Revisiting the Elitzur-Vaidman bomb paradox

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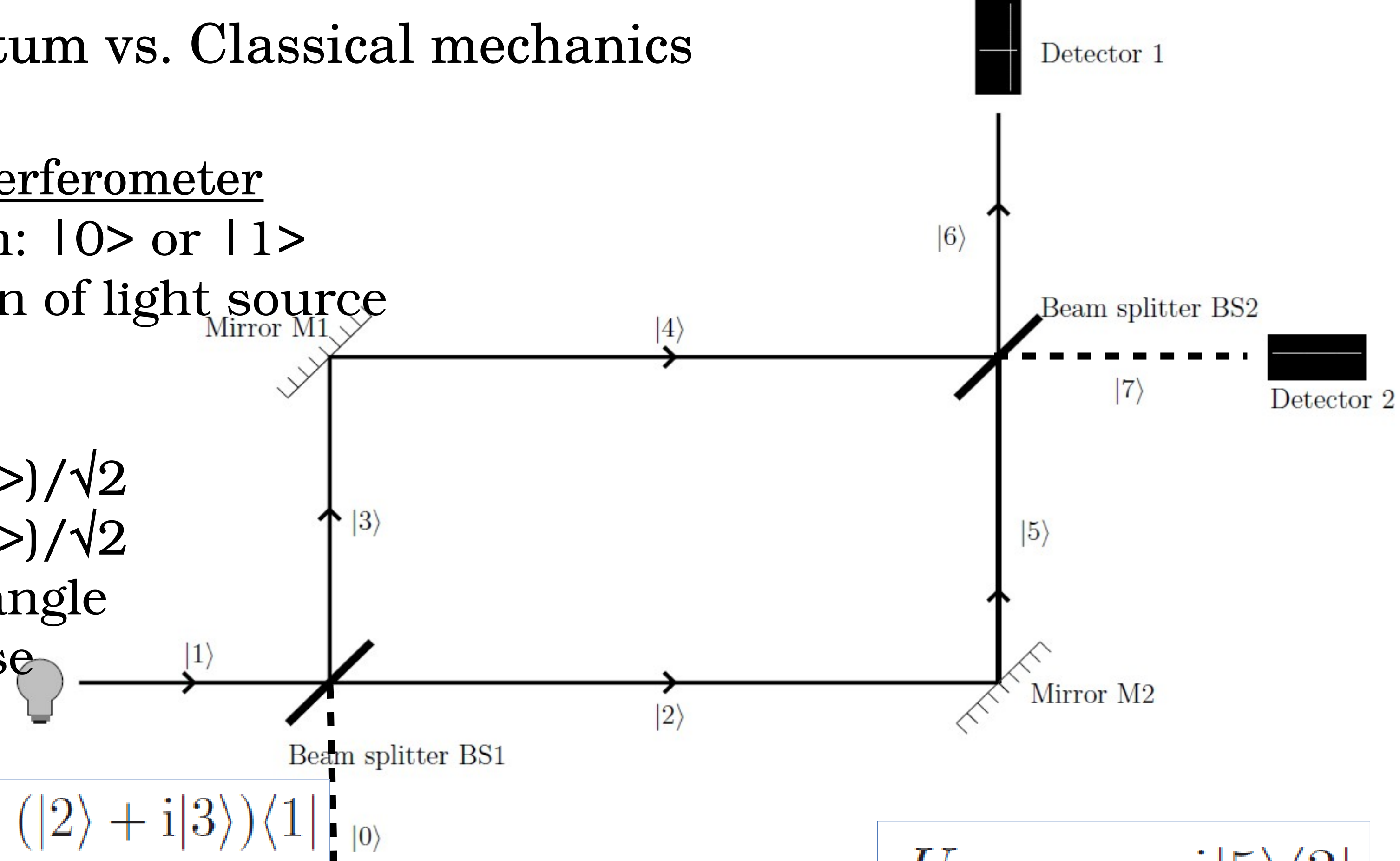
Feb. 18. 2015

Outline

- Mach-Zehnder interferometer
- Elitzur-Vaidman “Bomb” paradox
- Elitzur-Vaidman bomb paradox for electrons
- Elitzur-Vaidman paradox as a probe for Majorana's

The Elitizur-Vaidman bomb paradox problem is a thought experiment applied to photons in a Mach-Zehnder interferometer which brings to the fore neatly the fact that interaction free measurement can take place. In this work we apply this to electrons and analyze the consequences.

Quantum vs. Classical mechanics



The Mach Zehnder interferometer

- Initial state of photon: |0> or |1> depending on position of light source

- Beamsplitter BS1:
  $|0\rangle \longrightarrow (i|2\rangle+|3\rangle)/\sqrt{2}$
  $|1\rangle \longrightarrow (|2\rangle+i|3\rangle)/\sqrt{2}$
  reflection by a right angle generates a p/2 phase

$$U_{\mathrm{BS1}} = (\mathrm{i}|2\rangle + |3\rangle)\langle 0| + (|2\rangle + \mathrm{i}|3\rangle)\langle 1|$$

$$U_{M2} = \mathrm{i}|5\rangle\langle 2|$$
$$U_{M1} = \mathrm{i}|4\rangle\langle 3|$$

- Mirrors reflect by right angles generating another $\pi/2$ phase

- State after mirror reflections:

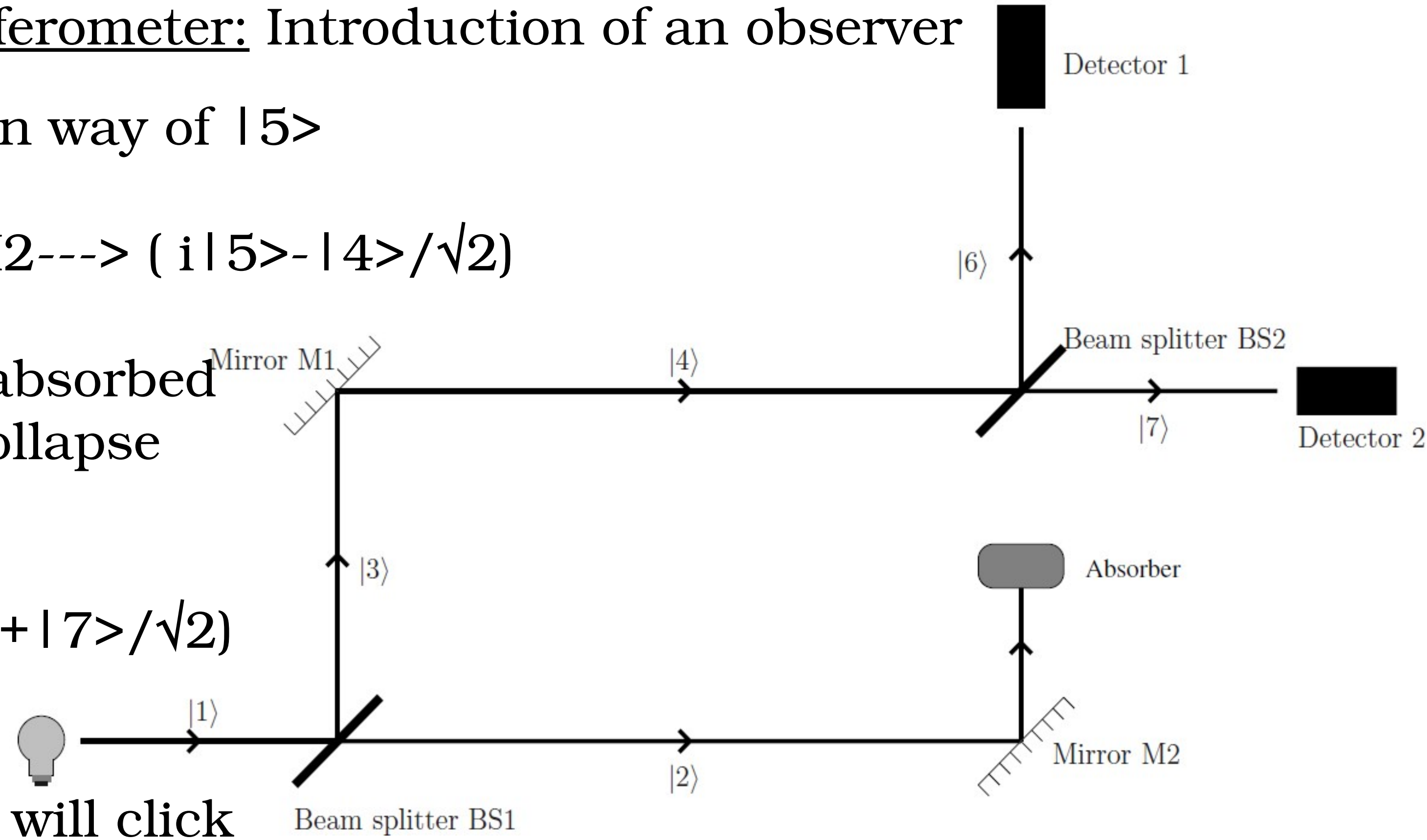
$$|0\rangle \xrightarrow{U_{BS1}} \frac{1}{\sqrt{2}}(i|2\rangle+|3\rangle) \xrightarrow{U_{M1}+U_{M2}} = \frac{(i|4\rangle-|5\rangle)}{\sqrt{2}}$$

$$|1\rangle \xrightarrow{U_{BS1}} \frac{1}{\sqrt{2}}(|2\rangle+i|3\rangle) \xrightarrow{U_{M1}+U_{M2}} = \frac{(-|4\rangle+i|5\rangle)}{\sqrt{2}}$$

- Beamsplitter BS2: $|4\rangle \longrightarrow (|7\rangle+i|6\rangle)$ and $|5\rangle \longrightarrow (|6\rangle+i|7\rangle)$

$$|0\rangle \to U_{BS1} \to U_{M1}+U_{M2} \to U_{BS2} \to -|6\rangle$$
$$|1\rangle \to U_{BS1} \to U_{M1}+U_{M2} \to U_{BS2} \to -|7\rangle$$

The Mach Zehnder interferometer: Introduction of an observer

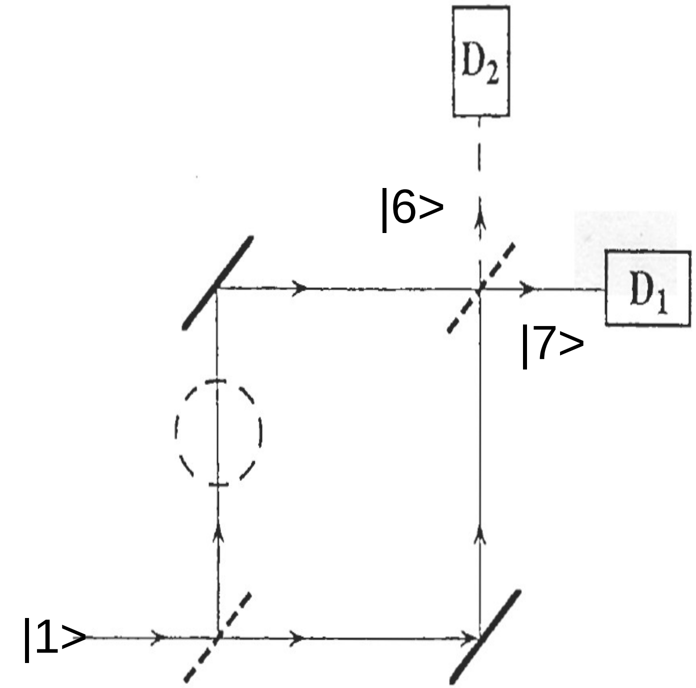


- An observer is placed in way of |5>
- |1> ---> BS1--->M1+M2---> (i|5>-|4>/√2)
- 50% probability to be absorbed or if not absorbed to collapse into |4>.
- |4> --> BS2---> (i|6>+|7>/√2)
- Either detectors 1 or 2 will click with 25 % probability each.
- Moral: Somehow, the possible presence of a photon at |5> (when not absorbed) prevents photon at |4> from reaching detector 1.
- What is the absorber operator?

Mach-Zehnder: Interaction free measurement
(or the “Elitzur-Vaidman” Bomb paradox)

Vistas in Astronomy, Vol. 37, pp. 253–256, 1993
Printed in Great Britain.

0083–6656/93 $24.00



IS IT POSSIBLE TO KNOW ABOUT SOMETHING WITHOUT EVER INTERACTING WITH IT?

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We place a bomb in such a way that its sensor is located in one of the possible routes of the photon inside the interferometer. We send photons one by one through the interferometer until either the bomb explodes or detector D_2 detects the photon. If neither of the above happens, we stop the experiment after a large number of photons have passed the interferometer. In the latter case we can conclude that this given bomb is not good, and we shall try another one. If the bomb is good and exploded, we shall also start all over again with the next bomb. If, however, D_2 clicks, then we achieved what we promised: we know that this bomb is good and we did not explode it.

The probability for such a success is $p = \frac{1}{4}$. By repeating our procedure in cases D_1 has clicked, the probability increases to $p = \frac{1}{3}$. We have showed (Elitzur and Vaidman 1991) that by an appropriate modification one can reach $p = \frac{1}{2}$.

	only D1 (|7>) lights up	only D2 (|6>) lights up	No lights
Bad Bomb	100%	0 %	0%
Good Bomb	25%	25%	50% (EXPLOSION)

- Two main difficulties with electrons:
  1. Electrons cant be absorbed unlike Photons which can
  2. Single electron emitters are hard to design

LETTER 30 OCTOBER 2014 | VOL 514 | NATURE | 603
doi:10.1038/nature13821

Quantum tomography of an electron

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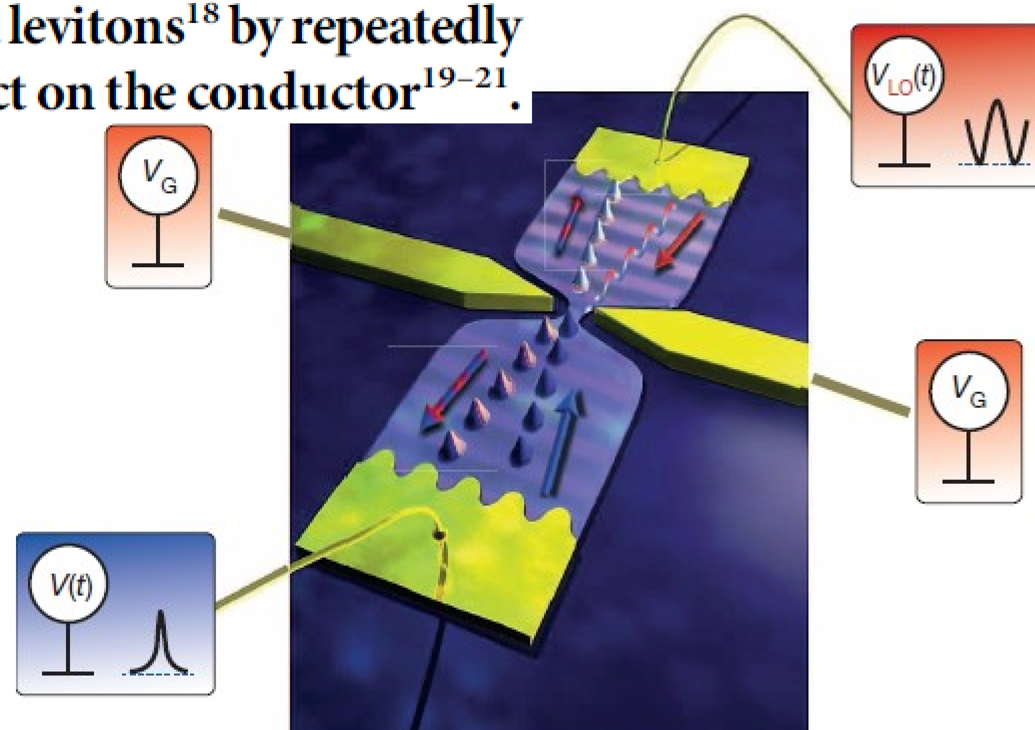
injected into a ballistic conductor. Many identical electrons are prepared in well-controlled quantum states called levitons[18] by repeatedly applying Lorentzian voltage pulses to a contact on the conductor[19–21].



Elitzur-Vaidman bomb paradox for electrons

Many-body manifestation of interaction-free measurement: the Elitzur-Vaidman bomb

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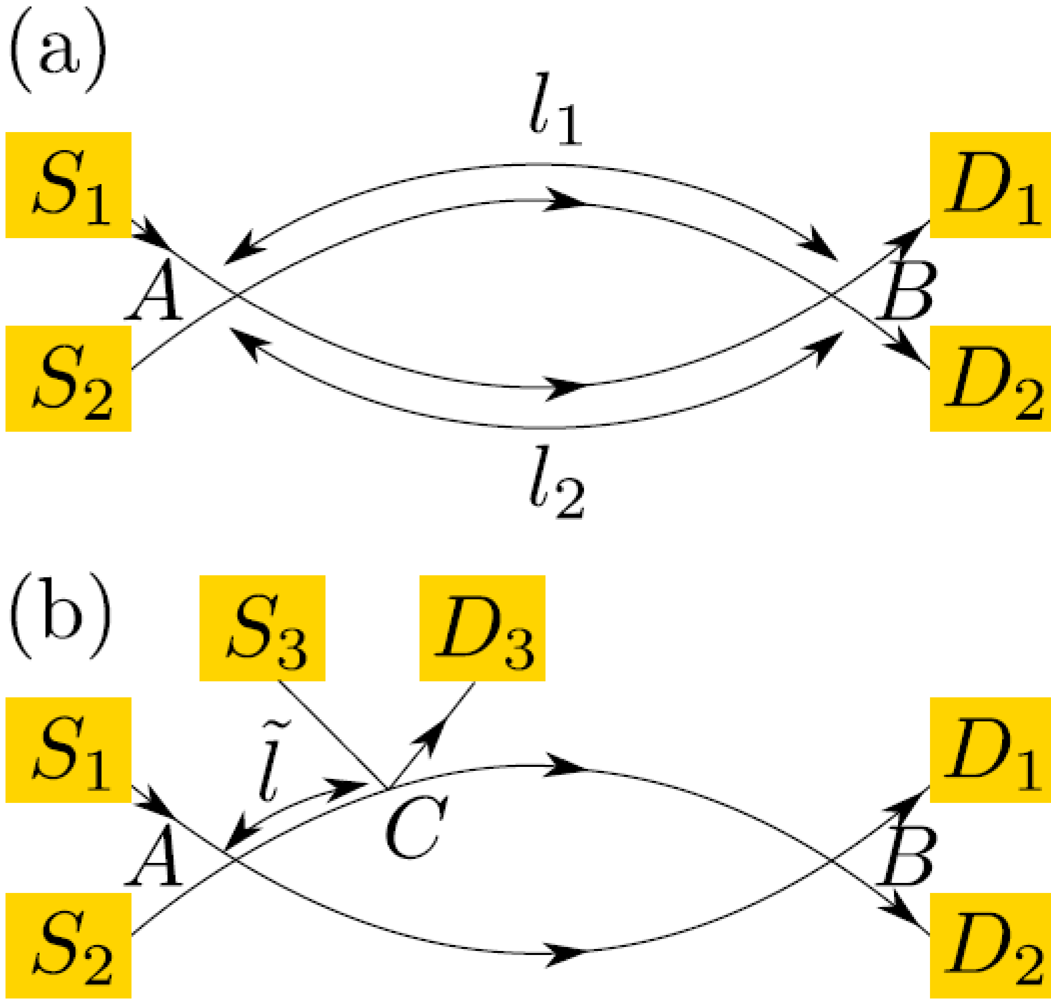
(Dated: December 4, 2015)

We consider an implementation of the Elitzur-Vaidman bomb experiment in a DC-biased electronic Mach-Zehnder interferometer with a leakage port on one of its arms playing the role of a "lousy bom". Many-body correlations tend to screen out manifestations of interaction-free measurement. Analyzing the correlations between the current at the interformeter's drains and at the leakage port, we identify the limit where the originally proposed single-particle effect is recovered. Specifically, we find that in the regime of sufficiently diluted injected electron beam and short measurement times, effects of quantum mechanical wave-particle duality emerge in the cross-current correlations.

arXiv:1512.01086v1



$$|i\rangle = r_A\,|1\rangle + t_A\,|2\rangle$$

$$\mathbf{b}_k = \mathcal{S}_A \mathbf{a}_k,\ \mathbf{c}_k = \mathcal{S}_C \mathbf{b}_k,\ \mathbf{d}_k = \mathcal{S}_B \mathbf{c}_k,$$

with

$$\psi_{km}(x) = e^{ikx} \begin{cases} a_{km}, & x < 0; \\ b_{km}, & 0 < x < \tilde{l}; \\ c_{km}, & \tilde{l} < x < l_2; \\ d_{km}, & l_m < x. \end{cases}$$

$$\mathcal{S}_i = \begin{pmatrix} r_i & t_i & 0 \\ -t_i^* & r_i & 0 \\ 0 & 0 & 1 \end{pmatrix}\ ;\ i = A, B,$$

$$\mathcal{S}_C = \begin{pmatrix} r_C & 0 & -t_C \\ 0 & 1 & 0 \\ t_C & 0 & r_C^* \end{pmatrix}.$$

<u>Without 'absorber':</u>

$$P_0(i \rightarrow D_1) = |\langle D_1| |i\rangle|^2$$

$$|i\rangle = r_A |1\rangle + t_A |2\rangle$$

$$|D_1\rangle = r_B |1\rangle + t_B e^{i\phi} |2\rangle$$

$$P_0(i \rightarrow D_1) = |r_A|^2|r_B|^2 + |t_A|^2|t_B|^2 +$$

$$2|r_A r_B t_A t_B| \cos(\phi + \phi_T), \text{ where } \phi_T = \arg(r_A r_B^\star t_A t_B^\star).$$

<u>With 'absorber':</u>

$P(i \rightarrow D_3) = |r_A|^2|t_C|^2$. 'Bomb goes off'

Upon detection of the injected electron in D3 , we declare the interference experiment void. In such a "partial collapse" the state |1> is projected out of the space spanned by |1> and |2>.

If bomb does not go off:

If such a projection-out does not take place (i.e. the electron is not detected in D3), the original qubit state is rotated by the measurement's back-action into

$$|i_C\rangle = (1/\tilde{\mathcal{N}}) \left(r_A r_C |1\rangle + t_A |2\rangle \right)$$
$$\tilde{\mathcal{N}} = \sqrt{1 - P(i \to D_3)}$$

Consequently, the probability for the particle to subsequently arrive in drain D1 is

$P(i_C \to D_1) P(\overline{i \to D_3})$, where by overline we denote the complementary event, i.e. $P(\overline{i \to D_3}) = 1 - P(i \to D_3)$.

Note that $P(i_C \to D_1)$ can be written using the conditional probability $P(i \to D_1 | \overline{i \to D_3})$

As a result we obtain that the particle would reach drain D1 with the joint probability

$$P(i \to D_1) = P(i \to D_1, \overline{i \to D_3}) = |r_A|^2|r_B|^2|r_C|^2 + |t_A|^2|t_B|^2 + 2|r_C||r_A r_B t_A t_B| \cos(\phi + \phi_T + \phi_C)\,,$$

where $\phi_C = \arg(r_C)$. Note that due to causality $P(i \to D_1) = P(i \to D_1, \overline{i \to D_3})$ and similarly

$$P(i \to D_1, i \to D_3) = 0\,.$$

The fact that $P(i \to D_1) \neq P_0(i \to D_1)$ can be used to detect the presence of the leakage port. Specifically, if the MZI is tuned to have $P_0(i \to D_1) = 0$, the detection of a particle at D_1 in any single realization of the experiment indicates the presence of the leakage port without the particle having leaked out. If the particle is not detected at D_1, no conclusion on the presence of a leakage channel can be drawn. This is a manifestation of the EV-bomb detection scheme.

Majorana bound states

- Topological states- resistant to local perturbations-errors,decoherence
- One possibility: MBS found in superconducting states induced in Topological insulators
- Theoretically predicted, experimentally not unambiguously detected

C Benjamin & J K Pachos, PRB 81,085101 (2010)

Majorana Fermions - Particles and Antiparticles

- particles which are their own antiparticles(all neutral)
  - neutral pions (spin 0) Klein-Gordon equation
  - photons (spin 1) Maxwell equations (EM)
  - gravitons (spin 2) Einstein Equations (GR)

  } formulated using real numbers

particle created by operator / field: jj

particle = antiparticle:jɉ* jɉ* (real operator / field)
[neutron (spin ½) not it's own antiparticle (but neutral)]
[electrons, protons (spin ½) have distinct antiparticles]

- Dirac equation: complex numbers, complex fields, distinct antiparticles

- Majorana (Nuovo Cimento **5**, 171-184, 1937)
  -clever modification of Dirac eqn. using ONLY REAL numbers
  - spin ½ particles which are their own antiparticles
  -consistent with principles of relativity and quantum theory

Majorana Fermions in condensed matter

Excitons: bound electron – hole pair created by $c_j^\dagger c_k + c_k c_j^\dagger$
invariant under charge conjugation $c_j \leftrightarrow c_j^\dagger$
i.e. excitons are their own antiparticles

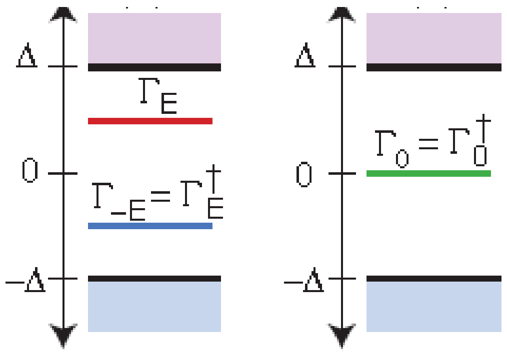
BUT: excitons are always bosons (integer spin, photon absorption)
so not Majoranas

In Superconductors:
How can one build Majorana Fermions from Electrons in solids?
(electrons are charged, antiparticles are holes)

superconductor: Cooper pairs, bosons, condensate

existence zero (energy) modes: equal mixtures of particles and holes, spin ½ $\gamma_j = c_j^\dagger + c_j$ invariant under $c \leftrightarrow c^\dagger$

Majorana fermions and TQC



Why Zero energy?

Finite energy pairs are not topologically protected, could be moved out of energy gap.

A single unpaired bound state at E=0 is protected as it cant move away.

A MF is half a fermion and thus a single fermion is associated with a pair.

MBS always come in pairs and a well separated pair defines a degenerate 2 level system (presence/absence of fermions), whose quantum state is stored non-locally.

The state cannot be measured by a local measurement on one bound state. ⟶ TQC

Condensed Matter: Majorana Candidates

not possible in ordinary superconductors, predicted in

-(px + ipy) wave superconductors, angular momentum 1

- **fractional quantum Hall effect**, =5/2 (Pfaffian / Moore-Read state)

- other **exotic superconductors**: strontium ruthenate
s-wave Cooper pairing if electrons in normal state obey Dirac-like equation

-**topological insulator surface** with proximity effect to regular superconductor or unconventional superconductor AND at ferromagnet-superconductor interfaces

- **semiconductor SOC superconductor**

TI- Unconventional Superconductor interface

$\Delta(\mathbf{k}) i \underline{\sigma}_y$

Hamiltonian for TI surface with dxy superconducting correlations

$$\hat{H} = \begin{pmatrix} \underline{H}_0(\mathbf{k}) & \underline{\Delta}(\mathbf{k}) \\ -\underline{\Delta}^*(-\mathbf{k}) & -\underline{H}_0{}^*(-\mathbf{k}) \end{pmatrix}$$

Nambu basis $\Psi = (\psi_\uparrow, \psi_\downarrow, \psi_\uparrow^\dagger, \psi_\downarrow^\dagger)$ $\quad \underline{H}_0(\mathbf{k}) = \upsilon_F(\underline{\sigma}_x k_x + \underline{\sigma}_y k_y) - \mu$

Zero energy bound state $\quad \varepsilon = \eta\sqrt{(\upsilon_F|\mathbf{k}| - \beta\mu)^2 + |\Delta(\mathbf{k})|^2}$

Particle-hole symmetry $\quad \Theta\hat{H}(\mathbf{k})\Theta = -\hat{H}^*(-\mathbf{k})$ $\quad \Theta = \begin{pmatrix} \underline{0} & \underline{1} \\ \underline{1} & \underline{0} \end{pmatrix}$

If $\psi_\varepsilon = [u_1(\mathbf{k}), u_2(\mathbf{k}), \upsilon_1(\mathbf{k}), \upsilon_2(\mathbf{k})]$ is an e.f. with e.v ε then

$$\Theta\psi_\varepsilon(-\mathbf{k})^* = \psi_{-\varepsilon}(\mathbf{k}) = [\upsilon_1^*(-\mathbf{k}), \upsilon_2^*(-\mathbf{k}), u_1^*(-\mathbf{k}), u_2^*(-\mathbf{k})]$$

is an e.f. with e.v.- ε

For $\varepsilon=0$, $\psi_\varepsilon = \psi_{-\varepsilon} \Longrightarrow u_1(\mathbf{k}) = \upsilon_1^*(-\mathbf{k})$

$$\gamma^\dagger(\mathbf{k}) = u_1(\mathbf{k})c_\uparrow^\dagger(\mathbf{k}) + u_2(\mathbf{k})c_\downarrow^\dagger(\mathbf{k}) + \upsilon_1(\mathbf{k})c_\uparrow(-\mathbf{k}) + \upsilon_2(\mathbf{k})c_\downarrow(-\mathbf{k})$$

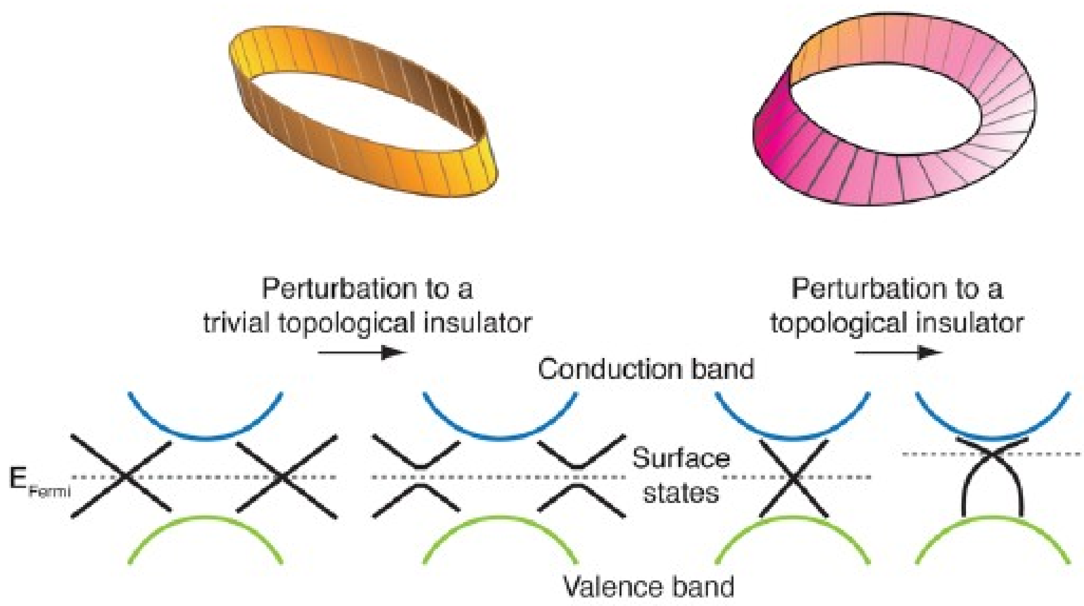
Majorana criterion $\gamma(\mathbf{k}) = \gamma^\dagger(-\mathbf{k})$

Why not in cuprates?

Now, the distinction between the zero-energy state in the cuprates and the present context of a TI is precisely the spin degeneracy which allows one to split up the 4×4 BdG equations to two separate 2×2 equations per spin. Because of the band structure on the surface of a TI, the $\varepsilon = 0$ solution is not spin-degenerate and we obtain only one zero-energy mode. As pointed out in Ref. [20], this guarantees the Majorana nature of the fermion. We re-emphasize that this is different from topologically trivial $\mathrm{N} \mid d_{xy}$-wave junctions, where the zero-energy solutions are spin degenerate, i.e., “double Majorana” modes.

J. Linder, et.al, PRL 104, 067001

TOPOLOGICAL INSULATOR



Theory

Dirac eqn. for Topological insulator:

$$[vp\tau_z\sigma_z + (eV - E_F + eA/\hbar c)\tau_z]\Psi = E\Psi$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

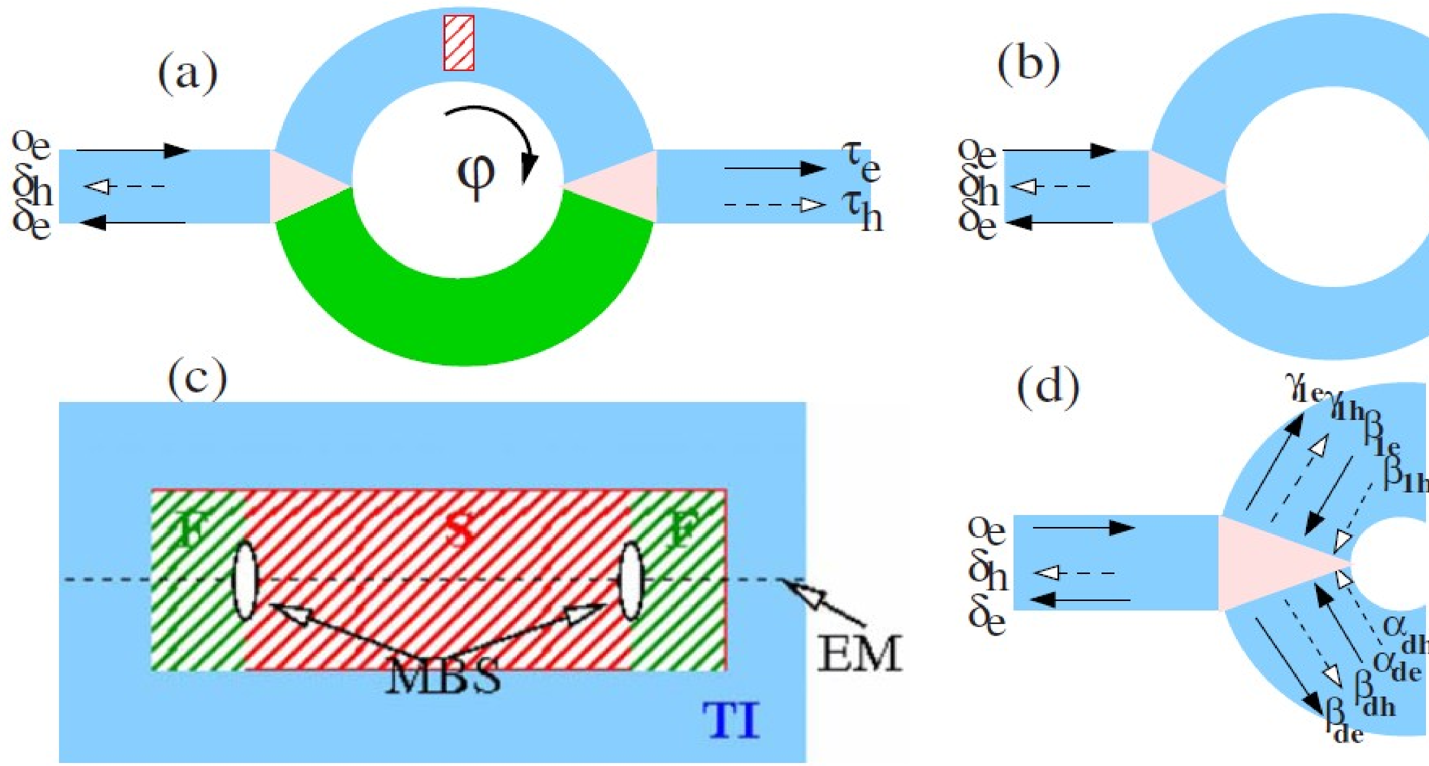
Majorana's idea: particles which are their own anti-particles

$$(i\tilde{\gamma}^\mu\partial_\mu - m)\tilde{\psi} = 0)$$

Hamiltonian for coupled Majorana bound states:

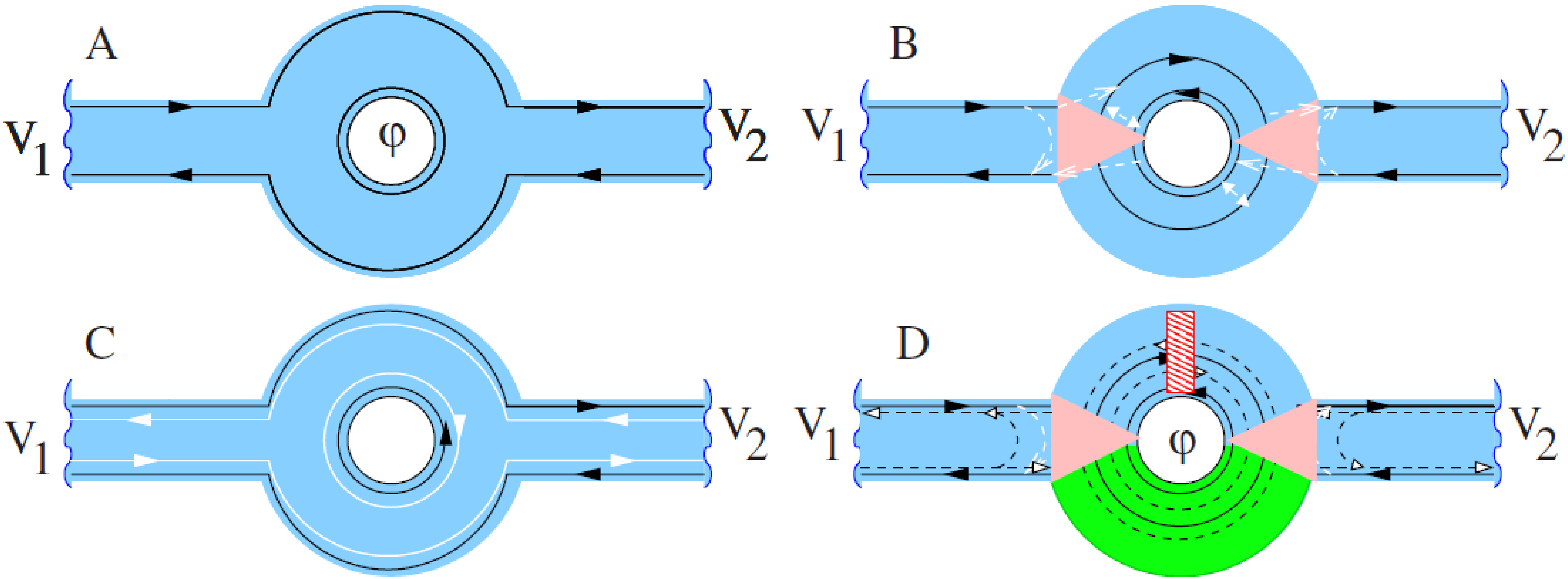
$$H_M = -\sigma_y E_M$$

Model



- Analogy with Elitzur-Vaidman:
  1. Bomb goes 'off'-
     Majorana present and electron–hole non-local scattering.
  2. Bomb does not go 'off'-
     (a) Majorana absent and electron-hole local scattering
     (b) Majorana present and absence of any electron-hole scattering

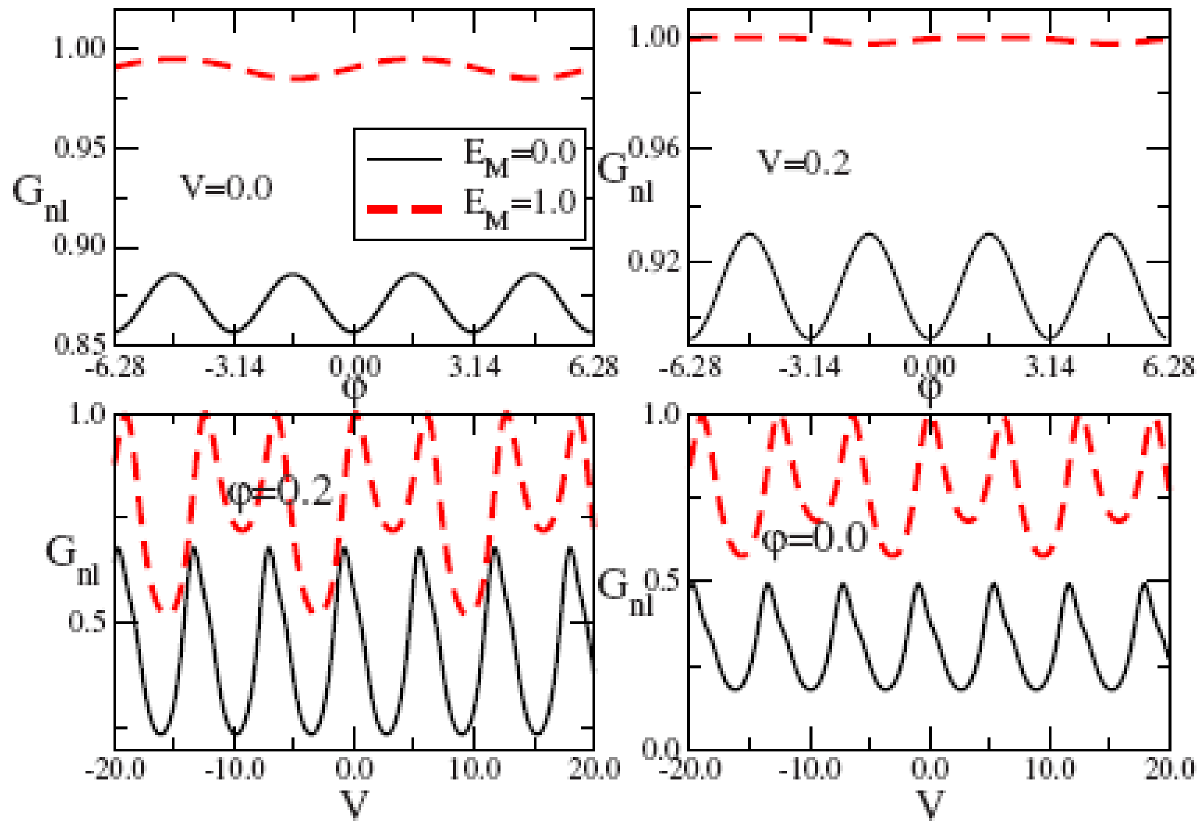
Edge modes



1. Magnetic/electric field asymmetry

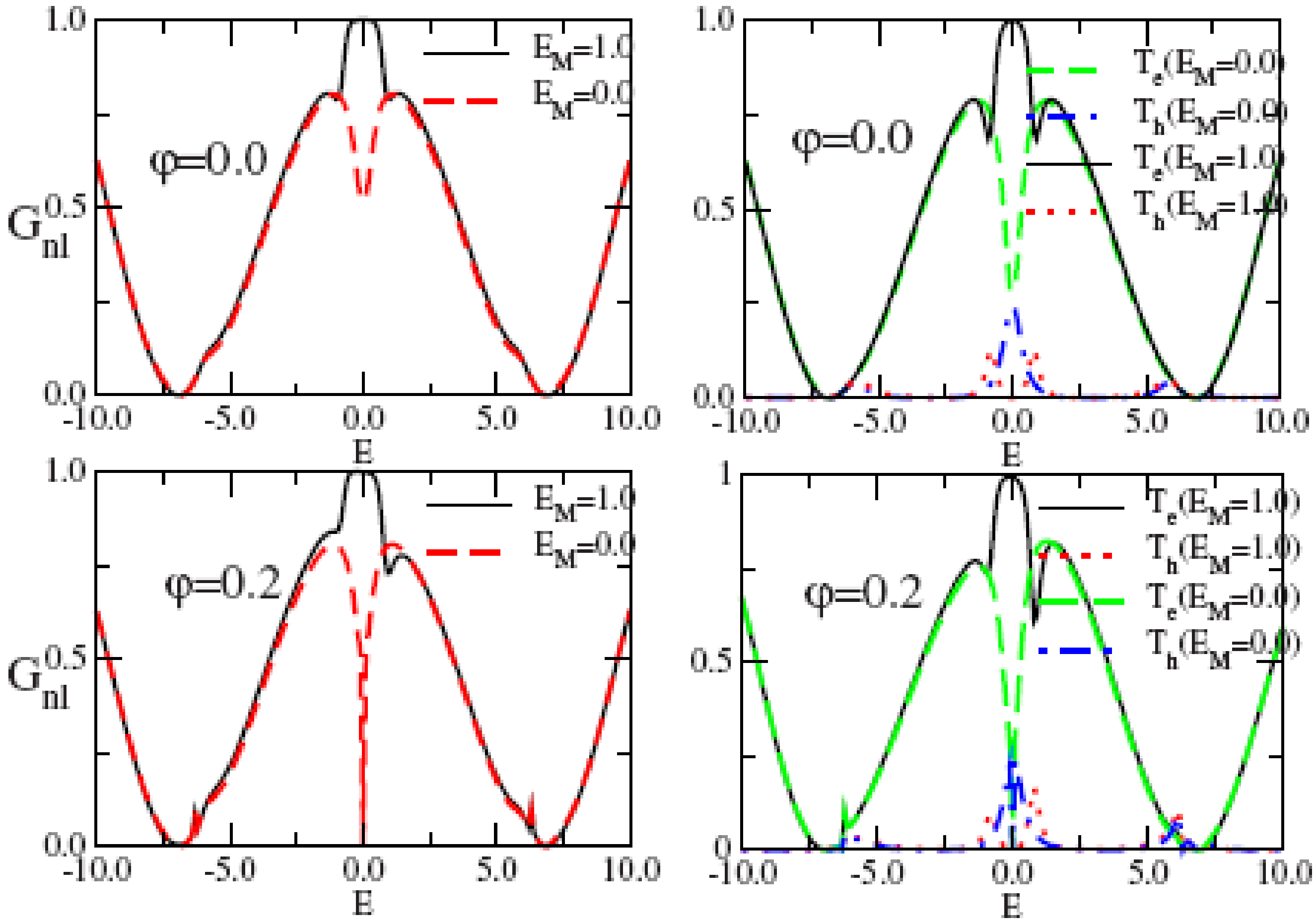
$$G_{nl} = (e^2/2\hbar)(1 - R_e - R_h + T_e + T_h)$$

$$\bar{E},\ \Gamma \ll \Delta$$



2. Gate voltage asymmetry

$\Gamma \ll \Delta$



Reasons

- Breaking of Time Reversal Symmetry for coupled MBS: $s_{12}^{ee} \neq s_{21}^{ee}$
- Breaking of Andreev Reflection Symmetry for either case:

$$(s_{11}^{eh*}(-E) = s_{11}^{he}(E)) \text{ instead of } s_{11}^{eh*}(-E) = -s_{11}^{he}(E)$$

Weak coupling: andreev reflection is negligible

TABLE I. Detecting MBS.

MBS	Magnetic field	Electric field/gate voltage
Present	$G(\phi) \neq G(-\phi)$	$G(E) = G(-E)$
Absent	$G(\phi) = G(-\phi)$	$G(V_1) \neq G(-V_1)$

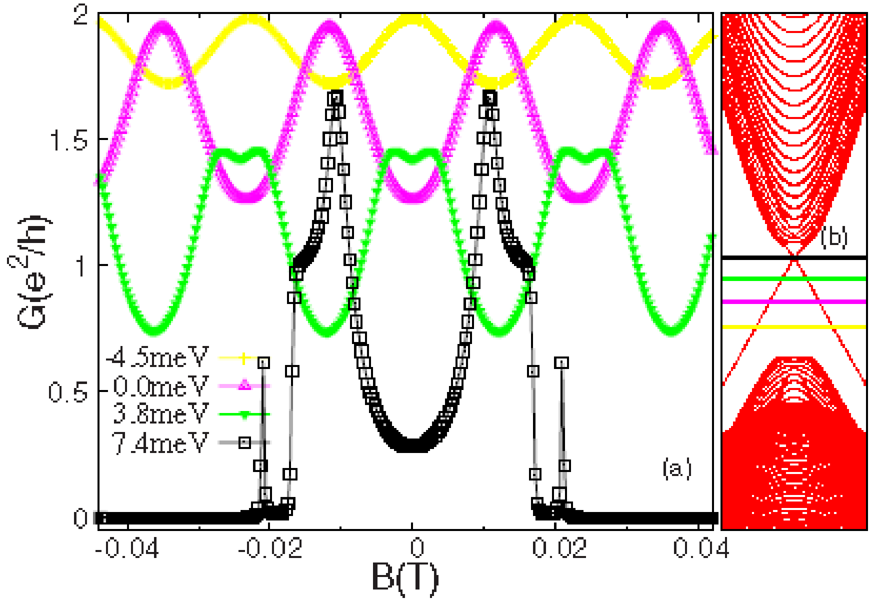
Note

- Presence of magnetic fields/impurities can break TRS too

- Another symmetry holds for magnetic fields/impurities:

T_up (B)=T_down(-B) $G(B) = \frac{e^2}{h}[T^{\uparrow}(B_{\perp}) + T^{\downarrow}(B_{\perp})]$

G symmetric with respect to field reversal



Coherent oscillations and giant edge magnetoresistance in singly connected topological insulators by R-L Chu, J Li, J. K. Jain and S-Q Shen Phys. Rev. B 80, 081102 (2009).