Some important recent results

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— ψ -Epistemic HVT model —

Knowledge of $\boldsymbol{\lambda}$ provides definite values for all possible observables.

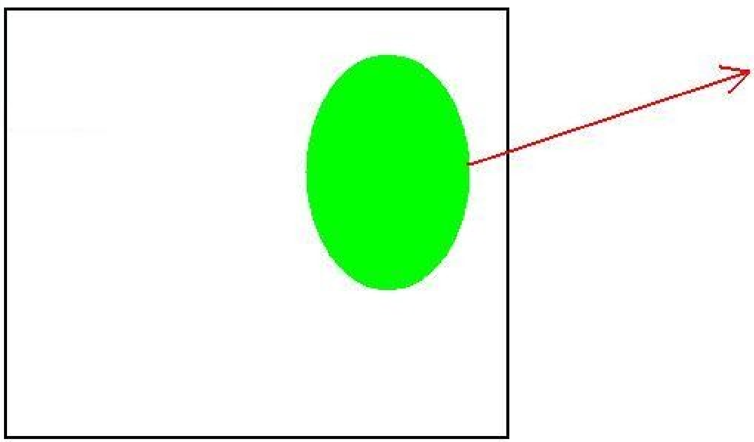
ψ Correcponds to specific distribution $\theta_{\psi}(\lambda)$ of $\boldsymbol{\lambda}$.

Different quantum states mean different distribution of $\boldsymbol{\lambda}$.

$$v_{\lambda}(A) = \textit{one of the eigenvalue of } A$$

$$< \psi|A|\psi > = \int \theta_{\psi}(\lambda) v_{\lambda}(A) d\lambda \quad \text{with} \quad \int \theta_{\psi}(\lambda) d\lambda = 1$$

$\lambda -$ space



Subset of $\lambda -$ space

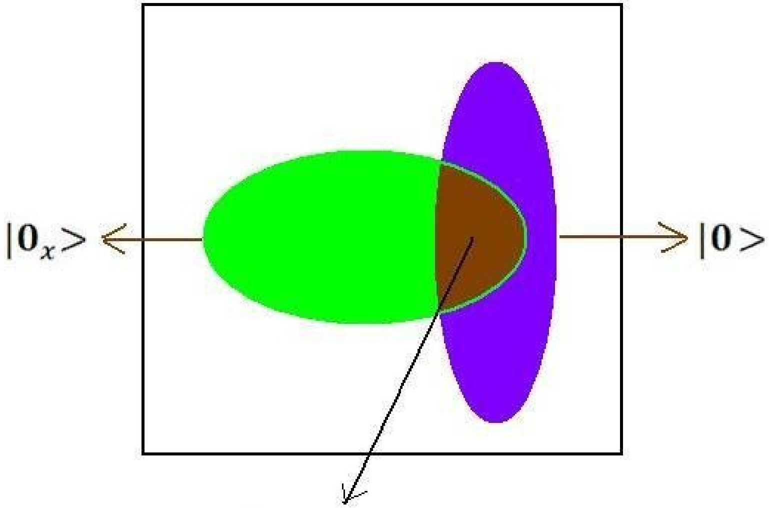
for which $\theta_{\psi}(\lambda) \neq 0$

Consider the following two states;

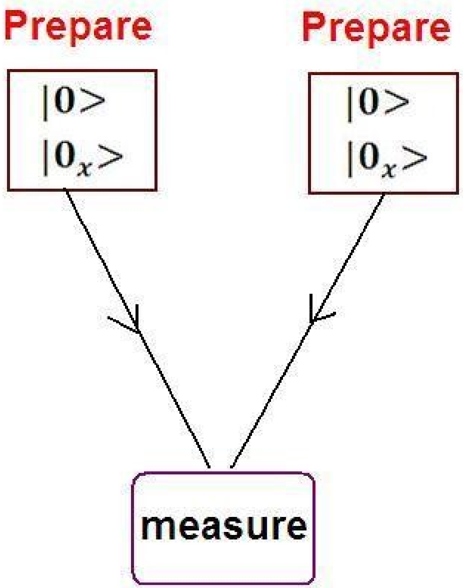
$$|0> \quad and \quad \frac{1}{\sqrt{2}}(|0> \; +|1>) \equiv |0_x>$$

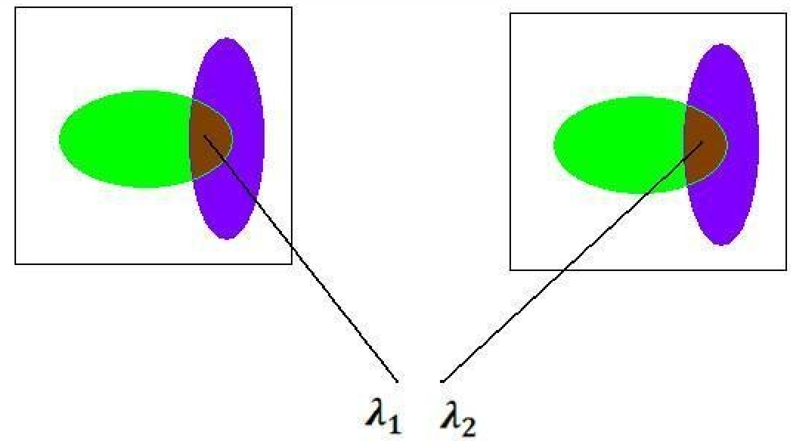
These two states can not be reliably distinguished by any operation in quantum mechanics

$\lambda -$ space



$$\theta_{|0>}(\lambda) \neq 0 \quad and \quad \theta_{|0_x>}(\lambda) \neq 0$$





For this joint HVT state those four states should not be reliably distinguished.

Possible quantum states: $|0\rangle|0\rangle$, $|0\rangle|0_x\rangle$, $|0_x\rangle|0\rangle$, $|0_x\rangle|0_x\rangle$

Measure in the orthogonal basis

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$$

$$|\psi\rangle_2 = \frac{1}{\sqrt{2}}(|0\rangle|1_x\rangle + |1\rangle|0_x\rangle)$$

$$|\psi\rangle_3 = \frac{1}{\sqrt{2}}(|0_x\rangle|1\rangle + |1_x\rangle|0\rangle)$$

$$|\psi\rangle_4 = \frac{1}{\sqrt{2}}(|0_x\rangle|1_x\rangle + |1_x\rangle|0_x\rangle)$$

Result	conclusion
$|\psi\rangle_1$	Not $|0\rangle|0\rangle$
$|\psi\rangle_2$	Not $|0\rangle|0_x\rangle$
$|\psi\rangle_3$	Not $|0_x\rangle|0\rangle$
$|\psi\rangle_4$	Not $|0_x\rangle|0_x\rangle$

Contradiction!

Information Causality Principle

and

Super quantum correlation

Consider the problem:

India vs England

To convey result of both the match **2 bits** is necessary.

India-junior vs Srilanka

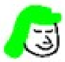
Encoding

WW	→	00
WL	→	01
LW	→	10
LL	→	11

Success with less than 2 bits would imply violation of causality

Information Causality principle

India vs England



But Alice communicates 1 bit.

India-junior vs Srilanka

Bob is asked to tell either the result of 1st match or the second match but not the both

Is the task possible with 1 bit of communication?

Information causality principle says the task is impossible.

– Technical version of Information Causality principle –

Alice sends m bits to Bob

Alice has n bits

$a_1 \; a_2 \; a_3 \dots \dots \dots . a_n$

Bob is given a number b where

$b \in \{1, 2, 3, \dots \dots . . n\}$

Bob has to determine the value of the b-th bit a_b

Let Bob's answer is β

The degree of success at this task is measured by

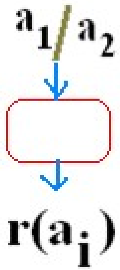
$$I = \sum_k I(a_k = \beta)|b = k)$$

where, I is the shannon mutual information between and $a_k = \beta$

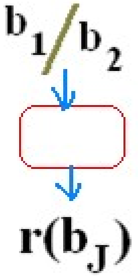
Infrmation causality principle tells,

$$I \leq m$$

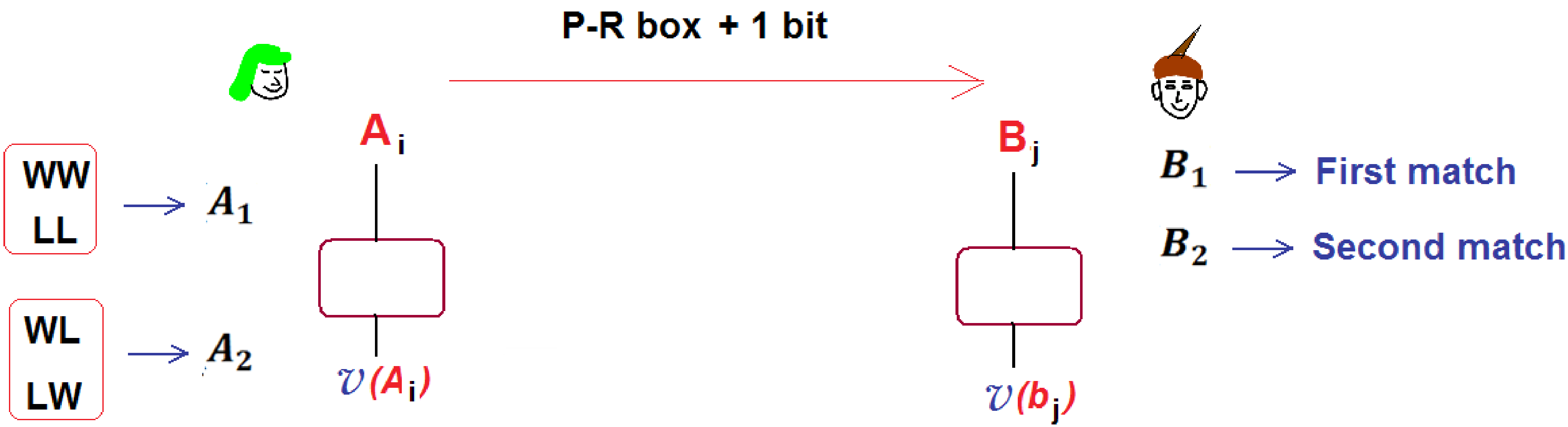
Non-signalling Correlation



$r(a_1)$	$r(b_1)$	Probability
+1	+1	$\frac{1}{2}$
−1	−1	$\frac{1}{2}$
$r(a_1)$	$r(b_2)$	
+1	+1	$\frac{1}{2}$
−1	−1	$\frac{1}{2}$
$r(a_2)$	$r(b_1)$	
+1	+1	$\frac{1}{2}$
−1	−1	$\frac{1}{2}$
$r(a_2)$	$r(b_2)$	
+1	−1	$\frac{1}{2}$
−1	+1	$\frac{1}{2}$



This is known as PR-Box correlation.



Alice sends the result if W appears first

and

flip the result and then sends if L appears first

by using one bit.

Bob's answer :

product of his result
and value sent by Alice

+1 → Win

-1 → Lost

– How it works –

Cases:

Result	Bob wants to learn			Bob's answer
		A_1	B_2	
WW	Second	+1	+1	+1
		-1	-1	+1
LL	first	A_1	B_1	
		+1	+1	-1
		-1	-1	-1
WL	Second	A_2	B_2	
		+1	-1	-1
		-1	+1	-1
LW	first	A_2	B_1	
		+1	+1	-1
		-1	-1	-1

- Quantum mechanics satisfies I-C principle.
- P-R box violates I-C principle.
- Any correlation that goes beyond Chirelsion bound violates I-C principle.