Residual Flavor Symmetries in the $\nu_\mu \nu_\tau$ Sector

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PLAN OF THE TALK

- INTRODUCTION
- NEUTRINO RESIDUAL FLAVOR SYMMETRIES
- NEUTRINO MIXING ANGLES AND PHASES
- NUMERICAL ANALYSIS
- NEUTRINOLESS DOUBLE BETA DECAY
- CP ASYMMETRY IN LONG-BASELINE OSCILLATIONS
- FLAVOR FLUX RATIOS AT NEUTRINO TELESCOPES
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1. INTRODUCTION

Regularities observed in neutrino mixing over the years:

1. Atmospheric mixing angle θ_{23} is close to maximal value $\pi/4$.
2. Solar mixing angle θ_{12} is not far from tribimaximal value $\sin^{-1}(\frac{1}{\sqrt{3}}) \sim 35.26^\circ$.
3. Reactor mixing angle θ_{12} not far from tribimaximal value 0°.
4. Dirac CP phase δ is close to the maximal value $3\pi/2$.

Current best-fit values

$$\theta_{12} = 33.82^\circ, \;\; \theta_{23} = 49.6^\circ(\mathrm{NO}), 49.8^\circ(\mathrm{IO}), \;\; \delta = 215^\circ(\mathrm{NO}), 284^\circ(\mathrm{IO})$$

$\Rightarrow$ Some kind of discrete symmetry in the flavor space of neutrinos.

2. NEUTRINO RESIDUAL FLAVOR SYMMETRIES

Work with Majorana neutrinos

$$-\mathcal{L}_\nu^{\rm mass} = \frac{1}{2}\overline{\nu_{\ell L}^C}(M_\nu)_{\ell m}\nu_m + \text{h.c.}, \quad (M_\nu)_{\ell m} = (M_\nu)_{m\ell}$$

$$U^T M_\nu U = M_d = \text{diag}(m_1, m_2, m_3), \text{ with } m_{1,2,3} \text{ assumed} > 0.$$

Perhaps there is a residual symmetry G with

$$G^T M_\nu G = ? \qquad \text{RHS can be} + M_\nu, -M_\nu, +M_\nu^*, -M_\nu^*.$$

G is a discrete symmetry. Questions:

1. What is G?
2. What are the characteristic phenomenological predictions of G?
3. How do these predictions compare with current experiment?
4. Will distinctive predictions from G be testable in future set-ups?
5. Can G be embedded in a larger symmetry group which in turn comes from a GUT?

We deal with exact symmetries: perturbations introduce too many parameters.

1. Historically, first $\mu\tau$ exchange symmetry ($\mu\tau S$);

Fukushima and Nishiura (1997), Review by King (2017)

Invariance under $\nu_{L\ell} \to G_{\ell m}\nu_{Lm}$

$$G = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } G^T M_\nu G = M_\nu. \text{ Obtainable from } S_4$$

Altarelli and Feruglio (2010)

$$M_\nu^{\mu\tau S} = \begin{pmatrix} x & a & -a \\ a & y & c \\ -a & c & y \end{pmatrix}, \quad x, a, c, y \text{ complex mass-dimensional}$$

$\Rightarrow \theta_{13} = 0$ ruled out at $10\sigma, \theta_{23} = \pi/4$ (disfavored).

No observable Dirac CP violation. Strong experimental hints to the contrary.

Abandoned!

2. Next, $\mu\tau$ exchange antisymmetry ($\mu\tau A$).

Grimus et al (2006)

Same G but $G^T M_\nu G = -M_\nu$. Obtainable from $\mathbb{Z}_4$

Altarelli and Feruglio (2010), Joshipura (2015)

$$\Rightarrow M_\nu^{\mu\tau S} = \begin{pmatrix} 0 & a & a \\ a & y & 0 \\ a & 0 & -y \end{pmatrix}$$

$\Rightarrow \theta_{13} = 0$ ruled out at $10\sigma, \theta_{23} = \pi/4$ (disfavored).

Same consequences as $\mu\tau S$ $+$ one massless and two degenerate neutrinos (ruled out by $\Delta m^2_{21} \neq 0 \neq \Delta m^2_{32}$). Proponents considered perturbations$\rightarrow$ too complicated.

Abandoned!

3. Now, CP extended $\mu\tau$ symmetry (CP$\mu\tau S$)

Harrison and Scott (2002)
Grimus & Lavoura (2004)
Mohapatra & Nishi (2015)

$G^T M_\nu G = M_\nu^*$. Obtainable from S_4

$$\Rightarrow M_\nu^{CP\mu\tau S} = \begin{pmatrix} x_1 & a & -a \\ a & y & c_1 \\ a^* & c_1 & y^* \end{pmatrix}$$

Symmetry transformation: $\nu_{L\ell} \to iG_{\ell m}\gamma^0\nu_{Lm}^C$.

Admits $\theta_{13} \neq 0$, Majorana phases 0 or π. $\theta_{23} = \pi/4$ and Dirac phase δ either $\pi/2$ or $3\pi/2$ (both in tension with latest data).

4. Next, CP extended $\mu\tau$ antisymmetry (CP$\mu\tau A$)

Samanta, PR, Ghosal (2018)

$$G^T M_\nu G = -M_\nu^*.$$

Obtainable from $\nu_{L\ell} \to -G_{\ell m}\gamma^0 \nu_{Lm}^C$

$$M_\nu^{CP\mu\tau S} = -iM_\nu^{CP\mu\tau S}$$

Phenomenology identical to that of $CP^{\mu\tau S}$. Leptogenesis with minimal seesaw (two heavy RH neutrinos N_1, N_2) worked out in detail by SRG.

5. Mixed (θ) $\mu\tau$ exchange symmetry ($\mu\tau\theta S$)
Introduces one extra parameter: mixing angle θ. Now,

$$G^\theta = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & \sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \text{ and } G^T M_\nu G = M_\nu.$$

$\theta \to \pi/2$ lets $\mu\tau\theta S \to \mu\tau S$.

$$M_\nu^{\mu\tau\theta S} = \begin{pmatrix} x & a & -a\frac{1-c_\theta}{s_\theta} \\ a & y & c \\ -a\frac{1-c_\theta}{s_\theta} & c & y + 2c\frac{c_\theta}{s_\theta} \end{pmatrix}.$$

Though $\theta_{23} \neq \pi/4, \theta_{13} = 0$.

Excluded!

A modification, proposed by Samanta, Sinha, Ghosal (2018) is still allowed.

6. Mixed (θ) $\mu\tau$ exchange antisymmetry ($\mu\tau\theta A$).

Same G^θ but $G^T M_\nu G = -M_\nu$.

$$M_\nu^{\mu\tau\theta A} = \begin{pmatrix} x & a & a\frac{1+c_\theta}{s_\theta} \\ a & y & y\frac{c_\theta}{s_\theta} \\ a\frac{1+c_\theta}{s_\theta} & y\frac{c_\theta}{s_\theta} & -y \end{pmatrix}.$$

$\theta \to \pi/2$ lets $\mu\tau\theta A \to \mu\tau A$. Once again, $\theta_{13} = 0$.

Excluded!

7. CP-transformed mixed $\mu\tau$ symmetry (CP$\mu\tau\theta S$).

Chen et al (2016)

$$\text{Now, } (G^\theta)^T M_\nu G^\theta = M_\nu^*.$$

$$M_\nu^{CP\mu\tau\theta S} = \begin{pmatrix} x_1 & a_1 + ia_2 & -a_1\frac{1-c_\theta}{s_\theta} + ia_2\frac{1+c_\theta}{s_\theta} \\ a_1 + ia_2 & y_1 + iy_2 & c_1 + iy_2\frac{c_\theta}{s_\theta} \\ -a_1\frac{1-c_\theta}{s_\theta} + ia_2\frac{1+c_\theta}{s_\theta} & c_1 + iy_2\frac{c_\theta}{s_\theta} & -iy_2 + 2c_1\frac{c_\theta}{s_\theta} \end{pmatrix}.$$

with $x, a_{1,2}, y_{1,2}$ and c_1 as real mass-dimensional parameters.
$\Rightarrow \theta_{13} \neq 0, \theta_{23} \neq \pi/4$ and δ not fixed:

$$\sin\delta = \pm\frac{\sin\theta}{\sin 2\theta_{23}}.$$

Note that, for $\theta \to \pi/2$, CP$\mu\tau\theta$S$\to$CP$\mu\tau$S.

8. CP-transformed mixed $\mu\tau$ antisymmetry (CP$\mu\tau\theta A$).

Sinha, Roy, Ghosal (2018)

Now, $(G^\theta)^T M_\nu G^\theta = -M_\nu^*$.
One obtains

$$M_\nu^{CP\mu\tau\theta A} = iM_\nu^{CP\mu\tau S}.$$

Phenomenology identical to that of $M_\nu^{CP\mu\tau\theta S}$.

Implications of leptonic CP violation in long-baseline experiments, $0\nu\beta\beta$ decay and flavor flux ratios at neutrino telescopes worked out in detail by SRG.

Lam's observation: Lam (2007)

$$\mathcal{G}^\theta U^* = U\tilde{d}, \quad \tilde{d} = \mathrm{diag}(\tilde{d}_1, \tilde{d}_2, \tilde{d}_3), \; d_{1,2,3} = \pm 1.$$

$$U = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})U_{\mathrm{PMNS}},$$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & e^{i\frac{\alpha}{2}}s_{12}c_{13} & s_{13}e^{-i(\delta-\frac{\beta}{2})} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & e^{i\frac{\alpha}{2}}(c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}) & c_{13}s_{23}e^{i\frac{\beta}{2}} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & e^{i\frac{\alpha}{2}}(-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}) & c_{13}c_{23}e^{i\frac{\beta}{2}} \end{pmatrix}.$$

Algebraic matching leads to

$$e^{i\alpha} = \tilde{d}_1\tilde{d}_2, \; e^{2i(\delta-\frac{\beta}{2})} = \tilde{d}_1\tilde{d}_3$$

$$\Rightarrow \alpha = 0 \text{ or } \pi, \; \text{and } \beta = 2\delta \text{ or } (2\delta - \pi).$$

Moreover, $\cot 2\theta_{23} = \cot\theta\cos(\phi_2 - \phi_3)$, $\sin\delta = \pm\sin\theta/\sin 2\theta_{23}$, i.e. $\theta \to \pi/2 \Rightarrow \theta_{23} \to \frac{\pi}{4}$. In general, $\theta_{23} \neq \pi/4$ and $\delta \neq 0$ or π.

Input mixing angles and mass-squared differences from latest global analysis. Esteban et al (2017)

Neutrino mass sum $m_1 + m_2 + m_3 < 0.17$ eV from Planck data. Aghanim et al (2016)

Table: Input 3σ ranges used in the analysis

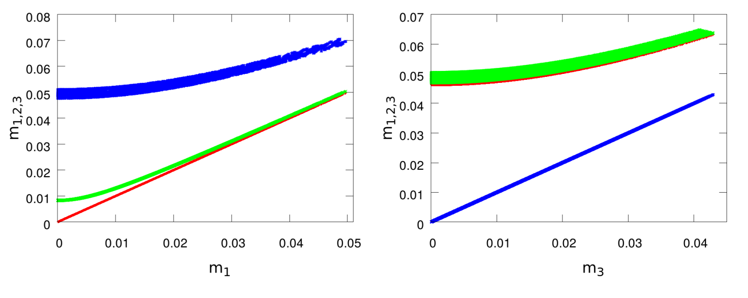
Values	θ_{12} degrees	θ_{23} degrees	θ_{13} degrees	Δm^2_{21} 10^{-5}eV^2	$\lvert\Delta m^2_{31}\rvert$ 10^{-3}(eV^2)
NO	31.42 to 36.05	40.3 to 51.5	8.09 to 8.98	6.80 to 8.02	2.399 to 2.593
IO	31.43 to 36.06	41.3 to 51.7	8.14 to 9.01	6.80 to 8.02	2.399 to 2.593

Table: Output values of the parameters of M_ν

Values	$10^3 x$	$10^3 a_1$	$10^3 a_2$	$10^3 y_1$	$10^3 y_1$	$10^3 c$	$\theta(^\circ)$
NO	-22 to 22	-45 to 45	-32 to 32	-35 to 35	-45 to 45	-35 to 35	12 to 164
IO	-25 to 25	-45 to 45	-4 to 4	-25 to 25	-35 to 35	-25 to 25	2 to 156

Table: Predictions on the light neutrino masses.

Normal Ordering ($m_3 > m_2$)			Inverted Ordering ($m_3 < m_1$)		
$10^3 m_1(\mathrm{eV})$	$10^{-3} m_2(\mathrm{eV})$	$10^3 m_3(\mathrm{eV})$	$10^3 m_1(\mathrm{eV})$	$10^3 m_2(\mathrm{eV})$	$10^3 m_3(\mathrm{eV})$
$8.4 \times 10^{-2} - 49$	$9 - 51$	$50 - 71$	$48 - 64$	$49 - 66$	$4.4 \times 10^{-2} - 42$



Neutrino masses for normal (left) and inverted (right) ordering against the lightest mass eigenvalue. The red, green and blue bands refer to m_1, m_2 and m_1 respectively.

5. NEUTRINOLESS DOUBLE BETA DECAY

$$(A, Z) \to (A, Z+2) + 2e^-$$

Half-life $T^{0\nu}_{1/2} = G_{0\nu}|\mathcal{M}|^2|M^{ee}_\nu|^2 m_e^{-2}$,
$G_{0\nu}$ = two-body phase space factor, $\mathcal{M}$ = nuclear matrix element,

$$M^{ee}_\nu = c^2_{12}c^2_{13}m_1 + s^2_{12}c^2_{13}m_2 e^{i\alpha} + s^2_{13}m_3 e^{i(\beta-2\delta)}$$

Four cases in our model.

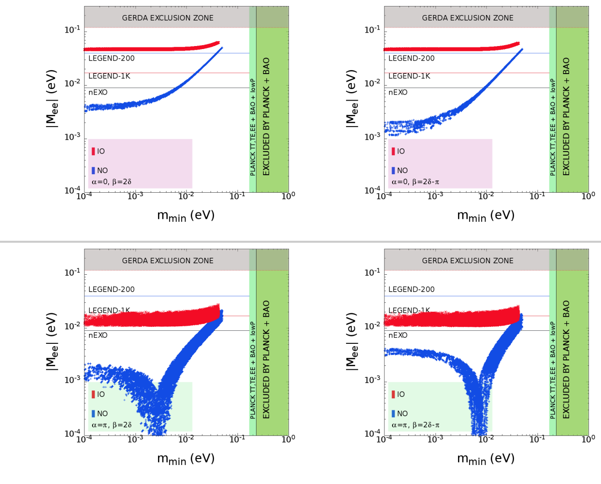
(i) $|M^{ee}_\nu| = c^2_{12}c^2_{13}m_1 + s^2_{12}c^2_{13}m_2 + s^2_{13}m_3$ for $\alpha = 0, \beta = 2\delta$,

(ii) $|M^{ee}_\nu| = c^2_{12}c^2_{13}m_1 + s^2_{12}c^2_{13}m_2 - s^2_{13}m_3$ for $\alpha = 0, \beta = 2\delta - \pi$,

(iii) $|M^{ee}_\nu| = c^2_{12}c^2_{13}m_1 - s^2_{12}c^2_{13}m_2 + s^2_{13}m_3$ for $\alpha = \pi, \beta = 2\delta$ and

(iv) $|M^{ee}_\nu| = c^2_{12}c^2_{13}m_1 - s^2_{12}c^2_{13}m_2 - s^2_{13}m_3$ for $\alpha = \pi, \beta = 2\delta - \pi$.

Plots of $|M_\nu^{ee}|$ versus the minimum neutrino mass m_{min}



The four plots correspond to four possible choices of α and β.

Predicted signal below the reach of GERDA phase II but reachable by LEGEND-200, LEGEND-1K and nEXO. Failure of nEXO to see any signal would rule out our model for IO.

Experimental CP asymmetry

$$A_{\mu e} = \frac{2\sqrt{P_{\rm atm}}\sqrt{P_{\rm sol}}\sin\Delta_{32}\sin\delta}{P_{\rm atm} + P_{\rm sol} + 2\sqrt{P_{\rm atm}}\sqrt{P_{\rm sol}}\cos\Delta_{32}\cos\delta}$$

with

$$\sqrt{P_{\rm atm}} \equiv s_{23}s_{13}\frac{\sin(\Delta_{31} - aL)}{\Delta_{31} - aL}\sin\Delta_{31},$$

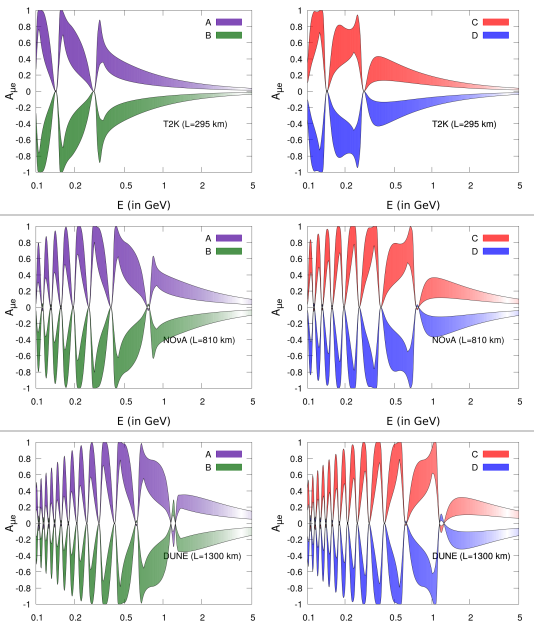
$$\sqrt{P_{\rm sol}} \equiv 2c_{12}s_{12}c_{23}c_{13}\frac{\sin(aL)}{aL}\sin\Delta_{21},$$

$\Delta_{ij} = \dfrac{\Delta m^2_{ij}L}{4E}$, $a = \dfrac{G_F N_e}{\sqrt{2}} \simeq 3500{\rm km}^{-1}$, $N_e =$ electron density in the medium

$\sin\delta$ and $\cos\delta$ can have four different combinations.

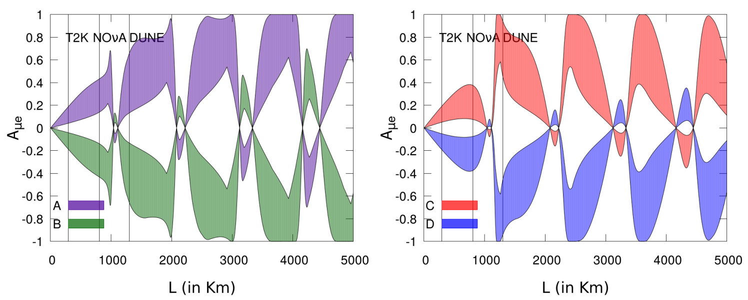
Table: Four possibilities for $A_{\mu e}$

Possibilities	$\sin\delta$	$\cos\delta$
Case A	$+\sin\theta(\sin 2\theta_{23})^{-1}$	$+(\sin 2\theta_{23})^{-1}\sqrt{\cos^2\theta\sin^2 2\theta_{23} - \sin^2\theta\cos^2 2\theta_{23}}$
Case B	$-\sin\theta(\sin 2\theta_{23})^{-1}$	$+(\sin 2\theta_{23})^{-1}\sqrt{\cos^2\theta\sin^2 2\theta_{23} - \sin^2\theta\cos^2 2\theta_{23}}$
Case C	$+\sin\theta(\sin 2\theta_{23})^{-1}$	$-(\sin 2\theta_{23})^{-1}\sqrt{\cos^2\theta\sin^2 2\theta_{23} - \sin^2\theta\cos^2 2\theta_{23}}$
Case D	$-\sin\theta(\sin 2\theta_{23})^{-1}$	$-(\sin 2\theta_{23})^{-1}\sqrt{\cos^2\theta\sin^2 2\theta_{23} - \sin^2\theta\cos^2 2\theta_{23}}$



Plots of $A_{\mu e}$ against beam energy E for different baselines lengths of T2K, NOνA and DUNE respectively.

The numerical distinction between NO and IO is insignificant for the 3σ range of θ_{23}.



CP asymmetry parameter $A_{\mu e}$ vs. baseline length L for cases A,B,C,D.

1. For a fixed beam energy of $E = 1\text{GeV}$.

2. Plots are practically indistinguishable for NO and IO.

3. The bands are due to 3σ range θ_{23} while the other parameters are kept at their best fit values.

7. FLAVOR FLUX RATIOS AT NEUTRINO TELESCOPES

Source: Cosmic pp collisions (TeV-PeV)
$\to \pi^+\pi^- \to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu \to e^+e^-2\nu_\mu 2\bar{\nu}_\mu\nu_e\bar{\nu}_e$

$$\Rightarrow \{\phi^S_{\nu_e}, \phi^S_{\bar{\nu}_e}, \phi^S_{\nu_\mu}, \phi^S_{\bar{\nu}_\mu}, \phi^S_{\nu_\tau}, \phi^S_{\bar{\nu}_\tau}\} = \phi_0\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, 0, 0\right\}.$$

Source: Cosmic $p\gamma$ collisions (GeV-10^2GeV)
$\to \pi^+ \to \mu^+\nu_\mu \to e^+\nu_e + \bar{\nu}_\mu$.

$$\Rightarrow \{\phi^S_{\nu_e}, \phi^S_{\bar{\nu}_e}, \phi^S_{\nu_\mu}, \phi^S_{\bar{\nu}_\mu}, \phi^S_{\nu_\tau}, \phi^S_{\bar{\nu}_\tau}\} = \phi_0\left\{\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3}, 0, 0\right\}.$$

With $\phi^S_\ell \equiv \phi^S_{\nu_\ell} + \phi^S_{\bar{\nu}_\ell}$,

$$\{\phi^S_e, \phi^S_\mu, \phi^S_\tau\} = \phi_0\left\{\frac{1}{3}, \frac{2}{3}, 0\right\}$$

for both sources, ϕ_0 =overall normalization.

Flux at source $S \to$ flux at telescope T changed by neutrino oscillations averaged over many periods.

Effectively, $P(\nu_m \to \nu_\ell) = P(\bar{\nu}_m \to \bar{\nu}_\ell) \simeq \sum_i |U_{ei}|^2 |U_{mi}|^2$ and

$$\phi_\ell^T = \sum_i \sum_m \phi_m^S |U_{\ell i}|^2 |U_{mi}|^2 = \frac{\phi_0}{3} \sum_i |U_{\ell i}|^2 (|U_{ei}|^2 + 2|U_{\mu i}|^2).$$

It follows from the unitarity of U that $\phi_\ell^T = \frac{\phi_0}{3}[1 + \sum_i |U_{\ell i}|^2 (|U_{\mu i}|^2 - |U_{\tau i}|^2)]$ which vanishes for exact $\mu\tau$ symmetry or antisymmetry, but is nonzero in general.

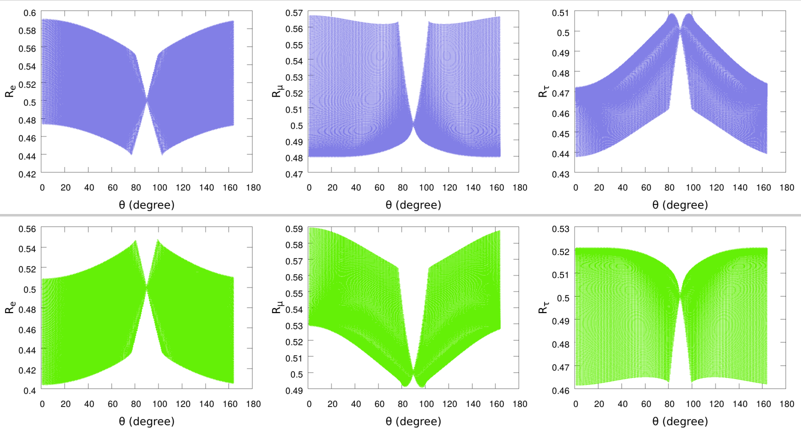
Neglect $\mathcal{O}(\sin^2\theta_{13}) \approx 0.01$ terms and define flavor flux ratios $R_e \equiv \phi_e(\phi_\mu + \phi_\tau)^{-1}$, $R_\mu \equiv \phi_\mu(\phi_e + \phi_\tau)^{-1}$, $R_\tau \equiv \phi_\tau(\phi_\mu + \phi_e)^{-1}$. Now,

$$R_e \approx \frac{1 + \frac{1}{2}\sin^2 2\theta_{12}\cos 2\theta_{23} + \frac{1}{2}\sin 4\theta_{12}\sin 2\theta_{23} s_{13}\cos\delta}{2 - \frac{1}{2}\sin^2 2\theta_{12}\cos 2\theta_{23} - \frac{1}{2}\sin 4\theta_{12}\sin 2\theta_{23} s_{13}\cos\delta},$$

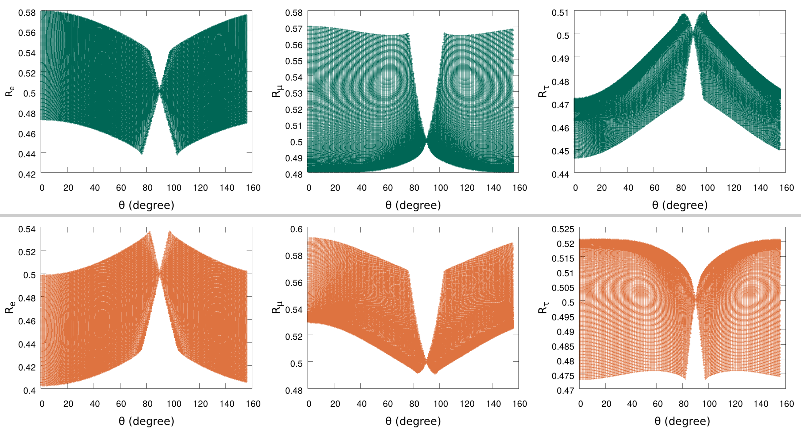
$$R_\mu \approx \frac{1 + \{c_{23}^2(1 - \frac{1}{2}\sin^2 2\theta_{12}) - s_{23}^2\}\cos 2\theta_{23} - \frac{1}{4}\sin 4\theta_{12}\sin 2\theta_{23} s_{13}\cos\delta(4c_{23}^2 - 1)}{2 - \cos^2 2\theta_{23} + \frac{1}{2}\sin^2 2\theta_{12}\cos 2\theta_{23} c_{23}^2 + \frac{1}{4}(3 - 4s_{23}^2)\sin 4\theta_{12}\sin 2\theta_{23} s_{13}\cos\delta},$$

$$R_\tau \approx \frac{1 + \{s_{23}^2(1 - \frac{1}{2}\sin^2 2\theta_{12}) + c_{23}^2\}\cos 2\theta_{23} - \frac{1}{4}\sin 4\theta_{12}\sin 2\theta_{23} s_{13}\cos\delta(4s_{23}^2 - 1)}{2 + \cos^2 2\theta_{23} + \frac{1}{2}\sin^2 2\theta_{12}\cos 2\theta_{23} c_{23}^2 + \frac{1}{4}(3 - 4c_{23}^2)\sin 4\theta_{12}\sin 2\theta_{23} s_{13}\cos\delta}.$$

Dependence on $\cos\delta$ makes R_ℓ double-valued except at $\theta = \pi/4$ ($\cos\delta = 0$ when $R_e = R_\mu = R_\tau = \frac{1}{2}$).



Flux ratios $R_{e,\mu,\tau}$ vs. θ for NO; range of θ: $12^\circ - 164^\circ$



Flux ratios $R_{e,\mu,\tau}$ vs. θ for IO; range of θ: $2^\circ - 156^\circ$

Continuous bands because of 3σ variation in input parameters.

Drastic change in R_e from $1/2$ (as θ moves away from $\pi/2$) can be used to pinpoint θ.

8. CONCLUSIONS

- Different aspects of flavor symmetries in the $\nu_\mu \nu_\tau$ sector outlined.
- CP transformed mixed ν_μ-ν_τ antisymmetry in M_ν proposed.
- With input neutrino neutrino mixing angles and mass-squared differences (3σ), ranges of values of neutrino masses for NO and IO given.
- Specific prediction on the $\beta\beta 0\nu$ process to be tested crucially by nEXO.
- Neutrino flavor flux ratios, when measured, will give information on θ.
- Specific predictions on neutrino-antineutrino flavor flux ratios to be measured in neutrino telescopes.