The Lepton Flavor Violation road to New Physics

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IMHEP 2019

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Introduction

Before the LHC started operating we all hoped for great discoveries...

LHC expectations

LHC results...

Introduction

Do we have a good reason to go Beyond the Standard Model?

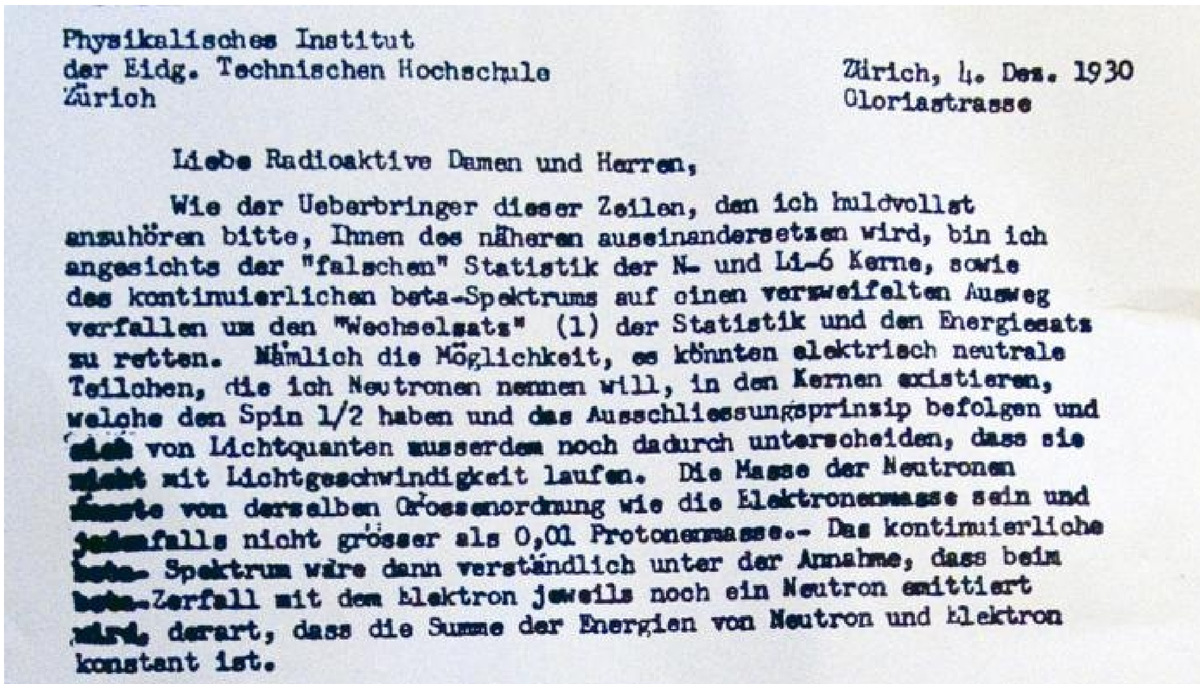
Introduction

Do we have a good reason to go Beyond the Standard Model?

Neutrinos!

The lepton sector is still
to be understood!

Dear radiactive Ladies and Gentlemen...



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst ansuhören bitte, Ihnen des näheren auseinandersetsen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen versweifelten Ausweg verfallen um den "Wechselsats" (1) der Statistik und den Energiesats zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Groessenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

December 4th, 1930
Letter to his colleagues in Tübingen

1930
Pauli's neutrino hypothesis

Open questions

What is the origin of neutrinos masses?

Are they Dirac or Majorana?

What is the absolute scale of neutrino masses?

What is the mass ordering?

Are there more than three neutrinos? Maybe sterile?

Is there CP violation in the lepton sector?

Lepton flavor violation

In the **Standard Model**, three copies of the leptonic SU(2) doublet are introduced

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Gauge and Yukawa interactions

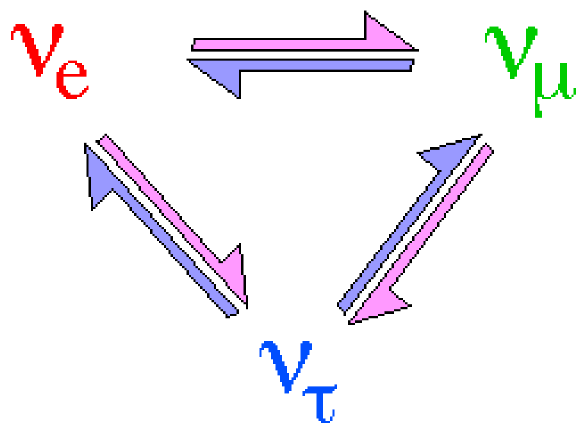
✗

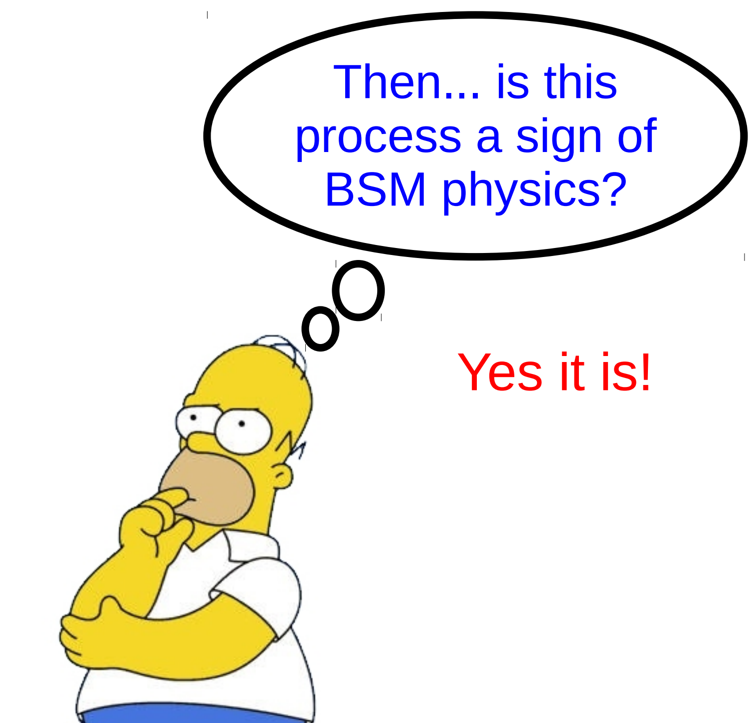
Is lepton flavor a conserved quantity?

Neutrino oscillations: LFV

We already know the answer: NO

Neutrino <u>flavor</u> oscillations: flavor violating process!





Yes it is!

What about cLFV?

In conclusion, lepton flavor is **not** conserved: there is **lepton flavor violation (LFV)**

However... what about **charged lepton flavor violation (cLFV)**?

$$\mu^- \to e^- \gamma \qquad h \to \mu^- \tau^+$$

$$\tau^- \to \mu^- \mu^+ \mu^- \qquad \pi^0 \to e^- \mu^+$$

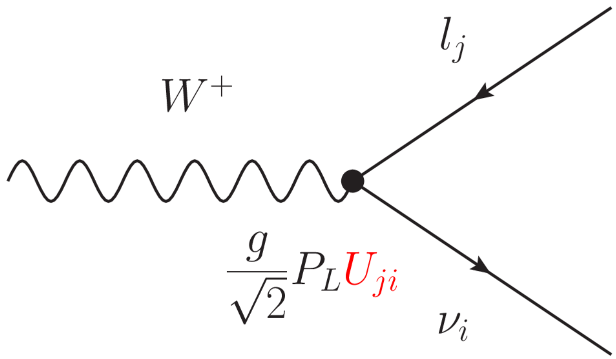
$$K_L^0 \to \pi^0 e^- \mu^+ \qquad \ldots$$

Never observed...

What about cLFV?

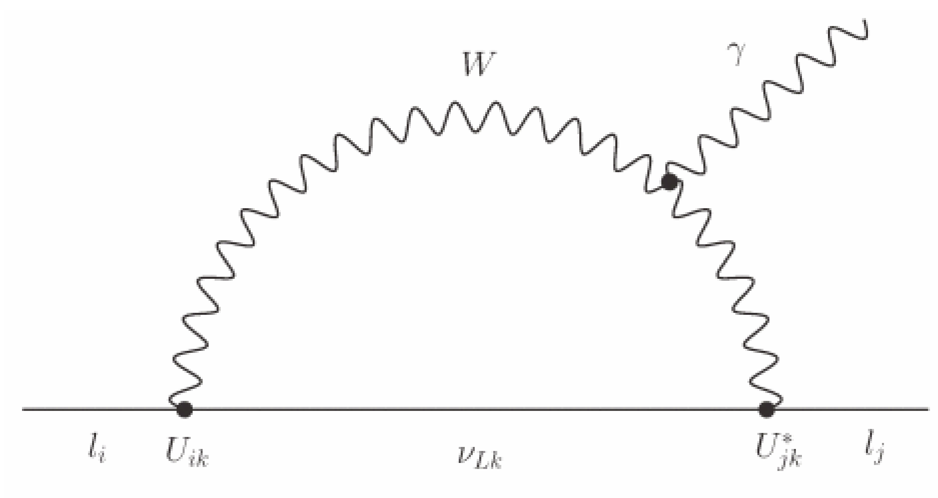
SM + Dirac neutrino masses

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} U_{ji} \bar{l}_j \gamma^\mu P_L \nu_i W^-_\mu + \text{c.c.}$$



U : lepton mixing matrix

[analog of the CKM matrix in the lepton sector]



$$\text{Br}(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_k U_{ek} U^*_{\mu k} \frac{m^2_{\nu k}}{m^2_W} \right|^2 \lesssim 10^{-54}$$

Since neutrino masses are the only source of LFV, all cLFV amplitudes are strongly suppressed (in fact, GIM suppressed)

Why do we care about LFV?

The observation of cLFV would be a clear signal of (non-trivial) physics beyond the Standard Model

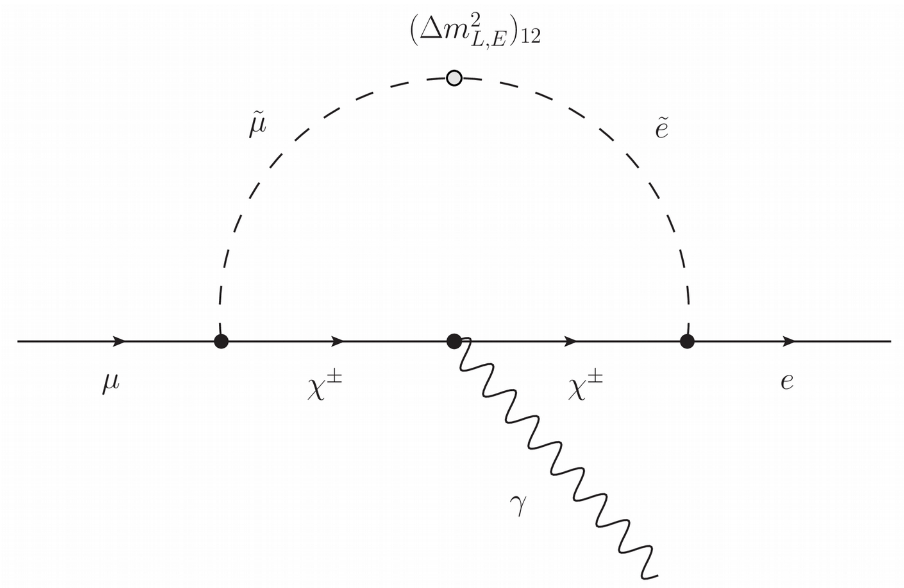
In fact, most BSM models predict large cLFV rates

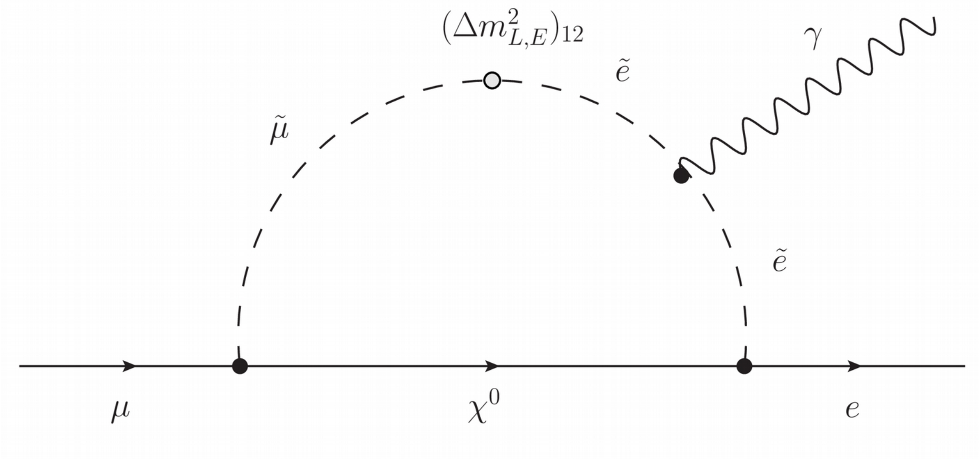
We can probe very high energy scales!

$$\mathcal{O} = \frac{c_{e\mu}}{\Lambda^2}\, \bar{\mu} e \bar{e} e \quad \Rightarrow \quad \frac{\Lambda}{\sqrt{c_{e\mu}}} \gtrsim 100\,\mathrm{TeV}$$

Why do we care about LFV?

Example 1: Supersymmetric models

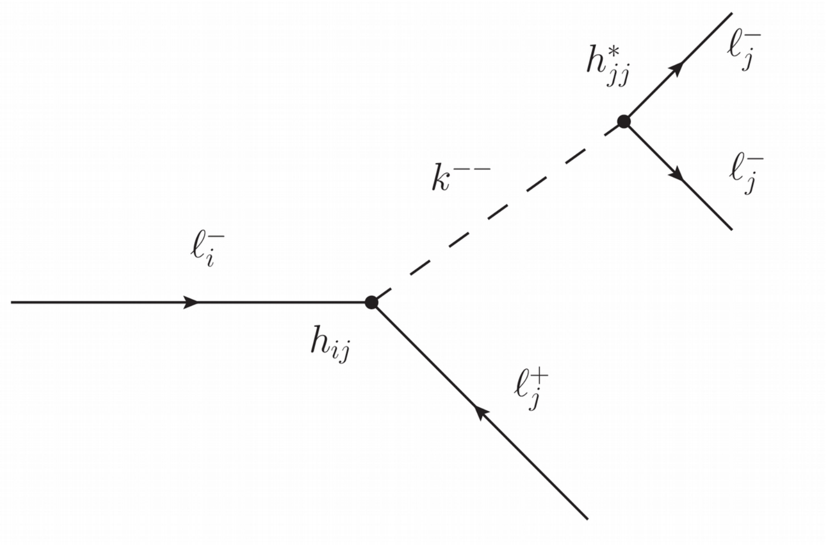




Sleptons: a whole new sector coupled to the SM leptons
Strong constraints on the off-diagonal soft terms

Why do we care about LFV?

Example 2: Babu-Zee model



$$\mathrm{BR} \sim \left| \frac{h_{ij} h^*_{jj}}{m_k^2} \right|^2$$

Small off-diagonal h couplings and/or heavy k's are required

Experimental projects

LFV Process	Present Bound	Future Sensitivity
$\mu \to e\gamma$	4.2×10^{-13}	6×10^{-14} (MEG)
$\tau \to e\gamma$	3.3×10^{-8}	$\sim 10^{-8} - 10^{-9}$ (B factories)
$\tau \to \mu\gamma$	4.4×10^{-8}	$\sim 10^{-8} - 10^{-9}$ (B factories)
$\mu \to 3e$	1.0×10^{-12}	$\sim 10^{-16}$ (Mu3e)
$\tau \to 3e$	2.7×10^{-8}	$\sim 10^{-9} - 10^{-10}$ (B factories)
$\tau \to 3\mu$	2.1×10^{-8}	$\sim 10^{-9} - 10^{-10}$ (B factories)
μ^-, Au $\to e^-$, Au	7.0×10^{-13}	—
μ^-, SiC $\to e^-$, SiC	—	2×10^{-14} (DeeMe)
μ^-, Al $\to e^-$, Al	—	$10^{-15} - 10^{-17}$ (COMET) $10^{-17} - 10^{-18}$ (Mu2e)
μ^-, Ti $\to e^-$, Ti	4.3×10^{-12}	$\sim 10^{-18}$ (PRISM/PRIME)

Experimental projects

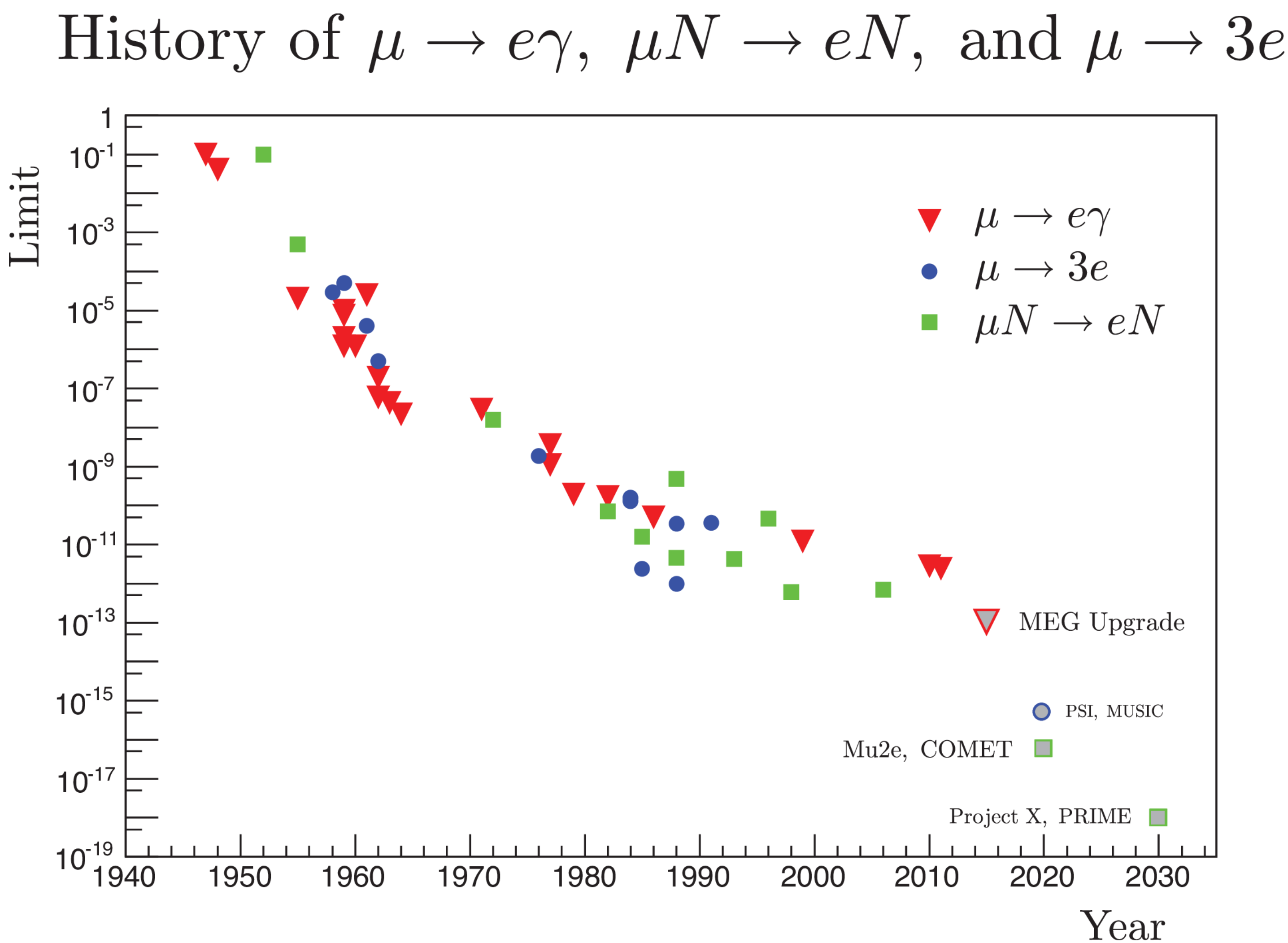
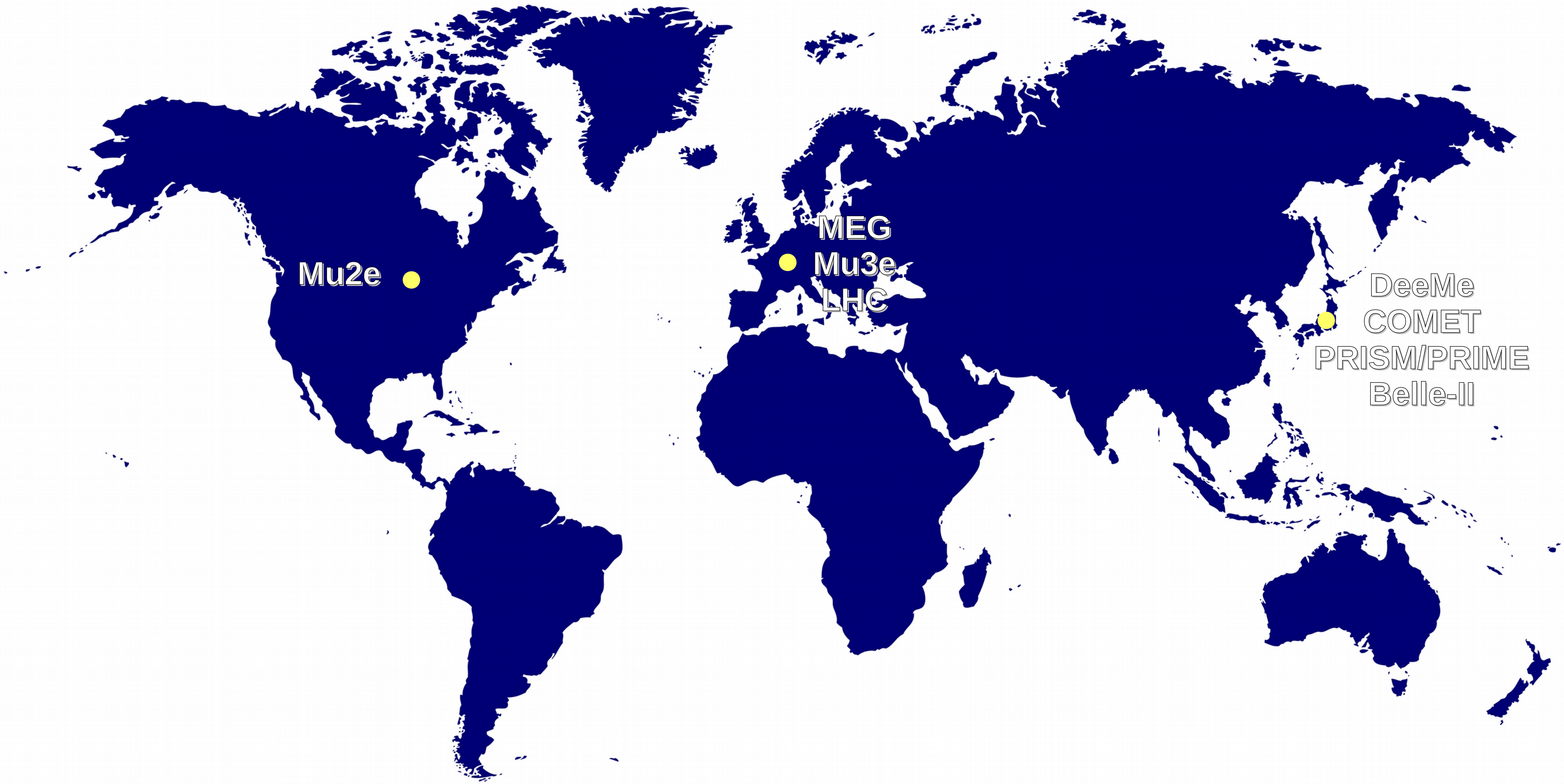


Figure taken from Bernstein & Cooper [arXiv:1307.5787]


Mu2e
MEG
Mu3e
LHC
DeeMe
COMET
PRISM/PRIME
Belle-II

LFV : Where to look for?

$\ell_i \to \ell_j \gamma$

$\ell_i \to 3\,\ell_j$

$\ell_i \to \ell_j \ell_k \ell_k$

$\mu - e$

conversion in nuclei

BSM

LFV at colliders

$M \to \ell_i \ell_j$

LFV : Where to look for?

Everywhere!

ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\text{Br}(\mu^- \to e^- e^+ e^-)}{\text{Br}(\mu \to e\gamma)}$	$0.02 \ldots 1$	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	$0.06 \ldots 2.2$
$\frac{\text{Br}(\tau^- \to e^- e^+ e^-)}{\text{Br}(\tau \to e\gamma)}$	$0.04 \ldots 0.4$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.07 \ldots 2.2$
$\frac{\text{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\text{Br}(\tau \to \mu\gamma)}$	$0.04 \ldots 0.4$	$\sim 2 \cdot 10^{-3}$	$0.06 \ldots 0.1$	$0.06 \ldots 2.2$
$\frac{\text{Br}(\tau^- \to e^- \mu^+ \mu^-)}{\text{Br}(\tau \to e\gamma)}$	$0.04 \ldots 0.3$	$\sim 2 \cdot 10^{-3}$	$0.02 \ldots 0.04$	$0.03 \ldots 1.3$
$\frac{\text{Br}(\tau^- \to \mu^- e^+ e^-)}{\text{Br}(\tau \to \mu\gamma)}$	$0.04 \ldots 0.3$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.04 \ldots 1.4$
$\frac{\text{Br}(\tau^- \to e^- e^+ e^-)}{\text{Br}(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8 \ldots 2$	~ 5	$0.3 \ldots 0.5$	$1.5 \ldots 2.3$
$\frac{\text{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\text{Br}(\tau^- \to \mu^- e^+ e^-)}$	$0.7 \ldots 1.6$	~ 0.2	$5 \ldots 10$	$1.4 \ldots 1.7$
$\frac{\text{R}(\mu\text{Ti} \to e\text{Ti})}{\text{Br}(\mu \to e\gamma)}$	$10^{-3} \ldots 10^{2}$	$\sim 5 \cdot 10^{-3}$	$0.08 \ldots 0.15$	$10^{-12} \ldots 26$

Table taken from Buras et al [arXiv:1006.5356]

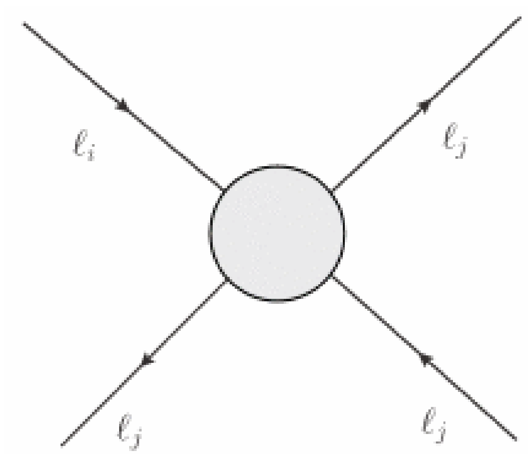
$\ell_i \to 3\,\ell_j$ vs $\ell_i \to \ell_j\gamma$

What contribution dominates $\ell_i \to 3\,\ell_j$?

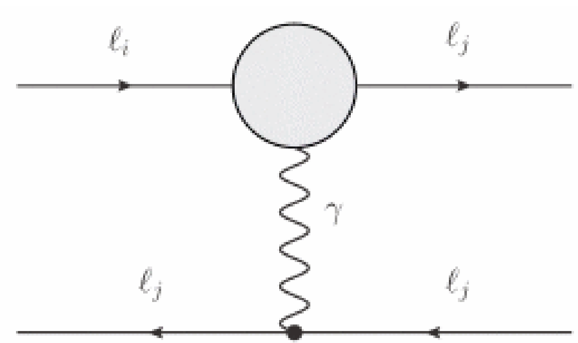
In many models of interest: Photonic dipole contributions

Most popular example: MSSM

[Hisano et al 1996; Arganda, Herrero 2006]



$\simeq$



Dipole dominance

$$\frac{BR(\ell_i \to 3\,\ell_j)}{BR(\ell_i \to \ell_j\gamma)} = \frac{\alpha}{3\pi}\left(\log\frac{m_{\ell_i}^2}{m_{\ell_j}^2} - \frac{11}{4}\right) \Rightarrow \quad BR(\ell_i \to \ell_j\gamma) \gg BR(\ell_i \to 3\,\ell_j)$$

The LFV program

In order to unravel the physics behind LFV (and perhaps neutrino masses!) we must:

- Search for LFV in as many observables as possible: they might have information about different sectors of the theory
- Study the relations among different observables (ratios, correlations, hierarchies...)
- Understand the origin of such relations: what is the underlying physics?

Outline of the talk

- Introduction: Lepton Flavor Violation
- Selected topics
  - LFV in low-scale seesaw models
  - LFV in B-meson decays
  - Master Majorana parametrization
- Final remarks

Chuck Norris fact of the day

Chuck Norris counted to infinity. Twice.

LFV in low-scale seesaw models

With Asmaa Abada, Manuel E. Krauss, Werner Porod, Florian Staub and Cédric Weiland

PRD 90 (2014) 013008 [arXiv:1312.5318]

JHEP 1411 (2014) 048 [arXiv:1408.0138]

Low-scale seesaw models

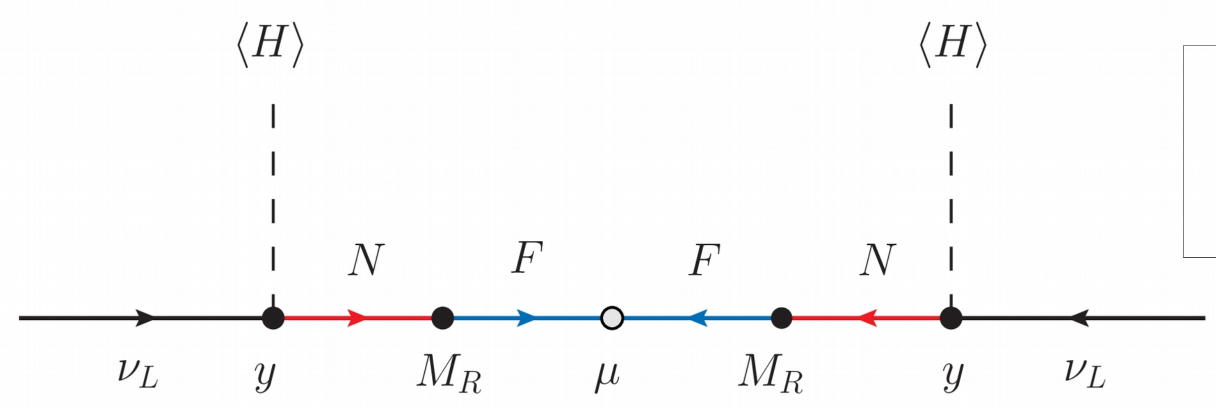
[Mohapatra, Valle, 1986]

The Inverse Seesaw

$$-\mathcal{L}_{IS} \supset y_{ij} \overline{N}_i L_j \widetilde{H} + M_{R_{ij}} \overline{N}_i F_j + \frac{1}{2} \mu_{ij} \overline{F_i^c} F_j$$

singlet states: $\#N = \#F = 3$

9x9 mass matrix $\longrightarrow$ $\mathcal{M} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} y^T v & 0 \\ \frac{1}{\sqrt{2}} y v & 0 & M_R \\ 0 & M_R^T & \mu \end{pmatrix}$



$$\boxed{m \simeq \frac{v^2}{2} y^T (M_R^T)^{-1} \mu M_R^{-1} y}$$

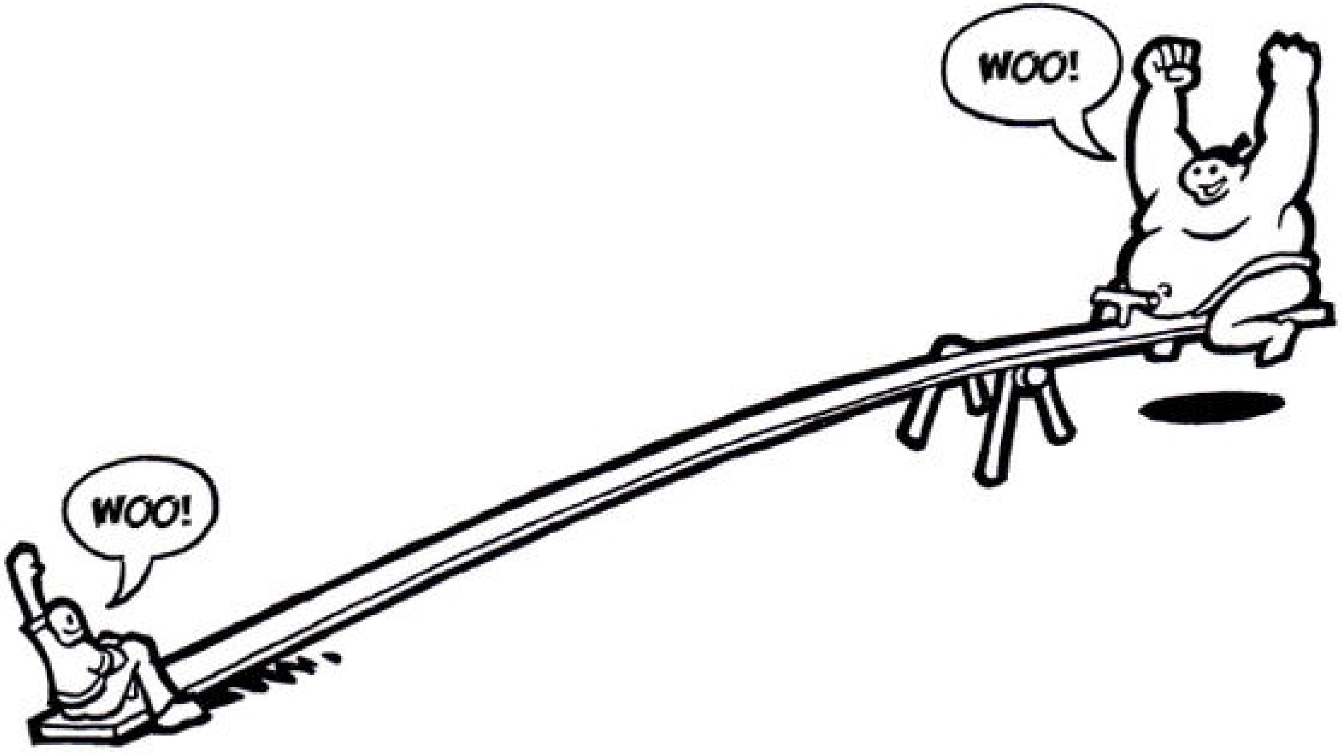
[if $\mu \ll yv \ll M_R$]

Standard vs Inverse Seesaw

Standard Seesaw

Standard vs Inverse Seesaw

Inverse Seesaw

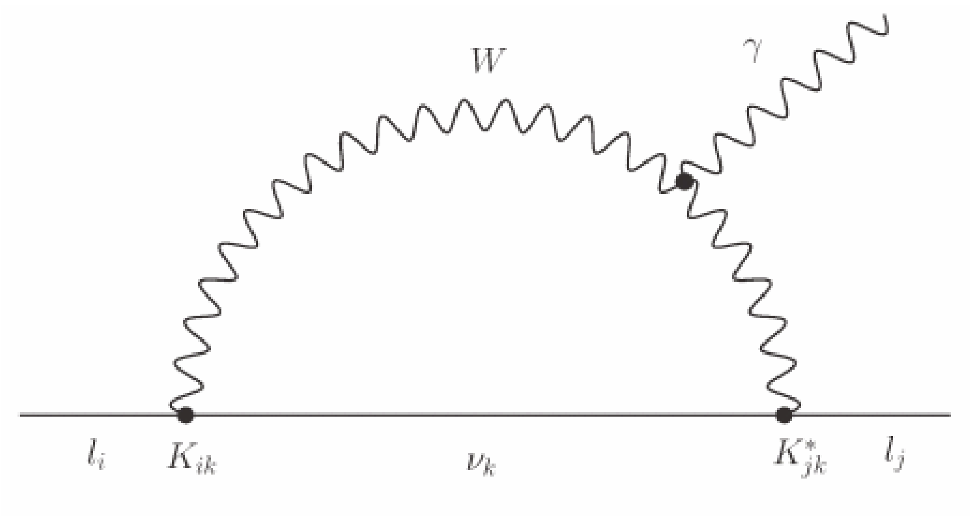


Penguins in the inverse seesaw

[Ilakovac, Pilaftsis, 1995; Deppisch, Valle, 2005]

$$\text{Br}(\mu \to e\gamma) = \frac{\alpha_W^3 s_W^2 m_\mu^5}{256\pi^2 m_W^4 \Gamma_\mu} \left| \sum_k K_{ek} K_{\mu k}^* G_\gamma \left(\frac{m_{\nu k}^2}{m_W^2} \right) \right|^2$$

Rectangular matrix

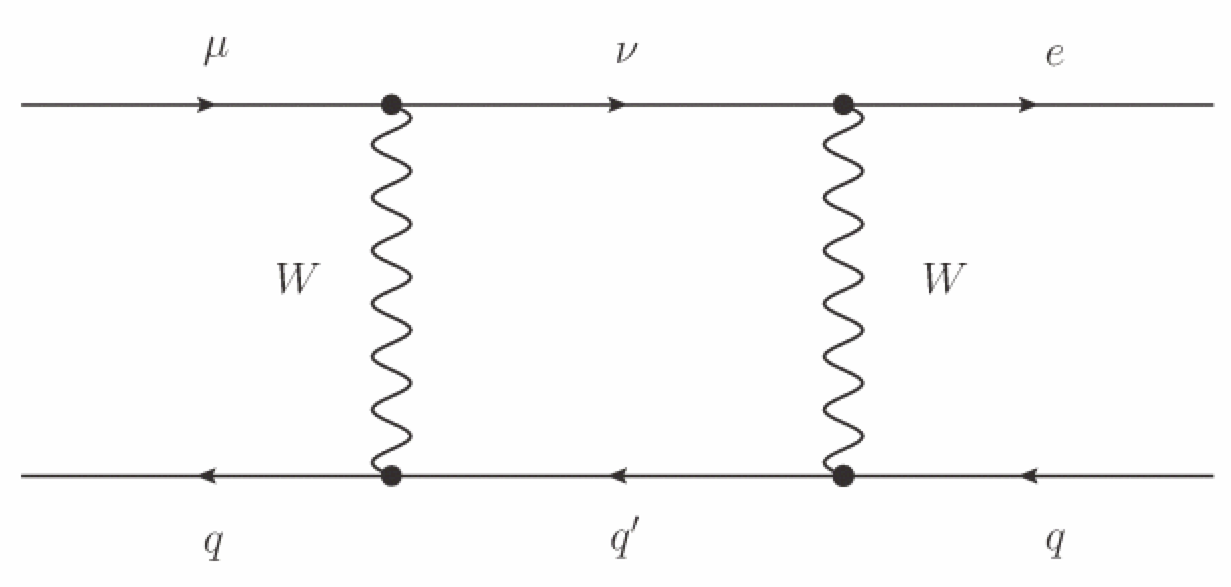


The GIM suppression is *spoiled* by the singlet neutrinos

Much larger rates expected!

Boxes in the inverse seesaw

Furthermore, for $\mu - e$ conversion in nuclei and $\ell_i \to 3\,\ell_j$ ...



[Ilakovac, Pilaftsis, 2009; Dinh, Ibarra, Molinaro, Petcov, 2012; Alonso, Dhen, Gavela, Hambye, 2013; Ilakovac, Pilaftsis, Popov, 2012]

- Non-supersymmetric contribution
- Relevant for light singlet neutrinos
- Large non-dipole contributions

Low-scale seesaw models

[Abada, Krauss, Porod, Staub, AV, Weiland, 2014]

75 pages paper

First complete study of all SUSY and non-SUSY contributions!

- Analytical and numerical study of

$$\ell_i \to \ell_j \gamma$$

$$\ell_i \to \ell_j \ell_k \ell_k$$

$\mu - e$ conversion in nuclei

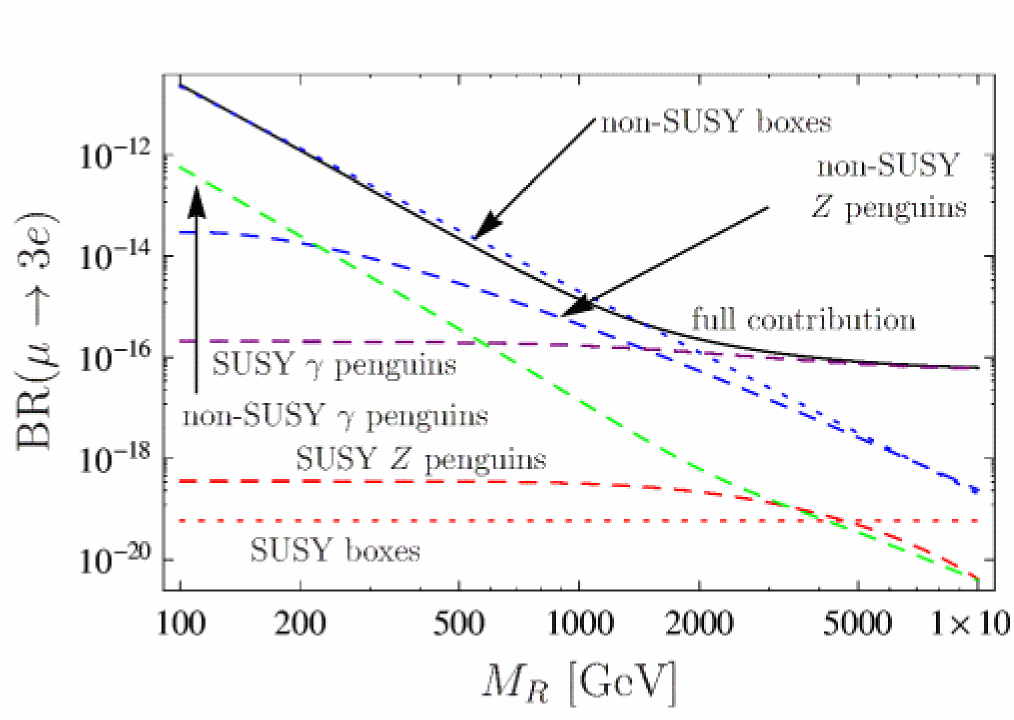
- All contributions included
- A few hundred Feynman diagrams $\longrightarrow$ How were they computed?

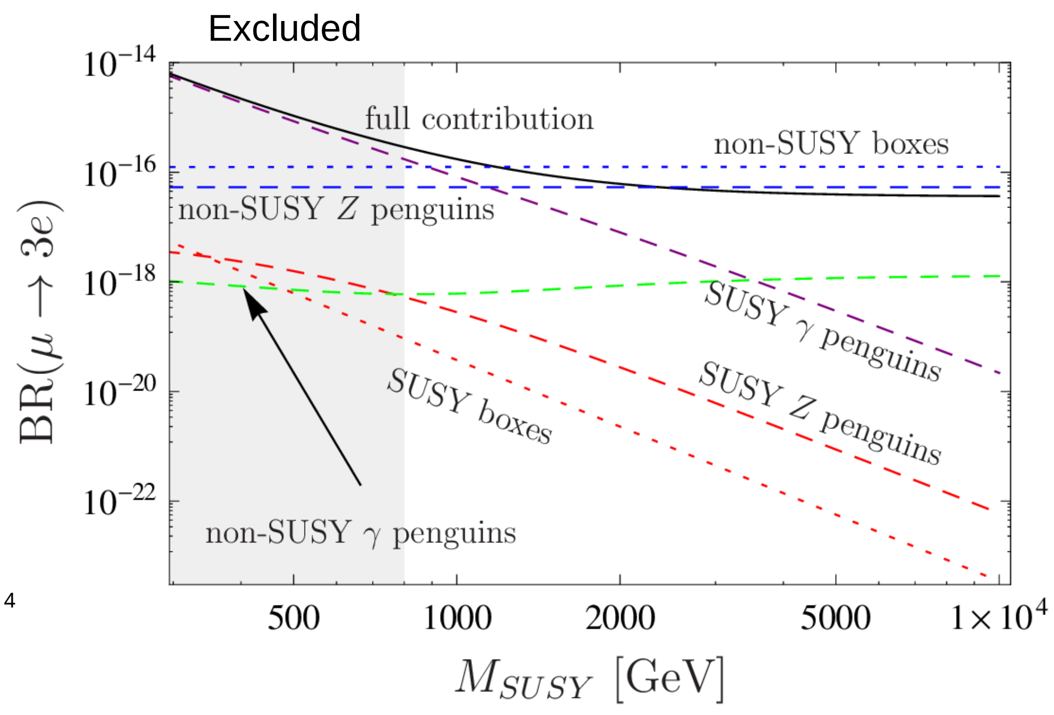
Later...

Low-scale seesaw models

$$\boxed{\ell_i \to 3\,\ell_j}$$

[Abada, Krauss, Porod, Staub, AV, Weiland, 2014]





The anatomy of LFV strongly depends on M_R and M_{SUSY}

FlavorKit

[Porod, Staub, AV, 2014]

A computer tool that provides automatized analytical and numerical computation of flavor observables. It is based on SARAH, SPheno and FeynArts/FormCalc.

Lepton flavor	Quark flavor
$\ell_\alpha \to \ell_\beta \gamma$	$B^0_{s,d} \to \ell^+ \ell^-$
$\ell_\alpha \to 3\,\ell_\beta$	$\bar{B} \to X_s \gamma$
$\mu - e$ conversion in nuclei	$\bar{B} \to X_s \ell^+ \ell^-$
$\tau \to P\,\ell$	$\bar{B} \to X_{d,s} \nu \bar{\nu}$
$h \to \ell_\alpha \ell_\beta$	$B \to K \ell^+ \ell^-$
$Z \to \ell_\alpha \ell_\beta$	$K \to \pi \nu \bar{\nu}$
	$\Delta M_{B_{s,d}}$
	ΔM_K and ε_K
	$P \to \ell \nu$

Not limited to a single model: use it for the model of your choice

Easily extendable

Many observables ready to be computed in your favourite model!

Manual: arXiv:1405.1434

Website: http://sarah.hepforge.org/FlavorKit.html

LFV in B-meson decays

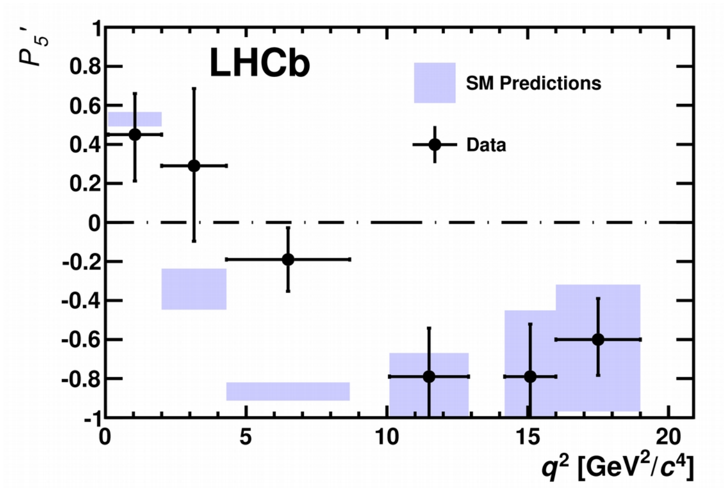
With Sofiane M. Boucenna, Paulina Rocha-Morán and José W. F. Valle

PLB 750 (2015) 367 [arXiv:1503.07099]

arXiv:1810.02135

Episode IV: A new hope

2013 : First anomalies found by LHCb



Episode V: LHCb strikes back

2014 : Lepton universality violation

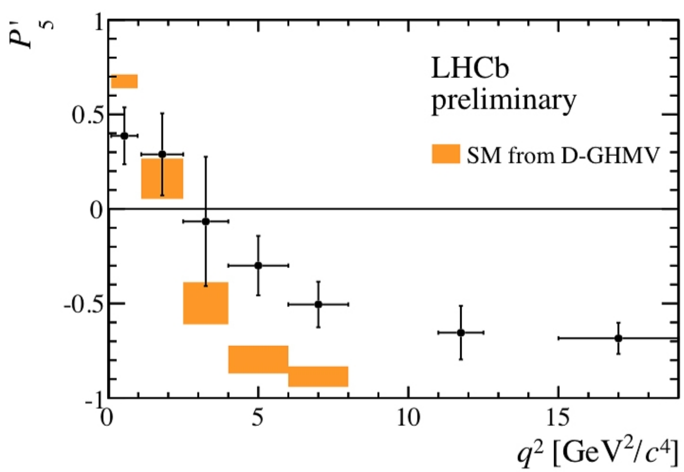
$$R_K = \frac{\mathrm{BR}(B \to K \mu^+ \mu^-)}{\mathrm{BR}(B \to K e^+ e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

$$R_K^{\mathrm{SM}} \sim 1.00 \pm 0.01$$

2.6σ away from the SM

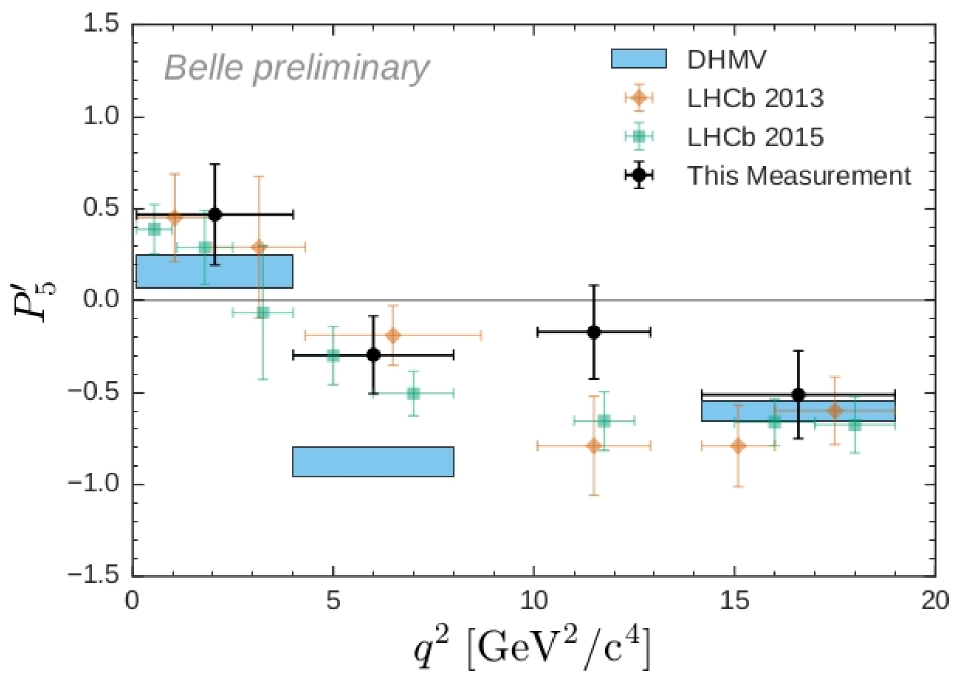
Episode VI: Return of the anomalies

2015 : LHCb confirms first anomalies



Episode I: The Belle menace

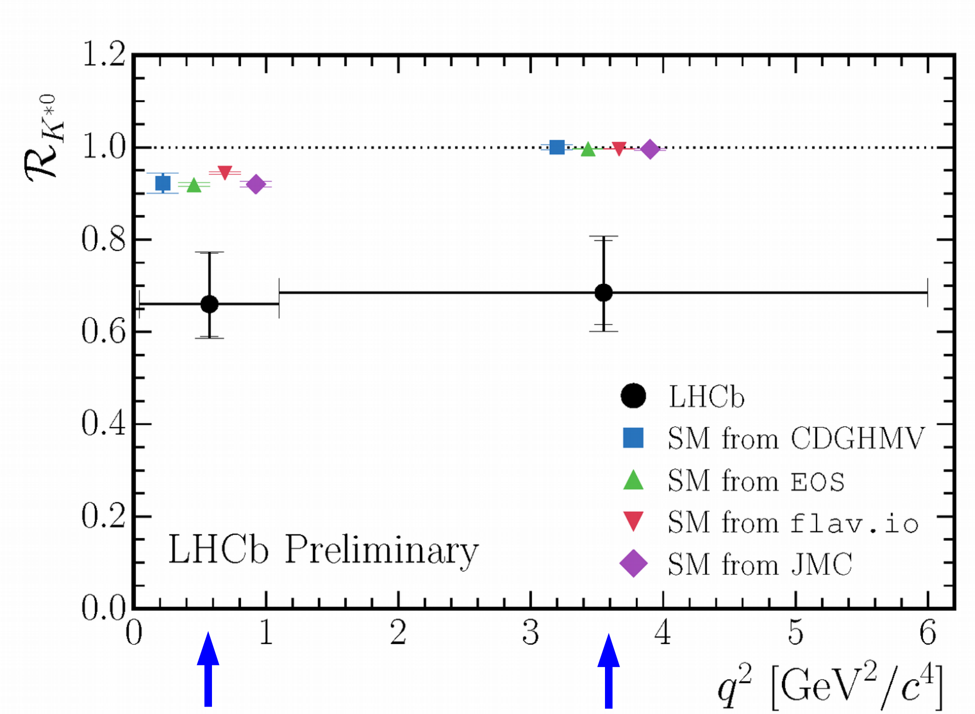
2016 : Belle finds additional hints



P_5' anomaly confirmed
+ little LFVU indication

Episode II: Attack of R_{K^*}

2017 : More universality violation in LHCb



low bin

central bin

[No new episode in 2018 though....]

Beyond the Standard Model

The $b \to s$ anomalies

Boring **S**izable corrections

(+ fluctuations)

LFV in B meson decays

What about LFV?

LFV in B meson decays

What about LFV?

[Glashow et al, 2014]

Lepton universality violation generically implies lepton flavor violation

Gauge basis

Mass basis

$$\mathcal{O} = \widetilde{C}^Q \left(\bar{q}'\gamma_\alpha P_L q'\right) \widetilde{C}^L \left(\bar{\ell}'\gamma^\alpha P_L \ell'\right) \longrightarrow \mathcal{O} = C^Q \left(\bar{q}\gamma_\alpha P_L q\right) C^L \left(\bar{\ell}\gamma^\alpha P_L \ell\right)$$

$$C^L = U_\ell^\dagger \, \widetilde{C}^L \, U_\ell$$

However: we must have a flavor theory in order to make predictions

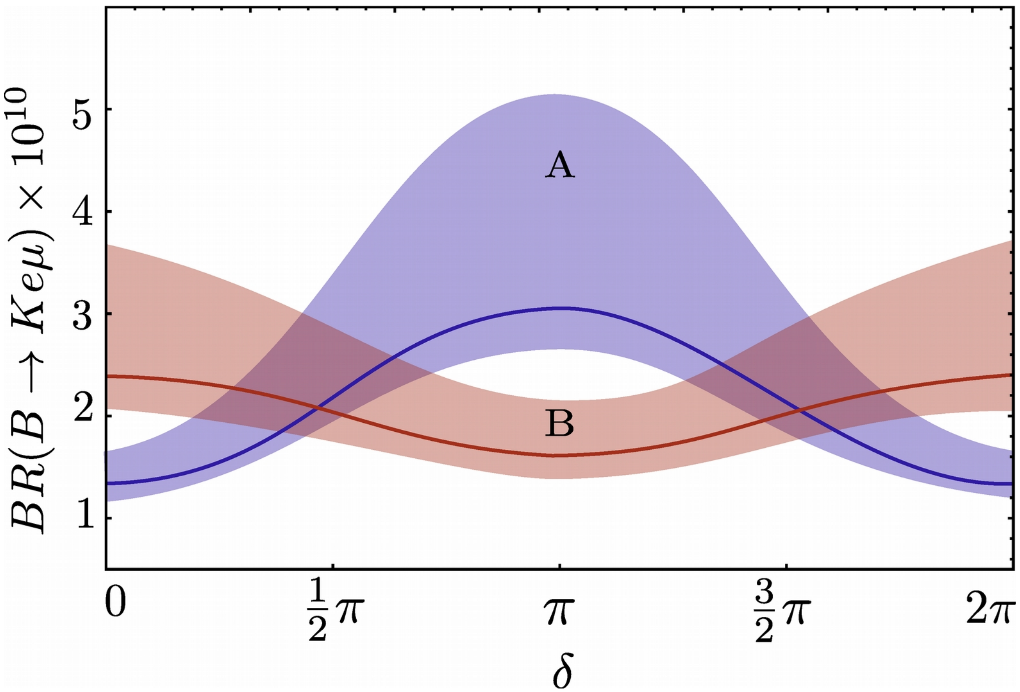
Are the LHCb anomalies related to neutrino oscillations?

Working hypothesis: What if $U_\ell = K^\dagger$? [Boucenna, Valle, AV, 2015]

Neutrino oscillations

Neutrinos ⟷ B-physics

Non-trivial link!



Lines: BF
Bands: 1σ

An explicit model

[Rocha-Moran, AV, 2018]
Extension of [Aristizabal Sierra, Staub, AV, 2015]

	generations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$
H	1	$\mathbf{1}$	$\mathbf{2}$	$1/2$	0
ϕ	1	$\mathbf{1}$	$\mathbf{1}$	0	2
S	1	$\mathbf{1}$	$\mathbf{1}$	0	-4
q_L	3	$\mathbf{3}$	$\mathbf{2}$	$1/6$	0
u_R	3	$\mathbf{3}$	$\mathbf{1}$	$2/3$	0
d_R	3	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	0
ℓ_L	3	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0
e_R	3	$\mathbf{1}$	$\mathbf{1}$	-1	0
$Q_{L,R}$	1	$\mathbf{3}$	$\mathbf{2}$	$1/6$	2
$L_{L,R}$	2	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	2
$F_{L,R}$	2	$\mathbf{1}$	$\mathbf{1}$	0	2

Gauge symmetry Z' boson

$U(1)_X$ breaking: $m_{Z'}$

Vector-like

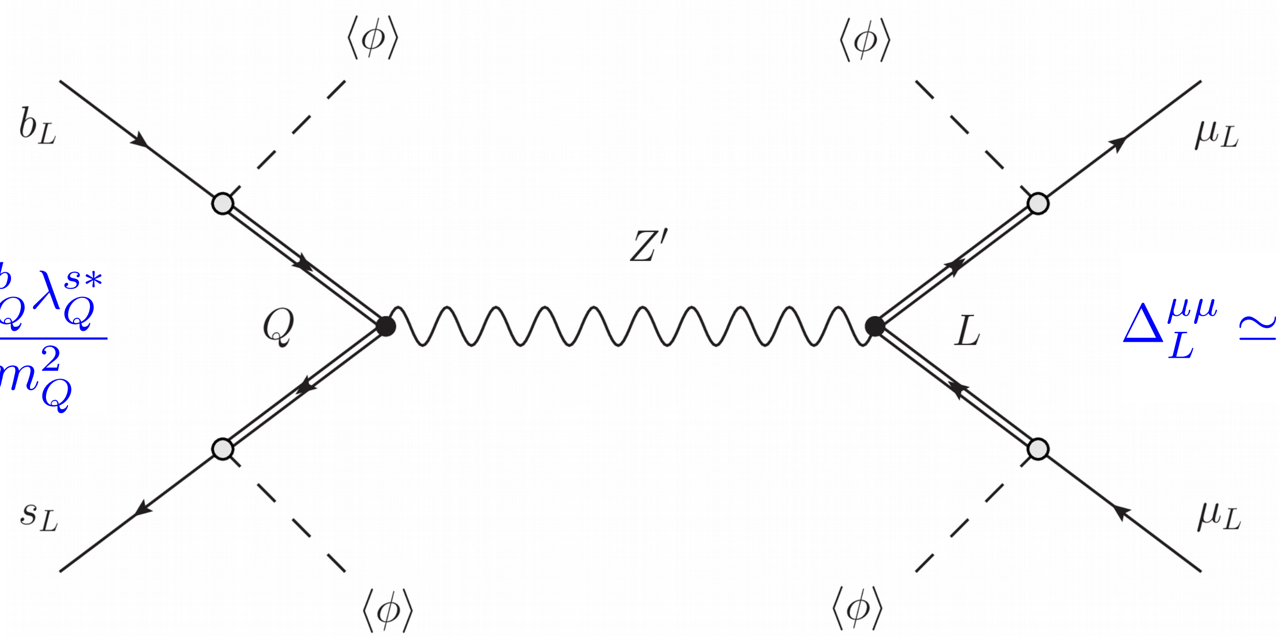
Vector-like = “joker” for model builders

Solving the $b \to s$ anomalies

[Aristizabal Sierra, Staub, AV, 2015]

Similar to Altmannshofer et al, Crivellin et al, 2014

$$\Delta_L^{bs} \simeq g_X v_\phi^2 \frac{\lambda_Q^b \lambda_Q^{s*}}{m_Q^2}$$



$$\Delta_L^{\mu\mu} \simeq \frac{2\, g_X |\lambda_L^\mu|^2 v_\phi^2}{2m_L^2 + |\lambda_L^\mu|^2 v_\phi^2}$$

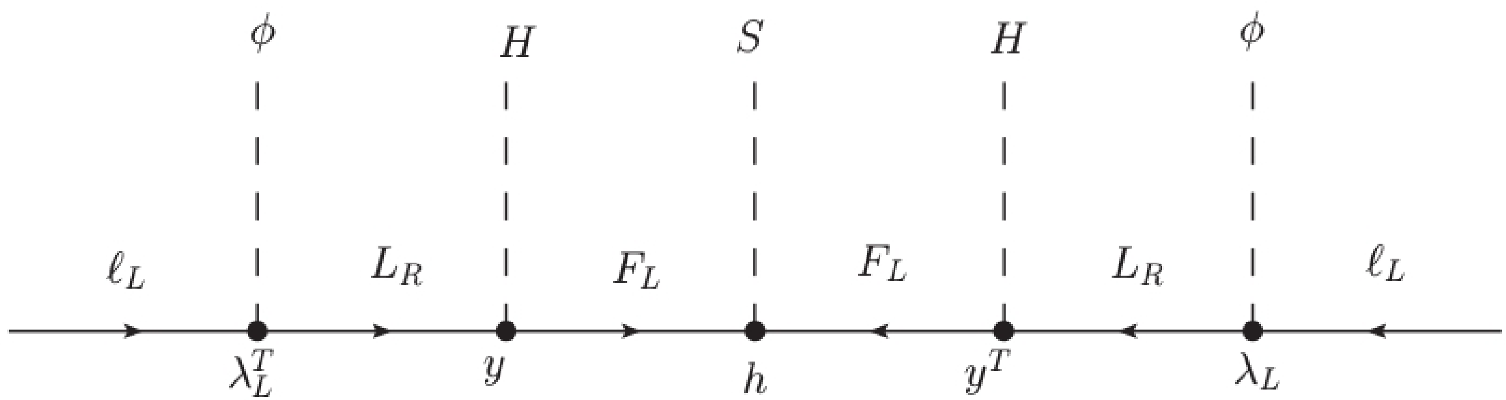
$$\mathcal{O} = (\bar{s}\gamma_\alpha P_L b)\ (\bar{\mu}\gamma^\alpha P_L \mu)$$

$$C_9^{\mathrm{NP}} = -C_{10}^{\mathrm{NP}}$$

Z' couplings induced by mixing with vector-like fermions

Neutrino mass generation

[Rocha-Morán, AV, 2018]



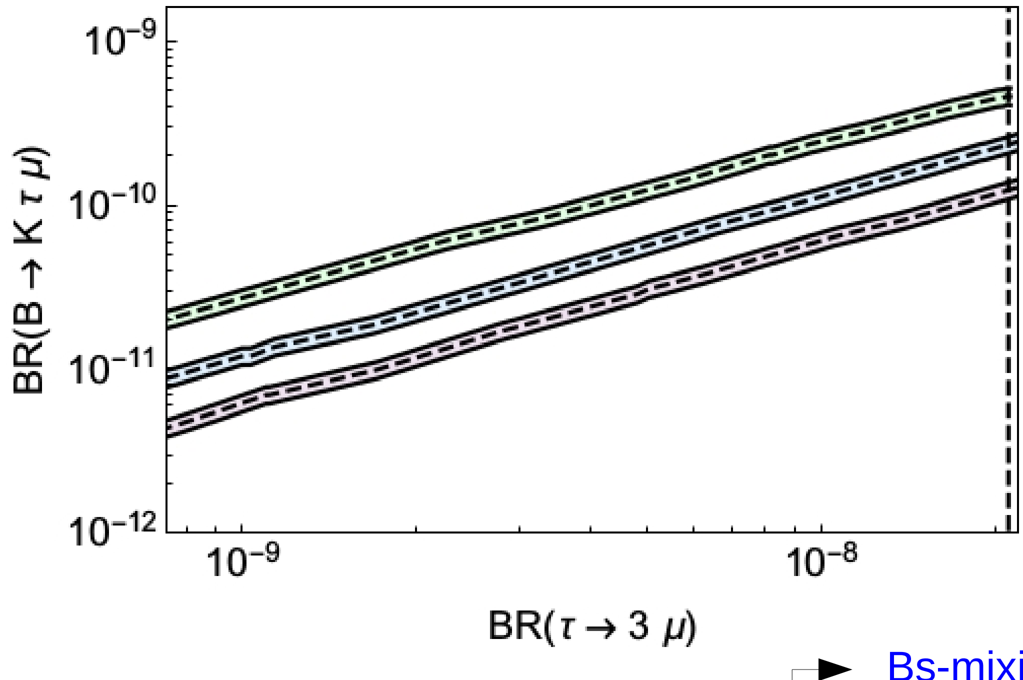
$$m \simeq \frac{v^2 v_\phi^2 v_S}{2\sqrt{2}} \, \lambda_L^T \, m_L^{-1} \, y \, m_F^{-1} \, h \, \left(m_F^{-1}\right)^T \, y^T \, \left(m_L^{-1}\right)^T \lambda_L$$

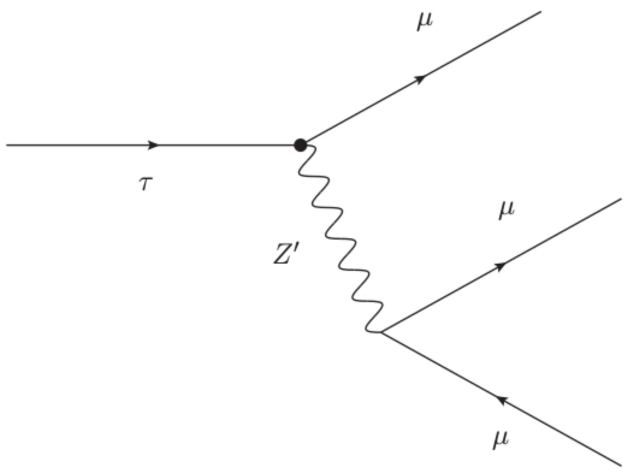
Inverse seeesaw-like mechanism

$$h \ll 1$$

LFV phenomenology

[Rocha-Morán, AV, 2018]





Bs-mixing

$$\frac{\mathrm{BR}(B \to K\tau\mu)}{\mathrm{BR}(\tau \to 3\,\mu)} = 1.7 \cdot 10^7\,\mathrm{TeV}^4 \left(\frac{\left|\Delta_L^{bs}\right|}{m_{Z'}}\right)^4 \frac{1}{\left|C_9^{\mu\mu,\mathrm{NP}}\right|^2}$$

Anomaly

$$\boxed{\mathrm{BR}(B \to K\tau\mu)_{\mathrm{max}} \lesssim 8 \cdot 10^{-10}}$$

Correlation is (almost)* unavoidable

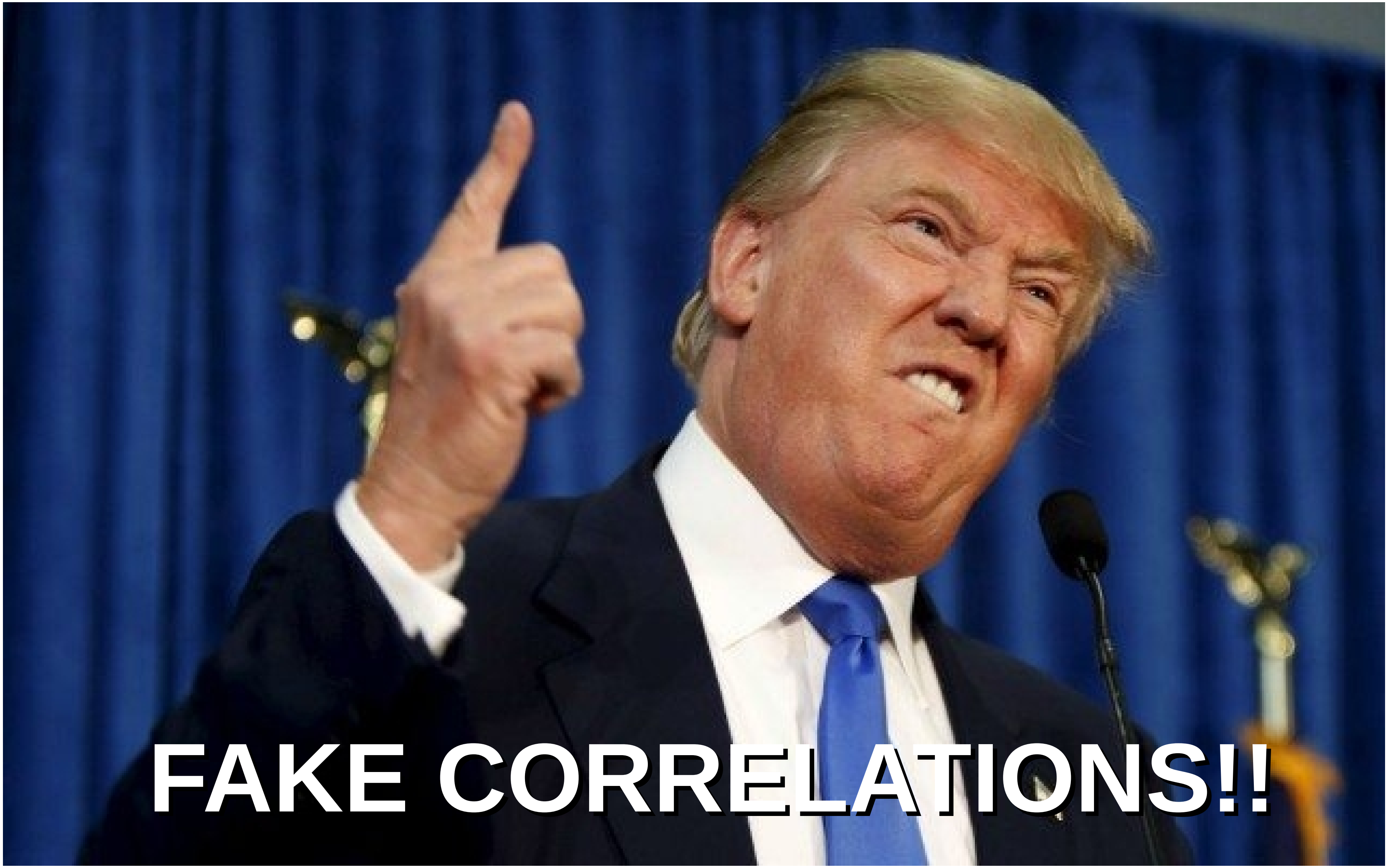
* : could be broken by loops (indeed possible!)

Master Majorana parametrization

With Isabel Cordero-Carrión and Martin Hirsch

arXiv:1812.03896

Motivation



The master formula

All Majorana neutrino mass models!

$$\boxed{m = f \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)}$$

m : symmetric 3×3 Majorana neutrino mass matrix

f : global factor [numerical factors, model parameters, mass ratios, ...]

y_1 : $n_1 \times 3$ Yukawa matrix

y_2 : $n_2 \times 3$ Yukawa matrix $[n_1 \geq n_2]$

M : $n_1 \times n_2$ matrix [dimensions of mass]

Towards a master parametrization

$$m = f \; \underbrace{\left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)}$$

Neutrino oscillation experiments

Model parameters

Goal:

To establish a <u>general</u>, <u>complete</u> and <u>programmable</u> parametrization of the y_1 and y_2 Yukawa matrices

Particular case: Type-I seesaw

Casas-Ibarra parametrization

[Casas, Ibarra, 2001]

$\Longrightarrow$ **Master parametrization**

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

Summary:

Experimental input:	**Model input:**	**Free Yukawa parameters:**
ν-data $\left\{ \begin{array}{l} \bar{D}_{\sqrt{m}} \\ U \end{array} \right.$	f $M \left\{ \begin{array}{l} \Sigma \\ V_{1,2} \end{array} \right.$	$\left. \begin{array}{cc} \widehat{W} & T \\ X_{1,2,3} & K \\ & (C_2) \end{array} \right\} \begin{array}{c} A \\ \widehat{B} \end{array}$

The master parametrization

Matrix	Dimensions	Property	Real parameters
X_1	$(n_2 - n) \times 3$	Absent if $n = n_2$	$6(n_2 - n)$
X_2	$(n_1 - n_2) \times 3$	Absent if $n_1 = n_2$	$6(n_1 - n_2)$
X_3	$(n_2 - n) \times 3$	Absent if $n = n_2$	$6(n_2 - n)$
W	$n \times r$		$r(2n - r)$
T	$r \times r$	Upper triangular with $(T)_{ii} > 0$	r^2
K	$r \times r$	Antisymmetric	$r(r-1)$
$\bar{B}$	$(n - r) \times 3$	Absent if $n = r$	$6(n - r)$
C_1	$r \times 3$	Case-dependent	0 or 2
C_2	3×3	Case-dependent	-

$$\#_{\text{free}} = \#_{X_1} + \#_{X_2} + \#_{X_3} + \#_T + \#_W + \#_K + \#_{\bar{B}} + \#_{C_1}$$

BNT model

	generations	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Φ	1	$\mathbf{1}$	$\mathbf{4}$	$3/2$
$\psi_{L,R}$	3	$\mathbf{1}$	$\mathbf{3}$	-1

$$\Phi = \begin{pmatrix} \Phi^{+++} \\ \Phi^{++} \\ \Phi^{+} \\ \Phi^{0} \end{pmatrix}$$

$$-\mathcal{L} \supset y_\psi \, \overline{L} \, H \, \psi_R + y_{\bar{\psi}} \, \overline{L^c} \, \Phi \, \psi_L + M_\psi \overline{\psi} \, \psi$$

An example of $y_1 \neq y_2$

$$\psi_{L,R} = \begin{pmatrix} \psi^{0} \\ \psi^{-} \\ \psi^{--} \end{pmatrix}_{L,R}$$

$$\mathcal{V} \supset \lambda_\Phi \, H^3 \, \Phi$$

$$\nearrow \quad \langle \Phi^0 \rangle = \frac{\lambda_\Phi v^3}{2\sqrt{2} M_\Phi^2}$$

$$\searrow \quad \Delta L = 2$$

Lepton number violation

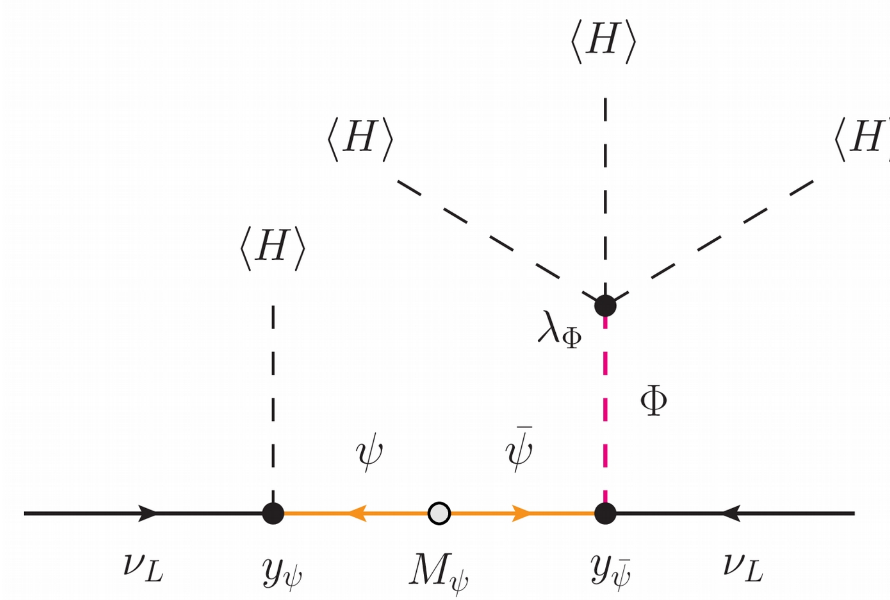
[Babu, Nandi, Tavartkiladze, 2009]

BNT model

$$\boxed{m = f \, \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)}$$

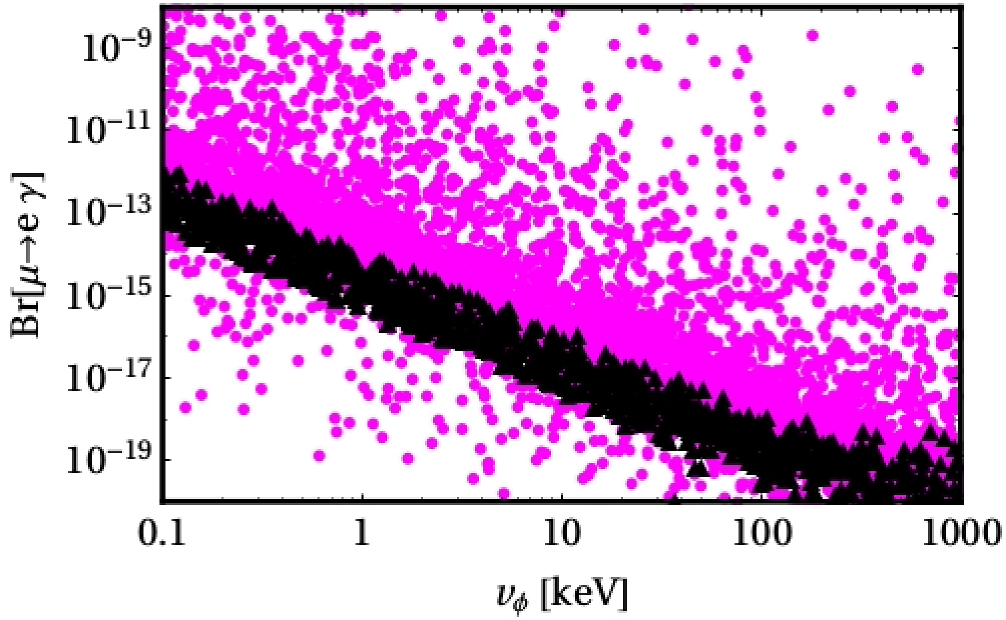
$$\left.\begin{aligned} f &= \frac{\lambda_\Phi \, v^2}{2 \, M_\Phi^2} \\ n_1 &= n_2 = 3 \\ y_1 = y_\psi &\neq y_2 = y_{\bar{\psi}} \\ M &= \frac{v^2}{2} M_\psi^{-1} \end{aligned}\right\} \Rightarrow$$

$$\boxed{m = \frac{\lambda_\Phi \, v^4}{4 \, M_\Phi^2} \left[y_\psi^T M_\psi^{-1} y_{\bar{\psi}} + y_{\bar{\psi}}^T (M_\psi^{-1})^T y_\psi \right]}$$



[Babu, Nandi, Tavartkiladze, 2009]

ν-data NH within 3σ $\quad M_\psi \in [0.5, 2]$ TeV $\quad W = \mathbb{I}$

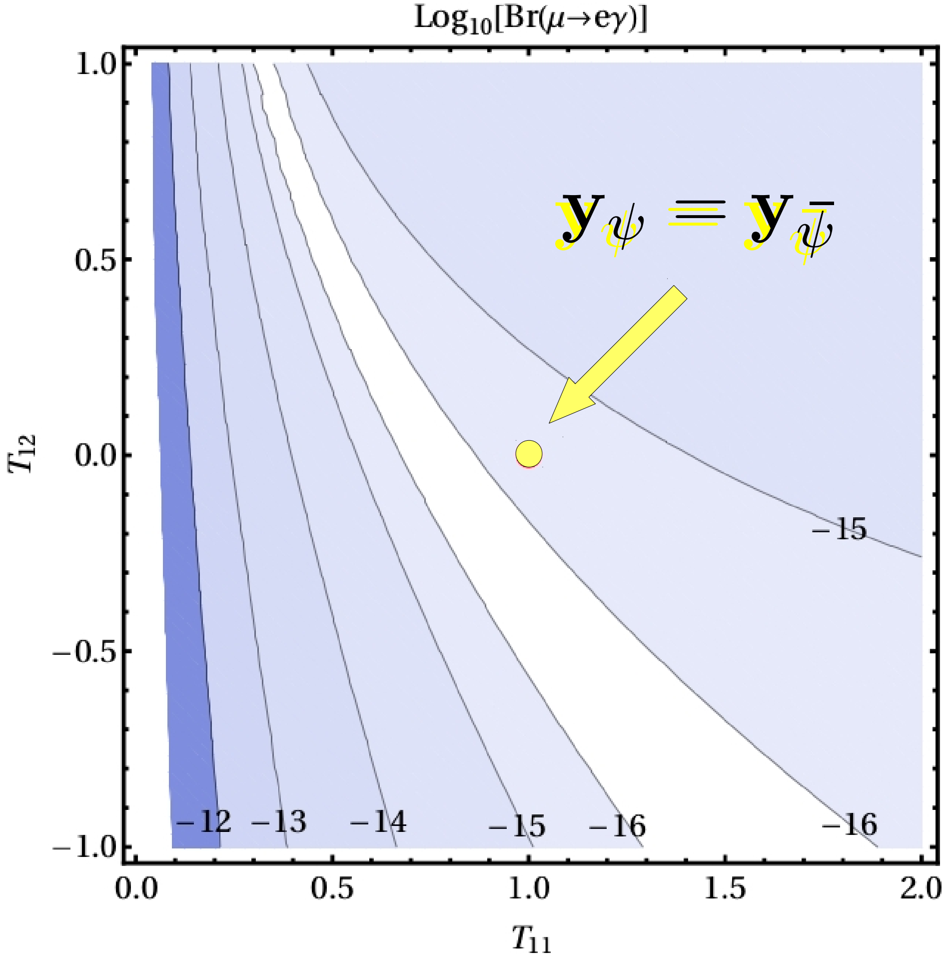


Black: Trivial scan $[T = \mathbb{I}$ & $K = 0\,]$

Purple: General scan

A large parameter space that can only be covered with the **master parametrization**

ν-data NH BFP $\quad v_\Phi = 10^{-5}$ GeV $\quad M_\psi = 0.5$ TeV



Model in **SARAH** and scans with **SSP** [Staub]

BR computed with **FlavorKit** [Porod, Staub, AV]

Final remarks

Final remarks

LFV is going to live a golden age

Many LFV observables. Correlations are not only possible, but in fact expected!

We must be ready: understand the LFV anatomy, patterns, correlations, hierarchies...

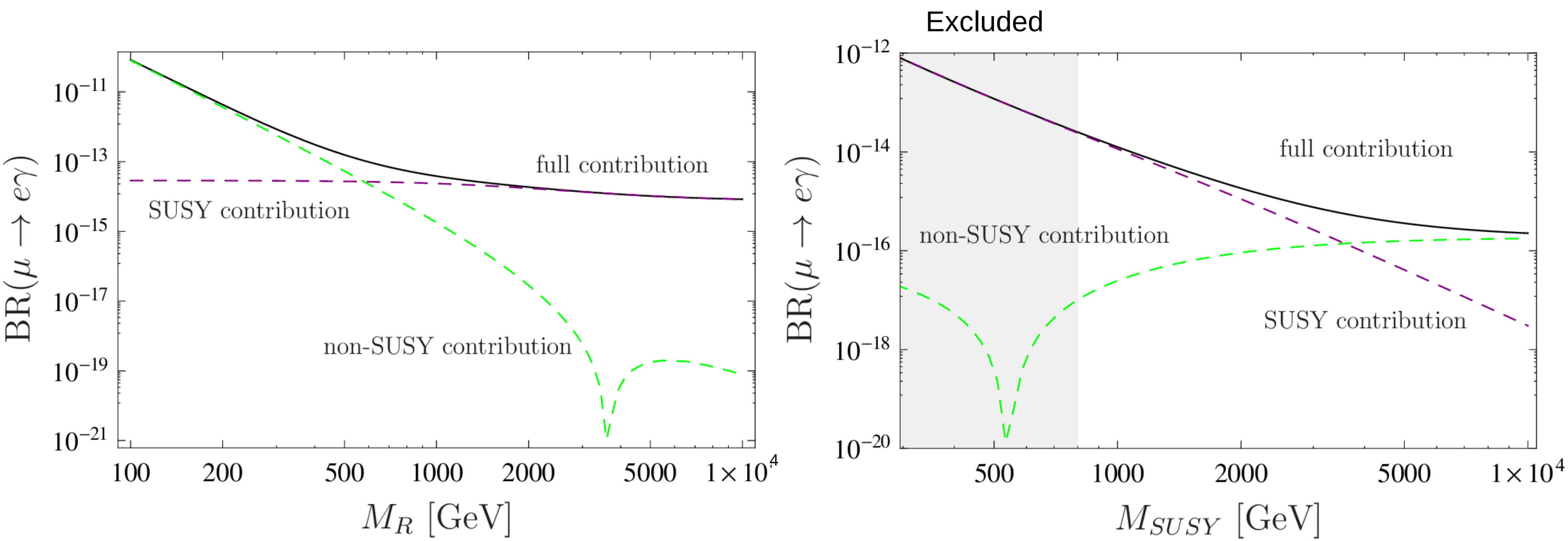

Thank you!

Backup slides

Low-scale seesaw models

$\boxed{\ell_i \to \ell_j \gamma}$

[Abada, Krauss, Porod, Staub, AV, Weiland, 2014]

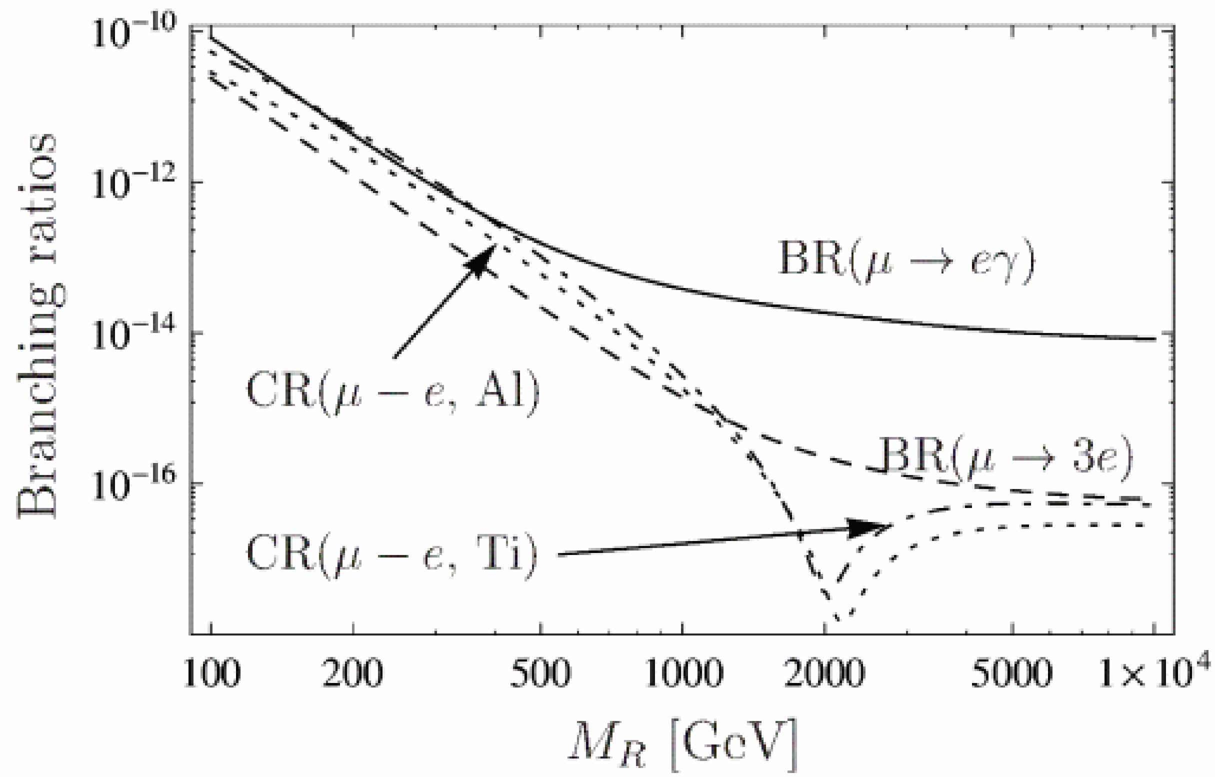


The anatomy of LFV strongly depends on M_R and M_{SUSY}

Low-scale seesaw models

$$\boxed{\ell_i \to 3\,\ell_j}$$

[Abada, Krauss, Porod, Staub, AV, Weiland, 2014]

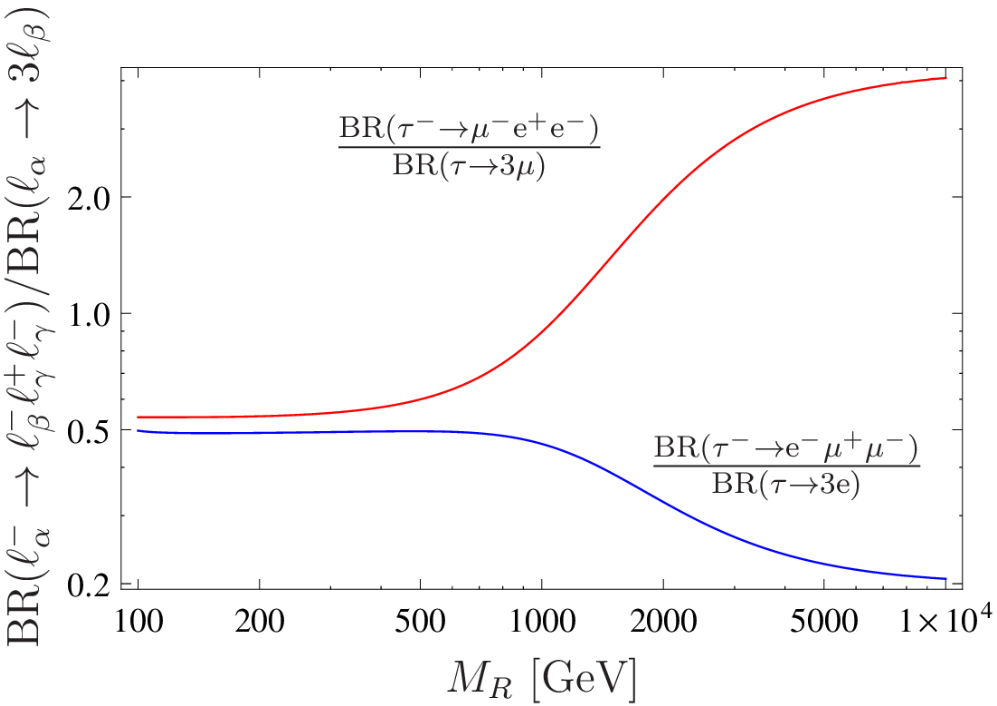


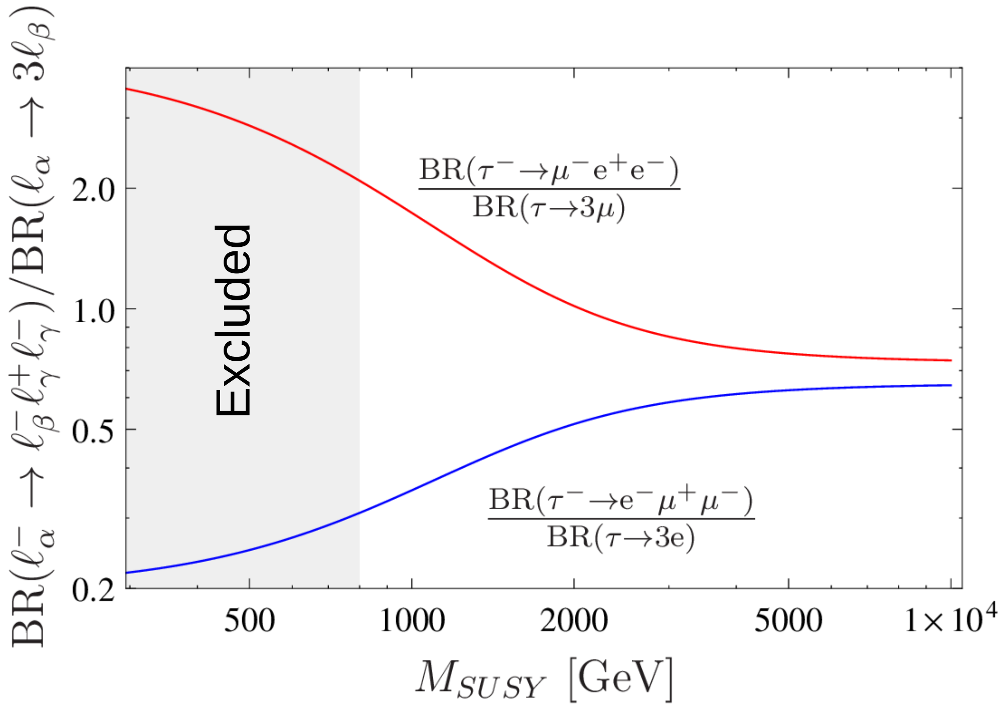
The dipole dominance is broken for low RH neutrino masses

Low-scale seesaw models

$$\boxed{\ell_i \to \ell_j \ell_k \ell_k}$$

[Abada, Krauss, Porod, Staub, AV, Weiland, 2014]





Tau LFV decay ratios provide information on the mass scales

Interpreting the anomalies

$$\boxed{\boldsymbol{b \to s}}$$

Effective hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i \left(C_i \mathcal{O}_i + C_i' \mathcal{O}_i' \right) + \text{h.c.}$$

C_i : Wilson coefficients

$\mathcal{O}_i$: Operators

$$\mathcal{O}_9 = (\bar{s}\gamma_\mu P_L b) \left(\bar{\ell}\gamma^\mu \ell\right) \qquad \mathcal{O}_9' = (\bar{s}\gamma_\mu P_R b) \left(\bar{\ell}\gamma^\mu \ell\right)$$

$$\mathcal{O}_{10} = (\bar{s}\gamma_\mu P_L b) \left(\bar{\ell}\gamma^\mu \gamma_5 \ell\right) \qquad \mathcal{O}_{10}' = (\bar{s}\gamma_\mu P_R b) \left(\bar{\ell}\gamma^\mu \gamma_5 \ell\right)$$

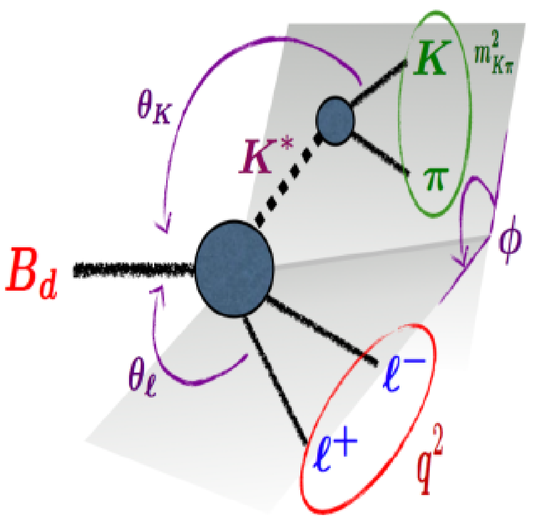
$$C_i = C_i^{\text{SM}} + C_i^{\text{NP}}$$

[analogous for primed operators]

The $b \to s$ anomalies

$B \to K^* (\to K\pi)\, \mu^+\mu^-$ differential angular distribution

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$
$$+J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi$$
$$+J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



[Figure borrowed from Javier Virto]

J_i : functions of q^2, C_i, FF

Optimized observables
[Descotes-Genon et al, 2012, 2013]

$$P_5' = \frac{J_5}{2\sqrt{-J_{2s}J_{2c}}}$$

LFV at LHCb

Lepton flavor violating decays at LHCb

$$\boxed{B^0_{d,s} \to \ell_i \ell_j}$$

[Aaij et al, LHCb collaboration, 2013]

Limits improved with respect to CDF

$$\mathrm{BR}(B^0 \to e\,\mu) < 2.8 \cdot 10^{-9}$$

$$\mathrm{BR}(B^0_s \to e\,\mu) < 1.1 \cdot 10^{-8}$$

$$\boxed{\tau \to 3\,\mu}$$

[Aaij et al, LHCb collaboration, 2014]

Impressive result in a hadronic machine

Large production of τ's, clean final state

$$\mathrm{BR}(\tau \to 3\,\mu) < 4.6 \cdot 10^{-8}$$ (at 90% CL)

To be compared with $2.1 \cdot 10^{-8}$ (Belle)

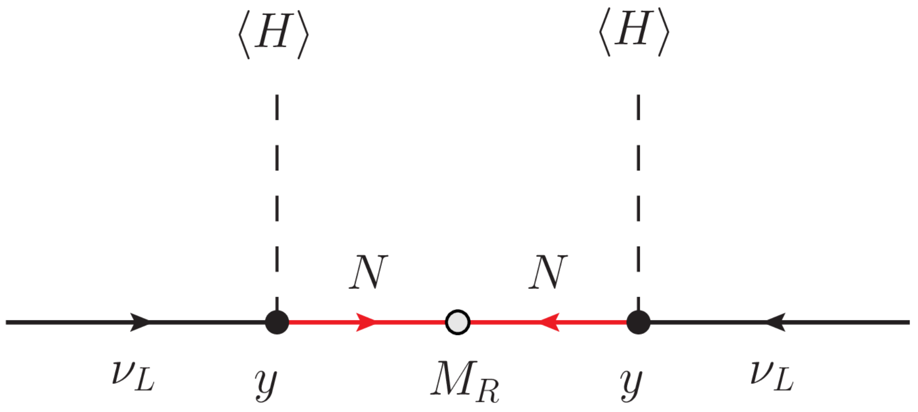
Type-I seesaw

everyone's model

$$\boxed{m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1\right)}$$

$$\left.\begin{array}{c} f = -1 \\ n_1 = n_2 = 3 \\ y_1 = y_2 = y/\sqrt{2} \\ M = \dfrac{v^2}{2} M_R^{-1} \end{array}\right\} \Rightarrow$$

$$\boxed{m = -\frac{v^2}{2} y^T\, M_R^{-1}\, y}$$

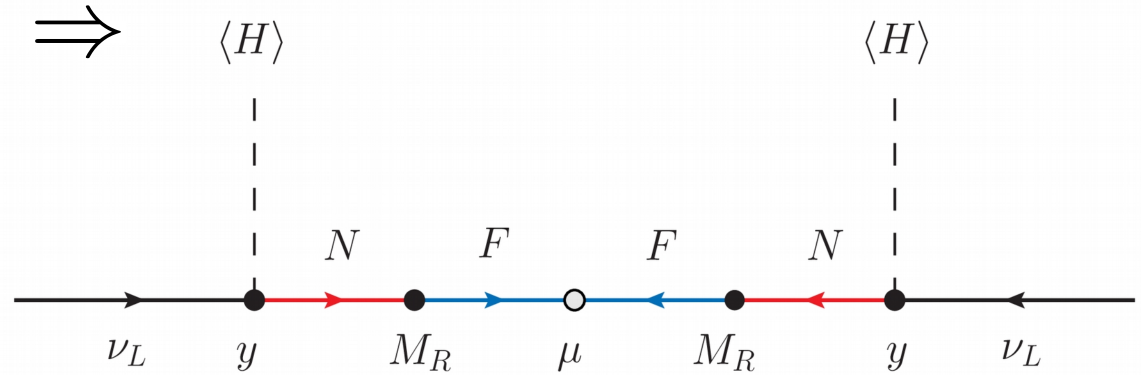


[Minkowski, 1977]

Inverse seesaw

$$\boxed{m = f \, \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)}$$

$$\left.\begin{array}{c} f = 1 \\ n_1 = n_2 = 3 \\ y_1 = y_2 = y/\sqrt{2} \\ M = \dfrac{v^2}{2} (M_R^T)^{-1} \mu \, M_R^{-1} \end{array}\right\} \Rightarrow \quad \boxed{m = \frac{v^2}{2} y^T \, (M_R^T)^{-1} \mu \, M_R^{-1} \, y}$$



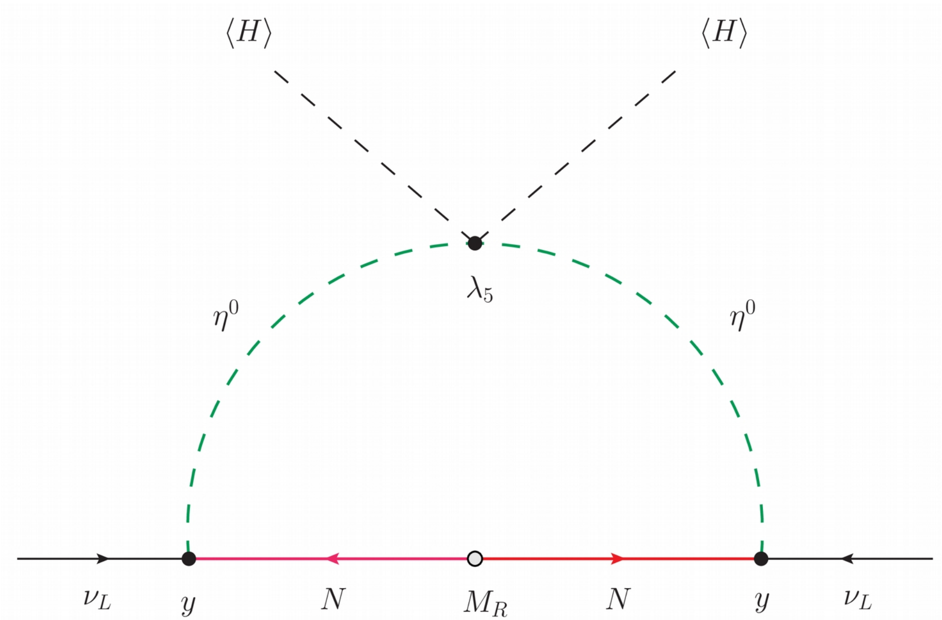
[Mohapatra, Valle, 1986]

Scotogenic model

$$\boxed{m = f \, \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)}$$

$$\boxed{m = \frac{\lambda_5 v^2}{32\pi^2} \, y^T \, M_R^{-1} f_{\text{loop}} \, y}$$

$$\left.\begin{aligned} f &= \frac{\lambda_5}{16\pi^2} \\ n_1 &= n_2 = 3 \\ y_1 &= y_2 = y/\sqrt{2} \\ M &= \frac{v^2}{2} M_R^{-1} f_{\text{loop}} \end{aligned}\right\} \Rightarrow$$



[Ma, 2006]

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$D_m = \mathrm{diag}\,(m_1, m_2, m_3) = U^T\, m\, U$$

$$\begin{array}{c} r_m = \mathrm{rank}(m) \\ 2 \,\mathrm{or}\, 3 \end{array} \left\{ \begin{array}{l} \bar{D}_{\sqrt{m}} = \mathrm{diag}\,(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}) \\ \bar{D}_{\sqrt{m}} = \mathrm{diag}\,(\sqrt{m_1}, \sqrt{m_2}, \sqrt{v}) \end{array} \right.$$

U : 3×3 unitary matrix

$$U^\dagger U = U U^\dagger = \mathbb{I}_3$$

Leptonic mixing matrix

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$M = V_1^T\, \widehat{\Sigma}\, V_2$$

Singular-value decomposition

$$\widehat{\Sigma} = \left(\begin{array}{cc} \Sigma & 0 \\ 0 & 0_{n_2-n} \\ \hline \multicolumn{2}{c}{0_{n_1-n_2}} \end{array}\right)$$

$$\widehat{\Sigma} \,:\, n_1 \times n_2$$

$$V_1 \,:\, n_1 \times n_1$$

$$V_2 \,:\, n_2 \times n_2$$

Unitary matrices

$$\Sigma \,:\, n \times n$$

$$\Sigma = \mathrm{diag}\left(\sigma_1, \sigma_2, \ldots, \sigma_n\right) \qquad (\sigma_i > 0)$$

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}} V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}} V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$X_1 \; : \; (n_2 - n) \times 3$$

$$X_2 \; : \; (n_1 - n_2) \times 3$$

$$X_3 \; : \; (n_2 - n) \times 3$$

Arbitrary matrices

[Note: absent if $n_1 = n_2 = n$]

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\,V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\,W\,A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\,U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\,V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\,\widehat{W}^*\,\widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\,U^\dagger$$

$$\widehat{W} = \begin{pmatrix} W & \bar{W} \end{pmatrix}$$

$$r = \mathrm{rank}(W) \leq \min(n,3)$$

$\widehat{W}$: $n \times n$ unitary matrix

$$\widehat{W}^\dagger \widehat{W} = \widehat{W}\widehat{W}^\dagger = \mathbb{I}_n$$

W : $n \times r$

$\bar{W}$: $n \times (n-r)$ $\longrightarrow$ Absent if $n = r$

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$A \, : \, r \times 3$

$A = T\, C_1$

$T \, : \, r \times r$

Invertible upper triangular matrix

$(T)_{ii} \in \mathbb{R}, \, (T)_{ii} > 0$

$C_1 \, : \, r \times 3$

Numerical matrix whose form depends on r_m and r

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\; A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\; \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$\widehat{B} = \begin{pmatrix} B \\ \bar{B} \end{pmatrix}$$

$\widehat{B}\ :\ n \times 3$

$B\ :\ r \times 3$

$$B = \left(T^T\right)^{-1} \left[C_1\, C_2 + K\, C_1\right]$$

$\bar{B}\ :\ (n-r) \times 3 \longrightarrow$ Absent if $n = r$

$C_2\ :\ 3 \times 3$

Matrix whose form depends on r_m and r

$K\ :\ r \times r$

antisymmetric

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

For $r_m = r = 3$:

$$C_1 = \mathbb{I}_3 \qquad C_2 = \mathbb{I}_3 + K_{12}\, \frac{T_{13}}{T_{11}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

[Different matrix forms for other values of (r_m, r). Ask me if you want to see them]

The Casas-Ibarra limit

Particular case: Type-I seesaw

$$m = -\frac{v^2}{2} y^T M_R^{-1} y$$

$$\Downarrow$$

$$\left.\begin{array}{c} f = -1 \\ n_1 = n_2 = n = 3 \\ r_m = r = 3 \\ y_1 = y_2 = y/\sqrt{2} \\ M = \frac{v^2}{2} M_R^{-1} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} V_1 = V_2 = V \ \ (= \mathbb{I} \text{ in mass basis}) \\ X_{1,2,3}\text{ , } \bar{W} \text{ and } \bar{B} \text{ are absent} \\ W^T W A = B \Rightarrow B = \left(A^T\right)^{-1} \\ \qquad \Rightarrow R = W A \\ \qquad \text{orthogonal } 3 \times 3 \\ \qquad R^T R = R R^T = \mathbb{I} \end{array}\right.$$

Casas-Ibarra parametrization [Casas, Ibarra, 2001]

$$\Uparrow$$

$$y = i\, \Sigma^{-1/2}\, R\, D_{\sqrt{m}}\, U^\dagger$$