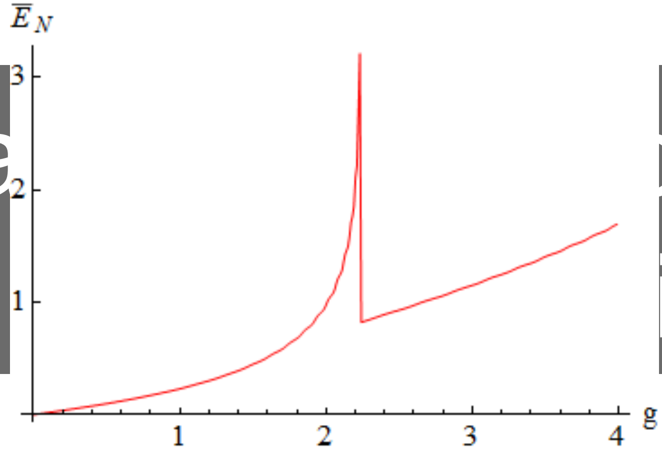
# Anoma our of
# strongly illators

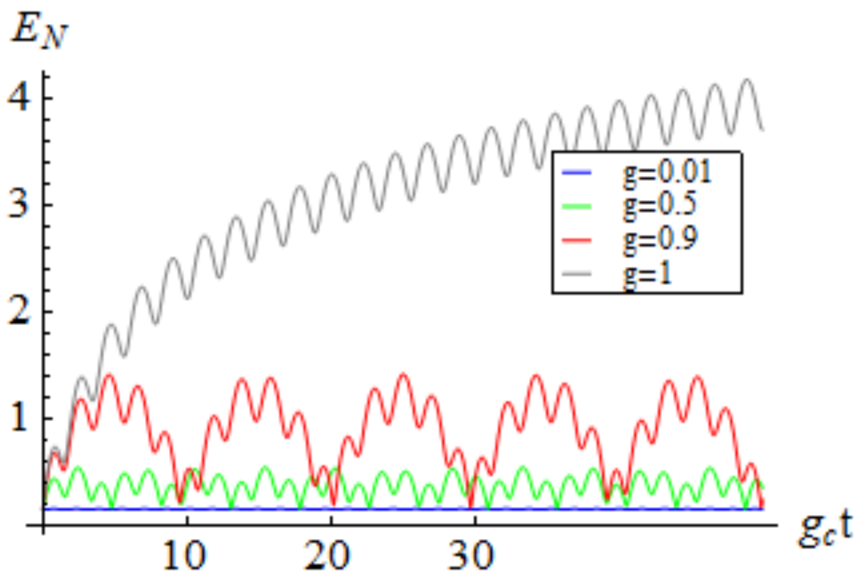



two coupled oscillators

– simplest interaction model

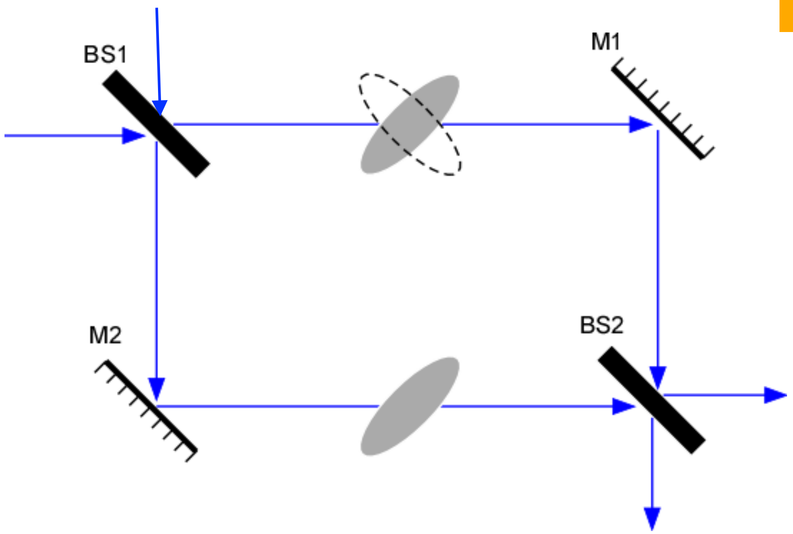
$$H = \frac{1}{2}\sum_{j=a,b}\omega_j\left(\hat{p}_j^2 + \hat{x}_j^2\right) - g\hat{x}_a\hat{x}_b$$

# Entanglement of oscillators



# If ω = $\omega_0$

The evolution is decomposed into $\hat{U} = e^{i\hat{H}t} = \hat{B}\hat{U}_1\hat{U}_2\hat{B}^+$




$\alpha^2$

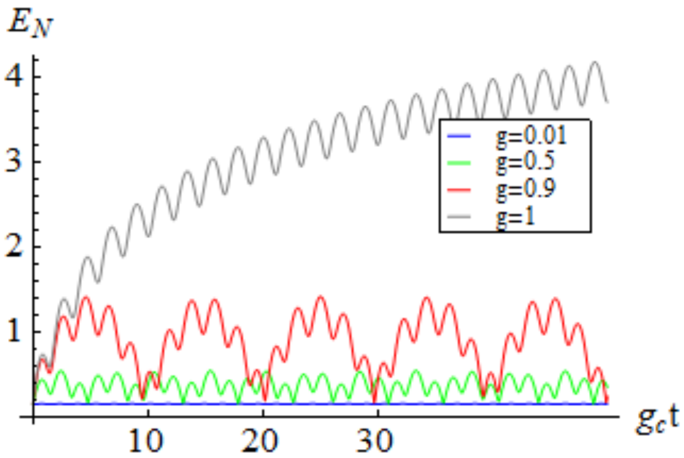
$$H_q = \tfrac{1}{2}\left(\omega\hat{p}_{1,2}^2 + (g_c \pm g)\hat{x}_{1,2}^2\right)$$

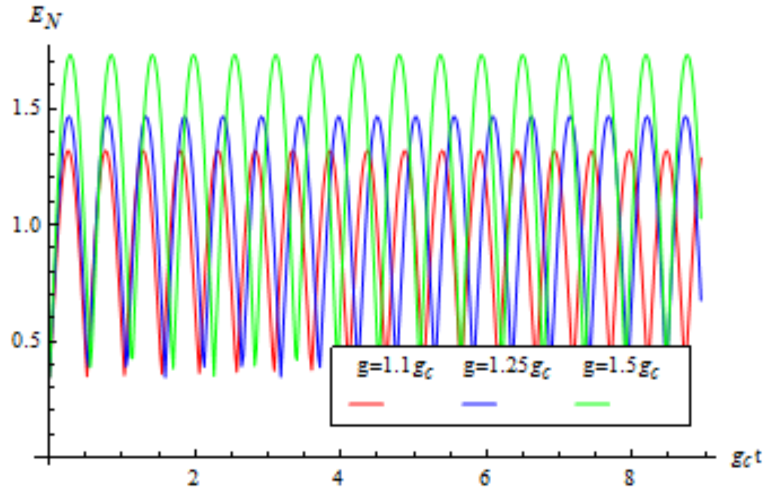
$\beta^2$

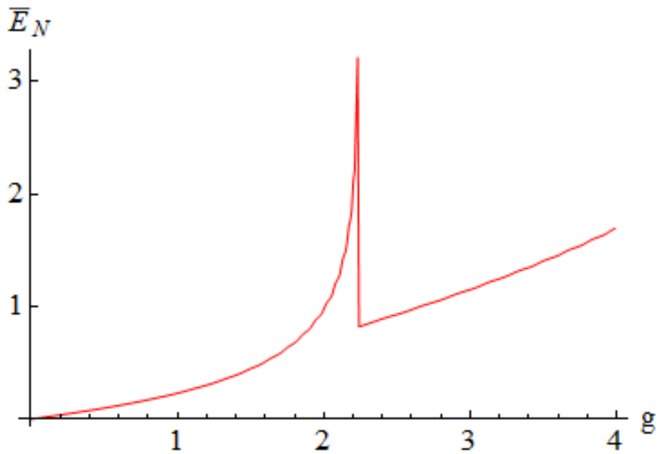
$$g_c = \sqrt{\omega\omega_0}$$

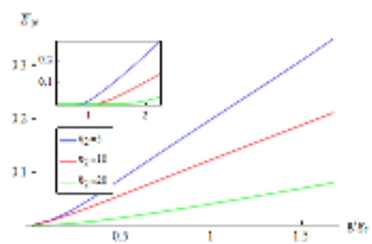
$$\begin{pmatrix} \hat{x}_q(t) \\ \hat{p}_q(t) \end{pmatrix} = \begin{pmatrix} \cos\alpha_q\beta_q t & \dfrac{-\beta_q}{\alpha_q}\sin\alpha_q\beta_q t \\ \dfrac{\alpha_q}{\beta_q}\sin\alpha_q\beta_q t & \cos\alpha_q\beta_q t \end{pmatrix} \begin{pmatrix} \hat{x}_q(0) \\ \hat{p}_q(0) \end{pmatrix}$$

# Entanglement of oscillators



# Coupled oscillators

- Hamiltonian

$$H = \omega_0 \hat{a}^+ \hat{a} + \omega \hat{b}^+ \hat{b} - g \hat{x}_a \hat{x}_b$$

- Normal modes $\Omega_\pm = A\hat{a} + B\hat{a}^+ + C\hat{b} + D\hat{b}^+$
- Energy separation of the normal modes

$$2E_\pm^2 = \omega_0^2 + \omega^2 \pm \sqrt{(\omega_0^2 + \omega^2)^2 + 4g_c^2(g^2 - g_c^2)}; \quad g_c = \sqrt{\omega_0 \omega}$$

- What if $g > g_c$? $2E_-^2 < 0$

# Super-radiance phase transition

- $N$ two-level atoms interacting with a single-mode field

$$H = \omega_0 \hat{a}^+ \hat{a} + \frac{\omega}{2} \sum_j \sigma_j^z + g' \sum_j \left( \hat{\sigma}_j^+ + \hat{\sigma}_j^- \right) \left( \hat{a}^+ + \hat{a} \right)$$

- Holstein-Primakoff transformation

$$\sum_j \sigma_j^- = \sqrt{2N} \sqrt{1 - \frac{\hat{b}^+ \hat{b}}{2N}} \hat{b} \quad ; \sum_j \sigma_j^z = N - \hat{b}^+ \hat{b}$$

- When $N$ is large,

$$H = \omega_0 \hat{a}^+ \hat{a} + \frac{\omega}{2} \hat{b}^+ \hat{b} + g \left( \hat{b}^+ + \hat{b} \right) \left( \hat{a}^+ + \hat{a} \right); \quad g = \sqrt{2N} g'$$

- When $g > g_c$, transition to super-radiance phase

# Hookian coupled oscillators

- Hamiltonian

$$H_C = \sum_{j=1}^{2}\left(\frac{\hat{P}_j^2}{2m} + \frac{1}{2}m\omega^2\hat{X}_j^2\right) + \frac{mG^2}{2}\left(\hat{X}_1 - \hat{X}_2\right)^2$$

- In dimensionless operators

$$H_C = \frac{\omega'}{2}\left[\left(\hat{p}_1^2 + \hat{x}_1^2\right) + \left(\hat{p}_2^2 + \hat{x}_{21}^2\right) - G'\,\hat{x}_1\hat{x}_2\right];$$

$$\omega' = \sqrt{\omega^2 + G^2} \quad \text{and} \quad \mathrm{G}' = \frac{G^2}{\omega^2 + G^2}$$

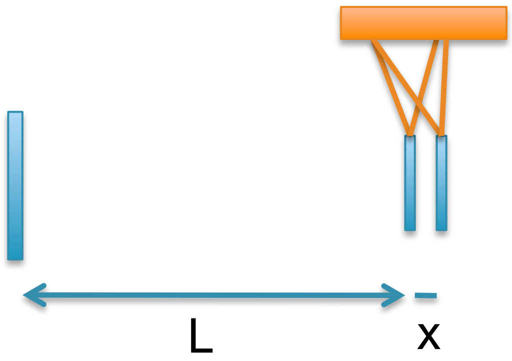
- Thus

$$G' < 1$$

# Optomechanics

- Challenging to find the Hamiltonian when the boundary moves
- Cavity resonance frequency

$$\omega_0 = \left(n + \frac{1}{2}\right)\frac{2\pi c}{L}$$




- Cavity Hamiltonian

$$H = \left(n + \frac{1}{2}\right)\frac{2\pi c}{L+x}\hat{a}^+\hat{a} = \omega_0 \frac{L}{L+x}\hat{a}^+\hat{a} \approx \omega_0\left(1 - \frac{x}{L}\right)\hat{a}^+\hat{a}$$

- Quantizing the mirror motion, the Hamiltonian is

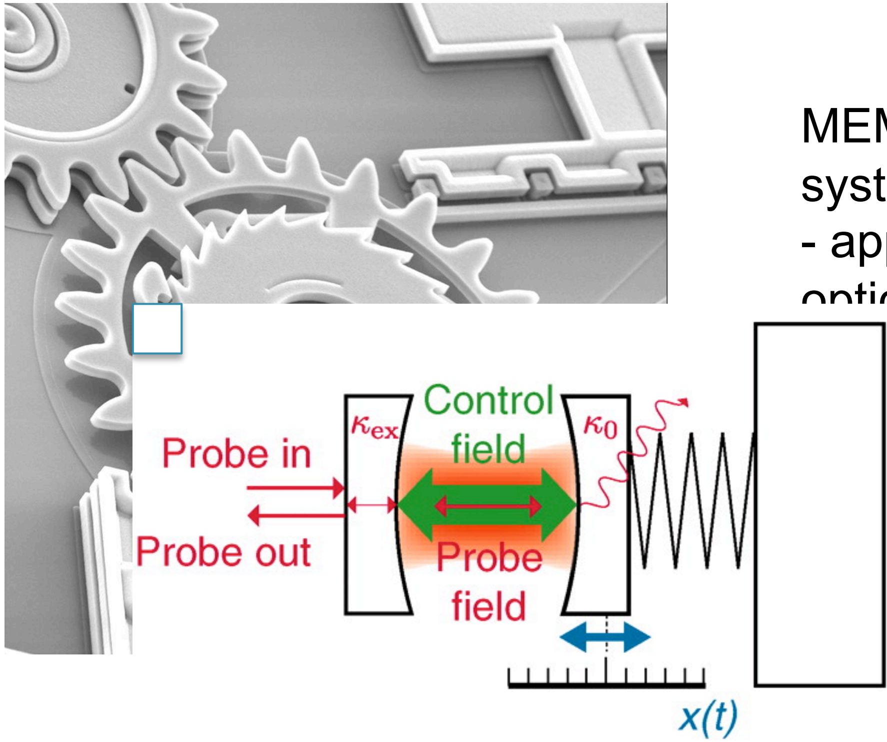
$$H = \omega_0\hat{a}^+\hat{a} + \omega\hat{b}^+\hat{b} - g'\hat{a}^+\hat{a}(\hat{b}^+ + \hat{b}); \;\; g' = \frac{\omega_0}{L}\left(\frac{2}{m\omega}\right)^{1/2}$$

$$H = \omega_0 \hat{a}^+ \hat{a} + \omega \hat{b}^+ \hat{b} - g' \hat{a}^+ \hat{a}(\hat{b}^+ + \hat{b}); \quad g' = \frac{\omega_0}{L}\left(\frac{2}{m\omega}\right)^{1/2}$$

- For an intense field

$$-g' \hat{a}^+ \hat{a}(\hat{b}^+ + \hat{b})$$
$$\rightarrow -g'(\hat{a}^+ + \alpha)(\hat{a} + \alpha)(\hat{b}^+ + \hat{b})$$
$$\rightarrow -g' \alpha(\hat{a}^+ + \hat{a})(\hat{b}^+ + \hat{b}) = -g \hat{x}_a \hat{x}_b; \quad g = g' \tfrac{\alpha}{2}$$

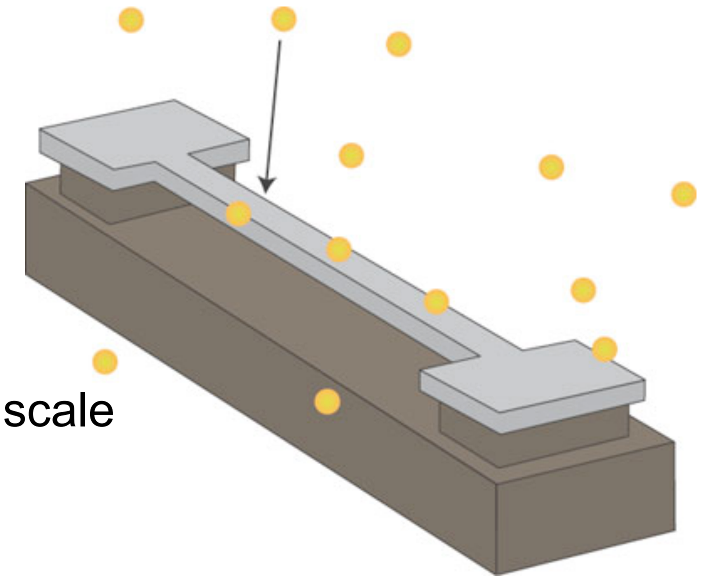
# Nanomechanical oscillators




MEMS (micro electromechanical systems) – well-developed technology
- applications: sensors, actuators, optical switching, inkjet printers etc

S. Weis et al, Science 330, 11520 (2010)

es the technique toward sub-micro regime



NEMS – a few hundreds to tens of nanometer scale
-extreme sensitivity, high Q values etc.