Leggett-type nonlocal realist inequalities without any constraint on the geometrical alignment of measurement settings

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Outline of the Talk

- Leggett's Nonlocal Realistic Model for Quantum Correlations.
- Experimental Tests of Leggett's Model :
  1. Leggett-type inequalities derived and tested to date
  2. Geometrical constraint on measurement settings
  3. Finite precision loophole
- New Forms of Leggett-type Inequalities :
  1. No geometrical constraint on measurement settings
  2. Enables closing finite precision loophole
- Concluding Remarks.

Leggett's Nonlocal Realistic Model: Motivation

Motivation

- By now, plethora of studies confirms experimental falsification of Bell-type inequalities, thereby ruling out the local realist models in favor of quantum mechanics (QM).
- The next issue is whether the question of compatibility between QM and its plausible *nonlocal realist models* can be subjected to a deeper scrutiny.
- Theoretical and experimental study of Leggett's nonlocal realist model for quantum correlations provides new physical insights and is a topical issue in the foundational study of Quantum mechanics.

Leggett's Nonlocal Realistic Model: Basic Features

Example: A source emitting photon-pairs.
Measurements: Polarization degree of freedom of photons.

Basic Features of the Model

1. Realism : All measurement outcomes are determined by pre-existing properties of particles independent of the measurement.
2. Physical states (Ensembles) : are statistical mixtures of subensembles with definite polarization.
3. Malus' law : Polarization is defined such that expectation values taken for each subensemble obey Malus' law.
4. Nonlocal : Measurement outcomes may depend on parameters in space-like separated region.

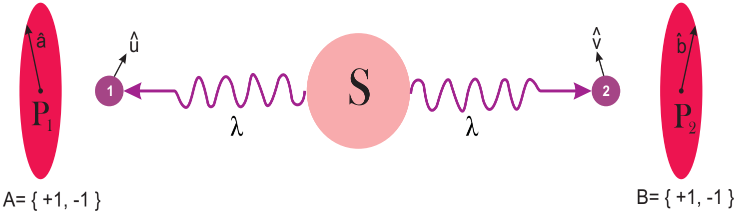
Leggett's Nonlocal Realistic Model: General Framework

General Framework

- Ensemble of photon-pairs : Disjoint union of subensembles.
- Subensemble : Characterized by definite hidden polarizations $\hat{u}$ and $\hat{v}$ of the respective photons of constituent photon-pairs.
- $D(\hat{u}, \hat{v})$: Distribution of subensembles.
- $\delta_{(\hat{u},\hat{v})}(\lambda)$: Distribution of hidden variables λ in a subensemble.
- Single measurement outcome : A / B is determined by
  1. Local hidden variable λ.
  2. Local hidden polarization $\hat{u}$ / $\hat{v}$.
  3. Local polarizer setting $\hat{a}$ / $\hat{b}$.
  4. Nonlocal parameter $\eta(\hat{b})$ / $\eta(\hat{a})$.
- Subensemble average satisfies Malus' law : $\overline{A} = \hat{u} \cdot \hat{a}$, $\overline{B} = \hat{v} \cdot \hat{b}$

Leggett's Nonlocal Realistic Model: Single Event

Single Event : Schematic Diagram



- $A = A(\lambda, \hat{u}, \hat{a}, \eta(\hat{b}))$
- $B = B(\lambda, \hat{v}, \hat{b}, \eta(\hat{a}))$

- Transmission $\rightarrow \quad +1$
- Absorption $\rightarrow \quad -1$

Leggett's Nonlocal Realistic Model: Experimental Observables

- At level of single event :
  $A = A(\lambda, \hat{u}, \hat{a}, \eta(\hat{b})); \quad B = B(\lambda, \hat{v}, \hat{b}, \eta(\hat{a}))$
- At level of subensemble :

  $\overline{A}(\hat{u}) = \int A\ \delta(\lambda)\ d\lambda = \hat{u} \cdot \hat{a};\ \ \overline{B}(\hat{v}) = \int B\ \delta(\lambda)\ d\lambda = \hat{v} \cdot \hat{b}$

  $\overline{AB}(\hat{u}, \hat{v}) = \int AB\ \delta(\lambda)\ d\lambda$

Experimental observables are ensemble averages

- Marginals : $\langle A \rangle = \int \overline{A}(\hat{u}) D(\hat{u}) d\hat{u}, \quad \langle B \rangle = \int \overline{B}(\hat{v}) D(\hat{v}) d\hat{v}$
- Correlation : $\langle AB \rangle = \iint \overline{AB}(\hat{u}, \hat{v}) D(\hat{u}, \hat{v}) d\hat{u} d\hat{v}$

Leggett's Nonlocal Realistic Model : Implications

Implications

- The model led to Leggett-type inequalities bounding certain combinations of observable correlation functions
- All the earlier experimental data falsifying local realistic models by violating Bell-type inequalities could be explained through Leggett's model
- However, for correlation between two spatially separated and entangled subsystems, quantum theory predicted violation of Leggett-type inequalities
- This motivated new kind of experiments for testing Leggett's model against QM predictions

Experimentally Tested Forms of Leggett-type Inequalities : Leggett / Gröblacher *et al.*

$$|\langle A_1^* B_1^* \rangle_{av*} + \langle A_2^* B_3^* \rangle_{av*}| + |\langle A_2^* B_2^* \rangle_{av*} + \langle A_2^* B_3^* \rangle_{av*}| \leq 4 - \frac{4}{\pi}\left|\sin\frac{\phi}{2}\right|$$

$*$ denotes an index for a continuum of suitable measurements.

- Alice's observable $\in \{\{\hat{a}_1, \hat{a}_2\}; \{\hat{a'}_1, \hat{a'}_2\}; \ldots\}$ and Bob's observable $\in \{\{\hat{b}_1, \hat{b}_2, \hat{b}_3\}; \{\hat{b'}_1, \hat{b'}_2, \hat{b'}_3\}; \ldots\}$
- $\langle A_i^* B_j^* \rangle_{av^*}$ is the uniform average of $\langle A_i^* B_j^* \rangle$ over $*$
- Set-up constraints underlying the derivation of the inequality
  1. $\hat{b}_3^* = \hat{a}_2^*$ and angle between $\hat{a}_i^*$, $\hat{b}_i^*$ for $i \in \{1, 2\}$ is ϕ
  2. Plane of $\hat{a}_1^*$, $\hat{b}_1^*$ is orthogonal to the plane of $\hat{a}_2^*$, $\hat{b}_2^*$

Experimentally Tested Forms of Leggett-type Inequalities : Gröblacher *et al.* Experiment

- Testing the derived form of the inequality as such required performing infinite many measurement in order to obtain experimental values for $\langle A_i^* B_j^* \rangle_{av^*}$
- So, the actual test was done assuming rotational symmetry : $\langle A_i^* B_j^* \rangle_{av^*} = \langle A_i B_j \rangle$, which in turn enabled conducting the experiment with finitely many measurements
- Rotational symmetry of correlation functions for a singlet state is experimentally well founded
- Then, an experimental violation of the inequality for photon-pairs prepared in a singlet state was demonstrated

Experimentally Tested Forms of Leggett-type Inequalities : Without Assuming Rotational Symmetry

- Later on , Paterek *et al.* (2007), Branciard *et al.* (2007), Branciard *et al.* (2008) derived different forms of Leggett-type inequalities which involved finite number of measurement settings.
- Hence, the experimental tests of these inequalities did not required assuming rotational symmetry.
- Experiments performed demonstrated violation of these inequalities.
- However, like the inequality tested by Gröblacher *et al.*, derivation of these later inequalities also crucially hinge upon assuming certain geometrical constrains on the relevant measurement settings

Experimentally Tested Forms of Leggett-type Inequalities : Branciard *et al.* (2008)

For example, let us consider the Leggett-type inequality derived and tested by Branciard *et al.* (2008)

$$\frac{1}{3}\sum_{i=1}^{3}|\langle A_iB_i\rangle + \langle A_iB_i'\rangle| \leq 2 - \frac{2}{3}|\sin(\frac{\beta}{2})|$$

- Alice's measurements are along $\hat{a}_i$ and Bob's along $\hat{b}_i$ or $\hat{b}_i'$
- Set-up constraints underlying the derivation of the inequality
  1. Angle between $\hat{b}_i$ and $\hat{b}_i'$ is same, say β, $\forall\ i$
  2. Directions of $(\hat{b}_i - \hat{b}_i')$ are mutually orthogonal

Leggett-type Inequalities : A Characteristic Feature

- A generic property of the Leggett-type inequalities is that while the left hand side of any such inequality involves experimentally measurable quantities (correlation functions), the upper bound (the right hand side) of such an inequality, *unlike* any Bell-type inequality, is not just a number fixed by the general assumptions used in the relevant derivation; instead, it depends on the *choice* of the geometrical configuration of measurement settings.
- Due to the above mentioned feature tests of Leggett-type inequalities demands highly precise control (an ideal control in tests performed so far) of the experimental set-up.

Experimentally Tested Forms of Leggett-type Inequalities : An Undesirable Feature

Different forms of LNR inequalities that have been derived and tested to date hold good only if certain geometrical constraints are *exactly* satisfied by the spatial arrangement of the relevant measurement settings. For example, appropriate to any such inequality, relative orientations of the planes of the relevant measurement settings need to satisfy suitable conditions such as that of orthogonality.

Experimentally Tested Forms of Leggett-type Inequalities : Finite Precision Loophole

Finite Precision Loophole

- We saw the geometrical constraint on the relevant measurement settings assuming which the inequalities tested by Gröblacher *et al.* and Branciard *et al.* (2008) were derived.
- Similar constraints also besets the derivation of inequalities by Paterek *et al.* (2007) and Branciard *et al.* (2007).
- This undesirable feature induces a logical glitch while testing such forms of Leggett-type inequalities.
- Any achieved degree of precision in such tests cannot guarantee to satisfy the geometrical constraints assuming which the inequalities claimed to be tested were derived.

New Forms of Leggett-type Inequalities : Motivation

Motivation

- Due to the intrinsic property of Leggett-model, in any inequality derived as a consequence, bound on experimentally observable correlation function must depend on the measurement settings.
- However, for deriving such inequalities assuming an a priori constraint on the configuration of measurement settings is not necessary (and undesirable).
- Deriving new forms of inequalities with no constraints on (full degrees of freedom for) the geometrical configuration of measurement settings would enable more flexibility in performing and analyzing the results of the experiments.

New Forms of Leggett-type Inequalities : Generalization of Branciard *et al.* (2008)

$$\frac{1}{3}\sum_{i=1}^{3}|\langle A_i B_i\rangle + \langle A_i B_i'\rangle| \leq 2 - \frac{2}{3}\sin(\frac{\beta_*}{2}) \times L_n \qquad (1)$$

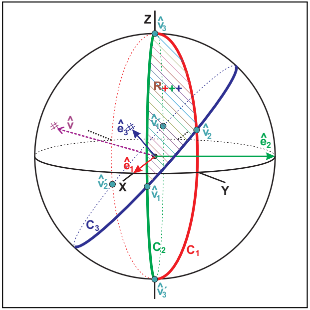
- $\hat{a}_i$ and $\{\hat{b}_i,\ \hat{b}'_i\}$ are respective measurement directions for Alice and Bob.
- $\beta_* = \min\{|\beta_1|, |\beta_2|, |\beta_3|\}$, where $\beta_i \in (-\pi, \pi)$ is the angle between the pair $(\hat{b}_i, \hat{b}'_i)$.
- L_n is a function of unit vectors $\hat{n}_i$ along $\{\hat{b}_i - \hat{b}'_i\}$.
- Alice and Bob are both free to arbitrarily choose their measurement settings.

New Forms of Leggett-type Inequalities : A novel inequality with fewer number of settings

$$\frac{1}{3}\sum_{i=1}^{3} |\langle A_i B_i\rangle - \langle A_i B_{i\oplus 1}\rangle| \leq 2 - \frac{2}{3}\cos(\frac{\delta^*}{2}) \times L_m \qquad (2)$$

- $\hat{a}_i$ and $\hat{b}_i$ are respective measurement directions for Alice and Bob, $\oplus$ is addition modulo 3.
- $\delta^* = \max\{|\delta_1|, |\delta_2|, |\delta_3|\}$ where $\delta_i \in (-\pi, \pi)$ is the angle between the pair $(\hat{b}_i, \hat{b}_{i\oplus 1})$.
- L_m is a function of unit vectors $\hat{m}_i$ along $\hat{b}_i + \hat{b}_{i\oplus 1}$.
- Alice and Bob are both free to arbitrarily choose their measurement settings.

New Forms of Leggett-type Inequalities : A Theorem Crucial to Our Analysis



Theorem.—On the Poincaré sphere, given three linearly independent unit vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ and a variable unit vector $\hat{v}$, the minimum value, say L, of the function

$$F(\hat{v}) = |\hat{e}_1 \cdot \hat{v}| + |\hat{e}_2 \cdot \hat{v}| + |\hat{e}_3 \cdot \hat{v}|$$

is given by the formula

$$L = \frac{|\hat{e}_1 \cdot (\hat{e}_2 \times \hat{e}_3)|}{\max\{|\hat{e}_1 \times \hat{e}_2|, |\hat{e}_2 \times \hat{e}_3|, |\hat{e}_3 \times \hat{e}_1|\}}.$$

New Forms of Leggett-type Inequalities : Formula for $L_{n,m}$

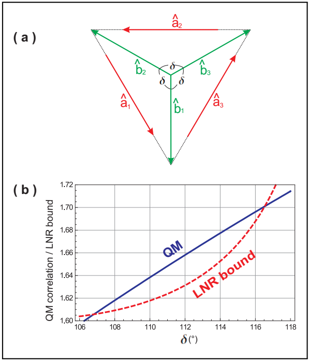
Our theorem provides the following formula for $L_{n,m}$

$$L_{n,m}(\alpha_{12}, \alpha_{23}, \alpha_{31}) = \frac{\left(1 - \sum_{\substack{1 \leq i \leq 3, \\ j = i \oplus 1}} \cos^2 \alpha_{ij} + \prod_{\substack{1 \leq i \leq 3, \\ j = i \oplus 1}} \cos \alpha_{ij}\right)^{\frac{1}{2}}}{\max\{\sin \alpha_{12}, \sin \alpha_{23}, \sin \alpha_{31}\}}$$

- $\alpha_{ij} \in (0, \pi)$ denotes the angle between a pair $\{\hat{n}_i, \hat{n}_j\}$ or $\{\hat{m}_i, \hat{m}_j\}$ for $i, j \in \{1, 2, 3\}$.

New Forms of Leggett-type Inequalities : QM violation

Form (1) of our inequality is known to be violated by quantum mechanics since it is a generalization of Branciard *et al.* (2008). Violation of form (2) is shown below

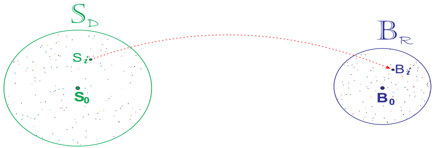


- (*a*) Measurement settings for observing QM violations for a singlet state.
- (*b*) Graph depicting violations.

New Forms of Leggett-type Inequalities : Advantages

- In order to test the new forms of inequalities, now all that is needed for an experimenter is to conveniently choose a neighborhood around a reference setting (preferably around one predicting a maximum violation) and just ensure that the settings in each run lie inside such a neighborhood. Then, a violation of the supremum of the set of bounds (each bound corresponds to some setting within the neighborhood) should be looked for.
- The inequality (2) involves $(3+3)$ number of settings which is the *least* number of settings achieved so far compared to all the LNR inequalities derived earlier; this may be convenient for future experiments.

New Tests of Leggett-type Inequalities



- S_0 is a reference setting and B_0 the corresponding numerical bound.
- S_i is any setting belonging to a suitably chosen neighborhood around S_0, and B_i is the corresponding numerical bound.
- In each run one just need to ensure that $S_i \in S_D$.
- Then, a violation of $\text{Sup}_i B_i$ would demonstrate a finite precision loophole free falsification of Leggett's model.

Concluding Remarks

The central significance of our new forms of Leggett-type inequalities lies in enabling a more robust (finite precision loophole free) tests of the LNR model than that has been hitherto possible, since the forms of the LNR inequalities derived in the present work are *free* from any constraint on the spatial alignment of the relevant measurement settings.

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