Spin based quantum computing
in Solid State Systems

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Outline

- Basic requirement on physical system to act as a quantum computer
- Quantum Dots: Basics, spin q-bits
- Controlled Spin-Orbit interaction in Nanosystem
- Scattering with SO interaction: Producing polarization from unpolarized source
- Coherent spin transport: Landauer-Buttiker theory
- Equilibrium and Non-Equilibrium spin current
  - Measurement of Spin currents
  - Equilibrium spin currents: Initializing solid state Q-bits
  - Increasing purity of state: Von-Neumann entropy
- Conclusion
Basic requirements: DiVincenzo Fortschr. Phys. 48, 771

- Information storage—the qubit: quantum property of a scalable physical system
- Initial state preparation: Initializing qubits to state 0
- Isolation: To avoid decoherence qubits must be free of all uncontrolled physical interaction, small system size
- Gate implementation: Manipulate the states of individual qubits as well to induce interaction between them in a controlled way; gate operation time $\tau_s \ll$ decoherence time $T_2$

$$\frac{\tau_s}{T_2} \ll r$$

r is the maximum error rate that can be tolerated for quantum error correction scheme to be effective

gating operation leads to time dependent inter-qubit interaction

- phase coherence far from thermodynamic equilibrium

- Readout: Measure the final state of qubits
Some Achievement in other areas

- **Cavity Quantum Electrodynamics**: P. Domokos et. al. PRA 52, 3554
  implementation of two-bit quantum logic gate using circular Rydberg
  atom and a superconducting millimeter wave cavity

- **Trapped ions**: Cirac J I and Zoller P PRL 74, 4091
  (ion mass = 105 * electron mass)

- **Nuclear Magnetic resonance**: Gershenfeld N A and Chuang I L
  Science 275, 350 (1997)

- **NMR liquid state implementation of Shor’s Algorithm**: Vandersypen et. al. Nature 414, 883 (2001)

- **Implementation of Grover’s algorithm**: Atomic Rydberg states

Most of these proposal rely on single multilevel system hence not scalable

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Solid State Nano-Systems ---- Scalabilty
Solid State Nanosystem: Quantum Dot

“Nano Box which can be filled with electrons”

FIG. 1. Schematic picture of a quantum dot in (a) a lateral geometry and (b) in a vertical geometry. The quantum dot (represented by a disk) is connected to source and drain reservoirs via tunnel barriers, allowing the current through the device $I$ to be measured in response to a bias voltage $V_{SD}$ and a gate voltage $V_G$.

- **2DEG at GaAs/AlGaAs interface**: 10nm thick sheet of electrons high mobility $10^5-10^7 \text{cm}^2/\text{Vs}$ and low electron density $(1-5) \times 10^{15} \text{m}^{-2}$

- Fermi Wave length 40nm, large screening length allows local depletion of 2DEG with an electric field

Hanson et. al. RMP 79, 1217 (2007) “Spin effects in Q-dots”
Spin qubits in Quantum Dots: Loss-DiVincenzo 1998, PRA 57, 120

- time dependent Heisenberg exchange coupling
  \[ H(t) = J(t) S_L \cdot S_R \]
  \[ \int J(t) dt / \hbar = J_0 \tau_s / \hbar = \pi (SWAP) \text{ for } t = \tau_s / 2 \text{ square-root swap} \]
  “pulsing of electrostatic barrier” controls \( J(t) \)

- time scale for rise/fall of \( J(t) \): \( \tau \gg 1/\omega \)
  \[ \hbar \omega \approx 1 \text{meV} \Rightarrow \tau \gg 1/\omega \approx 10^{-12} \text{sec} \]

- Array of exchange coupled q-dots

- Initialization: Applied magnetic field

- Single qubit operations: changing local Zeeman energy
**Mesoscopic Systems**: System Size $<<$ phase coherence length

discrete electronic spectrum

**Manifestation of Quantum effects in charge transport:**

AB Oscillation in rings, Conductance quantization, Coherent resonant Tunneling, Coulomb Blockade etc.

Conductance quantization:
(Wees et. al. PRL 60, 848)

\[
G = \frac{1}{R} = \frac{2}{r} = 2 \frac{e^2}{h}
\]

Conduction through two independent spin channels (classical)

\[
r_{\uparrow} = r_{\downarrow} = r = \frac{h}{e^2}
\]
Effect of electron Spin on transport:

In absence of external magnetic field it can only arise due to Spin-Orbit (SO) interaction, which is relativistic in origin.

In the rest frame of electron

\[ \vec{B} = -\frac{1}{c} \vec{v} \times \vec{E} = \frac{1}{m_0 c} \vec{E} \times \vec{p} \]

Energy = \[ -\vec{\mu} \cdot \vec{B} = -\frac{e h}{m_0^2 c^2} \bar{\sigma} \cdot (\vec{E} \times \vec{p}) \]
Spin-Orbit interaction in two dimensional system:

\[ H_{so} = \frac{\hbar}{8m_e c^2} [\sigma \cdot (p \times \nabla V) + \nabla V \cdot (\sigma \times p)] \]

Dimensionless parameter of spin orbit coupling:

- **Vacuum**
  \[ E(k)/m_0 c^2 \approx 10^{-6} \]

- **Semiconductor**
  \[ \Delta_{so}/E_G \approx 0.1 \]

**Two Dimensional Electron Gas:**

A 2DEG in xy plane: Confinement along z direction is strong, i.e.,

\[ \frac{dV}{dz} \gg \frac{dV}{dx}, \frac{dV}{dy} \]

\[ \nabla V \approx \hat{z} \frac{dV}{dz} \]

Electric field is parallel to z
if $V(z)$ is asymmetric with respect to reflection point $z=0$, then
\[
\alpha = \langle \psi(z) \frac{dV}{dz} | \psi(z) \rangle \neq 0
\]

Under these condition SO interaction becomes
\[
H_{so} = \frac{\hat{z}}{2\hbar} \left[ \alpha (\sigma \times p) + (\sigma \times p) \alpha \right] \equiv \alpha_R (\sigma_x k_y - \sigma_y k_x)
\]

Asymmetric confining potential in the direction perpendicular to 2DEG plane leads to SO interaction known as \textit{Rashba SO coupling} (\textit{Structure induced asymmetry})

Rashba So coupling can be tuned by an external gate voltage

- **Bulk Induced Asymmetry** (Dresselhaus)
  \[
  H_{so}^D = \alpha_D (\sigma_x k_x - \sigma_y k_y)
  \]

- **Impurity Induced SO coupling**
  \[\nabla V \rightarrow \text{due to heavy impurities}\]
Electronic properties of a 2DEG with Rashba effect

\[ H_R = \alpha (\vec{\sigma} \times \vec{k}) \]

spin-splitting in Rashba system
No magnetization
time-reversal symmetry

spin-splitting in a ferromagnet
Net magnetization  time reversal symmetry  is broken
Gate voltage control of Rashba coupling


\[ H = \frac{\hbar^2 k^2}{2m} + \alpha (\sigma_x k_y - \sigma_y k_x) \]

\[ \varepsilon(k)_{\pm} = \frac{\hbar^2 k^2}{2m} \pm \alpha k \]

\[ \Delta_R = 2\alpha k_F \]

• in III-V (InAs) \( \Delta_R \) is typically 3-5 meV

• in II-VI (HgTe) \( \Delta_R \) is typically 30 meV

(Molenkamp et. al. PRB 70, 115328 (2004))
Spin Field Effect Transistor: single qubit rotation

(Datta and Das, APL 56, 665(1990))

At a given energy two wave vectors due to spin-splitting phase difference

\[ H_{so} = \alpha (k \times \hat{z}) \cdot \sigma \equiv \alpha B_{eff} \cdot \sigma \]

\[ \theta = \Delta k L = \frac{2\alpha m L}{\hbar^2} \]

Transmission \( \propto \cos(2\theta) \)

Current can be modulated by tuning gate voltage
Prerequisites for realizing Spin transistor

• injection of spin polarized current
• spin coherent propagation
• induction of controlled spin precession
• spin selective collection

This works only if the spin is injected from injector ferromagnet

Injector and Detector Ferromagnet are metallic, Fe

• 2DEG is semiconductor InAs

Conductivity mismatch forbids spin injection

(G. Schmidt et. al. PRB 62, R 4790 (2000))
Conductivity mismatch can be overcome by using DMS (GAMnAs) (G. Bouzerar & T. P. Pareek, PRB 65, 153202 (2002))

Stray field of Ferromagnetic injector influences the spin dynamics which is undesirable.

can one avoid magnetic contacts and magnetic field to generate spin currents intrinsically?
"Generating spin currents intrinsically through SO coupling"

Scattering with SO interaction: \[ H = H_0 + V \]

Scattering operator in spin space \( M \): \( (\sigma, k_i, k_f) \)

Rotational invariance requires \( M \) to be scalar or pseudoscalar

\[
M = g_1 + \sigma \cdot (k_i \times k_f) g_2 + \sigma \cdot (k_i + k_f) g_3 + \sigma \cdot (k_i - k_f) g_4
\]

\[
M(k_f, k_i; s_1 s_2) = M(-k_f, -k_i; s_1, s_2) \quad \text{Reflection invariance, } g_3 = g_4 = 0
\]

\[
M(k_f, k_i; s_1 s_2) = M(-k_f, -k_i; -s_1, -s_2) \quad \text{Time reversal invariance, } g_4 = 0
\]
Polarization of scattered beam:

\[ P_f = \text{Trace}(\sigma \rho_f) \quad , \quad \rho_f \propto M \rho_i M^\dagger \]

Rashba \quad \mid \quad \text{Dressulhaus} \quad \mid \quad \text{Impurity induced}

\[ H_R = \lambda (\sigma_x k_y - \sigma_y k_x) \quad H_D = \lambda_d (\sigma_x k_x - \sigma_y k_y) \quad H_{\text{imp}} = \lambda_I \sigma_z (p_y \nabla_x V - p_x \nabla_y V) \]

time reversal invariance only scattered beam has polarization in the plane as well perpendicular to the scattering plane

scattered beam is polarized perpendicular to the scattering plane
If incident beam is polarized

Asymmetric scattering

\[ \text{scattering cross section} \propto (\mathbf{n} \times \mathbf{n}') \cdot \mathbf{P}_{\text{in}} \]

Left – Right asymmetry in scattering of up – down electrons

For two dimensional systems scattering plane is fixed

these effects of SO scattering is maximized
Landauer-Büttiker theory for spin transport:


- Scattering region connected to N reservoirs via perfect leads
  **Lead m:**
  - Number of Channels: $N_m^\sigma$ and $N_m^{-\sigma}$
  - Reservoir injects carriers with Fermi distribution: $f_m(E) = \frac{1}{e^{(E-\mu_m)/k_B T} + 1}$
  - Spin quantization axis $\hat{n}$, eigensate of operator $(\sigma \cdot \hat{n})$
  - Spin Resolved Transmission and Reflection coefficients: $T^n_\sigma m^\sigma$, $R^n_\sigma m^{-\sigma}$

- Charge current in spin channel $\sigma$ that impinges on sample from lead $m$
  \[ I_{m m}^\sigma = (e^2 / h) \left[ N_m^\sigma - (R_{m m}^\sigma + R_{m m}^{-\sigma}) \right] V_m \]

**Charge Conservation**  \( \Rightarrow \)  $I_{m m}^\sigma = \sum_{n \neq m, \alpha} I_{n m}^{\alpha \sigma} = (e^2 / h) \sum_{n} T_{n m}^{\alpha \sigma} \]

- Current leaves the sample through other leads
  \[ I_{m m}^\sigma = (e^2 / h) \sum_{n \neq m} T_{n m}^{\alpha \sigma} V_n \]

- Lead $n$ causes a current
  \[ I_{n m}^{\alpha \sigma} = (e^2 / h) \sum_{\alpha} \left[ T_{n m}^{\alpha \sigma} V_m - T_{m n}^{\alpha \sigma} V_n \right] \]

- Spin current $\sigma$ in lead $m$
  \[ I_{m m}^\sigma = (e^2 / h) \sum_{n \neq m, \alpha} \left[ T_{n m}^{\alpha \sigma} V_m - T_{m n}^{\alpha \sigma} V_n \right] \]
The net spin and charge currents are:

\[ I^s_m = I^\sigma_m - I^{-\sigma}_m \equiv e^2 / h \sum_{n \neq m, \alpha} \left\{ (T_{n m}^\sigma - T_{n m}^{-\sigma}) V_m + (T_{m n}^{-\sigma} - T_{m n}^\sigma) V_n \right\} \]  

(1)

\[ I^q_m = I^\sigma_m + I^{-\sigma}_m \equiv e^2 / h \sum_{n \neq m, \alpha, \sigma} (T_{n m}^\sigma V_m - T_{m n}^\sigma V_n) \]  

(2)

**Time reversal symmetry and Gauge invariance**

\[ \Rightarrow T_{n m}^{\alpha \sigma} = T_{m n}^{-\sigma -\alpha} \]
\n\[ \sum_n T_{n m} = \sum_n T_{m n} \]

**Absence of SO interaction:**

\[ T_{n m}^{\alpha \sigma} = T_{n m}^{-\alpha -\sigma} \text{ rotational symmetry in spin space} \]

\[ T_{n m}^{-\sigma \sigma} = 0 \quad (\text{spin flip transmission are zero}) \]

**In absence of SO interaction and magnetic element in the device spin currents are identically zero for all terminals**
Equilibrium spin currents: \( V_m = V_0 \ \forall m \)

**Time reversal and Gauge invariance** \( \Rightarrow T_{nm}^{\alpha \sigma} = T_{mn}^{-\sigma -\alpha} \)

\[
I_{m(eq)}^s(\hat{n}) = \frac{\hbar}{2} \sum_{n \neq m, \alpha} (T_{mn}^{\alpha \sigma} - T_{mn}^{-\sigma -\alpha}) V_0 \neq 0
\]

- Two terminal case:

\[
I_{1(eq)}^s(\hat{n}) = \frac{\hbar}{2} (T_{21}^{\uparrow \uparrow} - T_{21}^{\uparrow \downarrow} + T_{12}^{\downarrow \uparrow} - T_{12}^{\downarrow \downarrow}) V_0 \neq 0
\]

- Detection of Equilibrium spin current

- Consequences: \( \tau_{\text{spin}} = I_{eq}^s \hat{n} \)

**Torsional torque in nanoelectromechanical structure (NEMS)**
•Measurement of Equilibrium Spin Currents

suspended NEMS torsion balance to which the nanowire is rigidly attached

\[ J \frac{d^2 \theta}{dt^2} + \gamma \frac{d \theta}{dt} + K \theta = \tau_{\text{spin}} \]

\( J \) (moment of inertia), \( \gamma \) (frictional damping) \( K = (\pi G/2L)R^4 \)

\( G = \) shear modulus of oscillator, \( R \) (radius), \( L \) (length)

If nanowire is fabricated on top of torsion oscillator

torque generated in nanowire will translate to a torque in entire structure modifying mechanical parameter \( G \), \( J \) and resonance frequency

Twoelement torsional GaAS resonator:
lateral extent 25 (micrometer), thickness (800nm)


\[ L \approx 10 \mu m, n \approx 3 \times 10^{12} / cm^2 \Rightarrow \tau_{\text{spin}} \approx 10^{-23} Nm \]

minimum detectable torque at 4K is \( \approx 48 \times 10^{-23} Nm \)

•Eq. Spin currents are pure angular momentum transfer

Initialization of quantum dot Q-bits....
Y – Junction

Techniques:

• **Recursive Green's function method**

\[
G_{rp}^{L+R} = G_{r,p+1}^{L} V_{p+1,p} \left( 1 - G_{pp}^{L} \Sigma_{p}^{R} \right)^{-1} G_{pp}^{L}
\]

\[
G_{pp}^{L+R} = (1 - G_{pp}^{L} \Sigma_{p}^{R})^{-1} G_{pp}^{L}
\]

\[
\Sigma_{p}^{R} = V_{p,p+1} G_{p+1,p+1}^{R} V_{p+1,p}
\]

Transmission probability: \[ T^{\alpha\beta} = \text{Trace} \left( \Gamma_{L}^{\alpha} G_{LR}^{\alpha\beta} \Gamma_{R}^{\beta} G_{LR}^{\beta\alpha} \right) \]
• Non Equilibrium Pure Spin Current:

• Terminal 3 is a Voltage probe

\[ I_3^q = 0 \quad , \quad \frac{V_3}{V_2} = \frac{T_{32}}{T_{13} + T_{23}} \]

\[ I_3^s(\hat{n}) = \frac{\hbar}{2} \sum_{\alpha} \left[ (T_{13}^{\alpha\sigma} - T_{13}^{\alpha\bar{\sigma}} + T_{23}^{\alpha\sigma} - T_{23}^{\alpha\bar{\sigma}}) V_3 + (T_{32}^{\alpha\sigma} - T_{32}^{\alpha\bar{\sigma}}) V_2 \right] \neq 0 \]

• Terminal 3 is Non-Magnetic

\[ \tau_{so}/\tau_{el} \approx 5 - 10 \quad , \quad \lambda_{so} \approx 0.03 - 0.07 \text{ (impurity induced SO)} \]
Terminal 3 is Magnetic: \[ \frac{\Delta_{ex}}{E_F} = 0.5 \]
Scattering theory for Spin Density Matrix

Mixed states: Non Magnetic and Ferromagnetic contacts

\[ \psi_{in} = \left| n, \alpha \right> \]
\[ \psi_f = T \left| n, \alpha \right> \equiv \sum_{m, \beta} \left| m, \beta \right> \left< m, \beta \right| T \left| n, \alpha \right> \]

\[ \rho_f = \frac{1}{N} \left| \psi_f \right> \left< \psi_f \right| \]

Density Matrix:

\[ \rho_f^{n, \alpha} = \frac{1}{N} \left| \psi_f \right> \left< \psi_f \right| , \text{ incident density matrix} \]
\[ \rho_{in} = n_{\alpha} \left| \alpha \right> + n_{-\alpha} \left| \alpha \right> \]

\[ \Rightarrow \rho_f = n_{\alpha} \rho_f^{n, \alpha} + n_{-\alpha} \rho_f^{n, -\alpha} , \quad P_i = Tr(\sigma_i \rho_f) \text{ where } i = x, y, z \]

Non-Magnetic as well Magnetic systems can treated at same footing

Emergence of Classicality: Quantum to Classical Cross Over

Quantum information: Quantum Entropies

T. P. Pareek
PRB 75, 115308 (2007)
• Absolute Anisotropic Magnetoresistance and Non Equilibrium Spin Currents

Magnetoresistance:
TMR, GMR, BMR occurs in absence of SO interaction
relative orientation of FM1 and FM2

These Magnetoresistance vanishes if FM1 and FM2 are parallel

• Presence of SO interaction:
\[ \vec{M}_1 = \vec{M}_2 = M(\theta, \phi) \]
resistance depends on the absolute direction of magnetization

• Analogue electronics ???

T. P. Pareek
PRB, 75, 115308 (2007)
PRB 70, 033310 (2004)
PRB 66, 193301 (2002)
System with one FM (NM-NM-FM)

- Injected current is unpolarized
- Outgoing current gets polarized due to SO interaction
- Resistance will depend on absolute direction of magnetization
  - Resistance modulation will be proportional to the polarization of outgoing current
- Non Equilibrium Spin Current can be measured via electrical means in two terminal system
  - If both contact are non-magnetic:
    - It will give rise to torque in NEMS system
    - Mechanical means allows to detect spin current
Generation and measurement of non-equilibrium spin currents in two terminal system

<table>
<thead>
<tr>
<th>Injected currents is unpolrized</th>
<th>Outgoing current is polarized</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) NM Lead I</td>
<td>S. O.</td>
</tr>
<tr>
<td>(b) NM Lead I</td>
<td>S. O.</td>
</tr>
<tr>
<td>Magnetic I</td>
<td></td>
</tr>
</tbody>
</table>

\[
m(\theta, \varphi)
\]

\[
\begin{array}{c}
G(e/h) \\
\hline
25.6 \\
25.5 \\
25.4 \\
25.3 \\
25.2 \\
25.1 \\
\hline
\theta \text{ or } \varphi \text{(degree)}
\end{array}
\]

\[
\begin{array}{c}
\text{Polarization} \\
\hline
0.23 \\
0.225 \\
0.22 \\
0.215 \\
0.21 \\
0.205 \\
\hline
\theta \text{ or } \varphi \text{ (degree)}
\end{array}
\]

T. P. Pareek & A. M. Jayannavar/cond-mat-0707.1367
Magnetic Random Access Memory and SO induced torque:

In absence of SO interaction: For non collinear configuration of FM1 and FM2 torque acts on FM2 which leads to magnetization reversal.

- Fast Magnetic Memory: Magnetization switching time should be small.
- For collinear configuration: switching time goes to infinity for SO=0.
- In presence of SO coupling torque is non zero even for collinear configuration, switching time becomes finite.

- System with one FM:
  
  Pure charge current can switch the magnetization.

\[
\omega_m = 10^{11} \text{ / sec}
\]

\[
j = 10^{13} \text{ A/m}^2
\]
Von-Neumann Entropy and Generating Polarization:

- Polarization generation from unpolarized source
- Purity of outgoing beam is increased
- Entanglement is generated
Noise

``The Noise is Signal'' --- Rolf Landauer

Thermal Noise --- Fluctuation in Equilibrium; gives information about temperature $T$

Noise

Shot Noise: Discreteness of electrical charge
Provides information not contained in average quantities

Characterization:

$$S(f) = \frac{\langle \delta I(f)^2 \rangle}{\Delta f} \quad I \text{--- Current} \quad f \text{--- frequency}$$

if charge is transferred in independent units of $q \rightarrow S = 2qI_{av}$

Fano Factor, $F = \frac{S}{(2eI_{av})}$

Fractionality of elementary charge in fractional quantum Hall system was established through Shot noise measurement

- Symmetry of Wave function ⇔ Statistical properties of scattering

Noise in Spin transport?
• Noise in Charge transport:

Fluctuation spectra of incident, transmitted and reflected currents are correlated

\[ S_i = \frac{e^2}{\pi \hbar} \int dE f (1 - f), \]
\[ S_T = \frac{e^2}{\pi \hbar} \int dE T f (1 - T f), \]
\[ S_R = \frac{e^2}{\pi \hbar} \int dE R f (1 - R f) \]

\[ S_{TR} = \frac{e^2}{\pi \hbar} \int dE T f \ R f \]

• Noise in Spin transport:

Noise correlators:
- same lead different spin component
- different leads

\[ S_{m,n}^{\sigma \sigma'}(t - t') = \frac{1}{2} \langle \Delta I_m^\sigma(t) \Delta I_n^{\sigma'}(t') + \Delta I_n^{\sigma'}(t) \Delta I_m^\sigma(t) \rangle \]

pure spin exchange effect
• Entanglement Generation and Detection

  two particle property $\rightarrow$ multi terminal system
  spin shot noise correlation between different terminals

• Dissipation in Spin transport

Quantum To Classical Crossover: Spin density matrix

Power Dissipation: work done and heat dissipation with applied bias

  spin dynamics (LLG approach)  Jarzynski and other fluctuation theorem for systems far from equilibrium

• Coulomb interaction: Hubbard Model with SO interaction
Summary:

- Landauer-Buttiker theory for spin and charge transport
- SO scattering and its polarizing property
- Spin density matrix scattering theory: magnetic and nonmagnetic leads
- Intrinsic Spin Currents: Equilibrium and Non-Equilibrium
- Solid state Q-bit initialization using Equilibrium spin currents
- Increasing purity of state in solid state system
- Fast Magnetic memory: SO induced torque
Thank you!
EquiSpin Transport with Rashba coupling

\[ \phi = 90^\circ; \alpha_R = 0.157; W/L = 1.0 \]

\[ I_z, S, I^\uparrow, S^\uparrow, I^\downarrow, S^\downarrow \]

\[ \theta = 0, 90, 180, 270, 360 \]
Some other aspects which are being pursued

- Entanglement Generation and detection
- Power Dissipation: In irreversible computation minimum heat dissipated is \( k_B T \ln(2) \). This limit is based on equilibrium consideration. Our approach is based on Jarzynski and other inequalities appropriate for non-equilibrium situations.
- Solid State qbit gates using SO interaction
- Quantum to Classical crossover
Spin transport in sandwitched CNT:

\[
\frac{dI}{dV} \propto T (E_f)^2 \ n_1(E_f) \ n_2(E_f)
\]
• Landauer-Büttiker theory for spin transport

• Equilibrium spin currents and its measurement

• Non-Equilibrium Pure spin currents and its measurement

• Spin transport in CNT
Y – Junction:

1. NM
2. NM
3. NM or FM

Incident wave vector
Scattered wave vector
Polarization direction

y
z
x

\( d \)
\( d/2 \)