Monogamy of Quantum Correlations

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Can quantum correlations be created in well controlled environments between distinct quantum systems?

1950's inconceivable question

Early attempt to quantify the quantum correlations was through Bell's inequalities

"As it is well known John Bell's questioning orthodox quantum mechanics was just his *hobby*, and it is this hobby *John Bell* is most famous for."

Quantum [un]speakables: from Bell to quantum information

- R. A. Bertlmann & A. Zeilinger

Can quantum correlations be created in well controlled environments between distinct quantum systems?

1950's inconceivable question

Early attempt to quantify the quantum correlations was through Bell's inequalities

Theory of Entanglement

- Characterization
- Quantification
- Manipulation

❖Quantification → Concurrence

S. Hill and W.K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997)
W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998)

Let A and B be a pair of qubits and let the density matrix of the pair be ρ_{AB}

Concurrence: $\mathcal{C}_{AB} = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$

λ's are the square root of the eigen values of $\rho_{AB}\tilde{\rho}_{AB}$

$$\tilde{\rho}_{AB} = (\sigma_y \otimes \sigma_y)\rho^*_{AB}(\sigma_y \otimes \sigma_y)$$

$\mathcal{C}_{AB} = 0$ → Unentangled

$\mathcal{C}_{AB} = 1$ → Maximally entangled

In a multipartite setting, sharing entanglement between several parties is restricted by the monogamy of entanglement

Monogamy of Entanglement

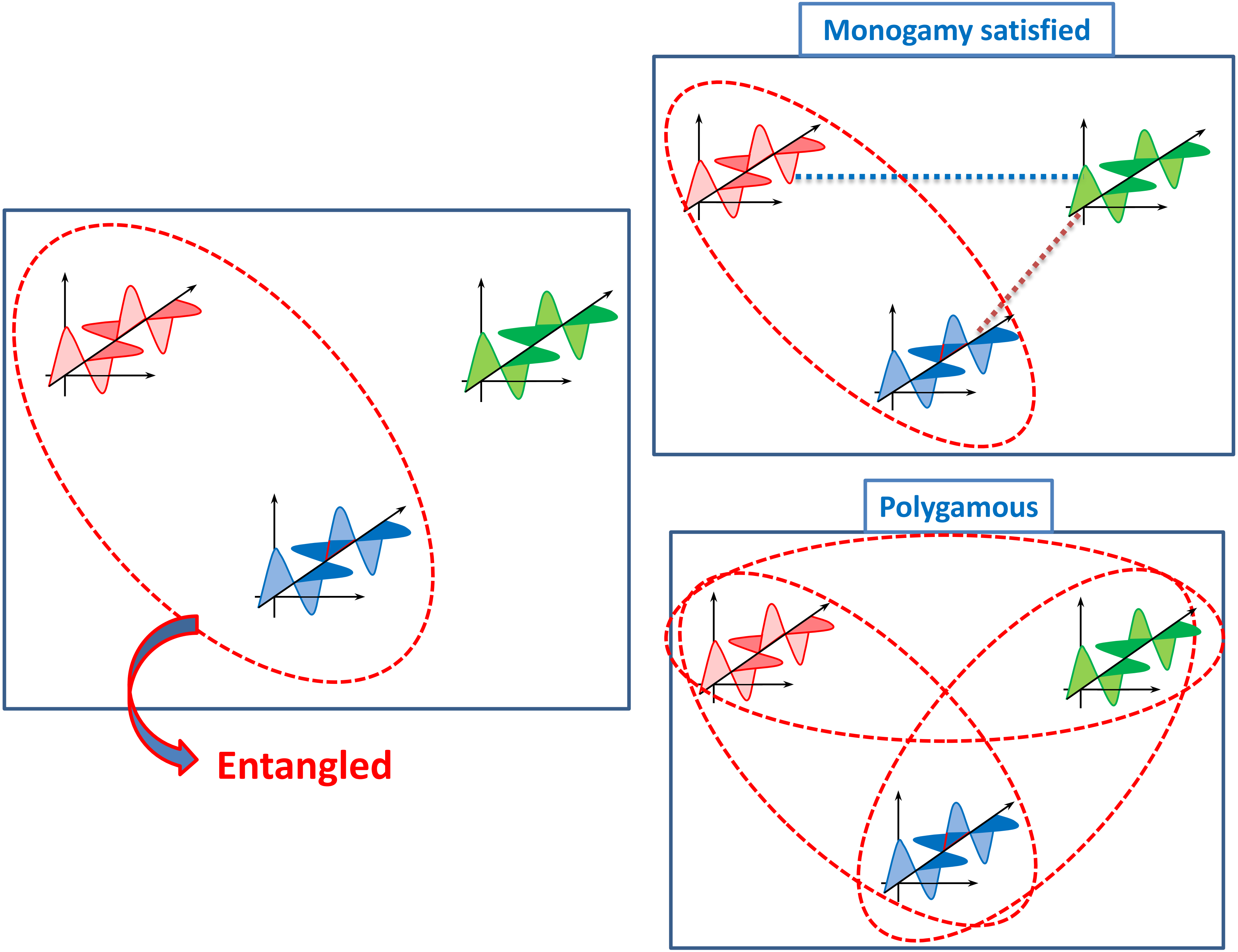
- If two qubits *A* and *B* have maximal quantum correlations, they cannot be correlated, at all with third qubit *C*

C.H. Bennett, et al., Phys. Rev. A **53**, 2046 (1996)

- Trade-off between the amount of entanglement between qubits *A* and *B* and the same qubit *A* and qubit *C*

V. Coffman, J. Kundu, and W.K. Wootters, Phys. Rev. A **61**, 052306 (2000)

- **In the classical world:** If *A* and *B* bits are perfectly correlated, then there are no constraints on correlations between bits *A* and *C*


Monogamy satisfied
Polygamous
Entangled

If two subsystems are highly entangled,
then they cannot share a substantial amount of
entanglement with other subsystems

Monogamy for Pure States

$$C_{AB}^2 + C_{AC}^2 \leq 4 \det \rho_A$$

Concurrence between qubit A and the pair BC: $C_{A(BC)}$

$$C_{AB}^2 + C_{AC}^2 \leq C_{A(BC)}^2$$

NOTE: Even though the state space of *BC* is four dimensional, only two of those dimensions are necessary to express the combined state ABC

V. Coffman, J. Kundu, and W.K. Wootters, Phys. Rev. A **61**, 052306 (2000)

Monogamy for Mixed States

Let $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ with set of all pure state decomposition $\{(\psi_i, p_i)\}$

$$C_{AB}^2 + C_{AC}^2 \leqslant (C^2)_{A(BC)}^{\min}$$

$$\min\left(\langle C_{A(BC)}^2\rangle = \sum_i p_i C_{A(BC)}^2(\psi_i)\right)$$

V. Coffman, J. Kundu, and W.K. Wootters, Phys. Rev. A **61**, 052306 (2000)

Recent!!

For more on monogamy of entanglement:

J.S. Kim, G. Gour, and B.C. Sanders arXiv:1112.1776

Concept of monogamy to an information-theoretic quantum correlation measure

Several quantum phenomena have been discovered in which entanglement is absent

- “quantum nonlocality without entanglement” – locally indistinguishable orthogonal product states

- Deterministic quantum computation with one quantum bit

Quantify quantum correlations independent of the entanglement-separability paradigm

- Quantum discord

L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001)
H. Ollivier and W.H. Zurek, Phys. Rev. Lett. **88**, 017901 (2002)

- Quantum deficit

J. Oppenheim, M, P, R, Horodecki,Phys. Rev. Lett. 89, 180402 (2002)
M, P, R, Horodecki, J. Oppenheim, A, U, Sen, and B. Synak-Radtke, Phys. Rev. A 71, 062307 (2005)
A.K.Rajagopal and R.W.Rendell, Phys. Rev. A 66,022104 (2002)

Quantum Discord

“Quantum discord is the difference between two classically equivalent expressions for the mutual information, when extended to the quantum regime.”

L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001)
H. Ollivier and W.H. Zurek, Phys. Rev. Lett. **88**, 017901 (2002)

Mutual information

CLASSICAL

One form:

$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

where X, Y are random variables

$$H(\{p_i\}) = -\sum p_i \log_2 p_i$$

Shannon entropy

Second form:

$$H(X:Y) = H(X) - H(X|Y)$$

where

$$H(X|Y) = \sum_y p(y) H(X|Y=y)$$

conditional entropy

QUANTUM

One form:

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$S(\sigma) = -\mathrm{tr}\,(\sigma \log_2 \sigma)$$

Von Neumann entropy

Second form:

$$H \Rightarrow S$$

Physical quantity that can be negative for some quantum states !!!

What next then?

Fixing second form:

Classical correlation: the difference between the von Neumann entropies before and after a complete measurement on the subsystem B

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

where quantum conditional entropy

$$S(\rho_{A|B}) = \min_{\{\Pi_i^B\}} \sum_i p_i S(\rho_{A|i})$$

minimization being over all projection-valued measurements performed on subsystem B

$$\rho_{A|i} = \frac{1}{p_i} \mathrm{tr}_B(\mathbb{I}_A \otimes \Pi_i^B \rho \mathbb{I}_A \otimes \Pi_i^B)$$

CLASSICAL

QUANTUM

Mutual information

CLASSICAL

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Von Neumann entropy

Second form:

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$$S(\rho_{A|B}) = \min_{\{\Pi_i^B\}} \sum_i p_i S(\rho_{A|i})$$

Q. conditional entropy

Quantum Discord

"Quantum discord is the difference between two classically equivalent expressions for the mutual information, when extended to the quantum regime."

$$D(\rho_{AB}) \;=\; I(\rho_{AB}) - J(\rho_{AB})$$

$I(\rho_{AB})$ → Total correlations

$J(\rho_{AB})$ → Cl. correlations

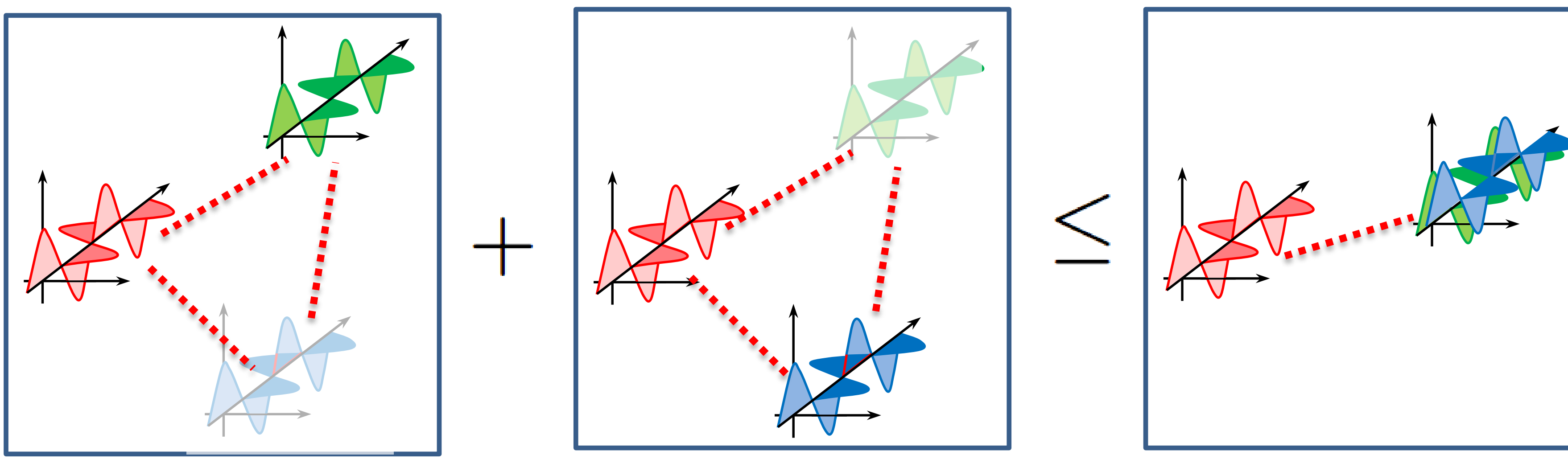
Questions

- Does quantum discord satisfy monogamy relation?
- Does the sharing of quantum discord follow the same broad guidelines that are followed by entanglement?

Monogamy of Discord

For tripartite state ρ_{ABC}

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$$



Monogamy relation
- satisfied → Monogamous
- violated → Polygamous

R. P, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$$

OR

$$\delta_M = D(\rho_{AB}) + D(\rho_{AC}) - D(\rho_{A:BC})$$

$\delta_M \leq 0$ → Q. correlations → Monogamous

$\delta_M > 0$ → Q. correlations → Polygamous

Interaction Information

The difference between the information shared by the subsystem AB when C is present, and when C is not present (traced out)

For tripartite state ρ_{ABC}

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

$$\mathcal{S}(\rho_{A|C}) - \mathcal{S}(\rho_{A|BC})$$

(Conditional mutual information)

Interaction Information

The difference between the information shared by the subsystem AB when C is present, and when C is not present (traced out)

Unmeasured

$\mathcal{II}$

Measurement

$\{\Pi_i^C, \Pi_j^{BC}\}, \{\Pi_k^B\}$

Interrogated

$\mathcal{II}$

Theorem 1

Tripartite states follow monogamy

iff

Interrogated $\mathcal{J}\mathcal{J} \leq$ unmeasured $\mathcal{J}\mathcal{J}$

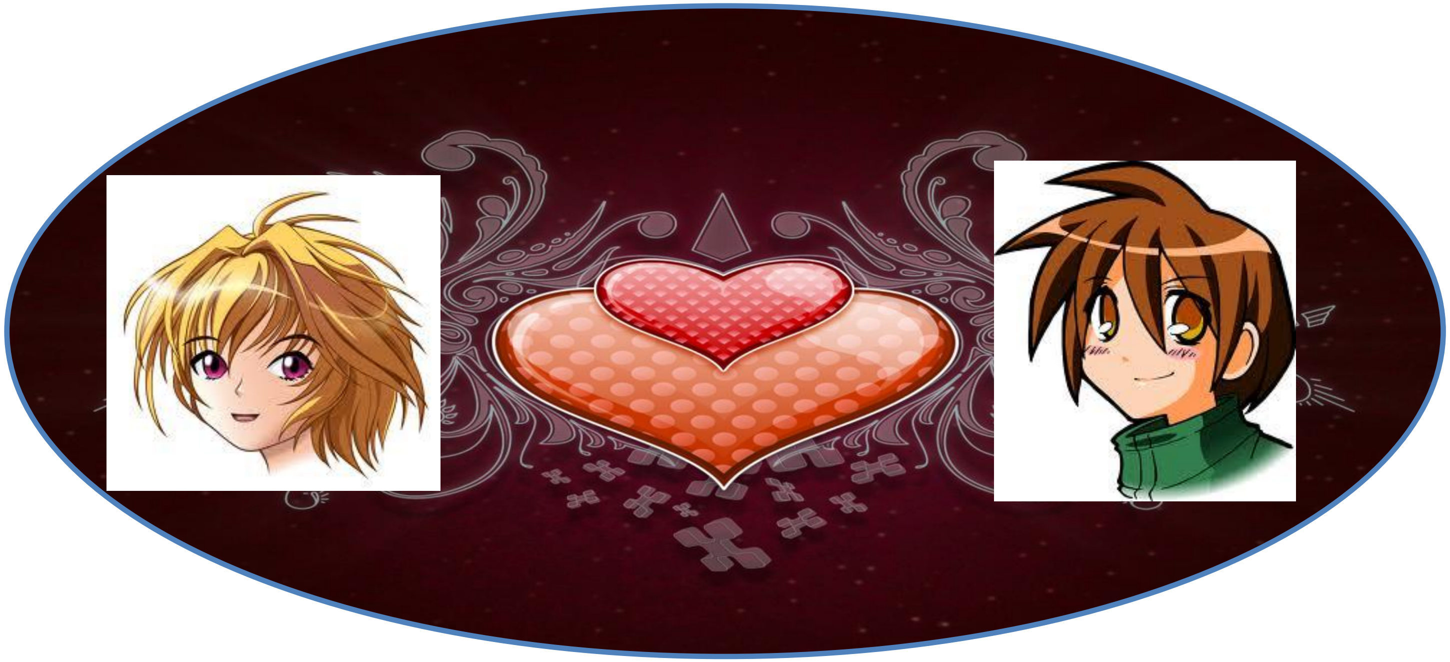
Theorem 2

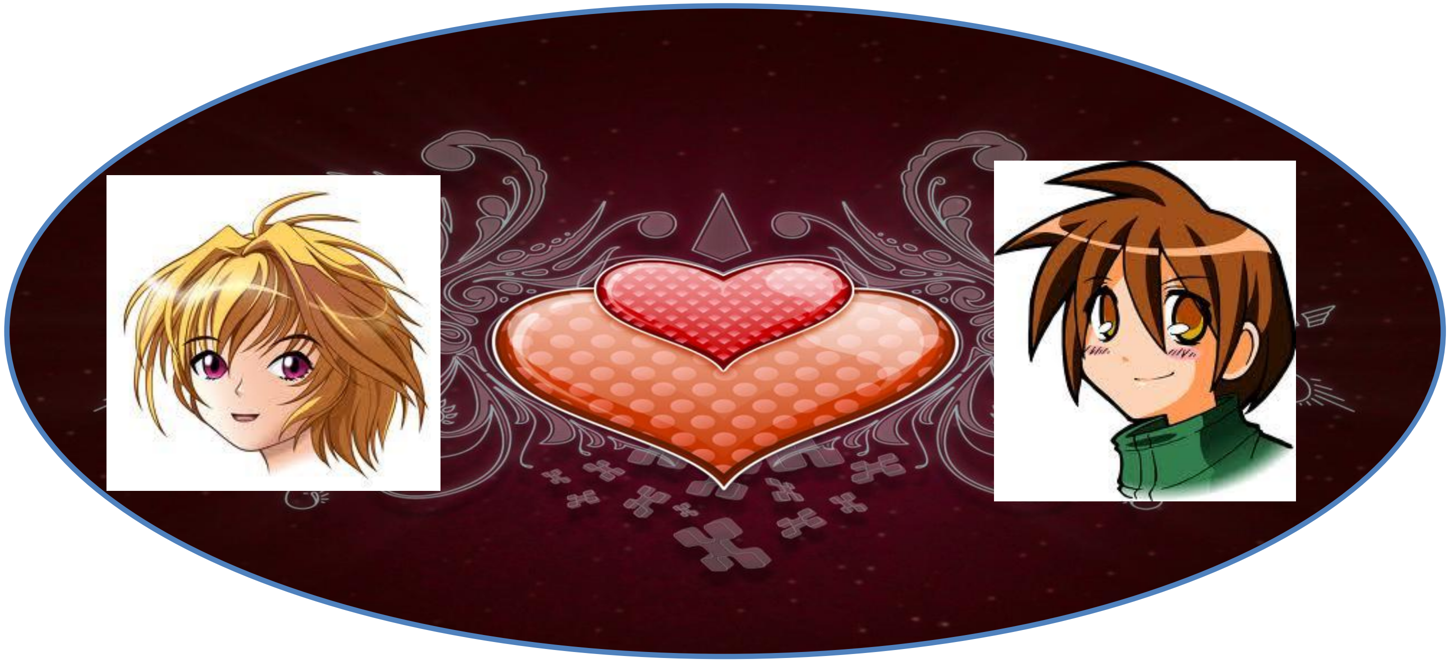
Unmeasured $\mathcal{J}\mathcal{J} < 0$

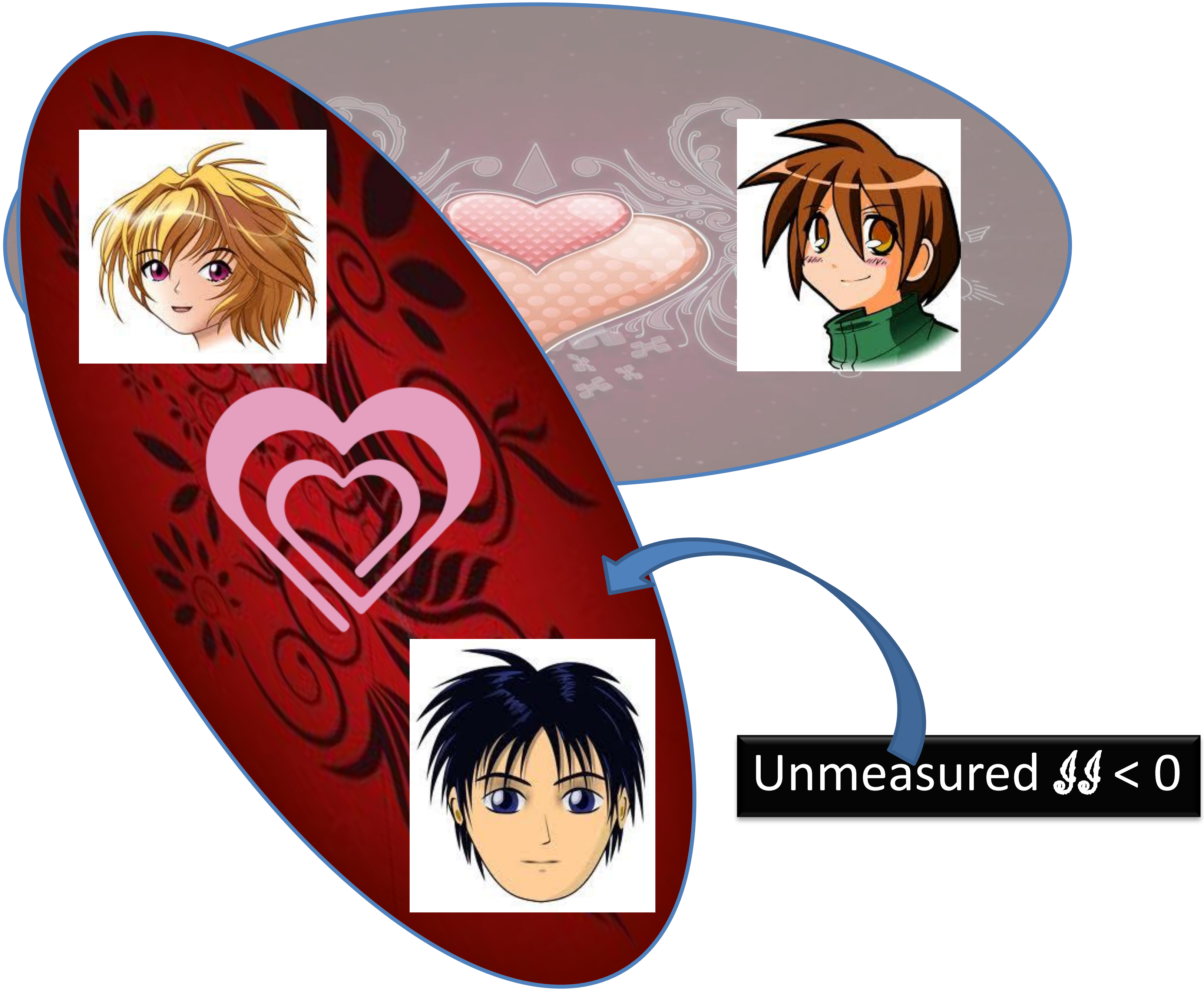
Interrogated $\mathcal{J}\mathcal{J} > 0$

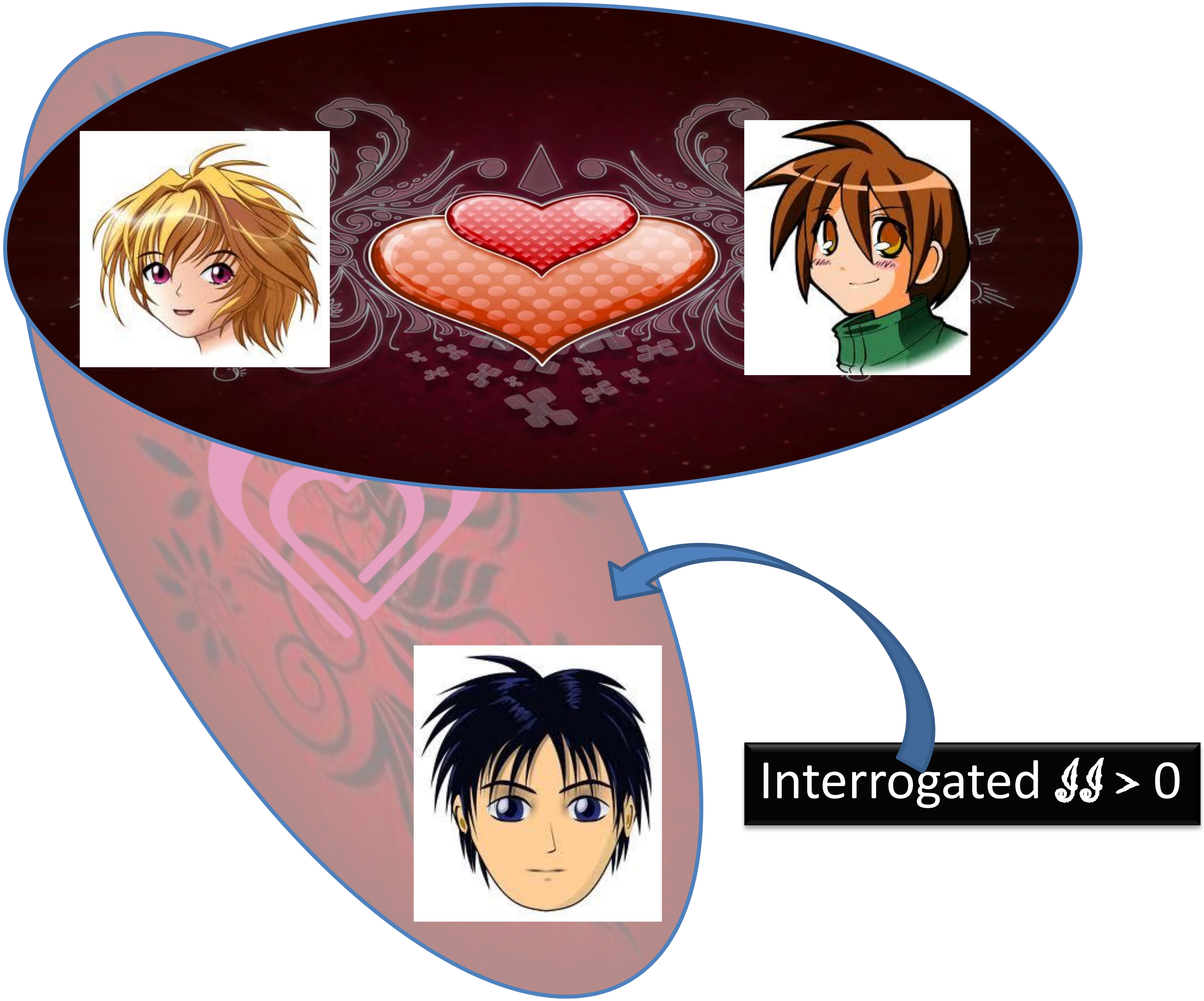
} Polygamous

R. P, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168






Unmeasured 𝒥𝒥 < 0


Interrogated $\mathscr{JJ} > 0$

Discord monogamy can be a test?

Generalized GHZ states

$$|\psi_{GHZ}\rangle_{ABC} = \cos\Phi|000\rangle + \sin\Phi|111\rangle$$

δ_M <0 → Monogamous

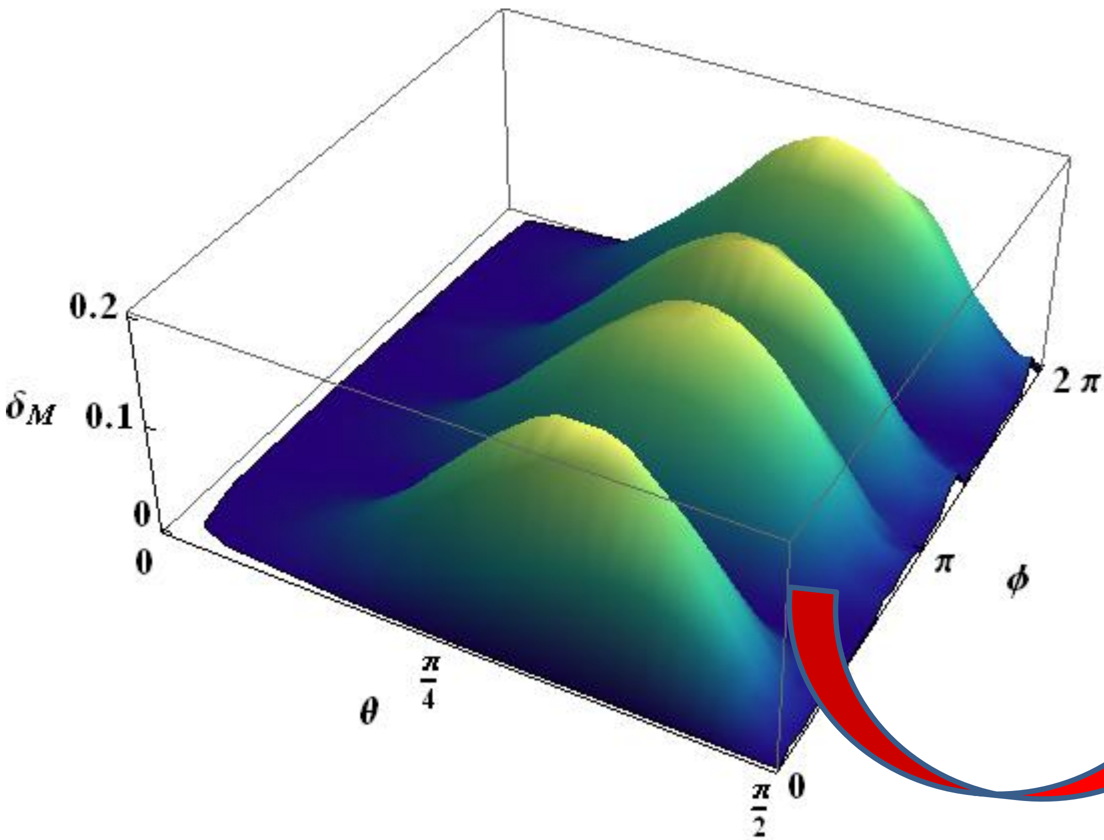
Generalized W states

$$|\psi_W\rangle_{ABC} = b|011\rangle + c|101\rangle + d|110\rangle$$

here $b = \sin\theta\cos\phi$

$c = \sin\theta\sin\phi$

$d = \cos\theta$



Polygamous

δ_M>0

GHZ class

$$|\psi_G\rangle = \cos\frac{\theta}{2}|000\rangle + |\psi_1\rangle|\psi_2\rangle|\psi_3\rangle$$

where $|\psi_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$

$$i = 1, 2, 3$$

W. Dur, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000)

75% → Monogamous

25% → Polygamous

W class

$$|\psi_{\rm W}\rangle = |a_1\rangle|b_1\rangle|c_1\rangle + |a_2\rangle|\phi_{BC}\rangle$$

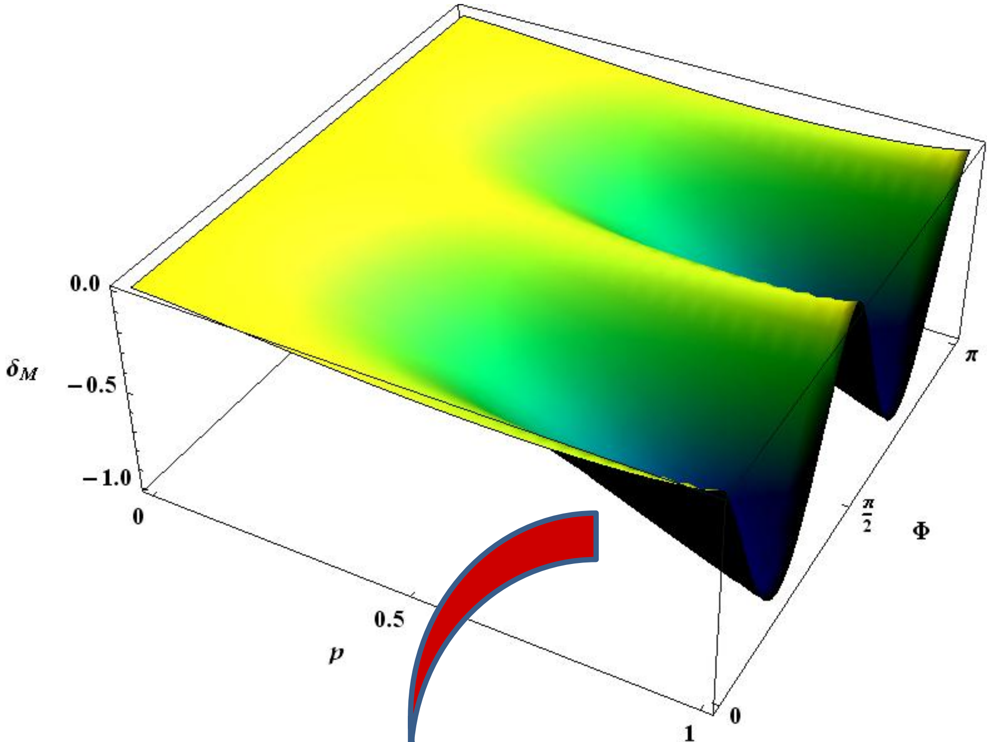
W. Dur, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000)

Numerical results confirm that these states always violate monogamy

100 % → Polygamous

Mixed state: Generalized GHZ states

$$\rho_{GHZ} = (1-p)\mathbb{I}/8 + p|\psi_{GHZ}\rangle\langle\psi_{GHZ}|$$

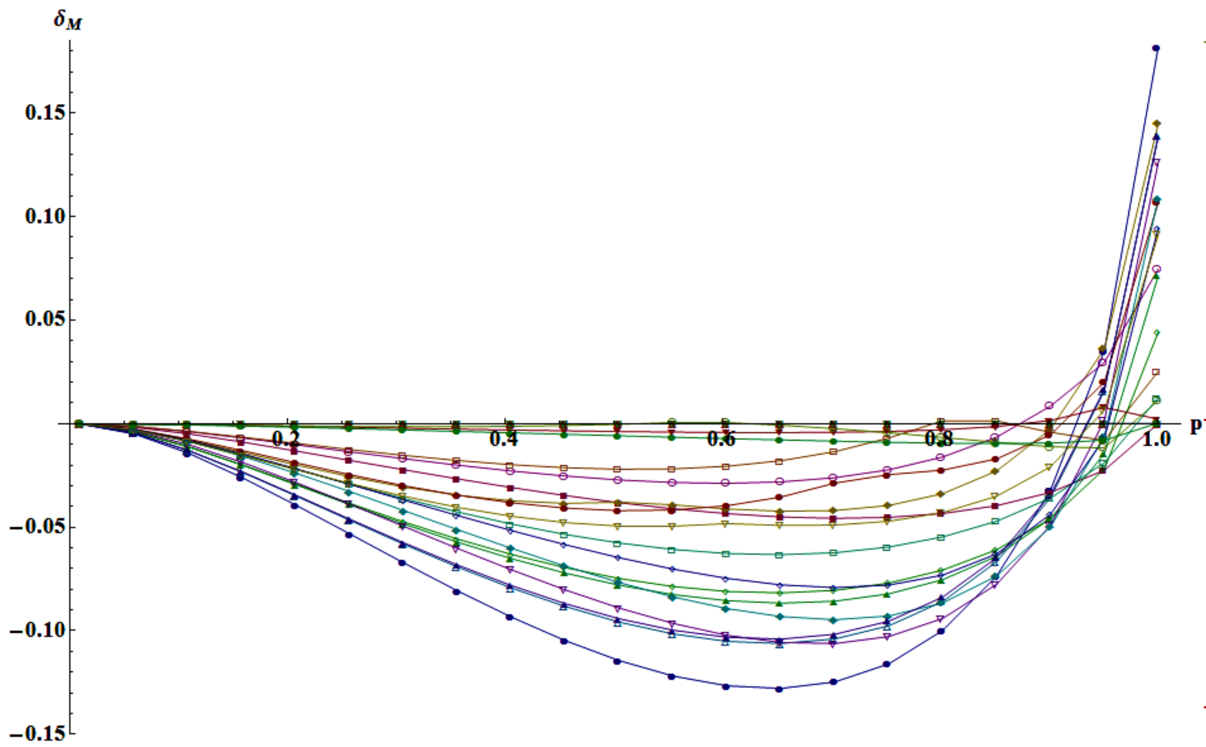


$\delta_M \leq 0$

Monogamous

Mixed state: Generalized W states

$$\rho_W = (1-p)\mathbb{I}/8 + p|\psi_W\rangle\langle\psi_W|$$



$\delta_M > 0$

Polygamous

$\delta_M \leq 0$

Monogamous

- Does quantum discord satisfy monogamy relation?

- Does the sharing of quantum discord follow the same broad guidelines that are followed by entanglement?

No !!

Results for discord monogamy test

Tripartite states	Discord Monogamy	Monogamy test result
Gen GHZ	< 0	Satisfy
Gen W	> 0	Violate
GHZ class	75%>0 0<25%	Satisfy Violate
W class	> 0	Violate

Thesis

When a state is subjected to test under monogamy of discord

Satisfying monogamy means the state belongs to GHZ class

Violating monogamy means the state belongs to **GHZ class** or W class

R. P, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168; arXiv:1109.4318

THANK YOU

for your attention