Teleportation and Broadcasting of continuous variable entanglement

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- *Teleportation of two-mode squeezed states*, S. Adhikari, ASM, N. Nayak, Phys. Rev. A **77**, 012337 (2008).
- *Broadcasting of continuous variable entanglement*, S. Adhikari, ASM, N. Nayak, arXiv:0708.1869; to appear in Phys. Rev. A (2008).

Continuous variable systems

- Quantum mechanics in infinite dimensional Hilbert spaces
  [First consequence of entanglement on quantum ontology--EPR]
- Essential technique for entanglement in quantum optical systems
  [Vast applicability for information processing through photonics]
- Rapid development of formalism and applications in recent years
  [e.g., coding, communication, quantification of entanglement]
- Recent state-of-art – interesting comparison with discrete variables
  [Progress in understanding and manipulating Gaussian states]

Manipulation of entanglement in continuous variable systems

- Quantum entanglement ***not*** freely shared by several systems:
  (entanglement swapping and monogamy) [Bennett (2003)]

- Copying of local information:
  (exact cloning of unknown state impossible) [Wootters, Zurek (1982)]
  Goal: specific input states leading to optimal output *fidelity*

- Various schemes for copying local continuous variable information
  (e.g., Duplication of coherent states) [Braunstein et al., (2001)]

- Purification of output modes by employing a number of copies
  (super-broadcasting using arrays of amplifiers, beam-splitters)

Transfer of continuous variable entanglement

- ***Broadcasting of entanglement:***

  Whether entanglement shared by two parties can be transmitted to two less entangled states by local operations ?

- *Process involves copying of local information*

  [Status for discrete variables: possible for restricted input states.

  Buzek (1997); Bandopadhyay, Kar (1999)]

- *Telecloning:* Clones generated locally and teleported to a distant location by previously shared entanglement [van Loock, Braunstein (2001)]

- ***Q***☺ : *Whether ideas for copying local information can be extended for mapping entangled and non-local states ?*

Teleportation of continuous variables

- ***Quantum teleportation:*** Replication of unknown quantum state at distant location using previously shared entanglement and LOCC

(vital quantum information processing task– can be combined with other operations to construct advanced quantum circuits)

- Schemes for teleportation of gaussian and non-gaussian states

  [Vaidman (1994); Braunstein, Kimble (1998); Adesso, Illuminati (2005)]
- Experimental continuous variable teleportation: *Furusawa et al. (1998)*

- Issue in practical teleportation: *Fidelity of entanglement*

  (loss of fidelity in non-maximally entangled channels)
- Suggested improvement of fidelity through various schemes

  [Kimble et al. (2003); Zhang et al. (2005)]

Transportation of entanglement

- Establishing entanglement between distant locations: challenging
  [quantum entanglement fragile: easily destroyed in distribution]

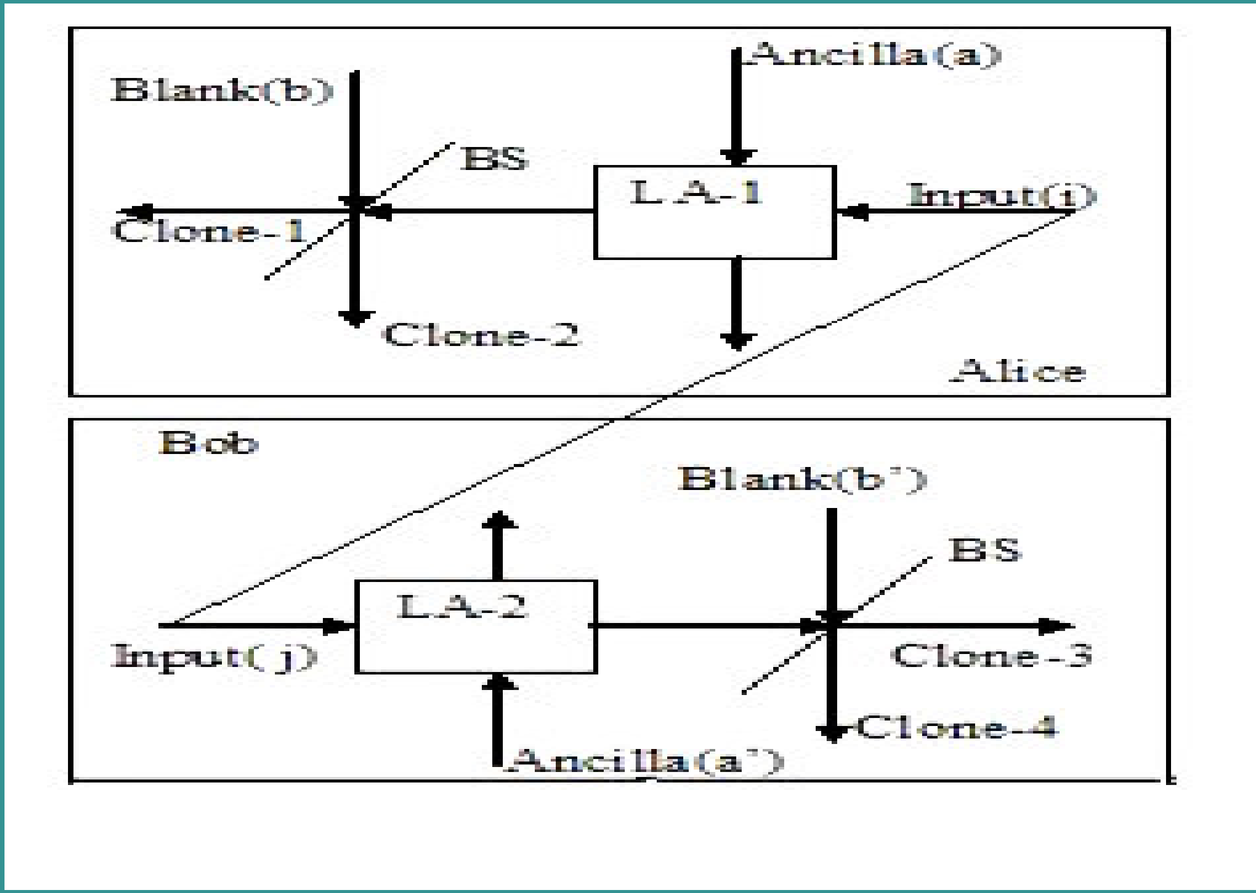
- ***Various methods:***

1. Entanglement swapping [Zeilinger et al. (1993)]
2. Quantum repeaters (swapping with purification) [Cirac et al. (1998)
3. Combining teleportation with cloning [van loock, Braunstein (2001)]

NO explicit protocol for teleportation of continuous variable Entanglement

[For discrete variables, c.f., Schumacher (1996)]

Broadcasting Protocol



Local cloning

- Single-mode squeezed vacuum state
  (r : squeezing parameter)
- Covariance matrix (CM)

$$S_i(r)=\begin{pmatrix} e^{r} & 0 \\ 0 & e^{-r} \end{pmatrix}$$

$$\sigma_i(r)=S_i(r)S_i^T(r)=\begin{pmatrix} e^{2r} & 0 \\ 0 & e^{-2r} \end{pmatrix}$$

- Mode i + Ancilla a
  ⇨ linear amplifier
  (φ: amplifier phase)

$$A_{ia}(r,\varphi)=[S_{ia}(r,\varphi)].[S_i \oplus I_a]$$

- Mode i + Ancilla a + Blank b ⇨ beam splitter
  ⇨ two clones

$$B_{iab}=\begin{pmatrix} \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} \end{pmatrix}$$

Covariance matrix formalism

- Two mode state (*x1,p1*), (*x2,p2*)
- Any CM Γ can be brought to the standard form

$$\Gamma=\begin{bmatrix} \mathrm{var}(x_1) & 0 & \mathrm{cov}(x_1,x_2) & 0 \\ 0 & \mathrm{var}(p_1) & 0 & \mathrm{cov}(p_1,p_2) \\ \mathrm{cov}(x_1,x_2) & 0 & \mathrm{var}(x_2) & 0 \\ 0 & \mathrm{cov}(p_1,p_2) & 0 & \mathrm{var}(p_2) \end{bmatrix}$$

  det(A), det(B), det(C), det Γ are invariants w.r.t local sympl-ectic transformations. (*A = B* for pure Gaussian states).

$$\Gamma=\begin{bmatrix} A & C \\ C^T & B \end{bmatrix}$$

- Canonical commutation relations in symplectic form:

$$[\hat{x}_i,\hat{x}_j]=2i\Omega_{ij} \quad \Omega=\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \quad \omega=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- Positivity of density matrix: Necessary and sufficient for Γ to represent a physical Gaussian state

$$\Gamma+i\Omega\geq 0$$

- In terms of symplectic eigenvalues:

$$\nu_i>1$$

Entanglement of Gaussian states

- *Take partial transpose of CM* Γ

  Γ^T can be obtained by changing the sign of the momentum of any one of the two modes
- *Compute symplectic eigenvalues*

$$\tilde{\nu}_{\pm}^{2} = \frac{\Delta\left(\Gamma^{T}\right) \pm \sqrt{\left[\Delta\left(\Gamma^{T}\right)\right]^{2} - 4\det\Gamma}}{2}$$

- ***PPT*** *criterion: State is entangled if*

$$\tilde{\nu}_{-} < 1$$

- *Measure of entanglement: Logarithmic negativity*

$$E_N = \max\left[0, -\log_2 \tilde{\nu}_{-}\right]$$

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Fidelity of clones

- CM of the output modes:

$$\sigma_{out} = \begin{pmatrix} P & 0 \\ 0 & M \end{pmatrix}$$

$$P = \left(e^{2r}(c - hs)^2 + k^2 s^2 + 1\right)/2$$

$$M = \left(e^{-2r}(c + hs)^2 + k^2 s^2 + 1\right)/2$$

$$c = \cosh(2r) \qquad s = \sinh(2r) \qquad h = \cos(2\varphi) \qquad k = \sin(2\varphi)$$

- Fidelity:

[Olivares et al (2006)]

$$F = \frac{1}{\sqrt{Det[\sigma_{in} + \sigma_{out}] + \delta} - \sqrt{\delta}}$$

$$\delta = 4\left(Det[\sigma_{in}] - 1/4\right)\left(Det[\sigma_{out}] - 1/4\right)$$

- State-dependent cloning:

F ⟹ *0* as *r* ⟹ ∞

F ⟹ *1* as *r* ⟹ *0*

Continuous variable entangled states

- Two-mode squeezed vacuum state: Bipartite entangled

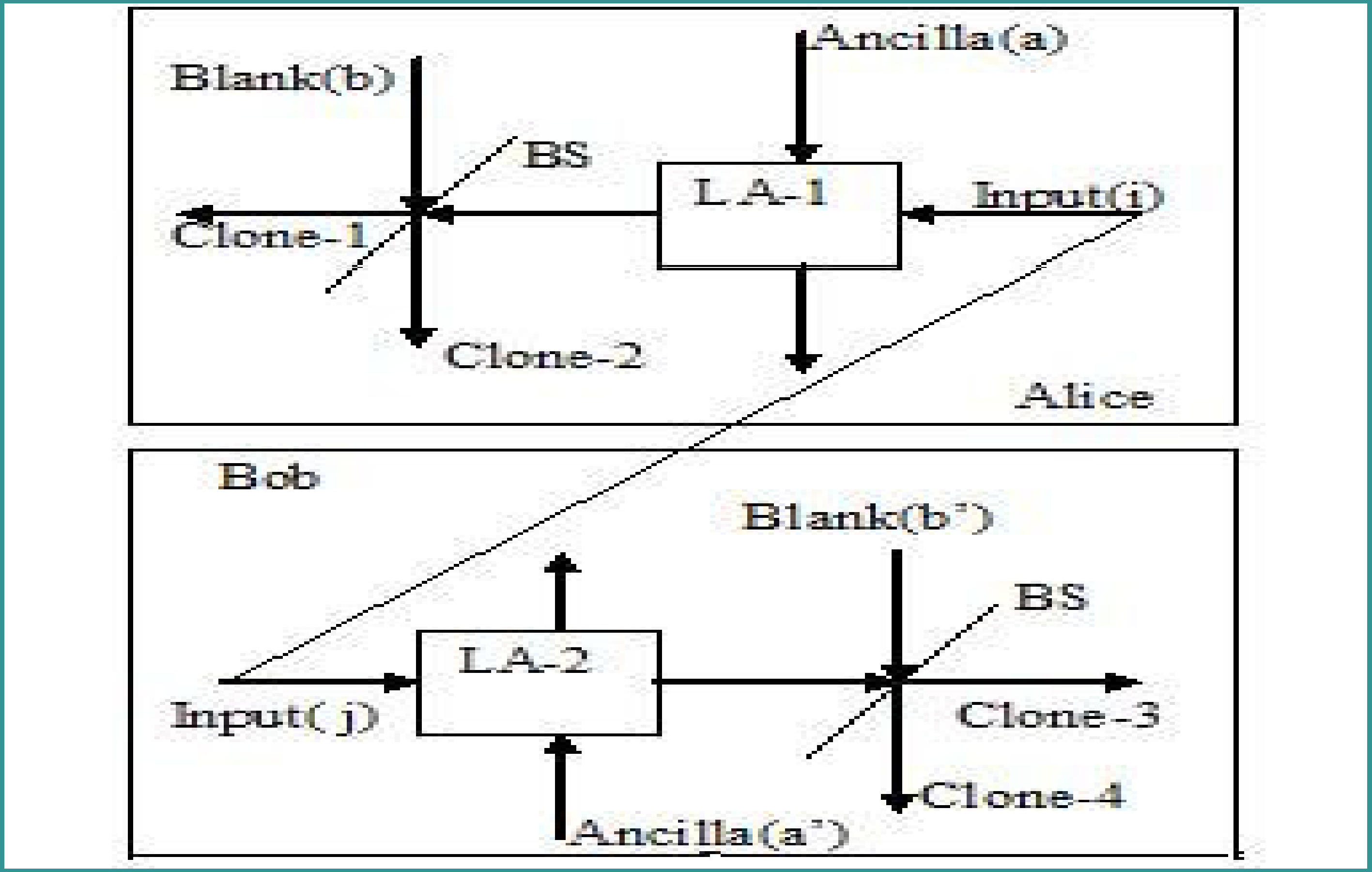
$$\sigma_{ij}(r) = \begin{pmatrix} c & 0 & s & 0 \\ 0 & c & 0 & -s \\ s & 0 & c & 0 \\ 0 & -s & 0 & c \end{pmatrix}$$

- Quadrature operators in Heisenberg picture

$$\hat{x}_i = \frac{e^r x_i^{(0)} + e^{-r} x_j^{(0)}}{\sqrt{2}}; \quad \hat{x}_j = \frac{e^r x_i^{(0)} - e^{-r} x_j^{(0)}}{\sqrt{2}}$$

$$\hat{p}_i = \frac{e^{-r} p_i^{(0)} + e^r p_j^{(0)}}{\sqrt{2}}; \quad \hat{p}_j = \frac{e^{-r} p_i^{(0)} - e^r p_j^{(0)}}{\sqrt{2}}$$

Broadcasting Protocol



Broadcasting of entangled states

- Input modes: Alice(*i*);Bob(*j*):Output modes:Alice(*i,a,b*);Bob(j,a′,b′)
- ***Implementation of broadcasting:***

Both nonlocal pairs of output modes (*i & b′*) and (*j & b*) entangled

Local pairs of output modes (*i & b* with Alice) and (*j & b′*) with Bob form separable states (*bipartite entanglement)*

Output modes are physical states (*obeying uncertainty principle)*

Joint (bi-)local cloning operation

$$A(r,\varphi)=\begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$$

- Two linear amplifiers

$$A_i=\begin{pmatrix} c-hs & 0 & ks & 0 \\ 0 & c+hs & 0 & -ks \\ ks & 0 & c+hs & 0 \\ 0 & -ks & 0 & c-hs \end{pmatrix}$$

$$B=\begin{pmatrix} B_{iab} & 0 \\ 0 & B_{ja'b'} \end{pmatrix}$$

- Two beam splitters

$$B_{iab}=\begin{pmatrix} \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} \end{pmatrix}$$

Output modes of broadcasting

- Local modes: Alice Bob

$$\sigma_{ib}^{local}(r,\varphi)=\sigma_{jb'}^{local}(r,\varphi) = \begin{pmatrix} \frac{G+1}{2} & 0 & \frac{G-1}{2} & 0 \\ 0 & \frac{H+1}{2} & 0 & \frac{H-1}{2} \\ \frac{G-1}{2} & 0 & \frac{G+1}{2} & 0 \\ 0 & \frac{H-1}{2} & 0 & \frac{H+1}{2} \end{pmatrix}$$

- Non-local modes:

$$\sigma_{ib'}^{nonlocal}(r,\varphi)=\sigma_{jb}^{nonlocal}(r,\varphi) = \begin{pmatrix} \frac{G+1}{2} & 0 & \frac{E}{2} & 0 \\ 0 & \frac{H+1}{2} & 0 & \frac{-E}{2} \\ \frac{E}{2} & 0 & \frac{G+1}{2} & 0 \\ 0 & \frac{-E}{2} & 0 & \frac{H+1}{2} \end{pmatrix}$$

$$G=(c-hs)^2c+ks^2$$
$$E=s(c-hs)^2$$
$$H=(c+hs)^2c+ks^2$$

Conditions for Broadcasting

- Entanglement of nonlocal modes (symplectic eigenvalues of partial transpose of output CM) *[Adesso, Illuminati (2007)]*

$$\tilde{\nu}_- = [(G+1)(H+1)+E^2-E(G+H+2)]/4 < 1$$

$$G \geq 1 \quad (H \geq 1)$$
$$G < H \quad (H < G)$$

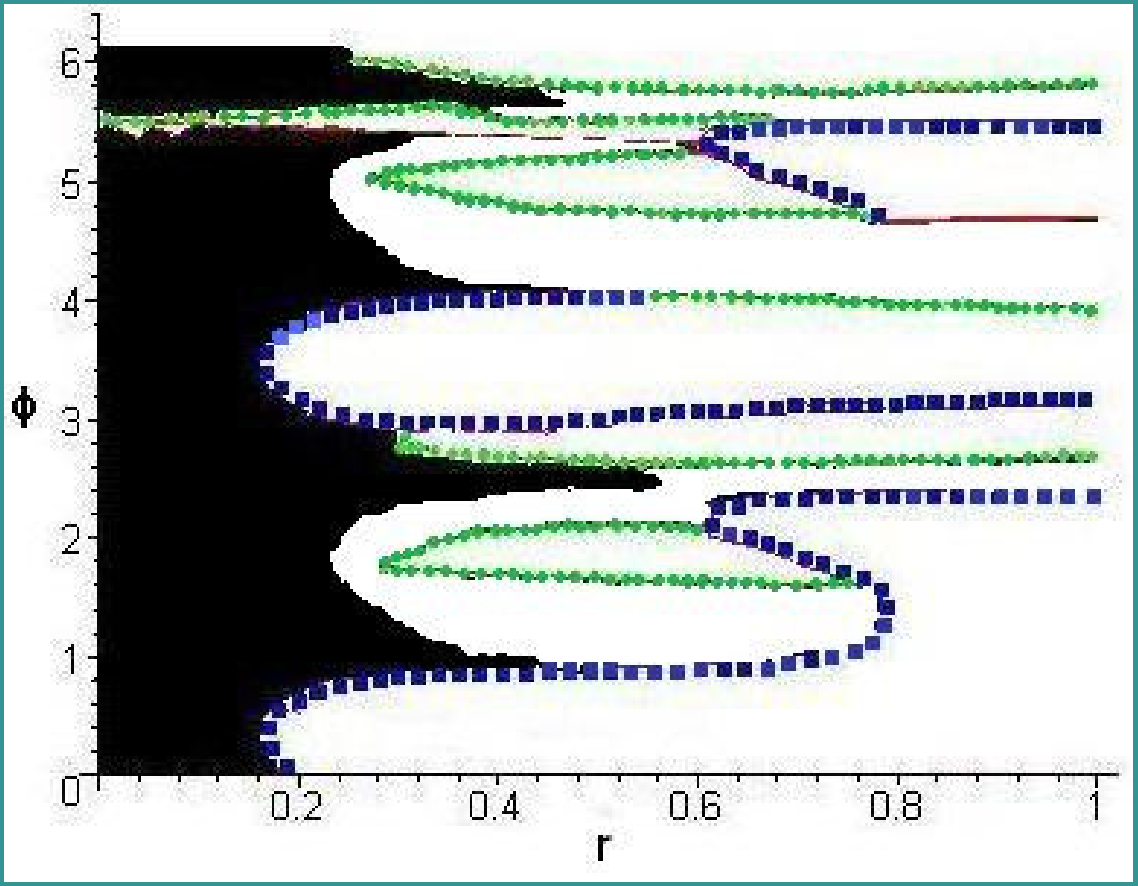
- Separability of local modes (PPT criterion) *[Simon (2000)]*

$$\sigma + iJ \geq 0 \qquad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- Physicality of output modes (*Ouput states **not** physical for certain values of squeezing and phase)*

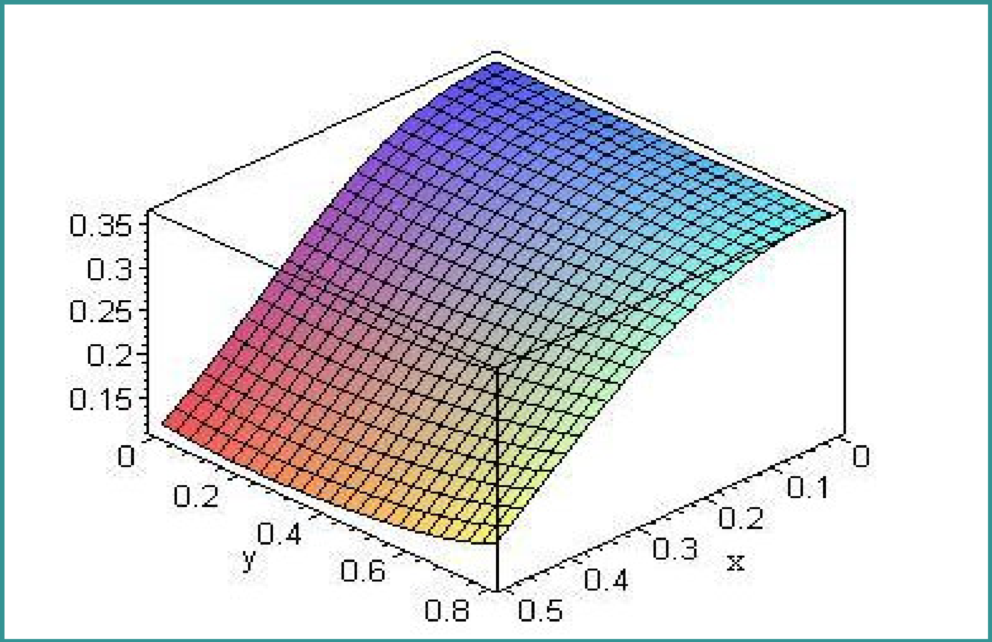
$$\nu_-^2 = [(G+1)(H+1)-E^2 \pm E(G-H)]/4 > 1$$
$$(H-G) \pm ve$$

Range of broadcasting



Fidelity of Broadcasting

- $F \Rightarrow 0$ as $r \Rightarrow \infty$
  $F \Rightarrow 0.36$ as $r \Rightarrow 0$

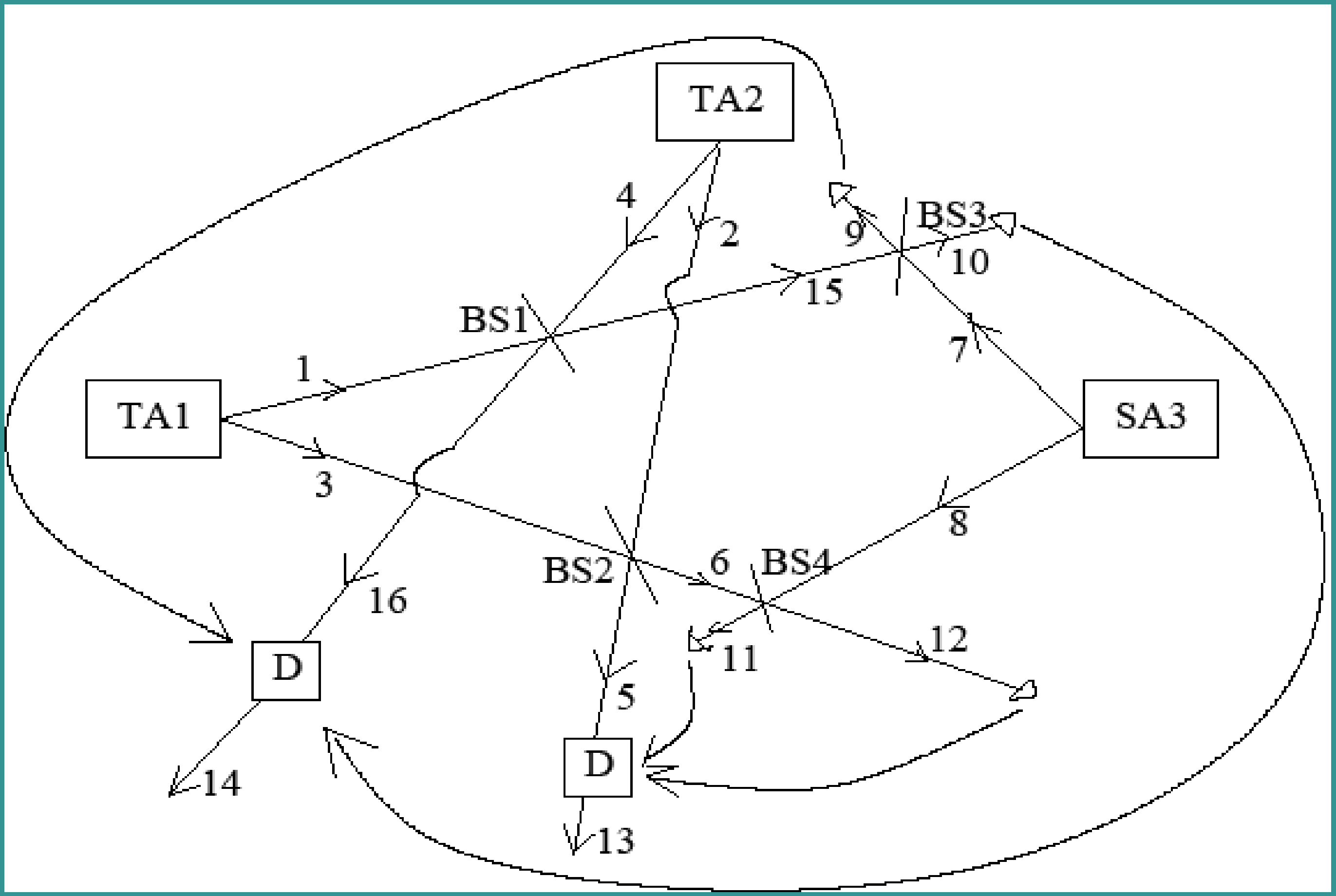


Teleportation of two-mode squeezed states

- Teleportation protocol: proceeds in the usual way
- Alice and Bob share four-mode entangled state
- Alice makes measurements on her side
- She communicates **four** bits of classical information to Bob

  [similar to teleporting two-qubit states: Yeo,Chua (2006)]
- Bob makes local operations to retrieve the two-mode squeezed state

Teleportation protocol



Generation of four-mode entangled state

- Input from Teleportation amplifiers:
  TA1: *x1,x3*   TA2: *x2,x4*

$$\sigma^{(1)(3)} = \sigma^{(2)(4)} = \begin{pmatrix} c-hs & 0 & ks & 0 \\ 0 & c+hs & 0 & -ks \\ ks & 0 & c+hs & 0 \\ 0 & -ks & 0 & c-hs \end{pmatrix}$$

- Beam-splitters BS1 & BS2

$$B_1 = B_2 = \begin{pmatrix} I_2/\sqrt{2} & 0 & 0 & I_2/\sqrt{2} \\ 0 & I_2/\sqrt{2} & I_2/\sqrt{2} & 0 \\ 0 & I_2/\sqrt{2} & -I_2/\sqrt{2} & 0 \\ I_2/\sqrt{2} & 0 & 0 & -I_2/\sqrt{2} \end{pmatrix}$$

- Output modes: *x5,x6,x15,x16*

x15,x6 → Alice

x5,x16 → Bob

$$\sigma^{(5)(6)(15)(16)} = B_1 \sigma^{(1)(2)(3)(4)} B_1^{+}$$

Teleportation procedure

$$\sigma^{(5)(6)(15)(16)(7)(8)} = \sigma^{(5)(6)(15)(16)} \oplus \sigma^{(7)(8)}$$

- Alice has entangled modes *x7* and *x8* from SA3 to teleport

  Combined six-mode state:

$$\sigma^{(7)(8)} = \begin{pmatrix} x-uy & 0 & vy & 0 \\ 0 & x+uy & 0 & -vy \\ vy & 0 & x+uy & 0 \\ 0 & -vy & 0 & x-uy \end{pmatrix}$$

Squeezing of input state

Phase of source amplifier

$$x = \cosh(2q) \quad y = \sinh(2q) \quad u = \cos(2\eta) \quad v = \sin(2\eta)$$

- Alice has two beam-splitters

BS3 & BS4 to combine the modes

$$B_2 = \begin{pmatrix} I_2/\sqrt{2} & 0 & 0 & 0 & I_2/\sqrt{2} & 0 \\ 0 & I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_2/\sqrt{2} & 0 & 0 & I_2/\sqrt{2} \\ 0 & 0 & 0 & I_2 & 0 & 0 \\ I_2/\sqrt{2} & 0 & 0 & 0 & -I_2/\sqrt{2} & 0 \\ 0 & 0 & I_2/\sqrt{2} & 0 & 0 & -I_2/\sqrt{2} \end{pmatrix}$$

LOCC by Alice and Bob

- Output modes with Alice: *x9,x10,x11,x12*
- *Alice performs measurements: X9, P10, X11,P12*
- *Alice communicates* ***four bits*** *to Bob*
- Modes with Bob: *x5, x16*
- *Bob displaces his modes by* **unitary operation:**

$$U = \begin{bmatrix} -\sqrt{2/3} & -\sqrt{1/3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{1/3} & 0 & 0 & 0 & \sqrt{2/3} & 0 \\ 0 & 0 & 0 & \sqrt{2/3} & \sqrt{1/3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{1/3} & 0 & -\sqrt{2/3} \end{bmatrix}$$

Combined physical operations

- Alice: Beam-splitters + measurements
- Bob: Unitary transformation

$$\sigma^{(13)(14)} = \left(UKB_2\right)\sigma^{(5)(6)(15)(16)(7)(8)}\left(UKB_2\right)^{+}$$

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Teleported state

- Final state with Bob:

$$\sigma^{(13)(14)} = \begin{pmatrix} \sigma_{11} & 0 & \sigma_{13} & 0 \\ 0 & \sigma_{22} & 0 & \sigma_{24} \\ \sigma_{13} & 0 & \sigma_{22} & 0 \\ 0 & \sigma_{24} & 0 & \sigma_{11} \end{pmatrix}$$

$$\sigma_{11} = [2c + 2ks + x - uy]/3 \qquad \sigma_{24} = vy/3 = -\sigma_{24} \qquad \sigma_{22} = [2c + 2ks + x + uy]/3$$

- Twice the level of vacuum noise is added to variances of input modes for *r =0* *[c.f., Tan (1999)]*

$$\sigma^{(13)(14)} = (\sigma')^{(7)(8)} = 2(c + ks)I$$

- Two-mode state perfectly teleported for ideal input squeezing [$r \Rightarrow \infty$] $k = -1$

$$\sigma^{(13)(14)} = (\sigma')^{(7)(8)}$$

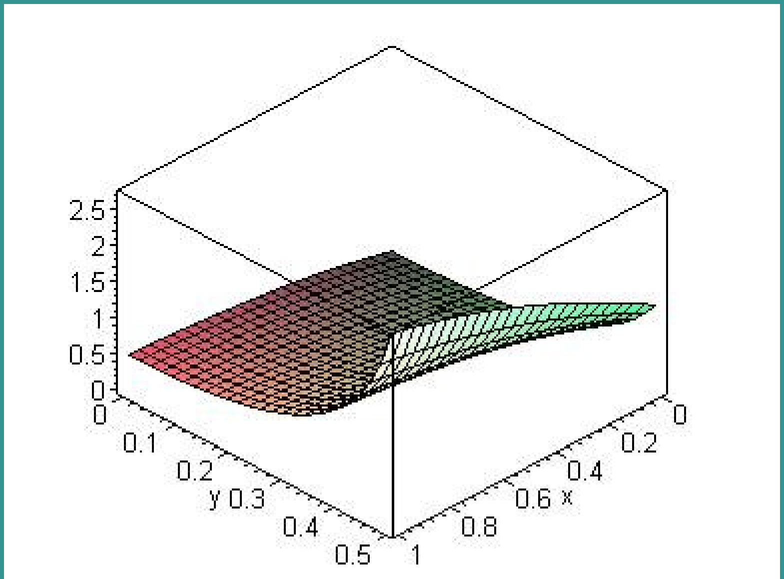
- Input state $\sigma^{(7)(8)}$ and $(\sigma')^{(7)(8)}$ related by local linear unitary Bogoliubov operations (LLUBO) *[Duan et al. (2000); Simon (2000)]*

Entanglement of output modes

- *Criterion for entanglement* : Smallest symplectic eigenvalue of partial transpose of output CM $\tilde{\nu}_{-} < 1$ [Adesso & Illuminati (2007)]

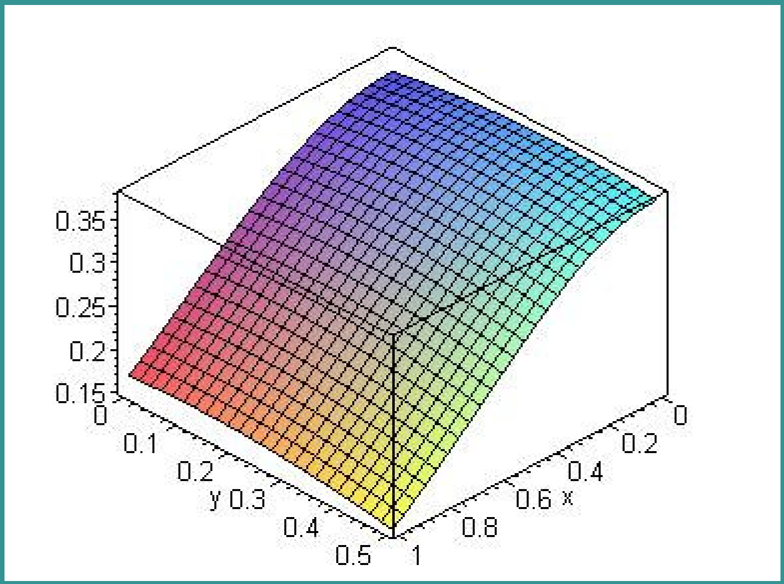
  Particular case: $u = 0 \quad v = 1 \quad h = k = 1/\sqrt{2}$ $\quad \tilde{\nu}_{-} = \left[\sqrt{(2c + x - y)^2 - 2s^2}\right]/3$

- *Magnitude of entanglement:* $E_N = \max\left[0, -\log_2 \tilde{\nu}_{-}\right]$



Fidelity of entanglement

- *Fidelity insensitive to squeezing by teleportation amplifiers– coherent states may be used to generate four-mode entangled states*
- *Fidelity decreases with squeezing for source amplifier – in contrast with entanglement of output modes*
- *Average fidelity may **not** be a good indicator of quality of teleportation. Other appropriate measures, e.g., "entanglement fidelity"* [Schumacher (1996); Braunstein & Kimble (1998)]



Teleportation and Broadcasting -- Summary

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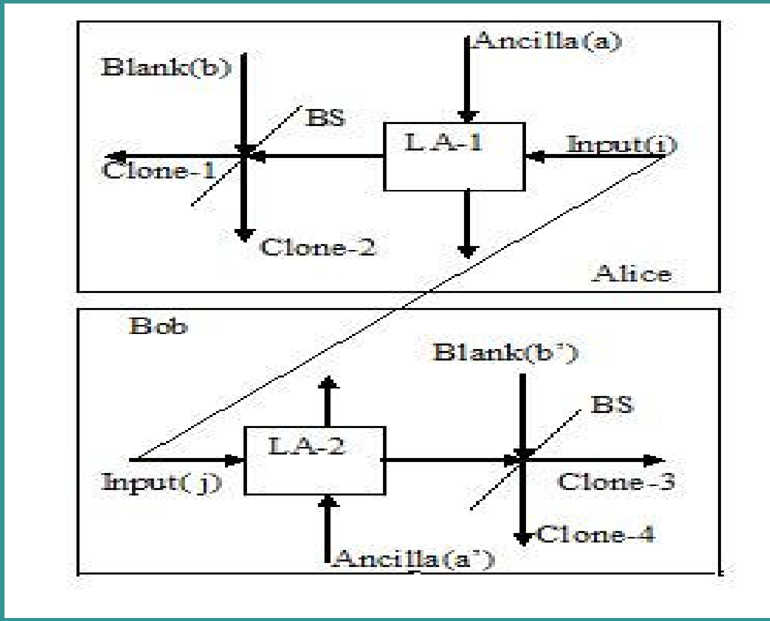

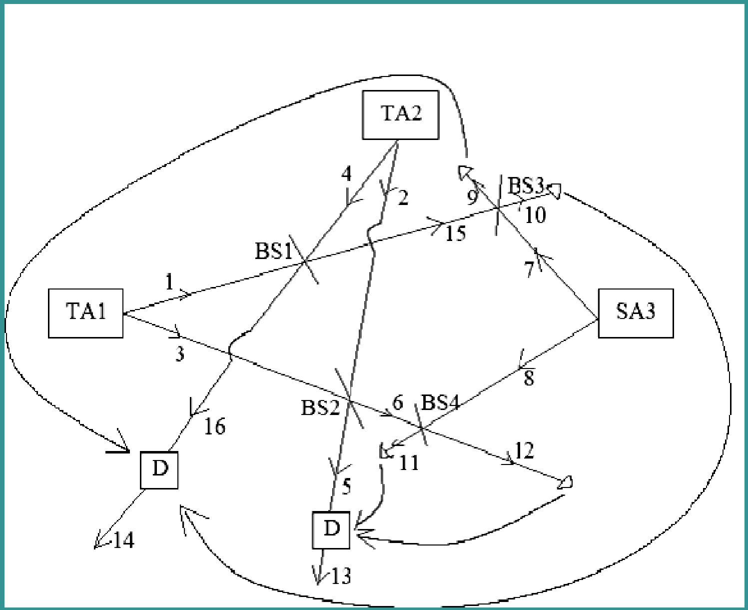

Broadcasting of continuous variable entanglement

- Entanglement shared by two parties transmitted to two less entangled states
- Protocol implemented through local cloning operations on the two modes
- Bipartite entanglement for physical output states for a range of parameter (squeezing, phase) values
- State dependence and phase sensitivity of cloning procedure

Teleportation of two-mode squeezed states

- First explicit scheme for teleportation of an unknown two-mode squeezed state
- Protocol implemented through generation of four-mode entangled state shared by Alice & Bob, and communication of four bits of classical information
- Perfect teleportation possible under ideal squeezing
- Entanglement of output modes increases with squeezing of input modes
- Loss of average fidelity with squeezing suggests possible use of other appropriate measures