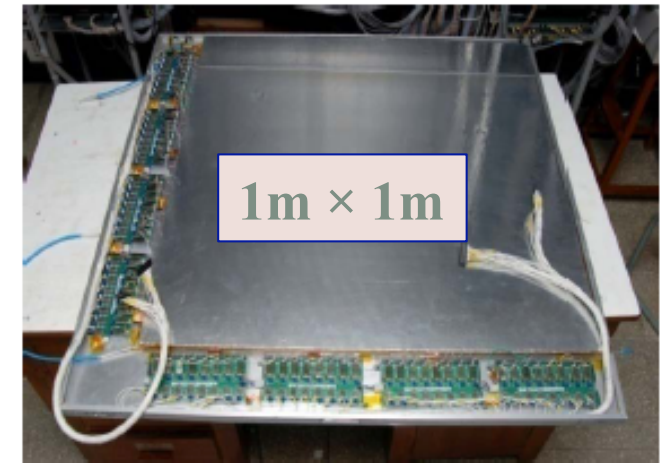
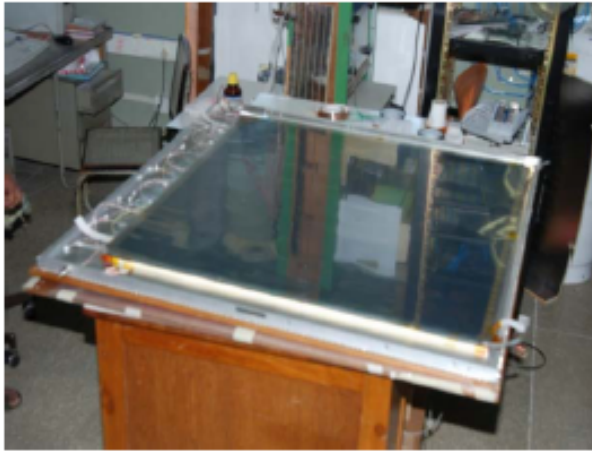
RPC (active detector element)



Specifications of the ICAL Detector

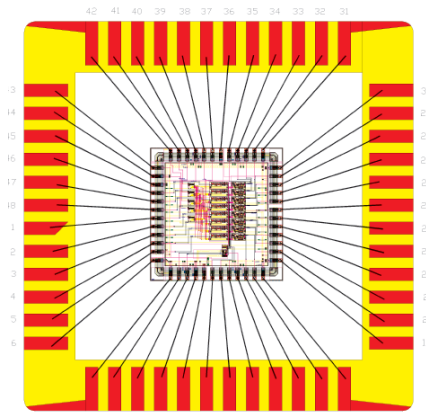
<i>No of modules</i>	<i>3</i>
<i>Module dimension</i>	<i>16 m X 16 m X 14.4m</i>
<i>Detector dimension</i>	<i>48.4 m X 16 m X 14.4m</i>
<i>No of layers</i>	<i>150</i>
<i>Iron plate thickness</i>	<i>5.6cm</i>
<i>Gap for RPC trays</i>	<i>4 cm</i>
<i>Magnetic field</i>	<i>1.4 Tesla</i>
<i>RPC unit dimension</i>	<i>195 cm x 184 cm x 2.4 cm</i>
<i>Readout strip width</i>	<i>3 cm</i>
<i>No. of RPCs/Road/Layer</i>	<i>8</i>
<i>No. of Roads/Layer/Module</i>	<i>8</i>
<i>No. of RPC units/Layer</i>	<i>192</i>
<i>Total no of RPC units</i>	<i>28800</i>
<i>No of Electronic channels</i>	<i>3.7 X 10⁶</i>

Fabricating Glass RPCs for ICAL

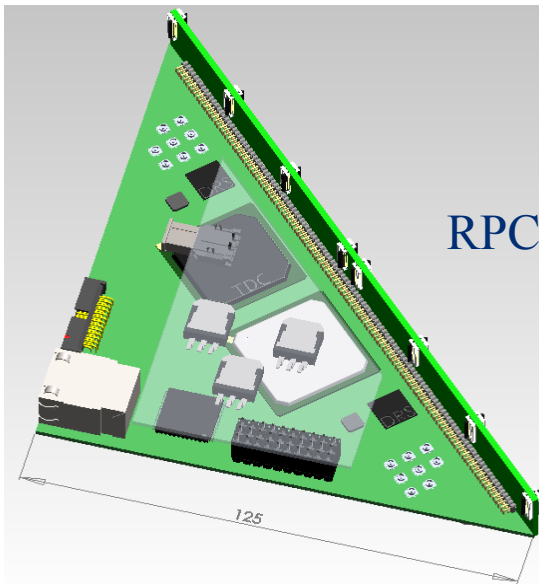


- *30 glass RPCs of 1m × 1m developed, tested for long in avalanche mode*
- *5 glass RPCs of 2m × 2m successfully assembled and tested*

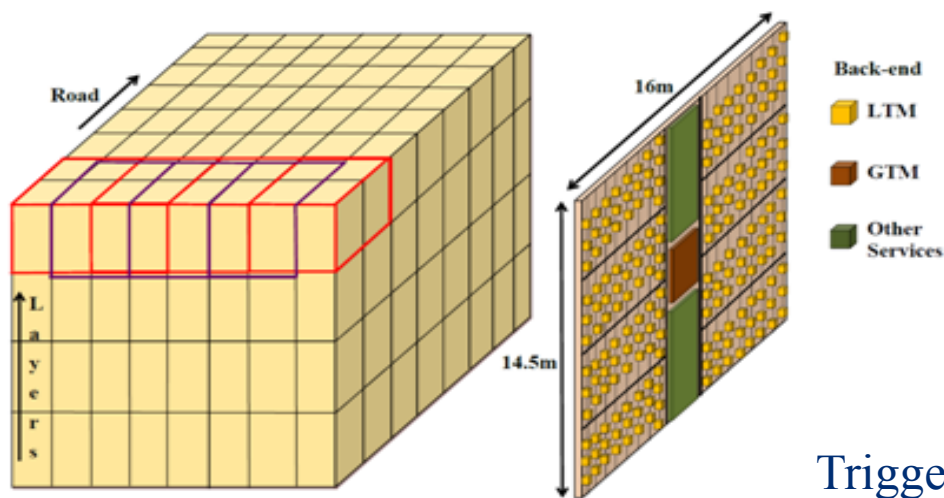
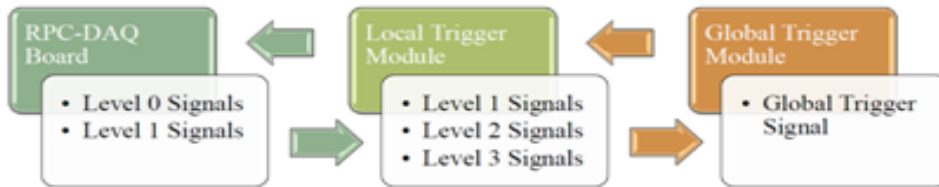
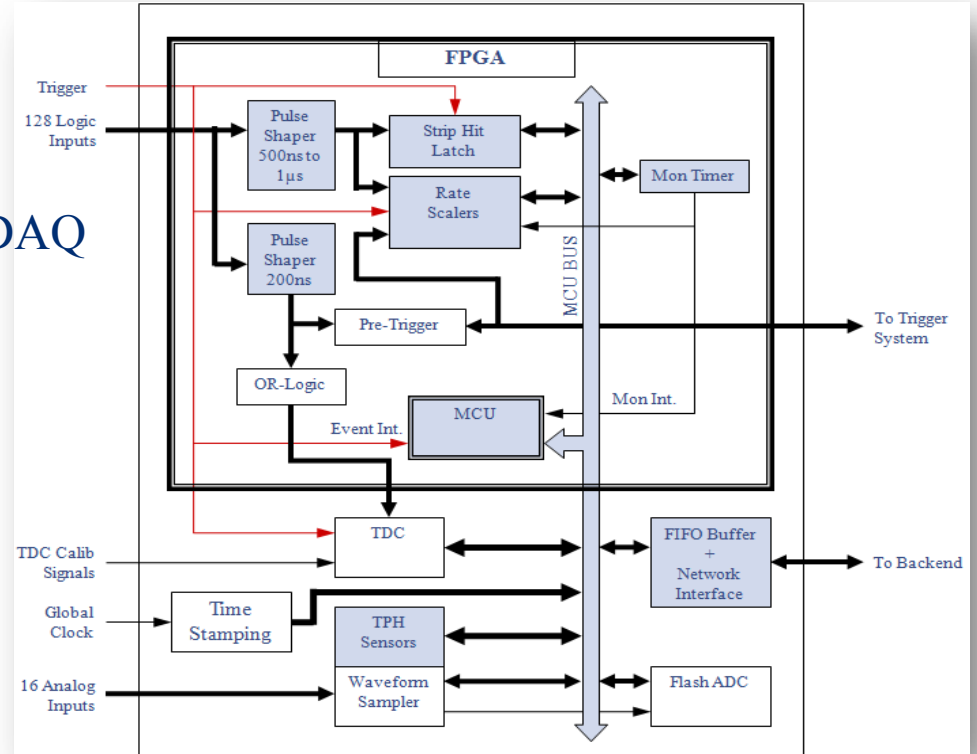
Various Components of ICAL Electronics



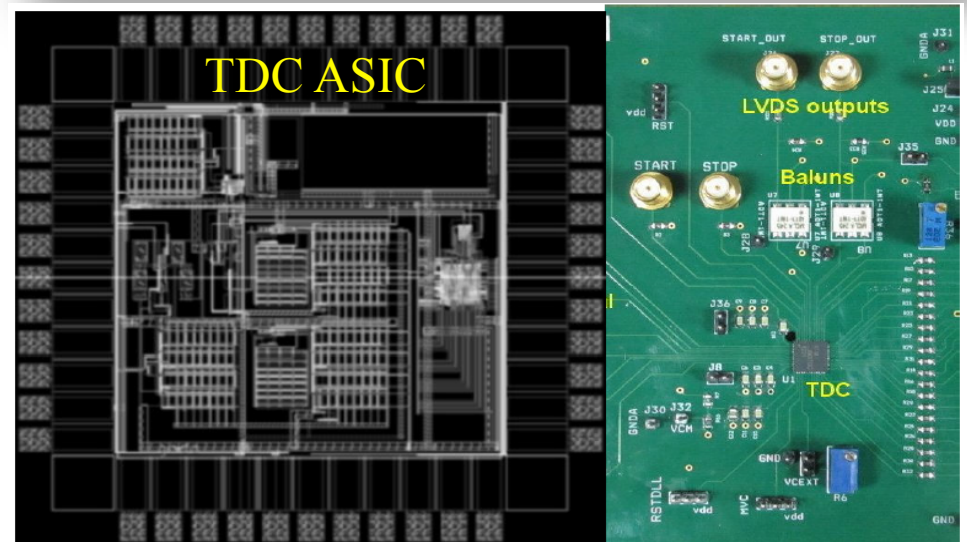
Front-end ASIC



RPCDAQ

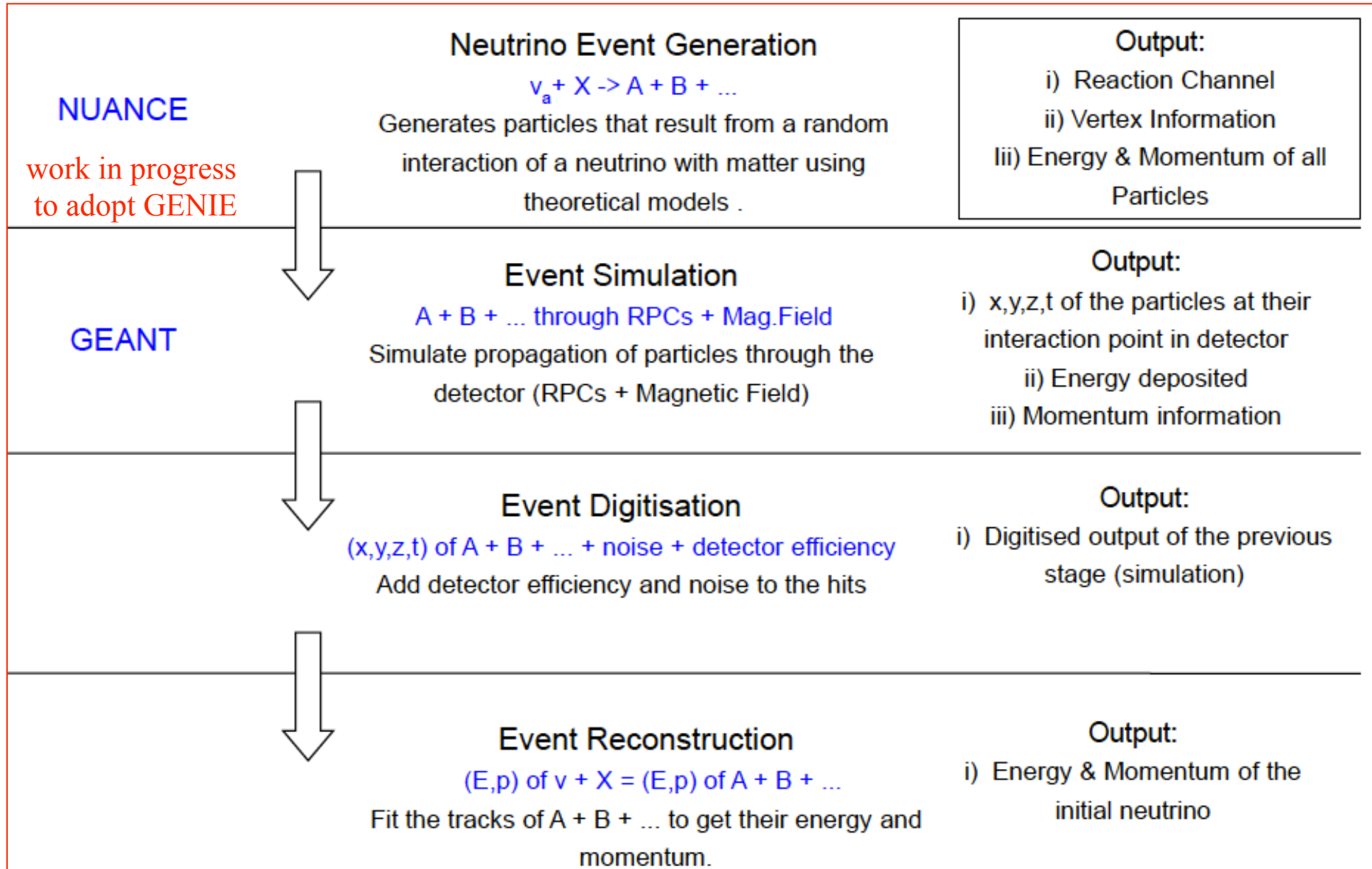


Trigger



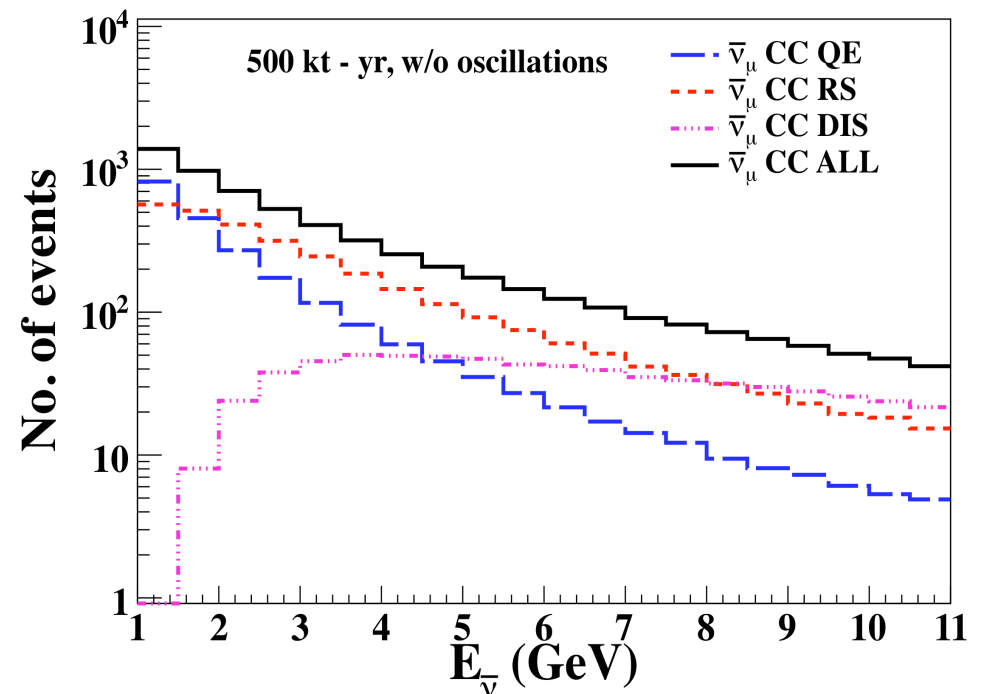
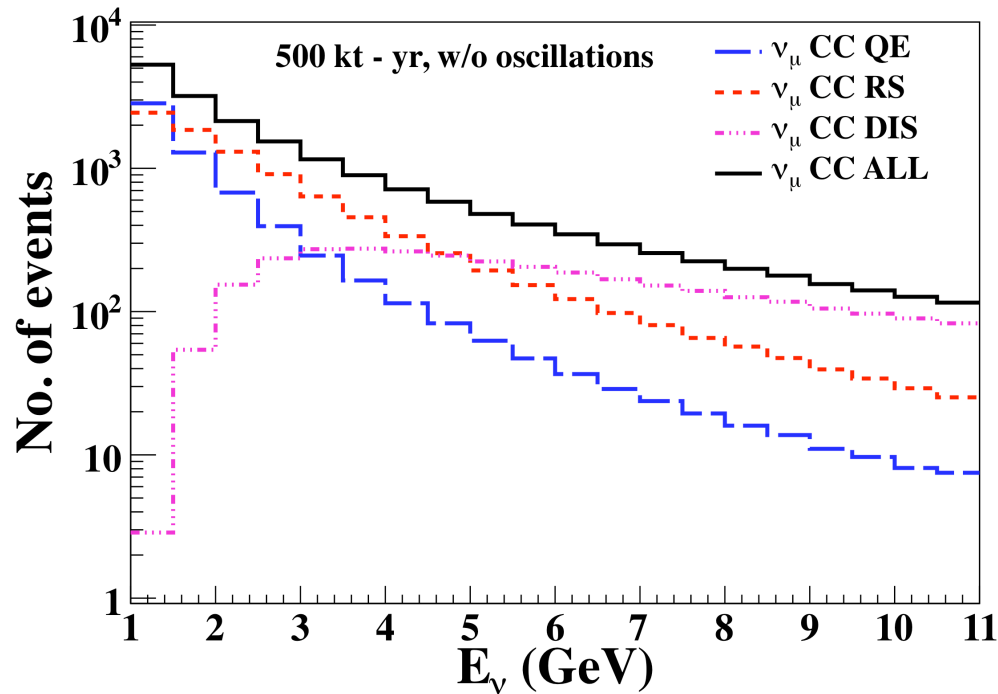
TDC ASIC

Overview of Simulation Framework



Simulation work is under progress in full swing!

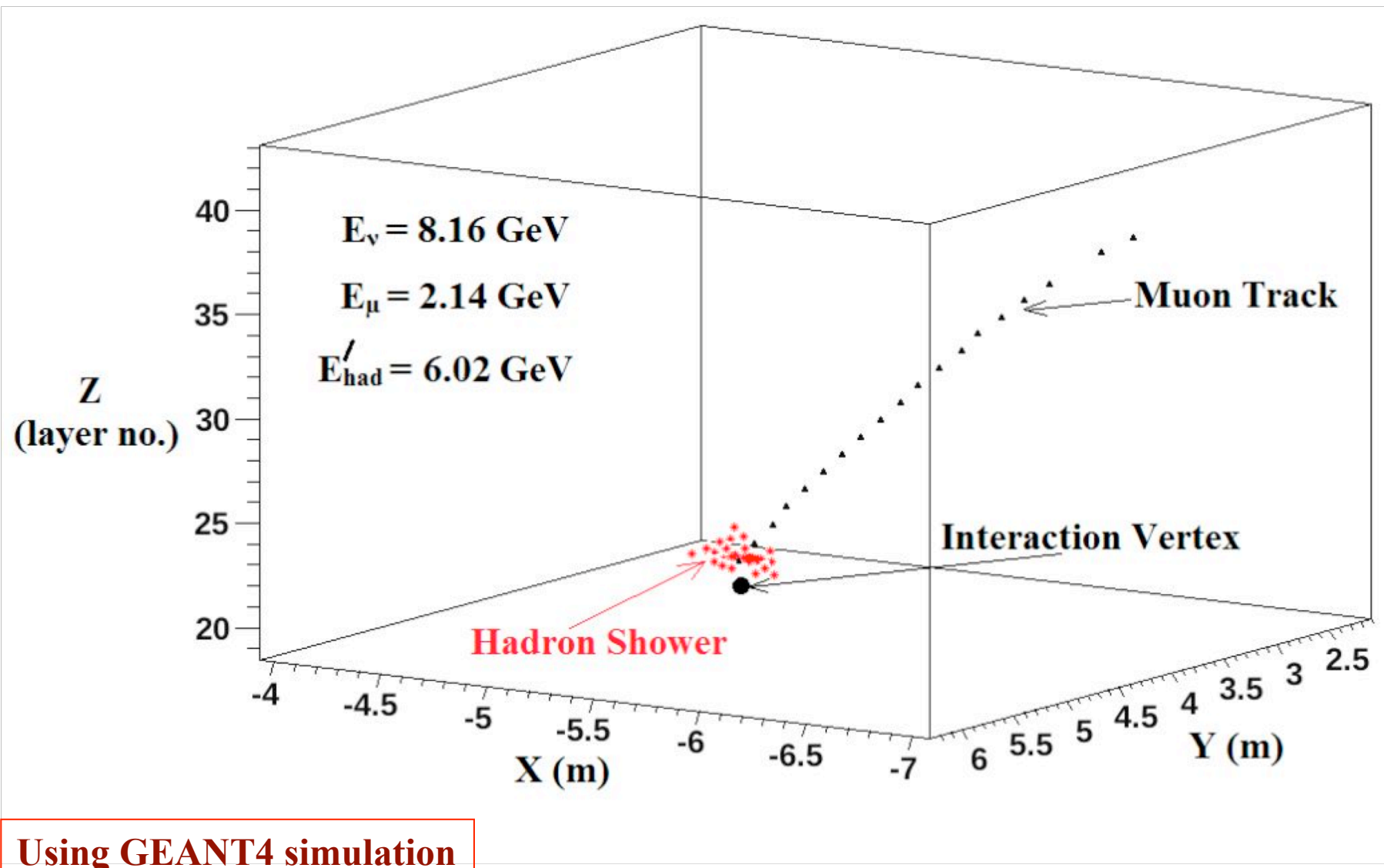
Events in Various Channels



Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

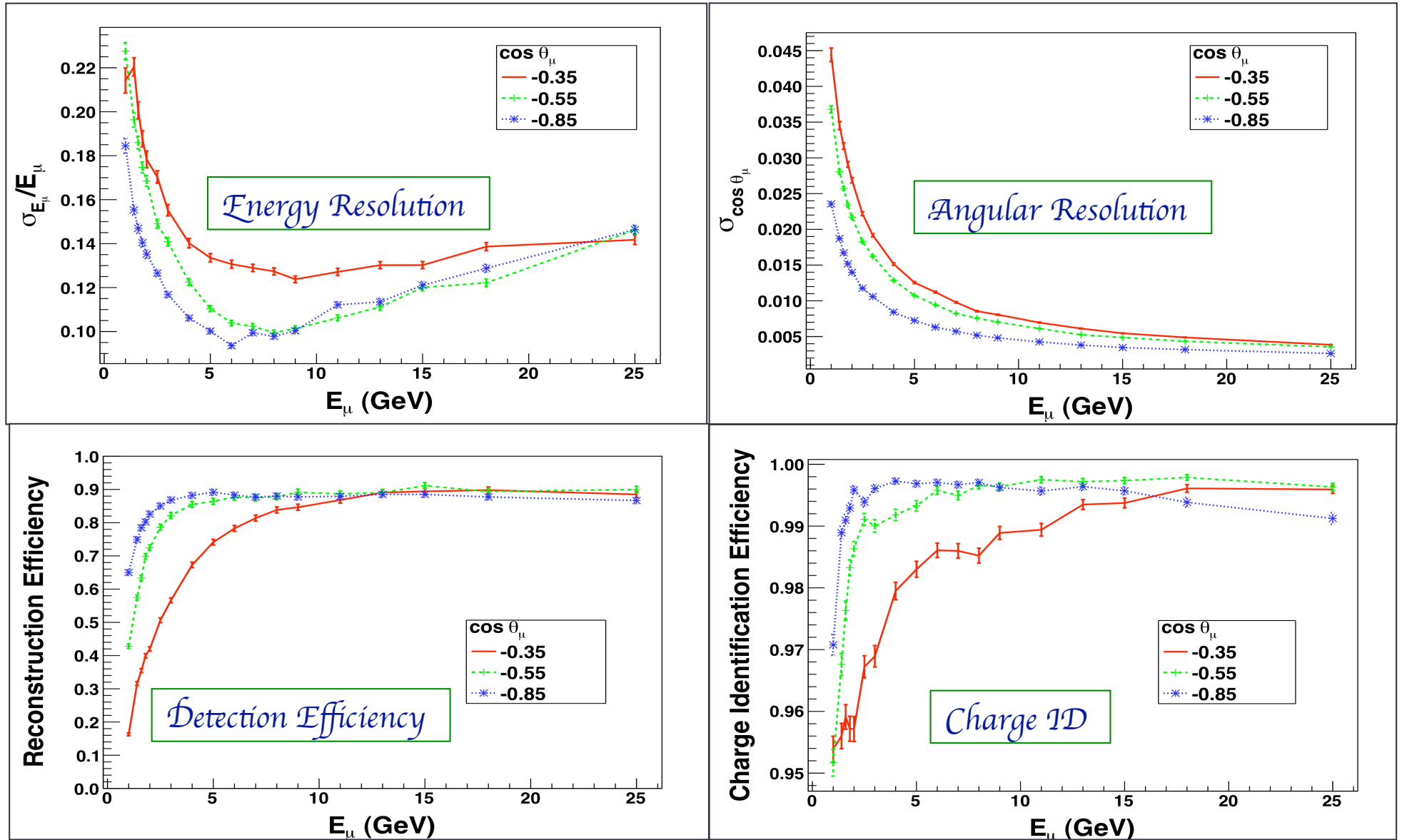
Relative contributions of three cross-section processes to the total events in the absence of oscillation and without detector efficiency and resolutions

Event Display Inside the ICAL Detector



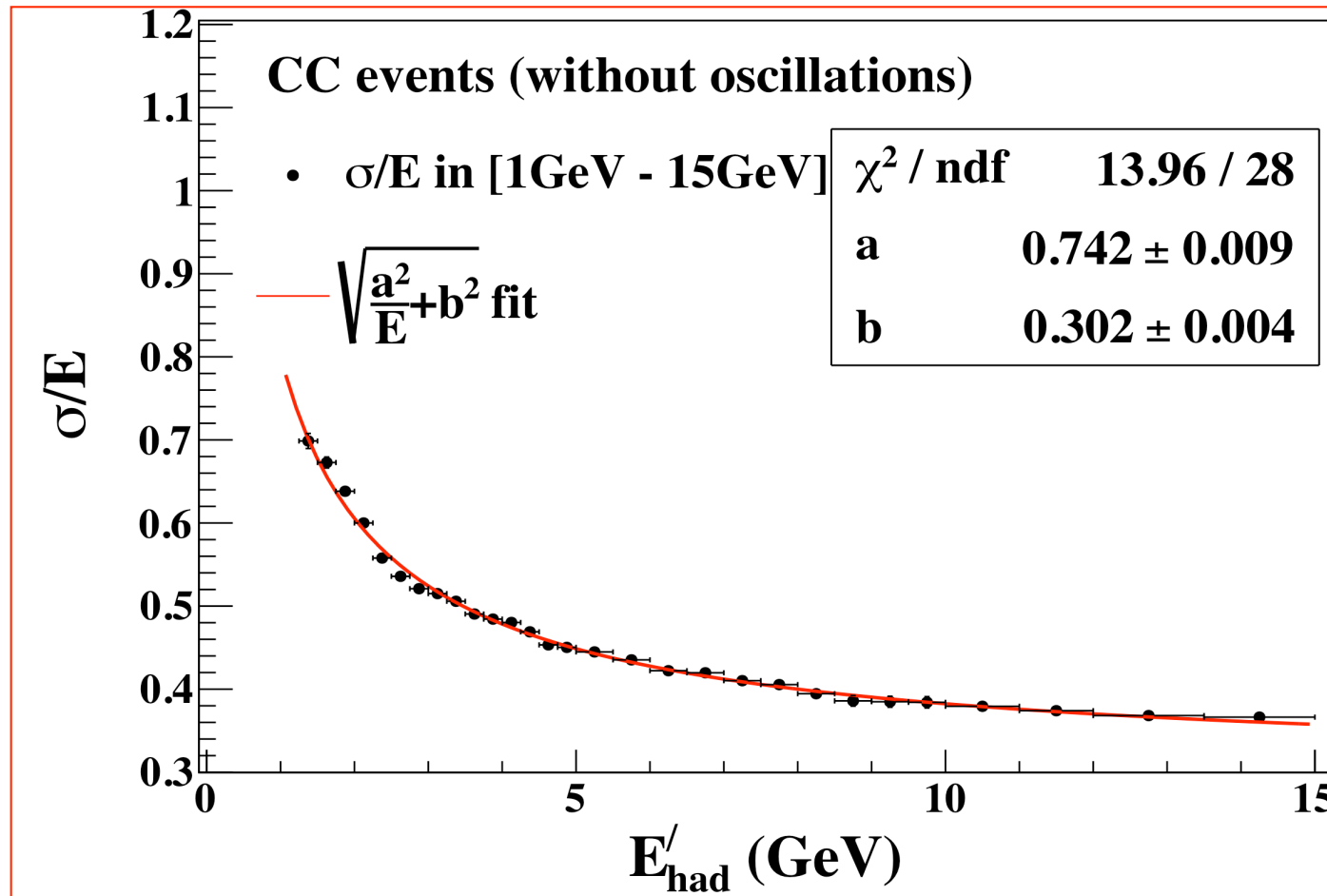
Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

Muon Efficiencies and Resolutions



Animesh Chatterjee, Meghna K.K., Kanishka Rawat, Tarak Thakore et al., arXiv:1405.7243 [physics.ins-det]

Hadron Energy Response of ICAL



$$E'_h = E_\nu - E_\mu \text{ (from hadron hit calibration)}$$

Hadron energy resolution: 85% at 1 GeV and 36% at 15 GeV

Moon Moon Devi, Anushree Ghosh, Daljeet Kaur, Lakshmi S. Mohan et al., JINST 8 (2013) P11003

The χ^2 Analysis

We define the Poissonian χ^2_- for μ^- events as :

$$\chi^2_- = \min_{\xi_l} \sum_{i=1}^{N_{E'_{\text{had}}}} \sum_{j=1}^{N_{E_\mu}} \sum_{k=1}^{N_{\cos \theta_\mu}} \left[2(N_{ijk}^{\text{theory}} - N_{ijk}^{\text{data}}) - 2N_{ijk}^{\text{data}} \ln \left(\frac{N_{ijk}^{\text{theory}}}{N_{ijk}^{\text{data}}} \right) \right] + \sum_{l=1}^5 \xi_l^2,$$

where

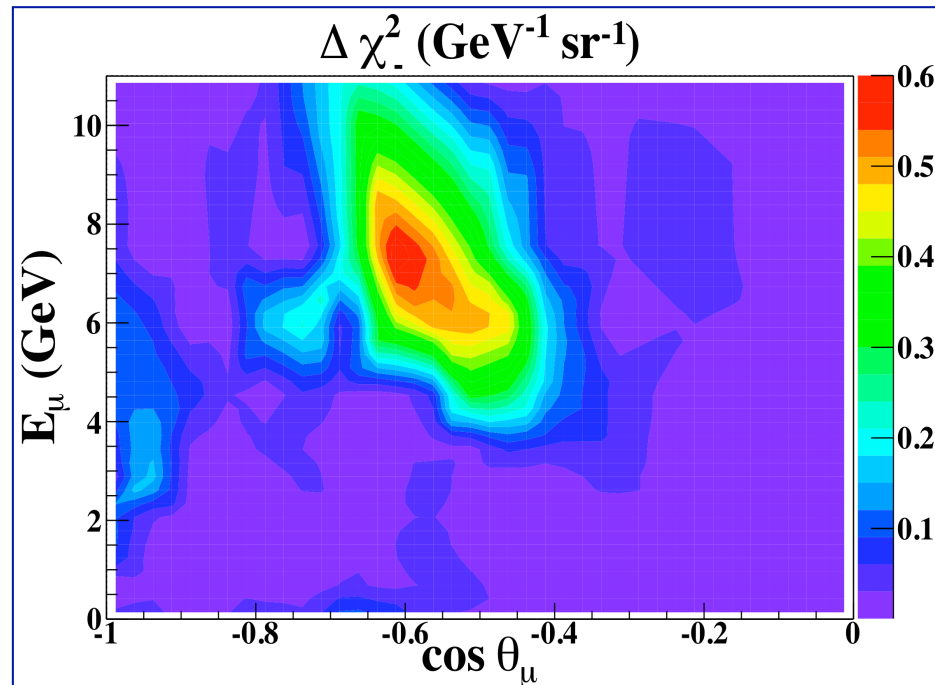
$$N_{ijk}^{\text{theory}} = N_{ijk}^0 \left(1 + \sum_{l=1}^5 \pi_{ijk}^l \xi_l \right).$$

Observable	Range	Bin width	Total bins
E_μ (GeV)	[1, 4)	0.5	6
	[4, 7)	1	3
	[7, 11)	4	1
$\cos \theta_\mu$	[-1.0, -0.4)	0.05	12
	[-0.4, 0.0)	0.1	4
	[0.0, 1.0]	0.2	5
E'_{had} (GeV)	[0, 2)	1	2
	[2, 4)	2	1
	[4, 15)	11	1

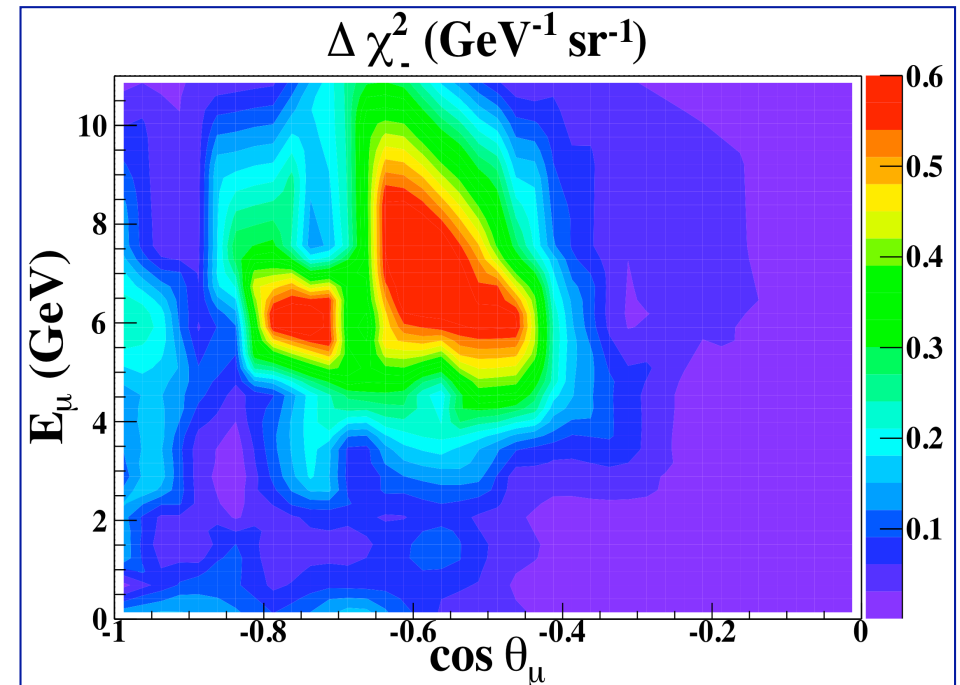
- 1) Overall 5% systematic uncertainty
- 2) Overall flux normalization: 20%
- 3) Overall cross-section normalization: 10%
- 4) 5% uncertainty on the zenith angle dependence of the fluxes
- 5) Energy dependent tilt factor:
 $\Phi_\delta(E) = \Phi_0(E) [E/E_0]^\delta \approx \Phi_0(E) [1 + \delta \ln E/E_0]$
 where $E_0 = 2$ GeV and
 δ is the 1σ systematic error of 5%

Neutrino Mass Hierarchy Discrimination

Distribution of $\Delta\chi^2$ [χ^2 (IH) - χ^2 (NH)] for mass hierarchy discrimination considering μ^- events



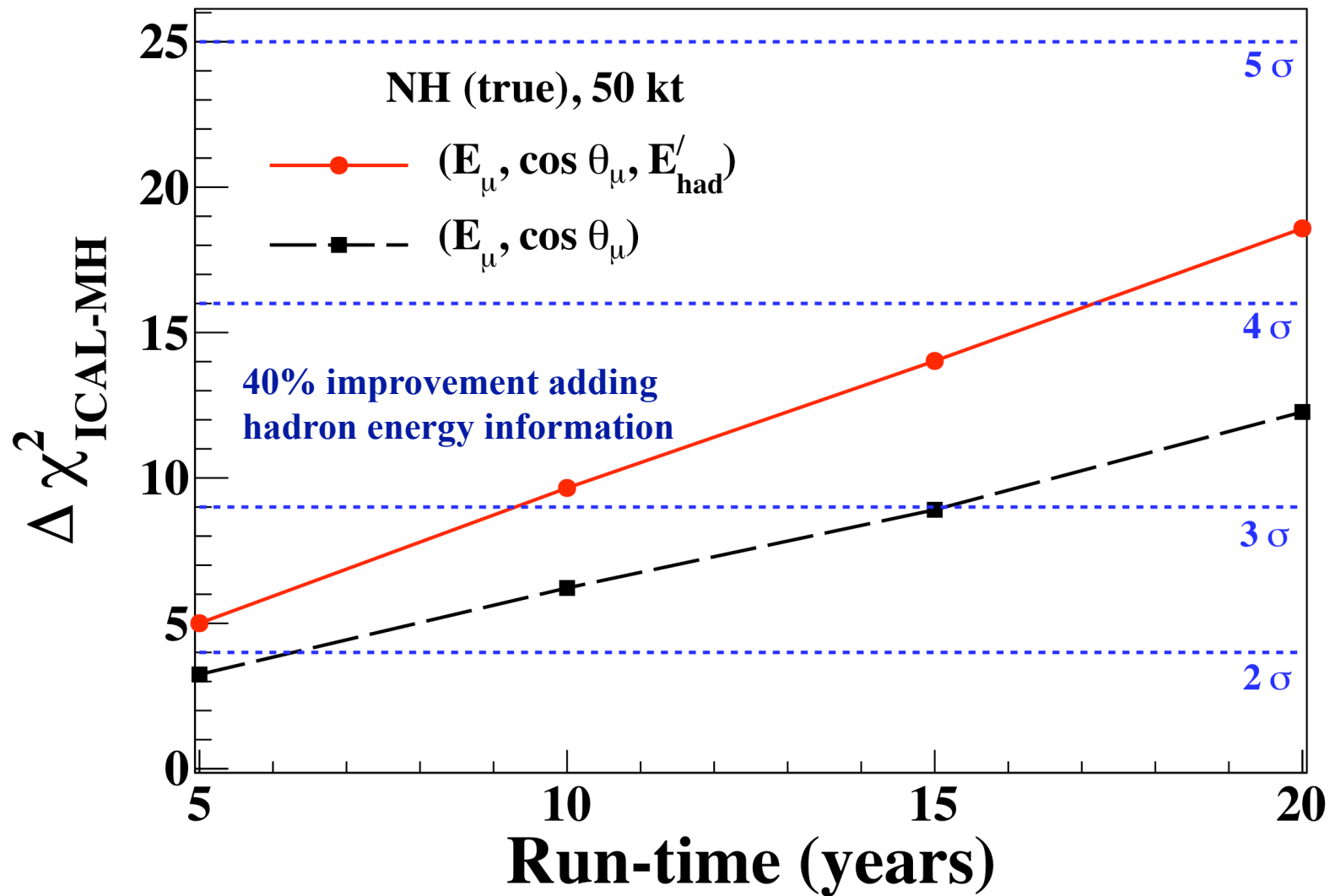
Hadron energy information not used



Hadron energy information used

- ⊙ Further subdivide the events into four hadron energy bins
- ⊙ Hadron energy carries crucial information
- ⊙ Correlation between hadron energy and muon momentum is very important

Identifying Neutrino Mass Hierarchy with ICAL

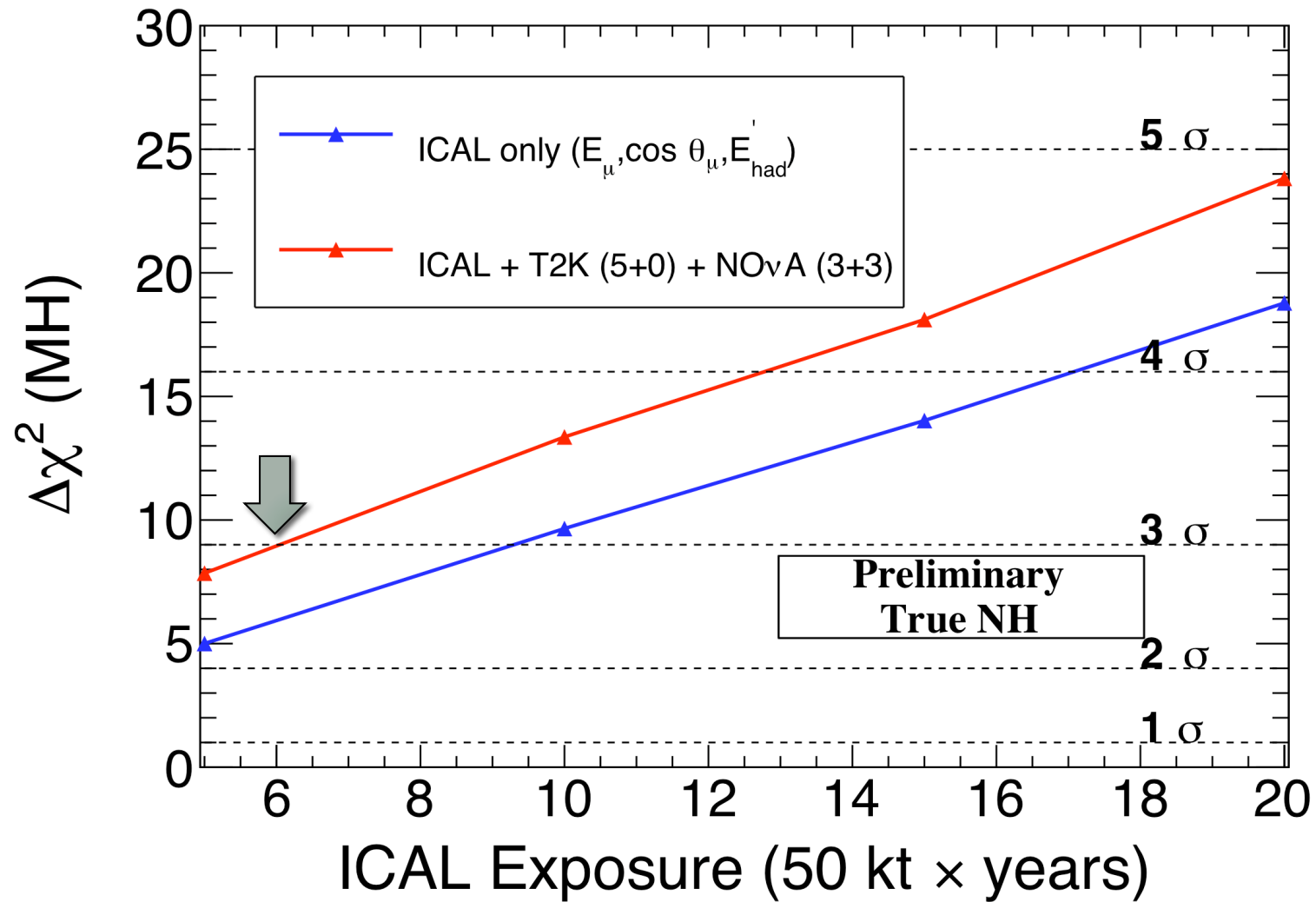


Median Sensitivity

Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

50 kt ICAL can rule out the wrong hierarchy with $\Delta\chi^2 \approx 9.5$ in 10 years

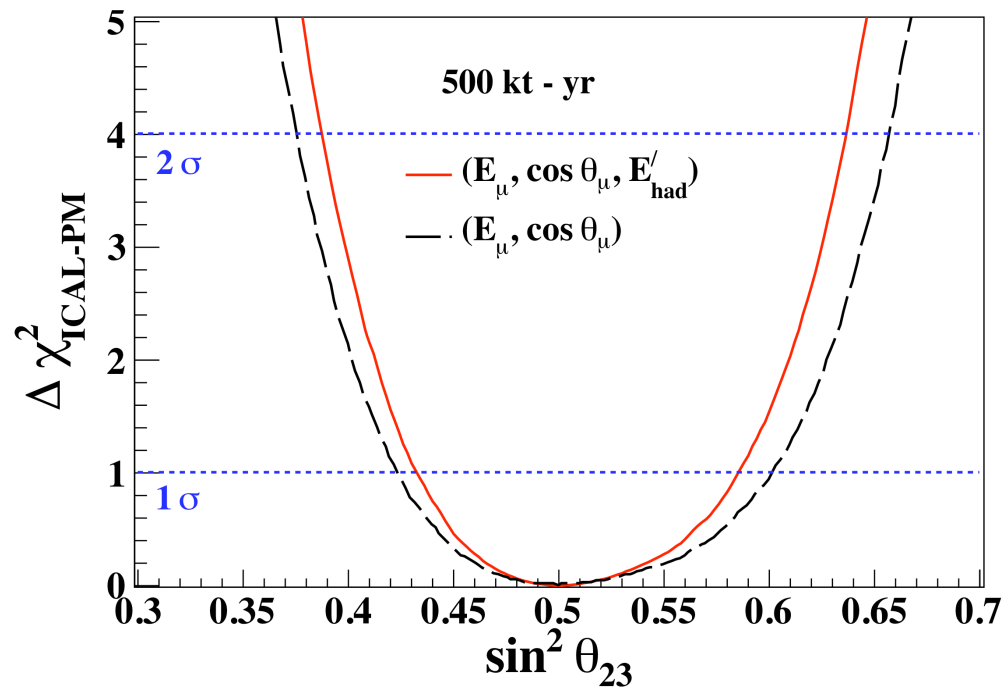
MH Discovery with ICAL+T2K+NOvA



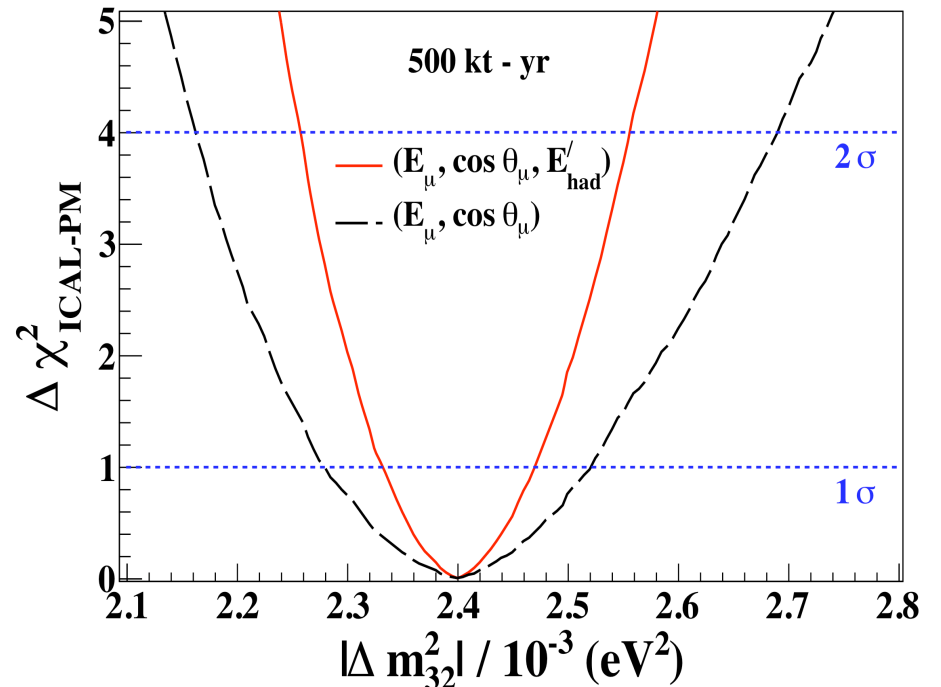
Agarwalla, Thakore, work in progress (INO Collaboration)

3σ median sensitivity can be achieved in 6 years

Precision of Atmospheric Oscillation Parameters



Relative 1σ precision: 12%

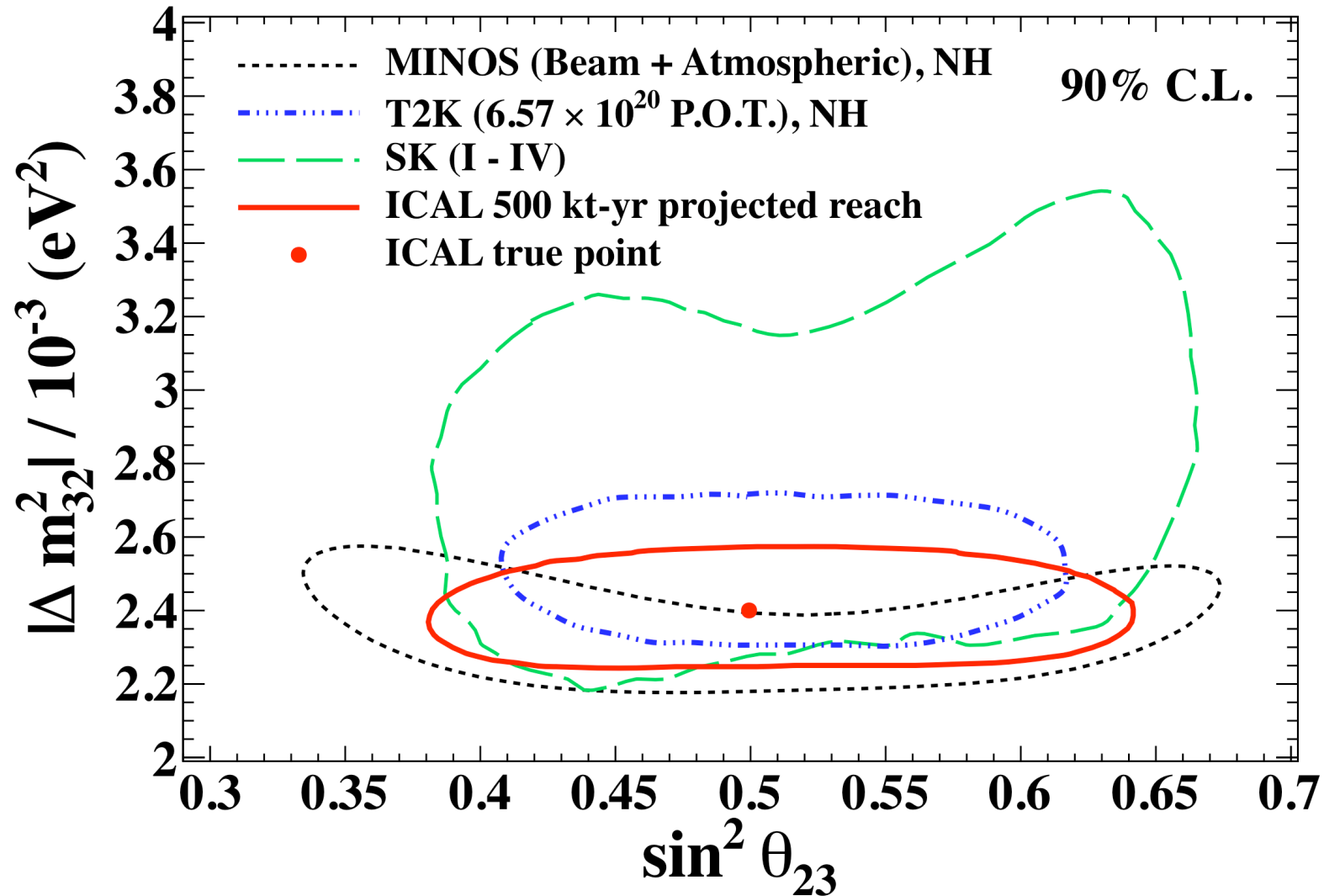


Relative 1σ precision: 2.9%

Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

Significant improvement in the precision measurement of atmospheric mass splitting by adding hadron energy information with muon momentum

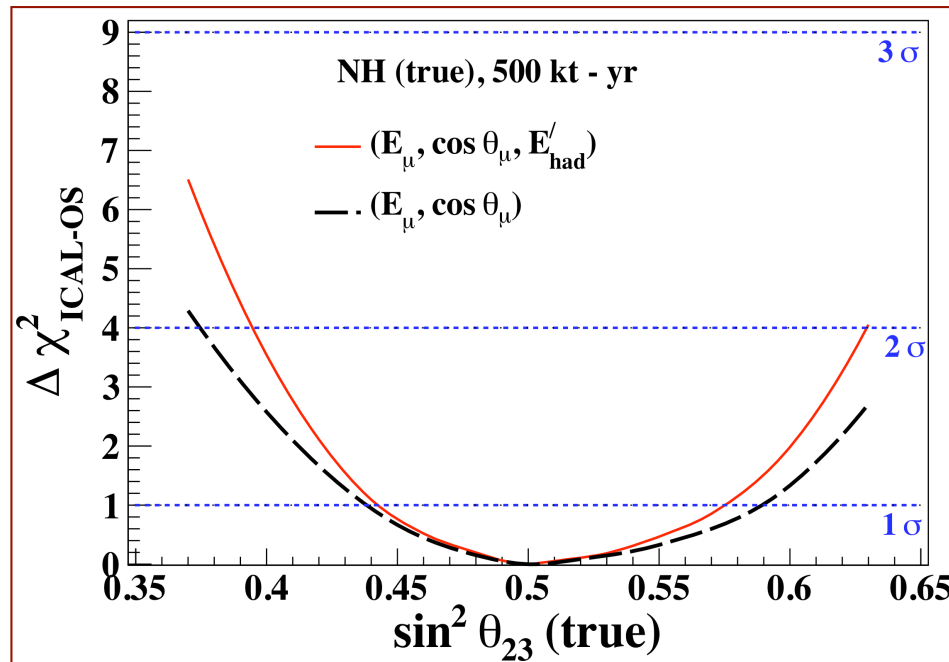
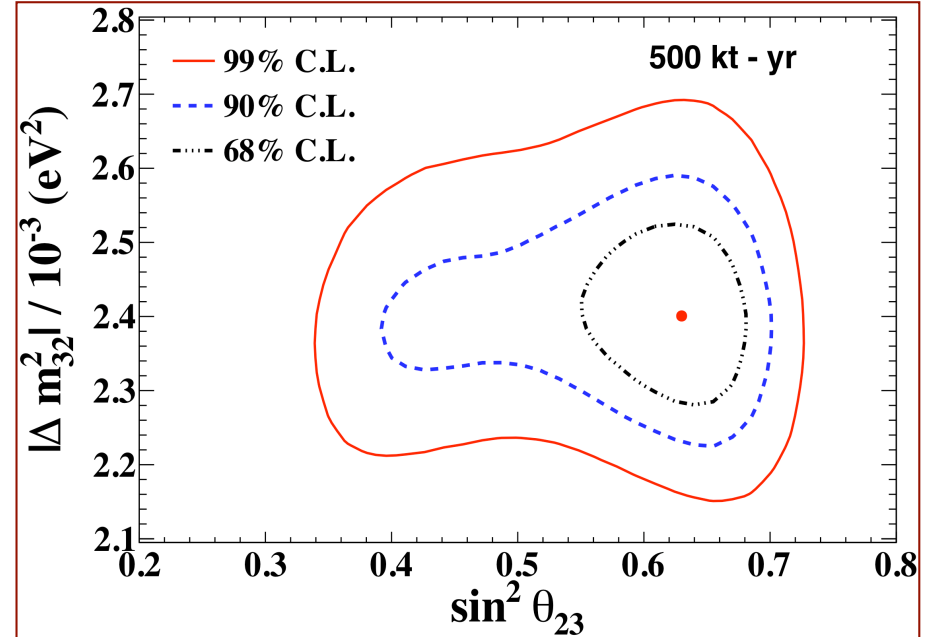
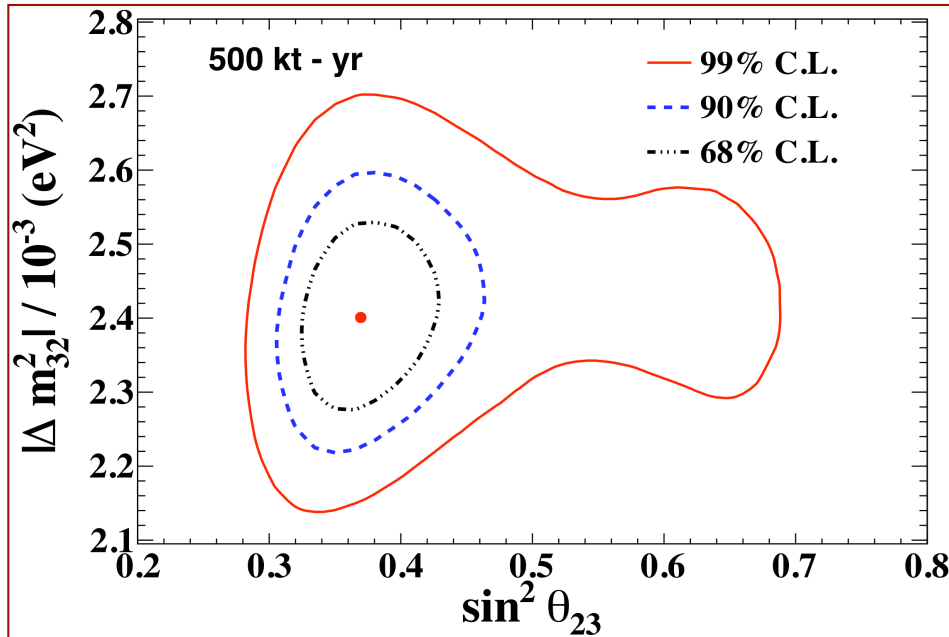
Precision Measurement of Atmospheric Parameters



Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

ICAL's expected precision on atmospheric mass splitting is far superior than SK

Octant of θ_{23} with ICAL-INO



Median 2 σ discovery of θ_{23} octant is possible if θ_{23} is sufficiently away from maximal value

Devi, Thakore, Agarwalla, Dighe, arXiv: 1406.3689 (INO Collaboration)

Current Status of INO

Pre-project activities started with an initial grant of ~ 15 M\$

- **Site infrastructure development**
 - **Development of INO centre at Madurai city (110 km from underground lab)
- Inter-Institutional Centre for High Energy Physics (ICHEP)**
 - **Construction of an 1/8th size engineering prototype module**
-
- **Detector R&D is now over**
 - **Detailed Project Report for Detector and DAQ system is ready**
 - **Soon go for industrial production of RPCs & associated front-end electronics**
 - **Full project approved by Indian Atomic Energy Commission
Waiting for approval from PM's cabinet committee to start construction**

Glimpse of Activities at the IICHEP Site



Glimpse of Activities at the INO Site

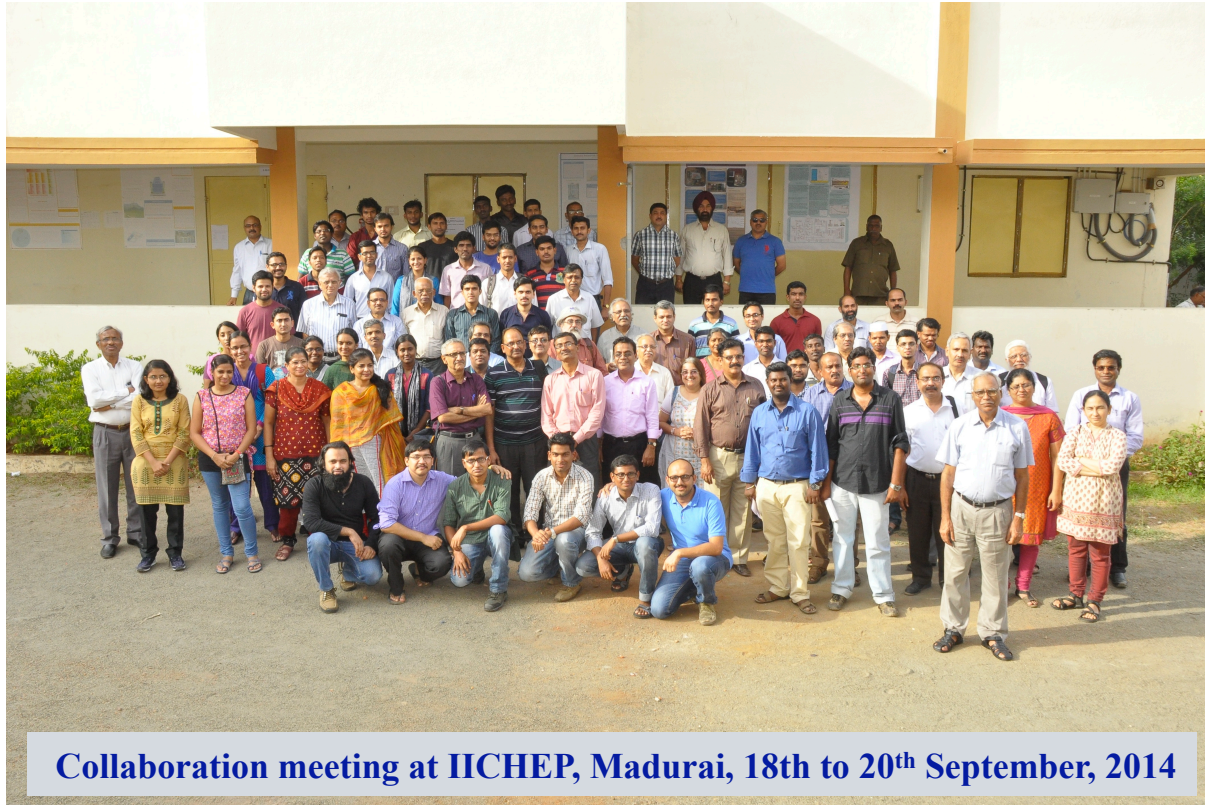


Human Resource Development and Training



- **INO Graduate Training Program started in August 2008, students are affiliated to HBNI**
- **At present students being trained for 1 year at TIFR in both experimental techniques & theory**
- **After completion of coursework, attached to Ph.D. guides at various collaborating institutions**
- **Many short/long term visits to RPC labs (Mumbai & Kolkata) of students & faculties from Universities in last several years**
- **Several students from 1st batch (2008) are at the final stage of writing their theses. Few of them have already received good post-doctoral offers from various experiments**
- **7th batch of 11 students have started their course work at TIFR in 2014**

Concluding Remarks



Collaboration meeting at IICHEP, Madurai, 18th to 20th September, 2014

Satisfactory progress
in all fronts in last
2 to 3 years

Strong support from
the community &
Funding agencies

All set to move ahead
with this mega-science
project

For more updates visit: <http://www.ino.tifr.res.in/ino/>

You can join us at: <https://www.facebook.com/ino.neutrino>

International collaboration most welcome

!! Looking Forward for Exciting Discoveries at INO !!

Thank You

Backup Slides

A Personal View of Sheldon Lee Glashow

Is observable CP violation confined to hadrons?

I would assign very high priority to experiments that could demonstrate the existence of CP violating effects in the neutrino sector

The other important mass-related issue is the binary choice between two orderings of neutrino masses

The accuracy with which oscillation parameters are already known surely suffices for the design of an experiment that can accomplish this goal

**Particle Physics in the United States
A Personal View
Sheldon Lee Glashow
arXiv:1305.5482v1 [hep-ph]**

!! Let us work together and resolve these fundamental issues !!

CKM vs. PMNS Precision

Recent discovery of θ_{13} signifies an important breakthrough in establishing the standard three flavor oscillation picture of neutrinos

It has opened up exciting possibilities for current & future oscillation experiments

At present, we have:

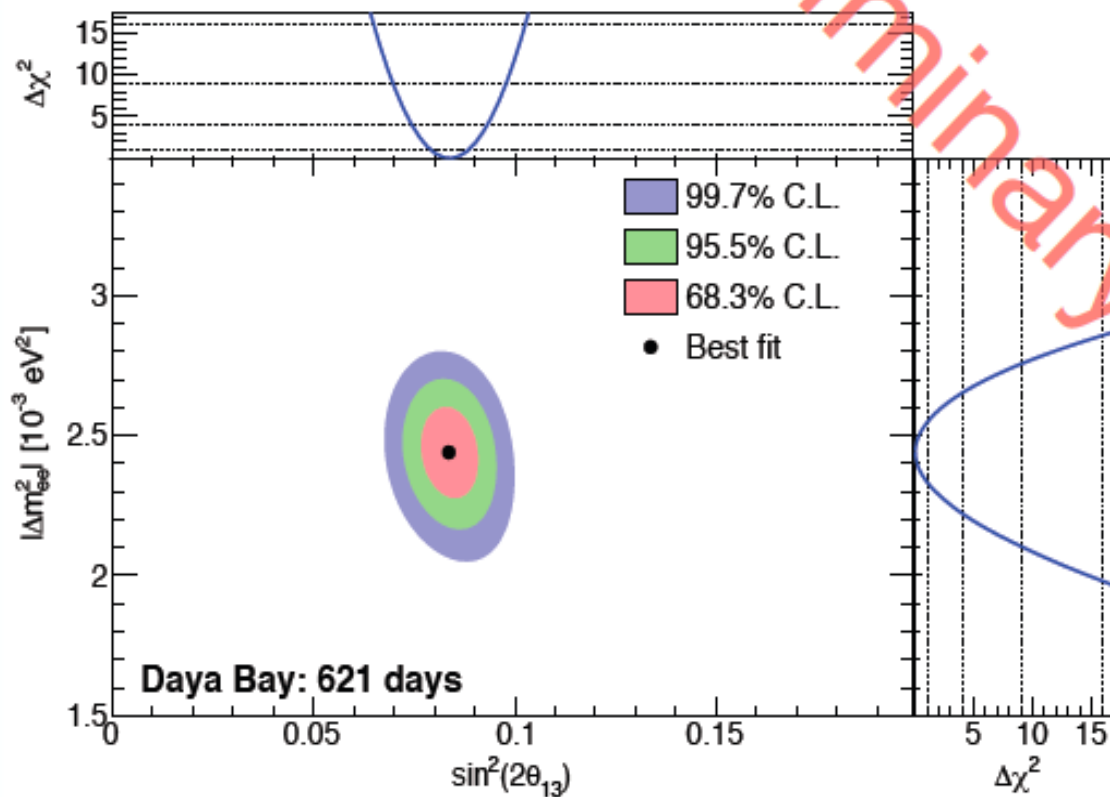
$$|U|_{\text{LEP}(3\sigma)} = \begin{pmatrix} 0.799 \rightarrow 0.844 & 0.515 \rightarrow 0.581 & 0.129 \rightarrow 0.173 \\ 0.212 \rightarrow 0.527 & 0.426 \rightarrow 0.707 & 0.598 \rightarrow 0.805 \\ 0.233 \rightarrow 0.538 & 0.450 \rightarrow 0.722 & 0.573 \rightarrow 0.787 \end{pmatrix}$$

Satisfactory progress in last 15 years but still very far from the ‘dream’ precision:

$$|V|_{\text{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2_{-5}^{+1.1}) \times 10^{-3} \\ (8.67_{-0.31}^{+0.29}) \times 10^{-3} & (40.4_{-0.5}^{+1.1}) \times 10^{-3} & 0.999146_{-0.000046}^{+0.000021} \end{pmatrix}$$

Latest Oscillation Results from Daya Bay

Rate + Shape Oscillation Results [Announced in Neutrino 2014]



$$\sin^2 2\theta_{13} = 0.084^{+0.005}_{-0.005}$$

$$|\Delta m_{ee}^2| = 2.44^{+0.10}_{-0.11} \times 10^{-3} \text{eV}^2$$

$$\chi^2/NDF = 134.7/146$$

6% precision in $\sin^2 2\theta_{13}$ achieved

Strong confirmation of oscillation-interpretation of observed $\bar{\nu}_e$ deficit

	Normal MH Δm_{32}^2 [10^{-3}eV^2]	Inverted MH Δm_{32}^2 [10^{-3}eV^2]
From Daya Bay Δm_{ee}^2	$2.39^{+0.10}_{-0.11}$	$-2.49^{+0.10}_{-0.11}$
From MINOS $\Delta m_{\mu\mu}^2$	$2.37^{+0.09}_{-0.09}$	$-2.41^{+0.11}_{-0.09}$

Present Understanding of the 2-3 Mixing Angle

Information on θ_{23} comes from: a) **atmospheric neutrinos** and b) **accelerator neutrinos**

In two-flavor scenario:
$$P_{\mu\mu} = 1 - \sin^2 2\theta_{\text{eff}} \sin^2 \left(\frac{\Delta m_{\text{eff}}^2 L}{4E} \right)$$

For accelerator neutrinos: relate effective 2-flavor parameters with 3-flavor parameters:

$$\Delta m_{\text{eff}}^2 = \Delta m_{31}^2 - \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \delta_{\text{CP}} \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23})$$

$$\sin^2 2\theta_{\text{eff}} = 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \quad \text{where} \quad \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = \tan^2 \theta_{23}$$

Nunokawa et al, hep-ph/0503283; A. de Gouvea et al, hep-ph/0503079

Combining beam and atmospheric data in MINOS, we have:

MINOS Collaboration: arXiv:1304.6335v2 [hep-ex]

$$\sin^2 2\theta_{\text{eff}} = 0.95_{-0.036}^{+0.035} (10.71 \times 10^{21} \text{ p.o.t})$$

$$\sin^2 2\bar{\theta}_{\text{eff}} = 0.97_{-0.08}^{+0.03} (3.36 \times 10^{21} \text{ p.o.t})$$

Atmospheric data, dominated by Super-Kamiokande, still prefers maximal value of $\sin^2 2\theta_{\text{eff}} = 1$ (≥ 0.94 (90% C.L.))

Talk by Y. Itow in Neutrino 2012 conference, Kyoto, Japan

Bounds on θ_{23} from the global fits

In ν_μ survival probability, the dominant term mainly sensitive to $\sin^2 2\theta_{23}$

If $\sin^2 2\theta_{23}$ differs from 1 (as indicated by recent data), we get two solutions for θ_{23} :
one in lower octant (LO: $\theta_{23} < 45$ degree), other in higher octant (HO: $\theta_{23} > 45$ degree)

In other words, if $(0.5 - \sin^2 \theta_{23})$ is +ve (-ve) then θ_{23} belongs to LO (HO)

This is known as the octant ambiguity of θ_{23}

Fogli and Lisi, hep-ph/9604415

Conferences	After Neutrino 2012	After NeuTel 2013	After TAUP 2013
$\sin^2 \theta_{23}$	$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$	$0.437^{+0.061}_{-0.031}$	$0.446^{+0.007}_{-0.007} \oplus 0.587^{+0.032}_{-0.037}$
3σ range	$0.34 \rightarrow 0.67$	$0.357 \rightarrow 0.654$	$0.366 \rightarrow 0.663$
1σ precision (relative)	13.4%	11.3%	11.1%

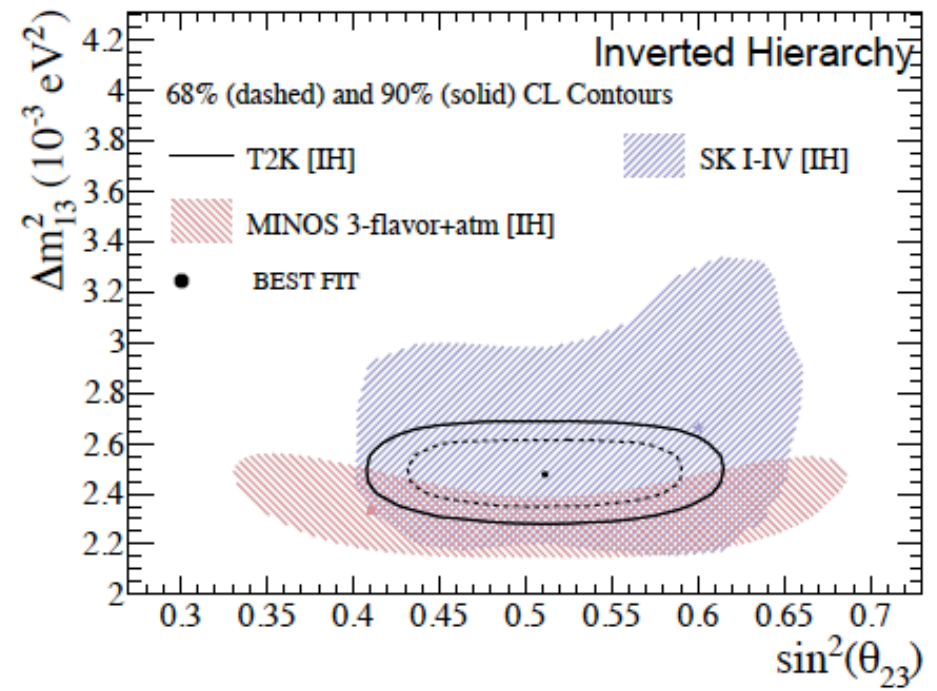
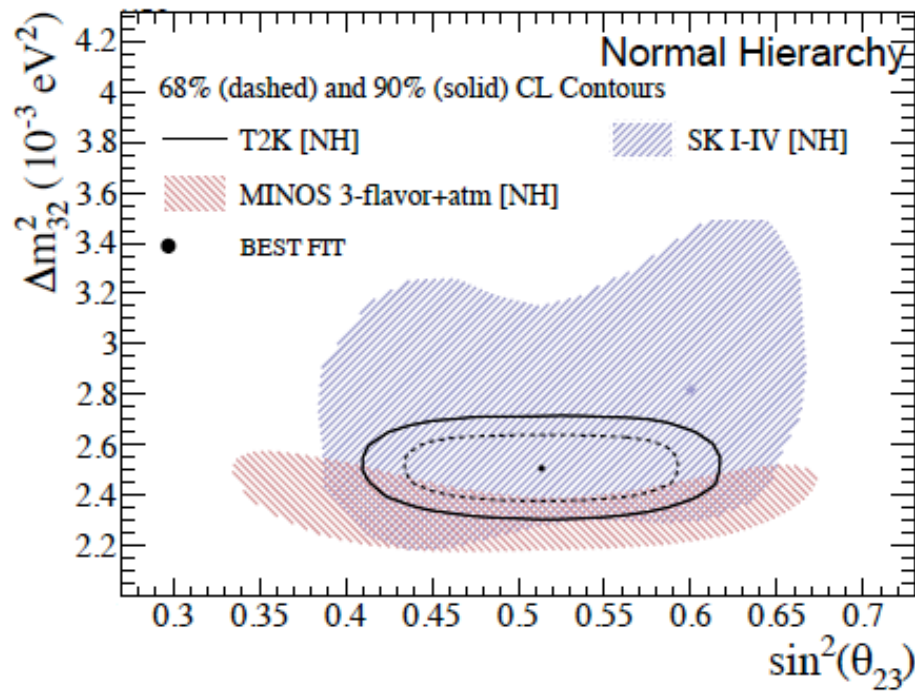
Based on Gonzalez-Garcia, Maltoni, Salvado, Schwetz, <http://www.nu-fit.org>

Global fit disfavors maximal 2-3 mixing at 1.4σ confidence level (mostly driven by MINOS)

ν_μ to ν_e oscillation data can break this degeneracy

The preferred value would depend on the choice of the neutrino mass hierarchy

New Measurements of Atmospheric Parameters

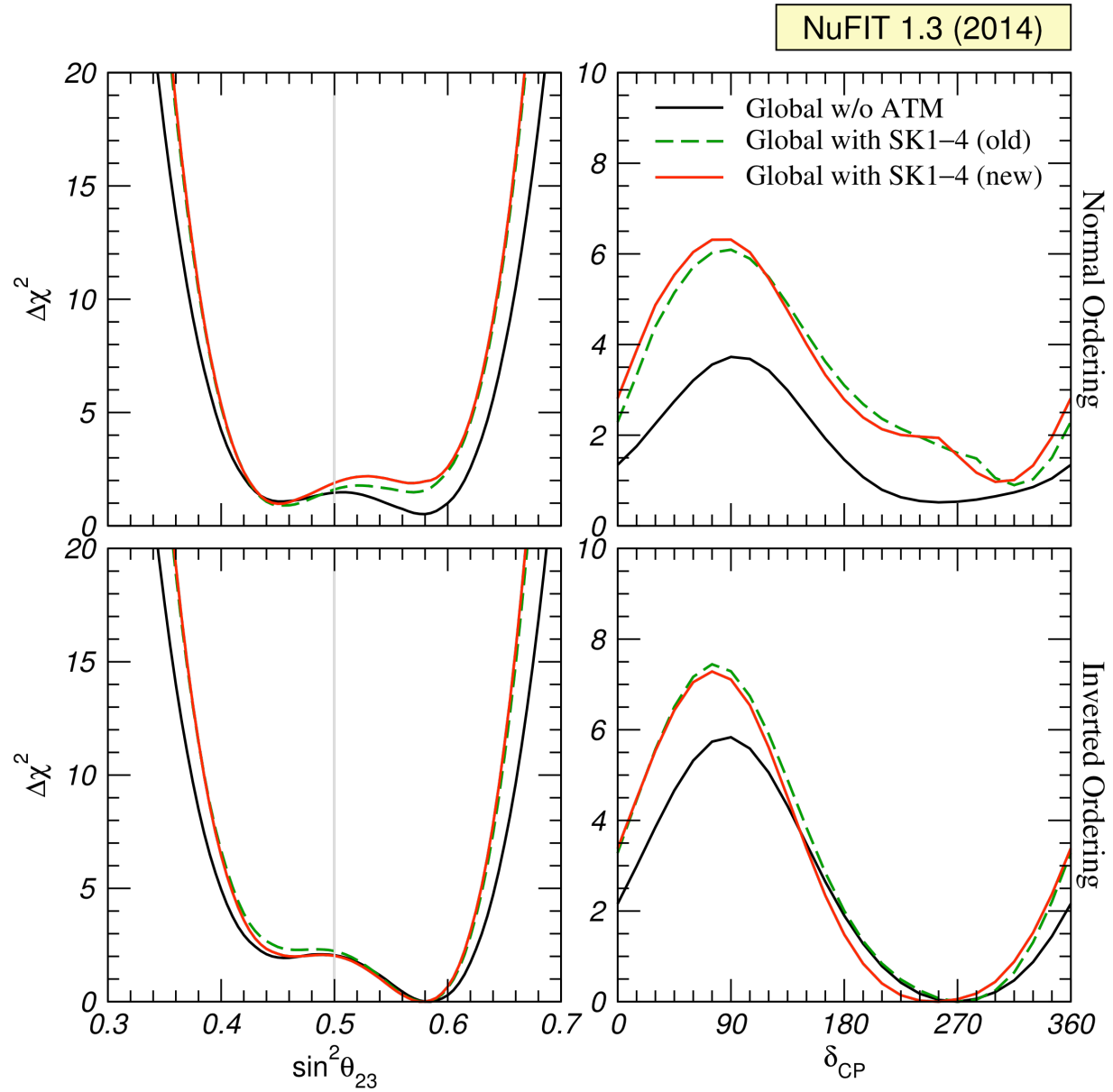


		Best-fit \pm FC 68% CL (Δm^2 units $10^{-3} \text{ eV}^2/c^4$)
NH	$\sin^2\theta_{23}$	$0.514^{+0.055}_{-0.056}$
	Δm^2_{32}	2.51 ± 0.10
IH	$\sin^2\theta_{23}$	0.511 ± 0.055
	Δm^2_{13}	2.48 ± 0.10

Already mixing angle is better constrained by T2K in comparison to SK and MINOS

Talk by C. Walter in Neutrino 2014

Role of Atmospheric Neutrinos in Global Fit



Based on the data available after Neutrino 2014 conference

Gonzalez-Garcia, Maltoni, Salvado, Schwetz, <http://www.nu-fit.org>

Three Flavor Effects in $\nu_\mu \rightarrow \nu_e$ oscillation probability

The appearance probability ($\nu_\mu \rightarrow \nu_e$) in matter, upto second order in the small parameters $\alpha \equiv \Delta m_{21}^2 / \Delta m_{31}^2$ and $\sin 2\theta_{13}$,

$$\begin{aligned}
 P_{\mu e} \simeq & \underbrace{\sin^2 2\theta_{13}}_{0.09} \underbrace{\sin^2 \theta_{23}}_{0.03} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2} \longrightarrow \theta_{13} \text{ Driven} \\
 & - \underbrace{\alpha \sin 2\theta_{13}}_{0.009} \xi \sin \delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \longrightarrow \text{CP odd} \\
 & + \alpha \sin 2\theta_{13} \xi \cos \delta_{CP} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \longrightarrow \text{CP even} \\
 & + \underbrace{\alpha^2}_{0.0009} \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}; \longrightarrow \text{Solar Term}
 \end{aligned}$$

where $\Delta \equiv \Delta m_{31}^2 L / (4E)$, $\xi \equiv \cos \theta_{13} \sin 2\theta_{21} \sin 2\theta_{23}$,
and $\hat{A} \equiv \pm(2\sqrt{2}G_F n_e E) / \Delta m_{31}^2$

changes sign with $\text{sgn}(\Delta m_{31}^2)$
key to resolve hierarchy!

changes sign with polarity
causes fake CP asymmetry!

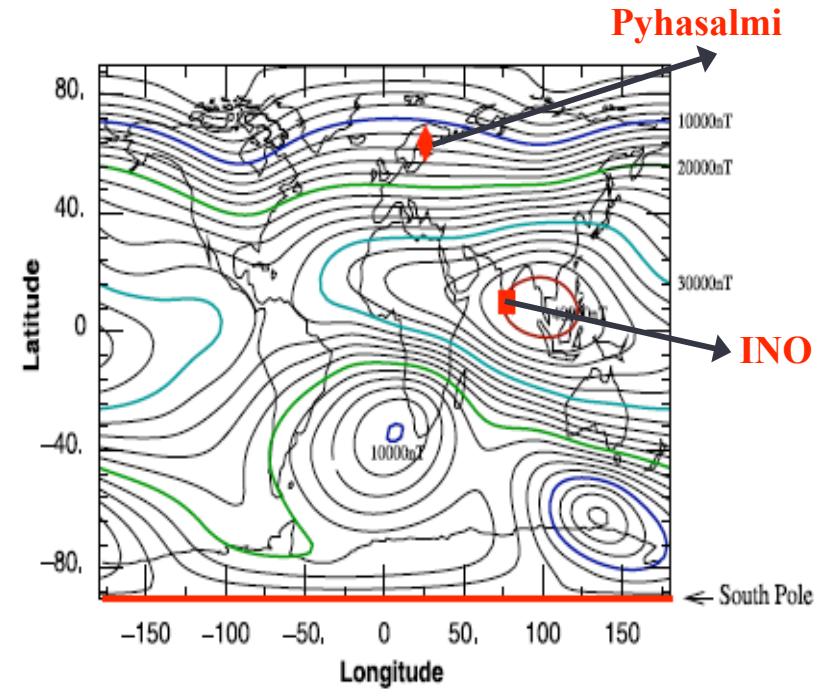
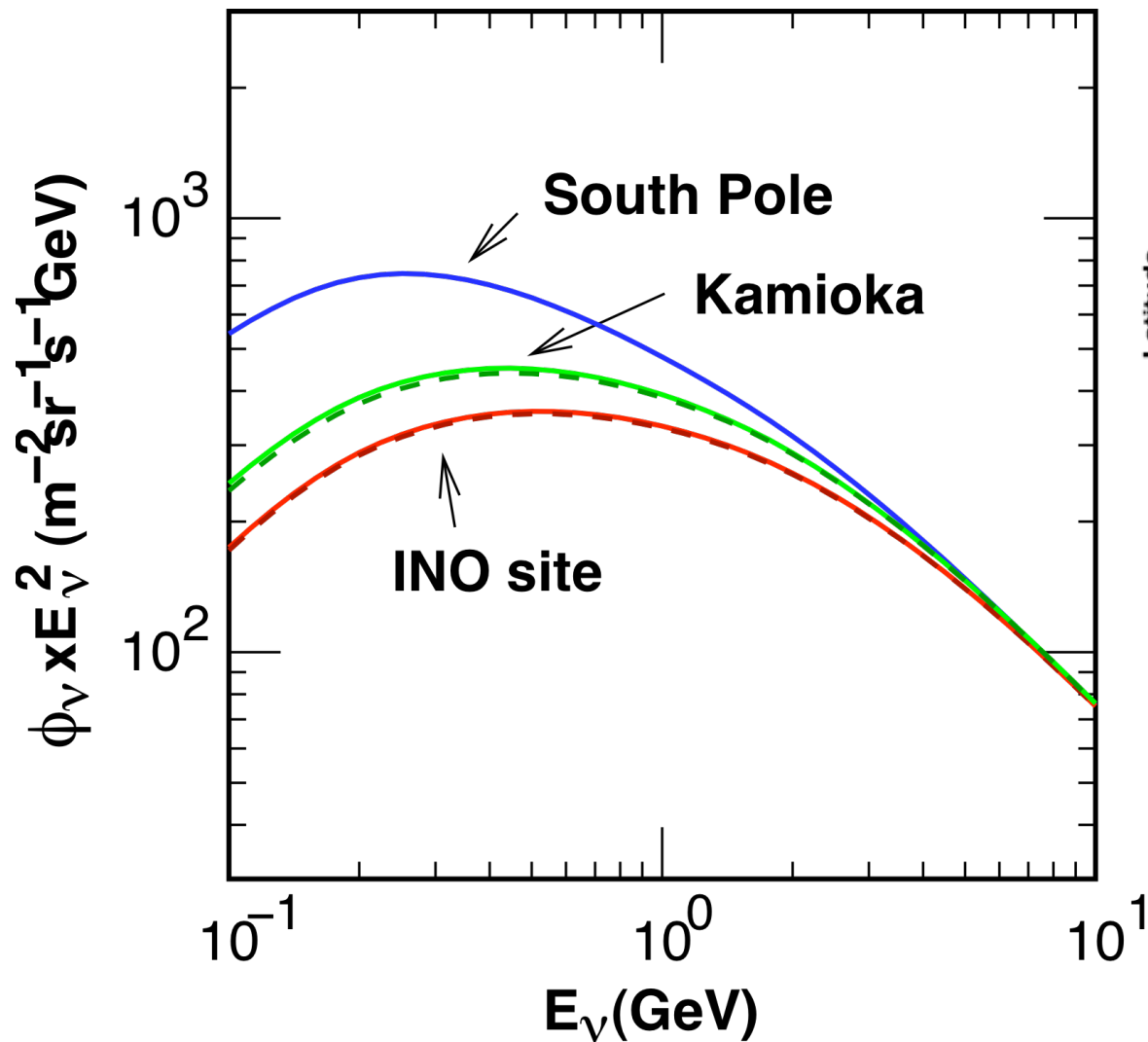
Cervera et al., hep-ph/0002108

Freund et al., hep-ph/0105071

See also, Agarwalla et al., arXiv:1302.6773 [hep-ph]

This channel suffers from: (Hierarchy – δ_{CP}) & (Octant – δ_{CP}) degeneracy! How can we break them?

Atmospheric Neutrino Flux



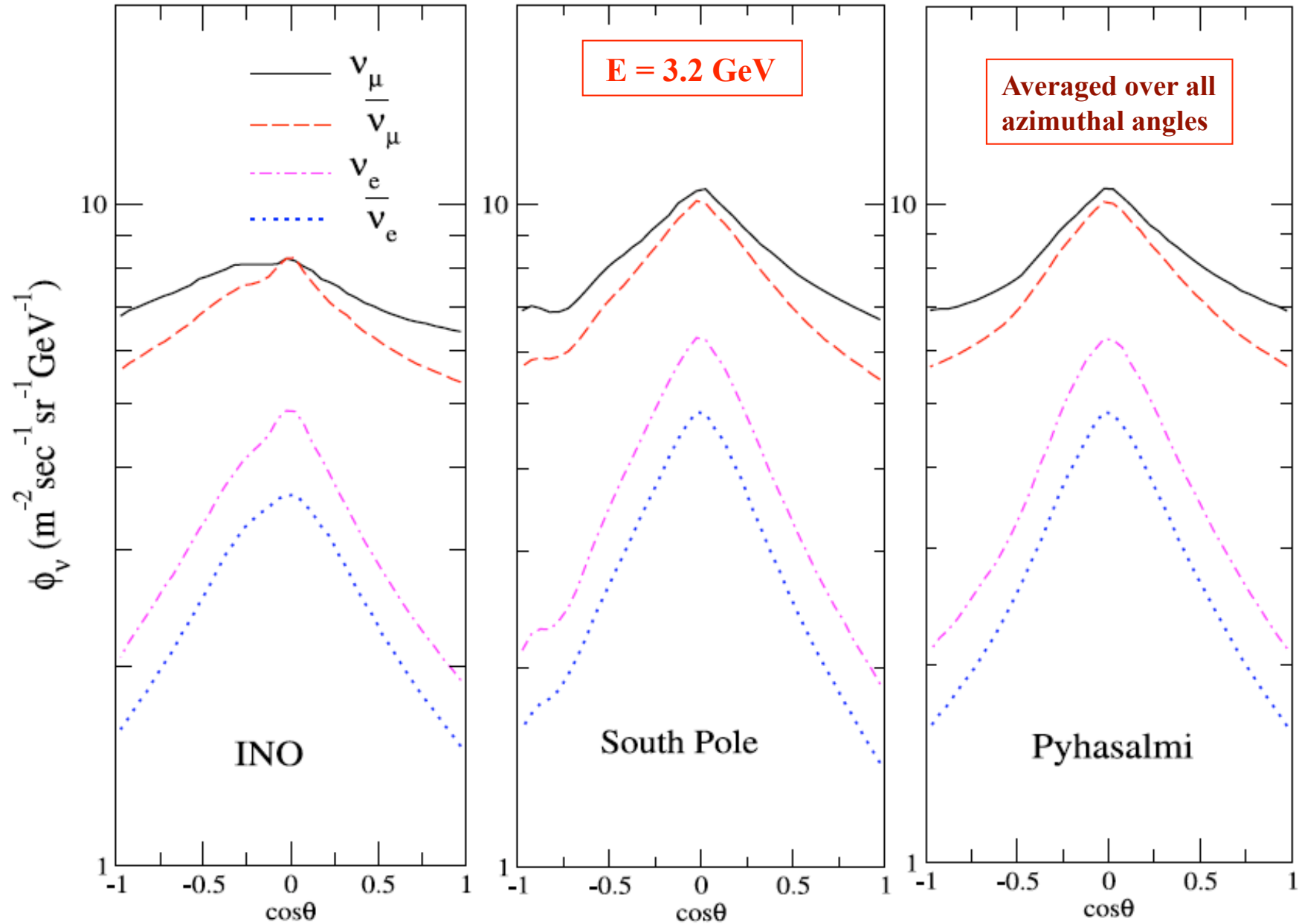
Horizontal component of the geomagnetic field

Magnitude at the Earth's surface ranges from 25 to 65 microtesla

Averaged over all directions
Summed over all flavors of neutrino and anti-neutrino

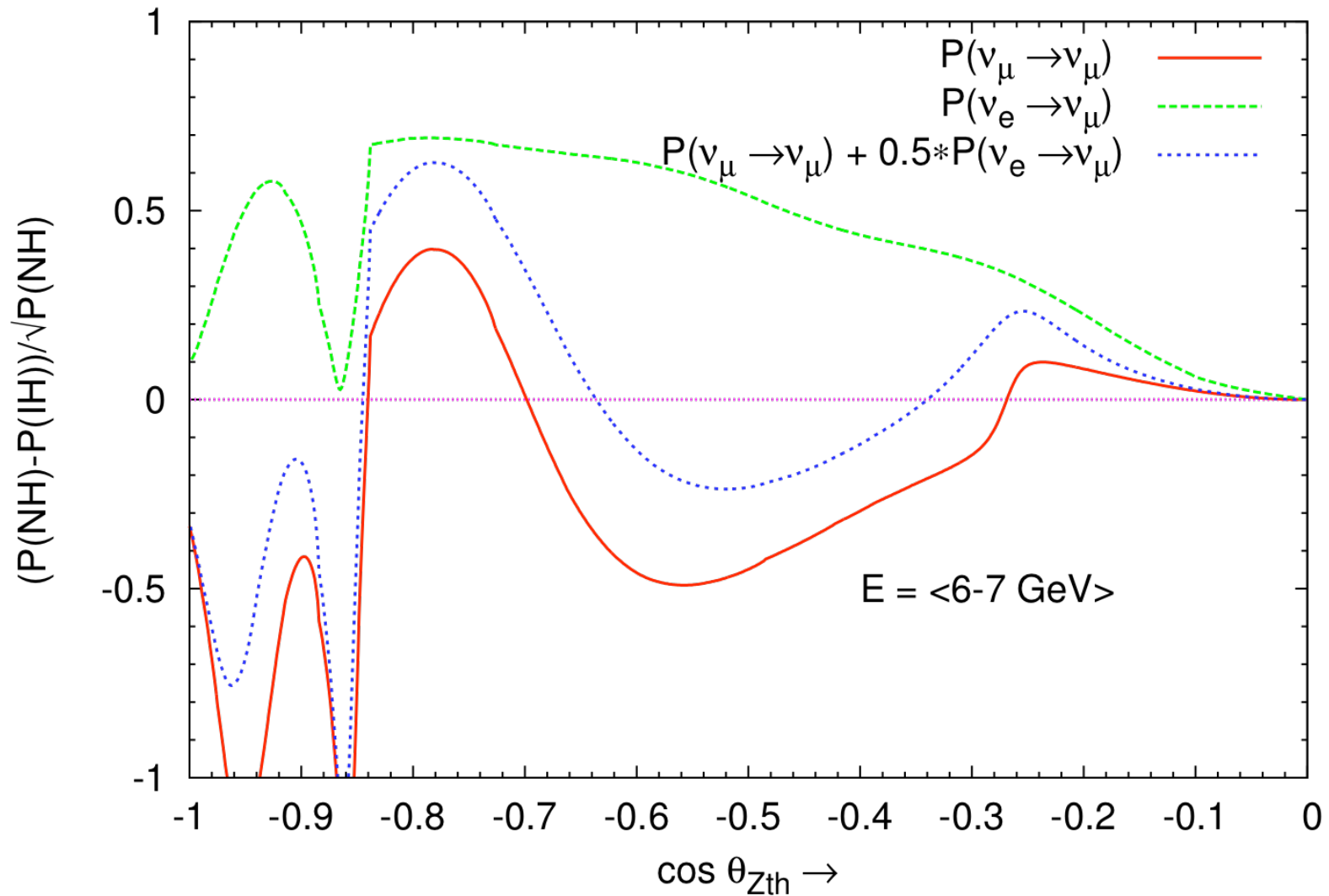
Athar, Honda, Kajita, Kasahara, Midorikawa, arXiv:1210.5154 [hep-ph]

Atmospheric Neutrino Flux



Athar, Honda, Kajita, Kasahara, Midorikawa, arXiv:1210.5154 [hep-ph]

Atmospheric Conspiracy

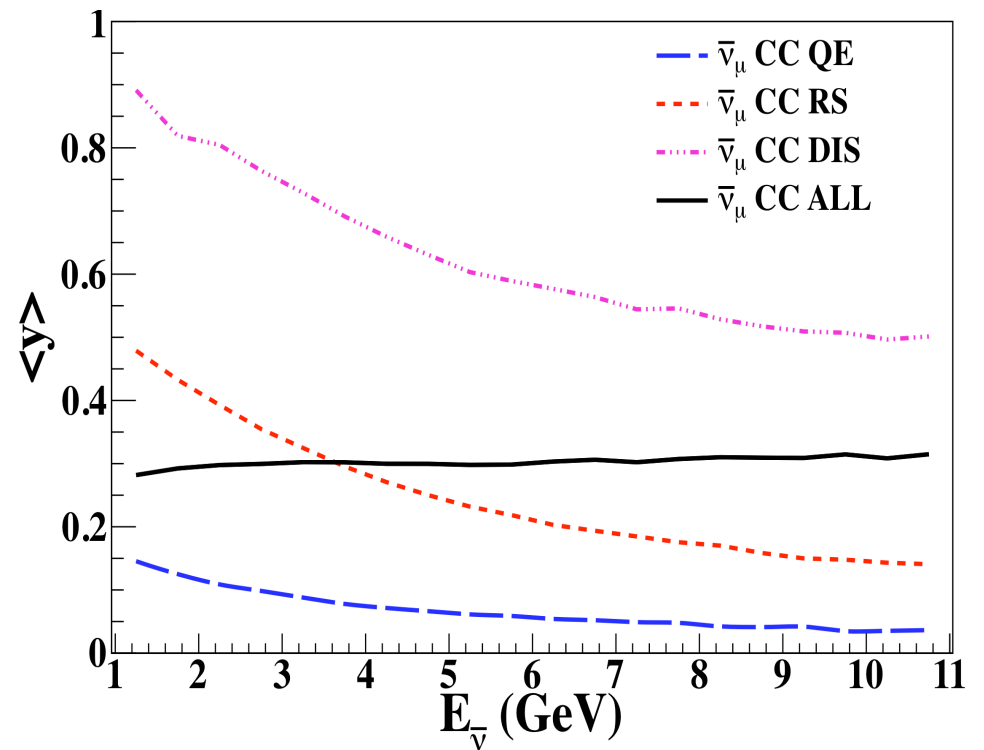
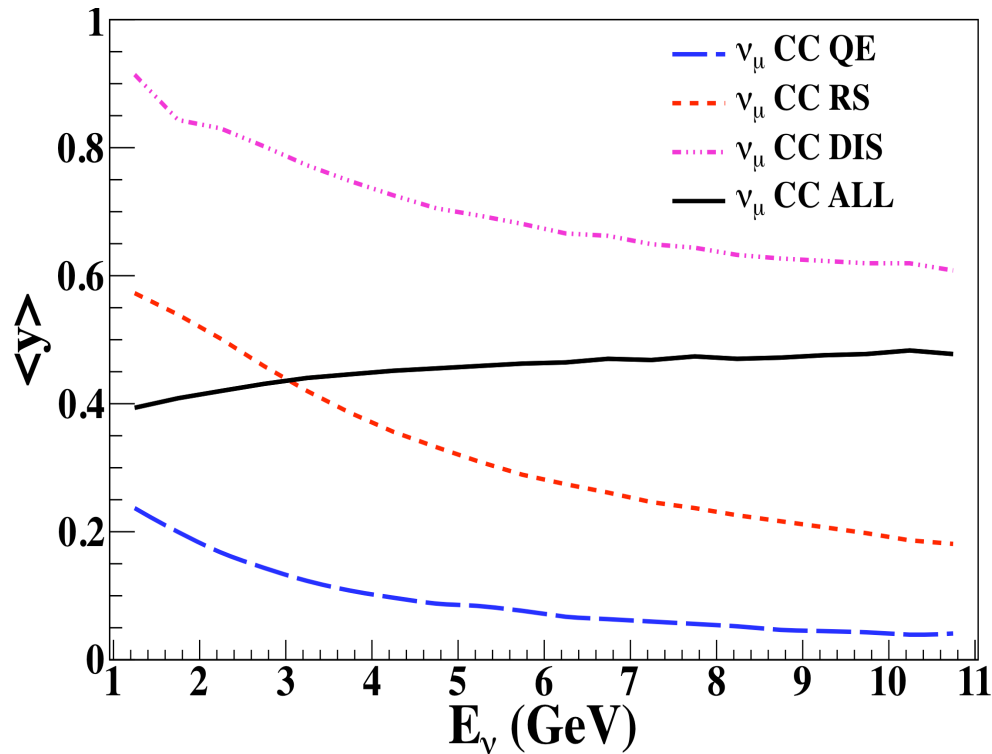


Presence of different flavors dilutes the MH effect in oscillation

Agarwalla, Chatterjee, Khatun, work in progress (INO Collaboration)

Average Inelasticities in Various Channels

$$y \equiv (E_\nu - E_\mu)/E_\nu = E'_{\text{had}}/E_\nu$$

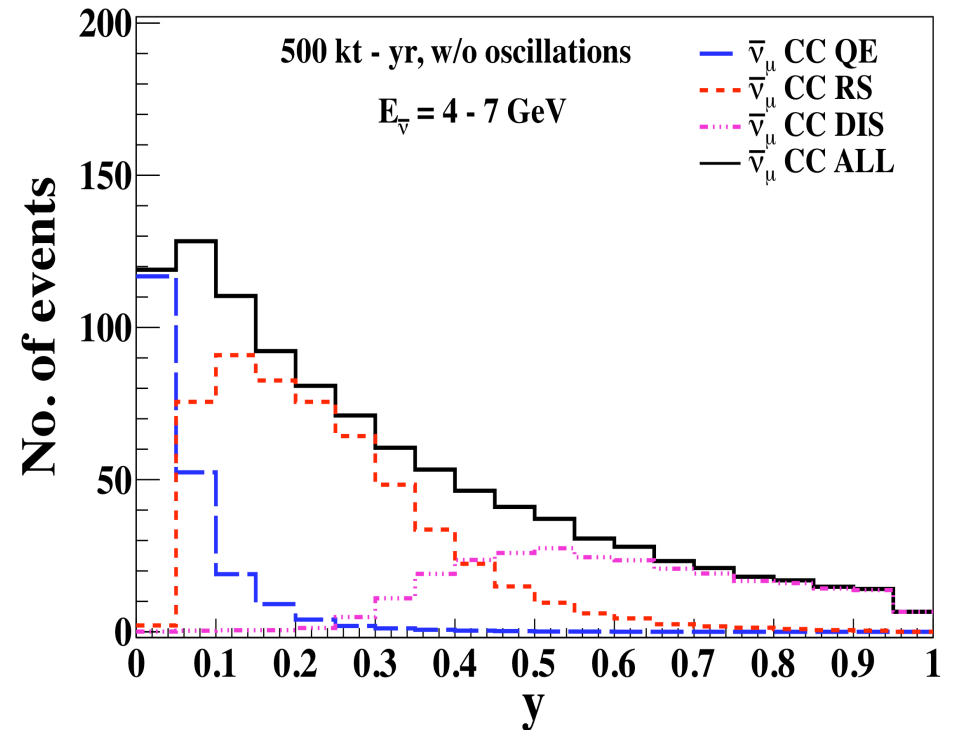
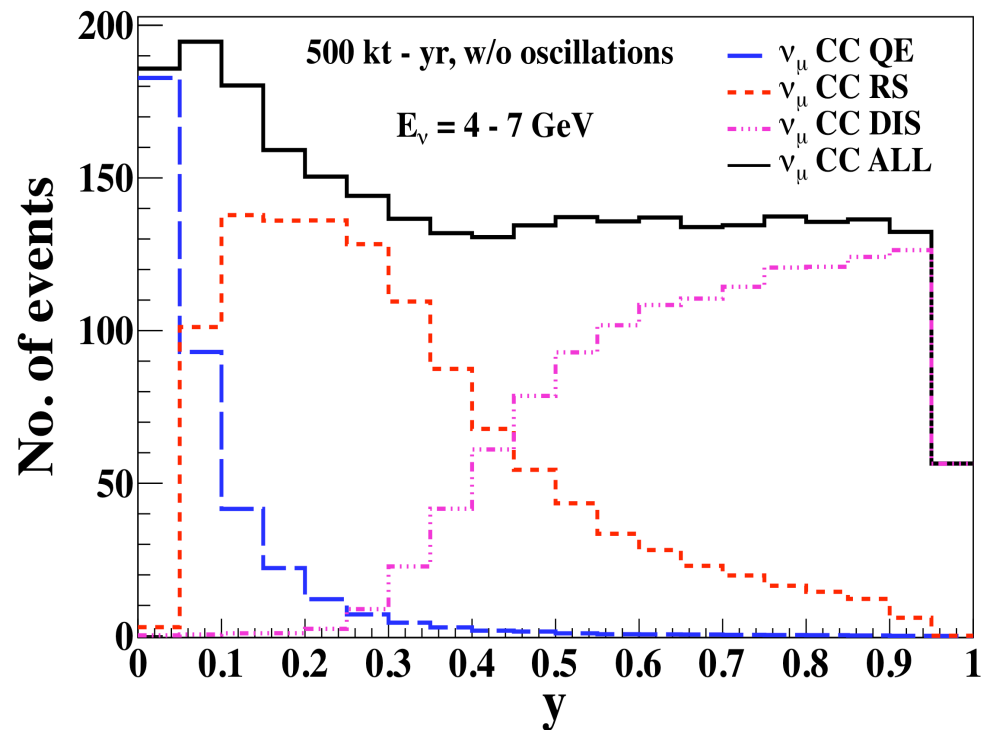


Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

Average Inelasticity in the deep-inelastic events is significant

Crucial for mass hierarchy identification

Distribution of Inelasticities in Events

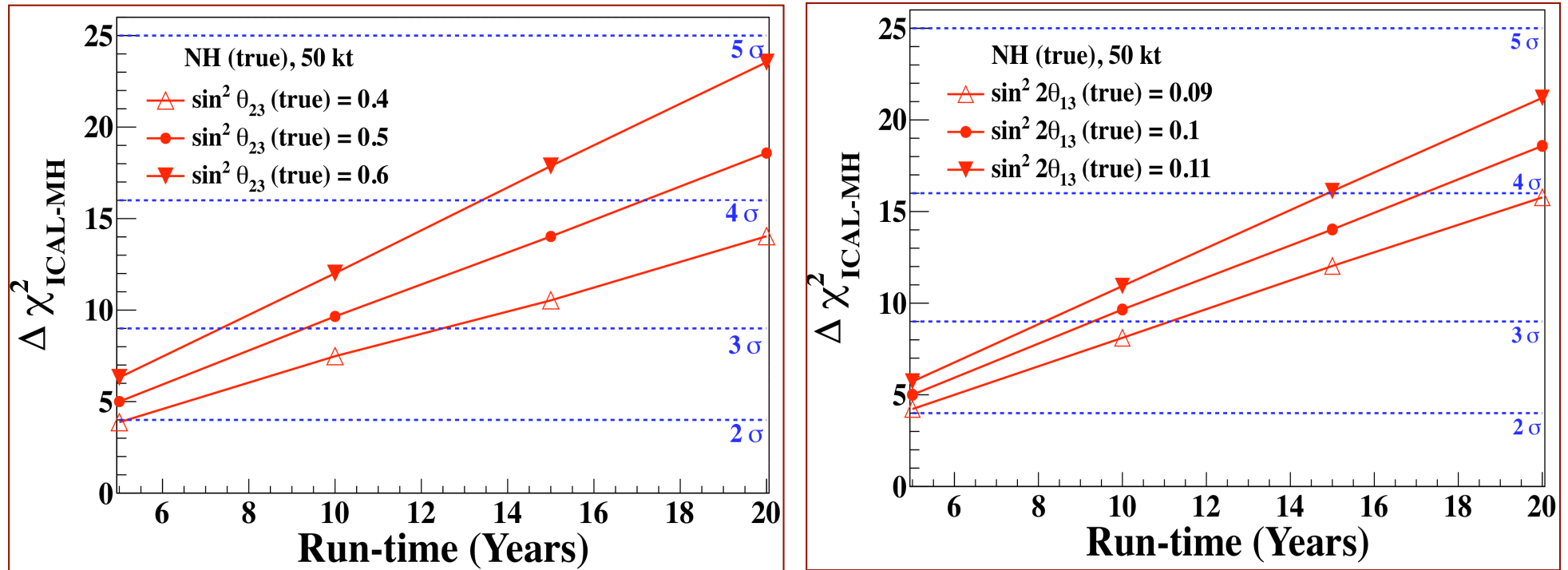


Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

Inelasticities in individual events have a wide distribution

Important to measure inelasticity in individual events

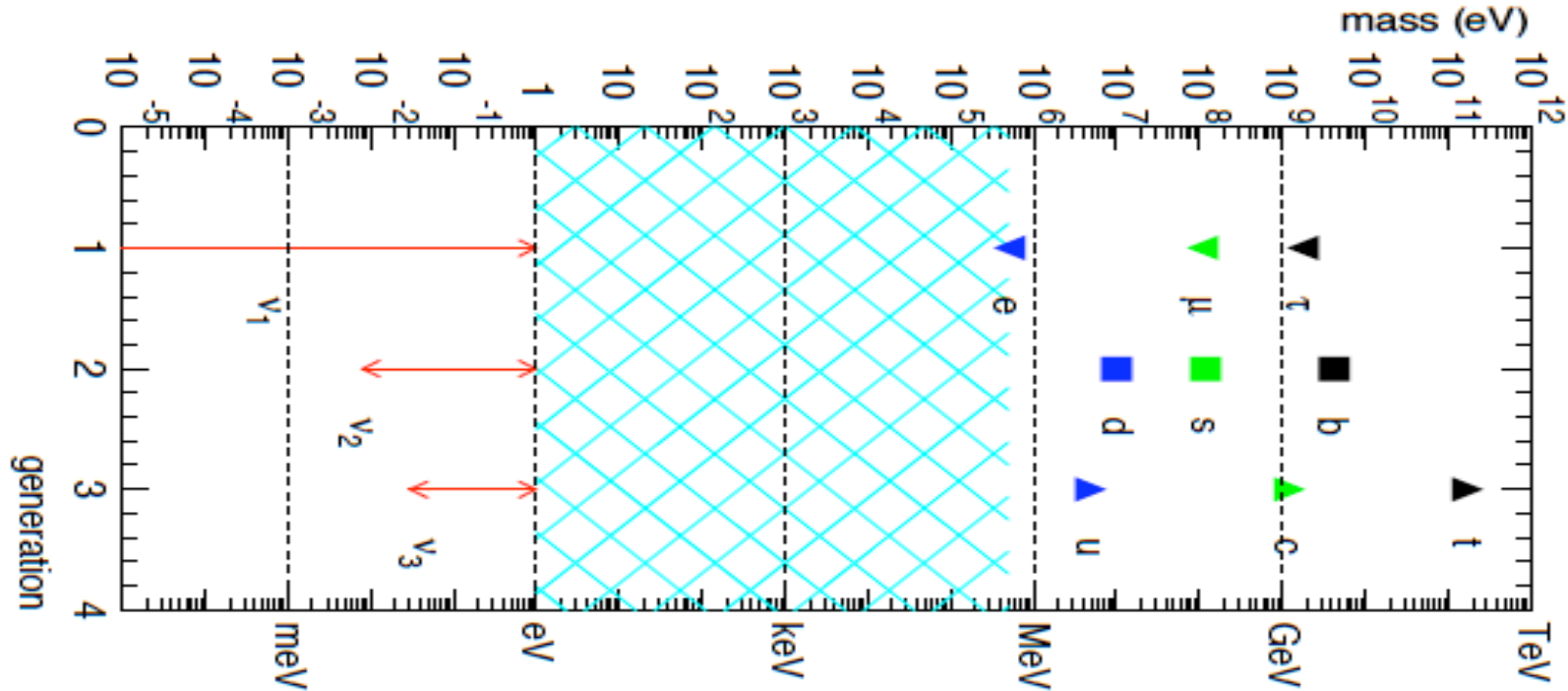
Impact of θ_{23} and θ_{13} on Mass Hierarchy



Devi, Thakore, Agarwalla, Dighe, arXiv:1406.3689 [hep-ph] (INO Collaboration)

50 kt ICAL can rule out the wrong hierarchy with median $\Delta \chi^2 \approx 7$ to 12 depending on the true values of θ_{23} and θ_{13} in 10 years

The Two Fundamental Questions



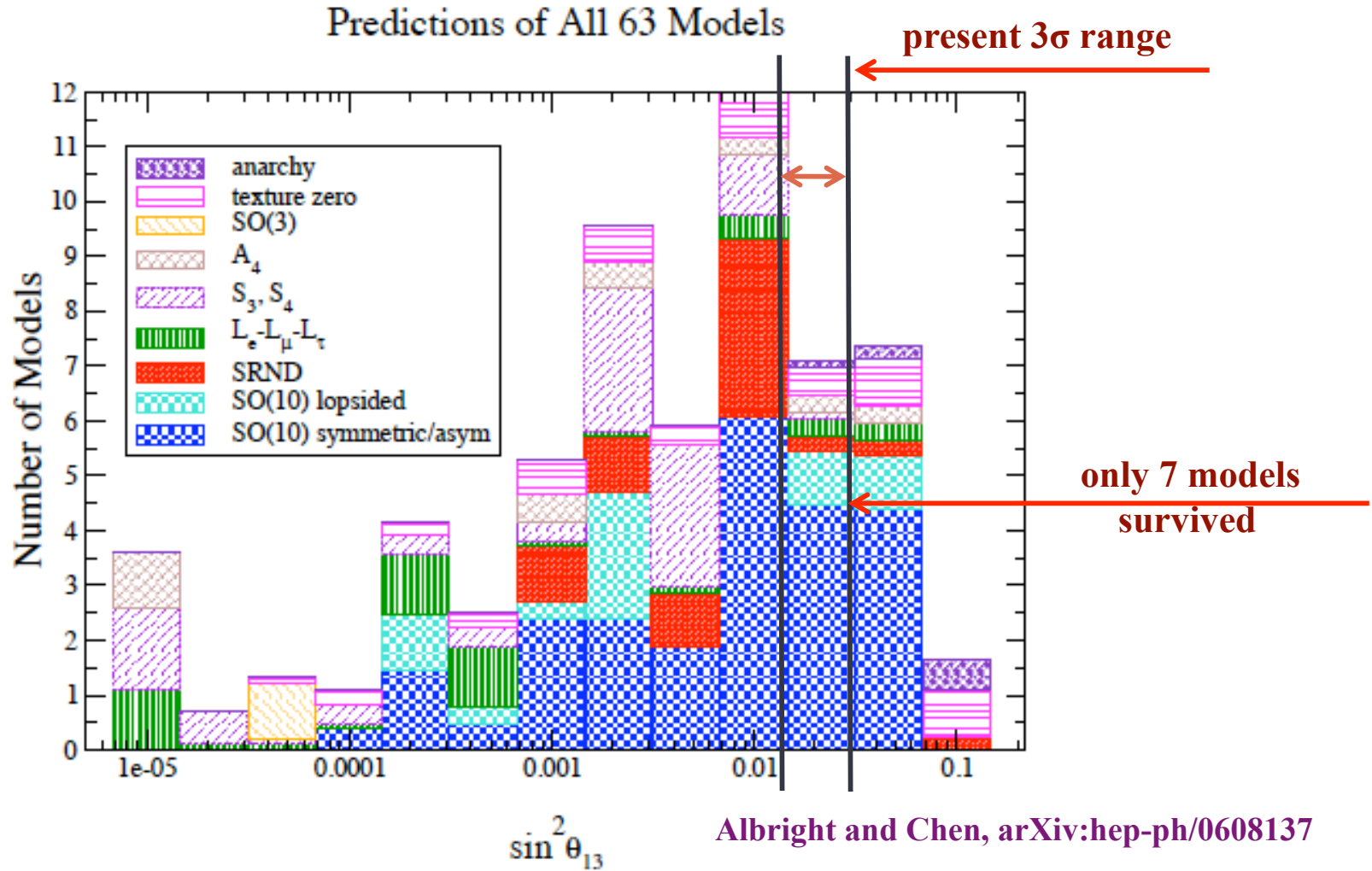
Why are neutrinos so light? The origin of Neutrino Mass!

	Neutrinos (PMNS)	Quarks (CKM)
θ_{12}	35°	13°
θ_{32}	43°	2°
θ_{13}	9°	0.2°
δ	unknown	68°

Why are lepton mixings so different from quark mixings?

The Flavor Puzzle!

Latest Results on θ_{13} : What happened to Mass models?



Survey of 63 ν mass models in June 2006 by Carl H. Albright and Mu-Chun Chen

Future high precision measurements of mixing angles, new information on neutrino mass ordering and CP phase will severely constrain these presently allowed models

Implications of Recent Measurement of θ_{13}

Simplest models that are ruled out!

- **Bimaximal mixing:** [Vissani (97), Barger, Pakvasa, Weiler, Whisnant (98)]

It predicts: $\theta_{12} = 45^\circ$, $\theta_{23} = 45^\circ$, and $\theta_{13} = 0^\circ$

predicted in flavor symmetry models with symmetry groups like A4, S4, A5

- **Tri-bimaximal mixing:** [Vissani (97), Harrison, Perkins, Scot (02)]

$$U_{\text{TBM}} = R_{32} \left(\theta_{32} = \frac{\pi}{4} \right) R_{13}(\theta_{13} = 0) R \left(\theta_{21} = \tan^{-1} \left(\frac{1}{\sqrt{2}} \right) \right) = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

- **Golden ratio:** [Datta, Ling, Ramond (03), Kajiyama, Raidal, Strumia (07)]

It predicts: $\theta_{12} = 31.7^\circ$, $\theta_{23} = 45^\circ$, and $\theta_{13} = 0^\circ$

Simplest models that are still alive!

- **Anarchy (ν mass matrix completely random):** [Hal, Murayama, Weiner (99), de Gouvea, Murayama (03, 12)]

It predicts: large θ_{13} , okay with observed value of θ_{13}

- **Quark-Lepton Complementarity:** [Minakata, Smirnov (94), Raidal (04)]

Based on observation: $\theta_{12} (\text{PMNS}) + \theta_{12} (\text{CKM}) = 45^\circ$

It predicts: $\sin\theta_{13} \approx \sin\theta_C / \sqrt{2} \approx 0.16$ (close to the observed value, other relations needs to be tested!)

The New Minimal Standard Model

- Minimal Extensions to give Mass to the Neutrino:

- * Introduce ν_R AND impose L conservation \Rightarrow Dirac $\nu \neq \nu^c$:

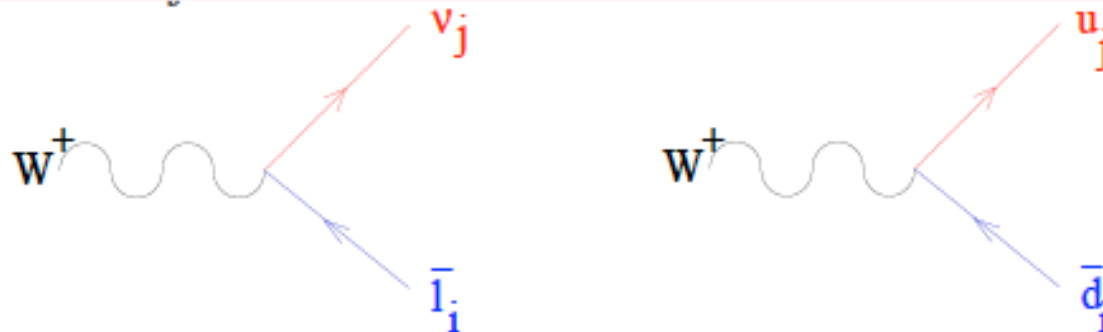
$$\mathcal{L} = \mathcal{L}_{SM} - M_\nu \bar{\nu}_L \nu_R + h.c.$$

- * NOT impose L conservation \Rightarrow Majorana $\nu = \nu^c$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_\nu \bar{\nu}_L \nu_L^C + h.c.$$

- The charged current interactions of leptons are not diagonal (same as quarks)

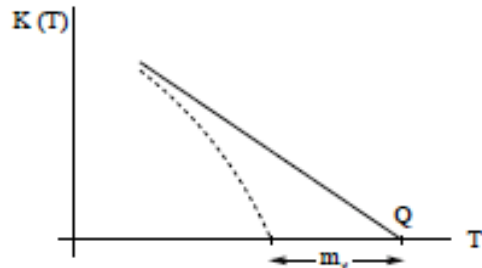
$$\frac{g}{\sqrt{2}} W_\mu^+ \sum_{ij} (U_{LEP}^{ij} \bar{\ell}^i \gamma^\mu L \nu^j + U_{CKM}^{ij} \bar{U}^i \gamma^\mu L D^j) + h.c.$$



Courtesy to Concha Gonzalez-Garcia

Neutrino Mass Scale

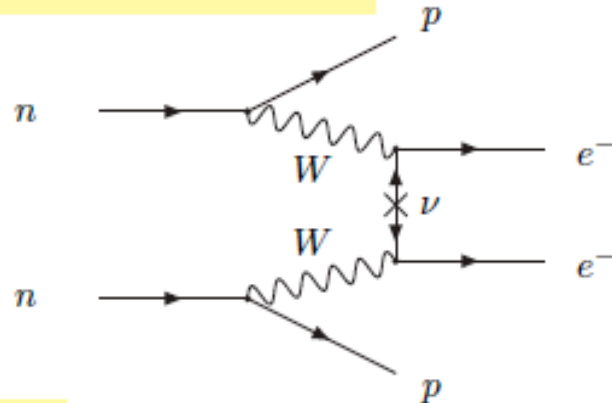
Single β decay : Dirac or Majorana ν mass modify spectrum endpoint



$$m_{\nu_e}^2 = \sum m_j^2 |U_{ej}|^2 = c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2$$

ν -less Double- β decay: \Leftrightarrow Majorana ν 's sensitive to Majorana phases

If m_ν only source of ΔL $(T_{1/2}^{0\nu})^{-1} \propto (m_{ee})^2$



$$m_{ee} = \left| \sum U_{ej}^2 m_j \right|$$

$$= \left| c_{13}^2 c_{12}^2 m_1 e^{i\eta_1} + c_{13}^2 s_{12}^2 m_2 e^{i\eta_2} + s_{13}^2 m_3 e^{-i\delta_{CP}} \right|$$

COSMO Neutrino mass (Dirac or Majorana) modify the growth of structures

$$\sum m_i$$

Backup Slides (Neutrinoless double beta decay)

Experimental Limits

Isotope	$0\nu\beta\beta$ half life	Experiment	$\langle m \rangle$ eV
^{48}Ca	$> 1.4 \cdot 10^{22}$ (90%CL)	ELEGANT-VI	$< 7 - 44$
^{76}Ge	$> 1.9 \cdot 10^{25}$ (90%CL)	Heidelberg-Moscow	< 0.35
^{76}Ge	2230^{+440}_{-310} (90%CL)	Subset of HM coll.	0.32 ± 0.03
^{76}Ge	$> 2.1 \cdot 10^{25}$ (90%CL)	GERDA†	$< 0.2 - 0.4$
^{82}Se	$> 2.1 \cdot 10^{23}$ (90%CL)	NEMO-3	$< 1.2 - 3.2$
^{100}Mo	$> 5.8 \cdot 10^{23}$ (90%CL)	NEMO-3	$< 0.6 - 2.7$
^{116}Cd	$> 1.7 \cdot 10^{23}$ (90%CL)	Solotvino	< 1.7
^{130}Te	$> 2.8 \cdot 10^{24}$ (90%CL)	Cuoricino	$< 0.41 - 0.98$
^{136}Xe	$> 1.9 \cdot 10^{25}$ (90%CL)	KamLAND-Zen††	$< 0.12 - 0.25$
^{136}Xe	$> 1.6 \cdot 10^{25}$ (90%CL)	EXO-200†††	$< 0.14 - 0.38$
^{150}Nd	$> 1.8 \cdot 10^{22}$ (90%CL)	NEMO-3	

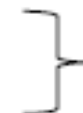
Courtesy to Liang Yang

[F. Avignone, S. Elliot, J. Engel, arXiv:0708: 1033v2 (2007)]

† [GERDA Collaboration, arXiv:1307.4720 (2013)]

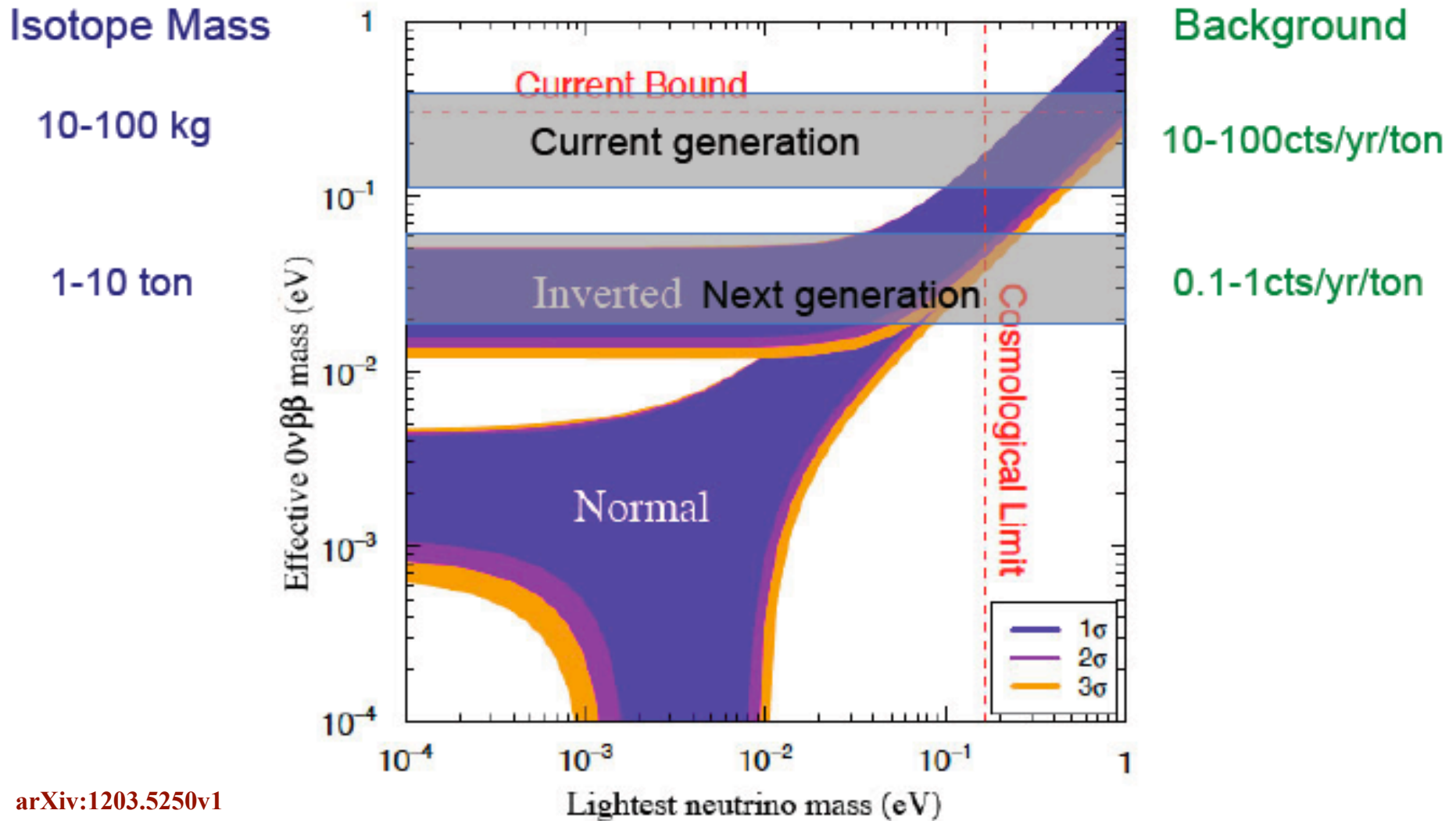
†† [KamLAND-Zen Collaboration, Phys. Rev. Lett. 110, 062502(2013)]

††† [EXO Collaboration, Phys. Rev. Lett.109, 0322505 (2012)]



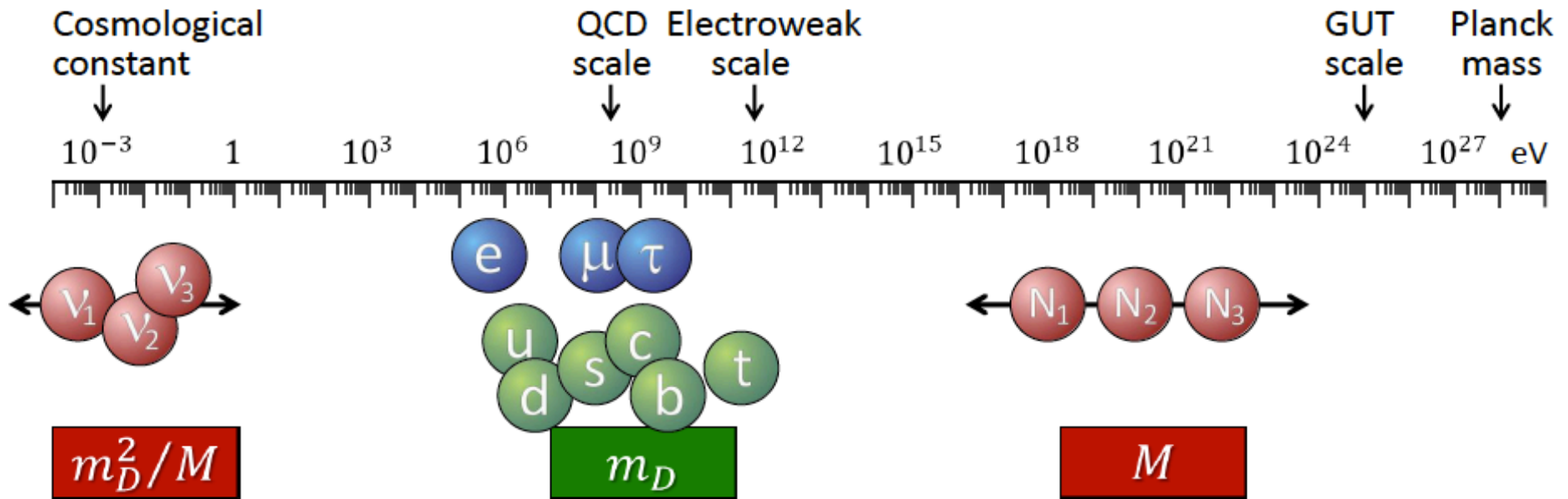
New results within
the last year!

Experimental Sensitivity to Neutrino Mass



Courtesy to Liang Yang

Backup Slides (See-Saw & Neutrino Mass)



Mass matrix for one family of ordinary and heavy r.h. neutrinos

$$(\bar{\nu}_L, \bar{N}_R) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix}$$

Diagonalization

$$(\bar{\nu}_L, \bar{N}_R) \begin{pmatrix} m_D^2/M & 0 \\ 0 & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix}$$

One light and one heavy Majorana neutrino



Courtesy to George Raffelt