Precision Physics at the LHC

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Precision Physics

Plan

- Why Precision Calculation (PC)
- Impact of PCs on discoveries
- Part-1

  Methods for

  Multi-leg processes
- Part-2

  Methods for

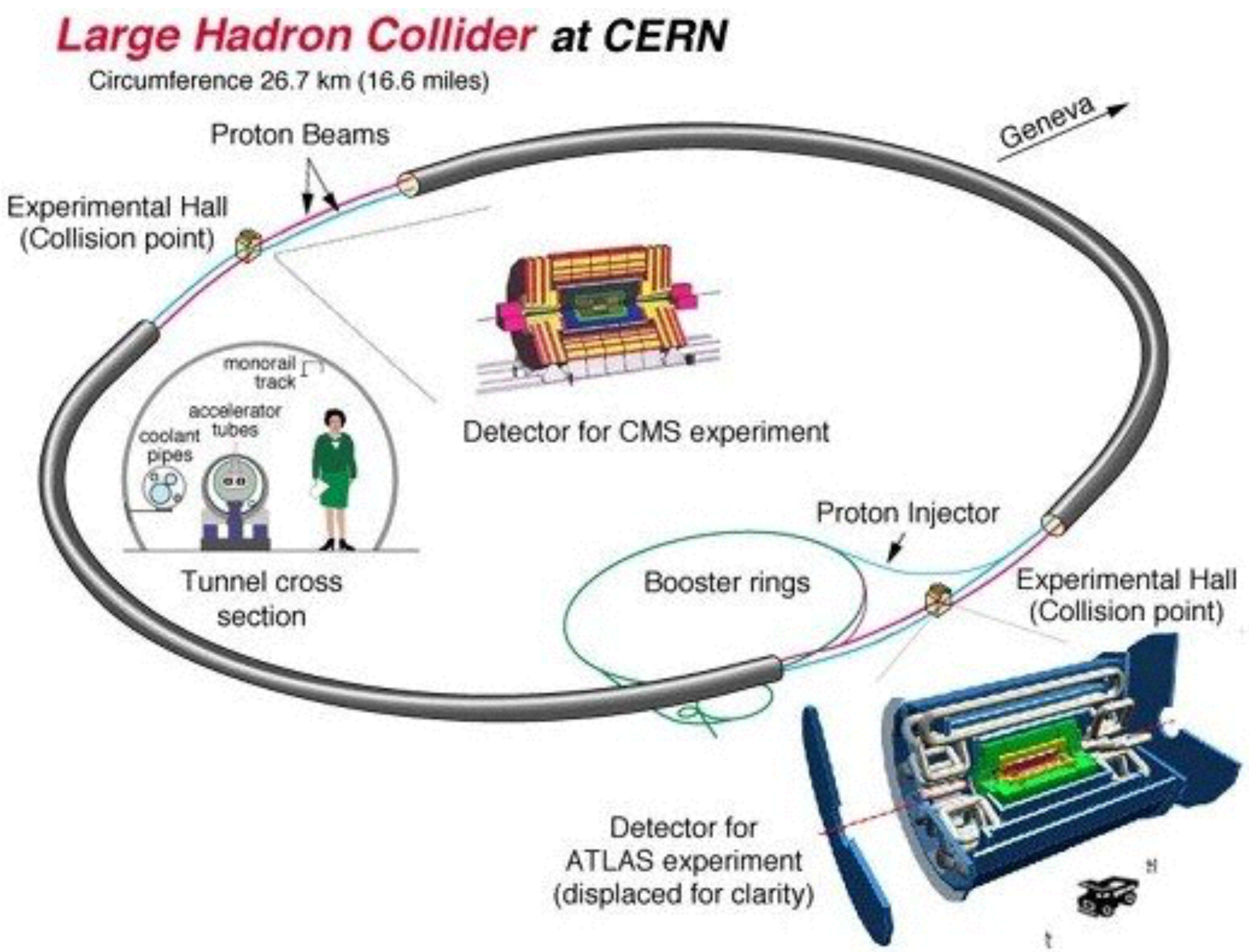
  Multi-loops processes
- Part-3

  Infrared physics

Large Hadron Collider

- Excellent Discovery Reach

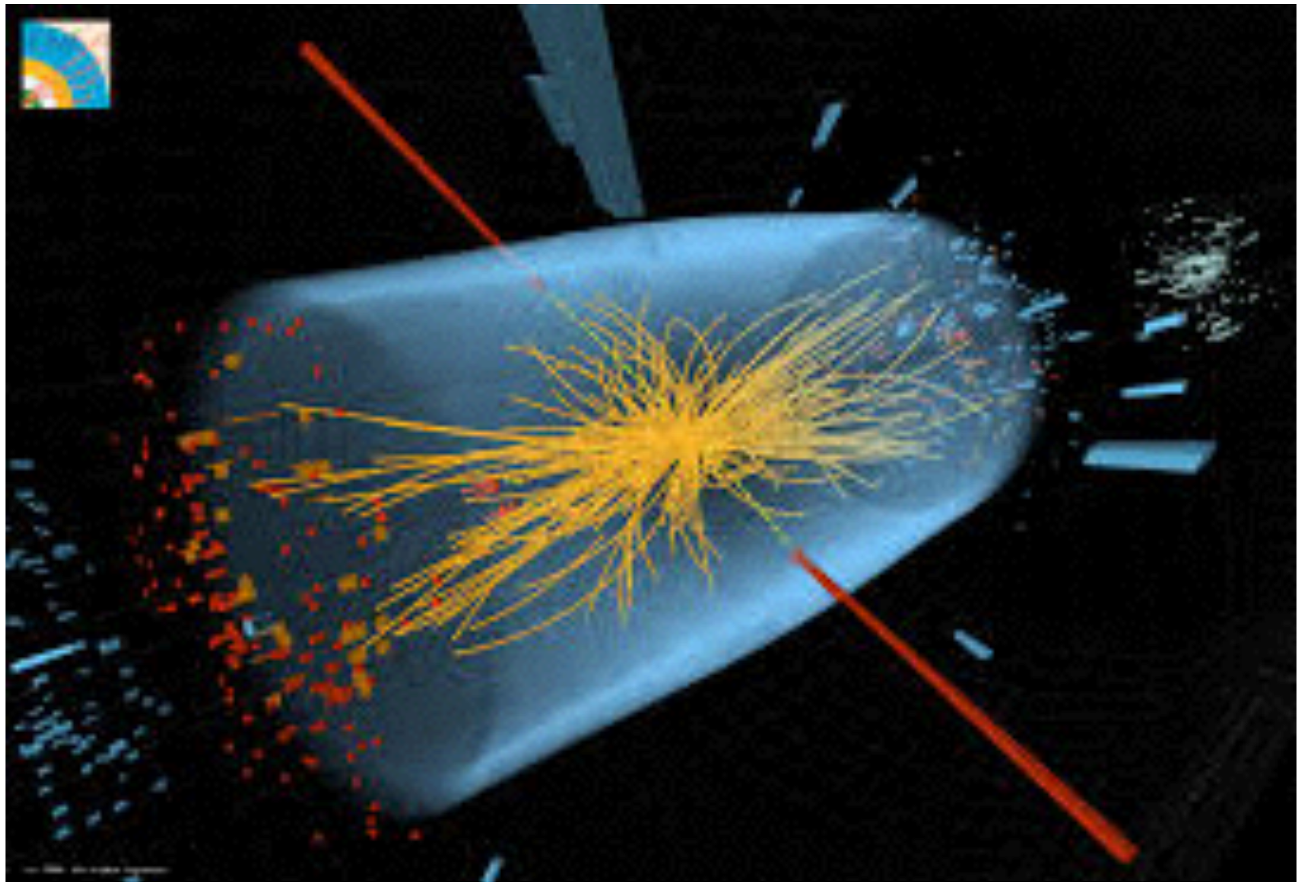
- Higgs
- Supersymmetry
- Extra-Dimensions
- Anything else



Large Hadron Collider

- Large amount of events

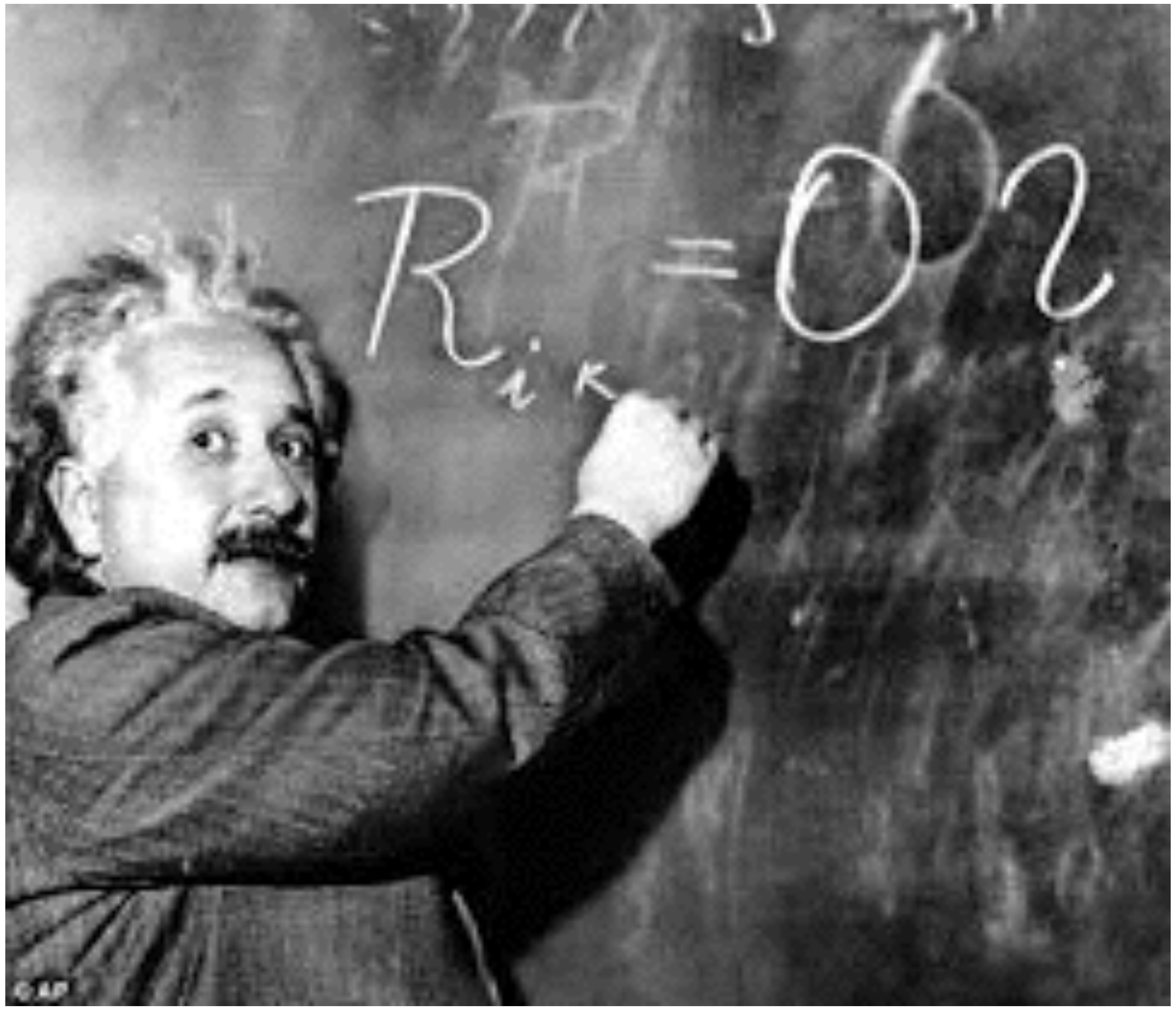
- $W \to e\nu$: 10^8 events
- $Z \to e^+e^-$: 10^7 events
- $t\bar{t}$ production 10^7 events
- Higgs production 10^5 events



Standard Model

- Theories

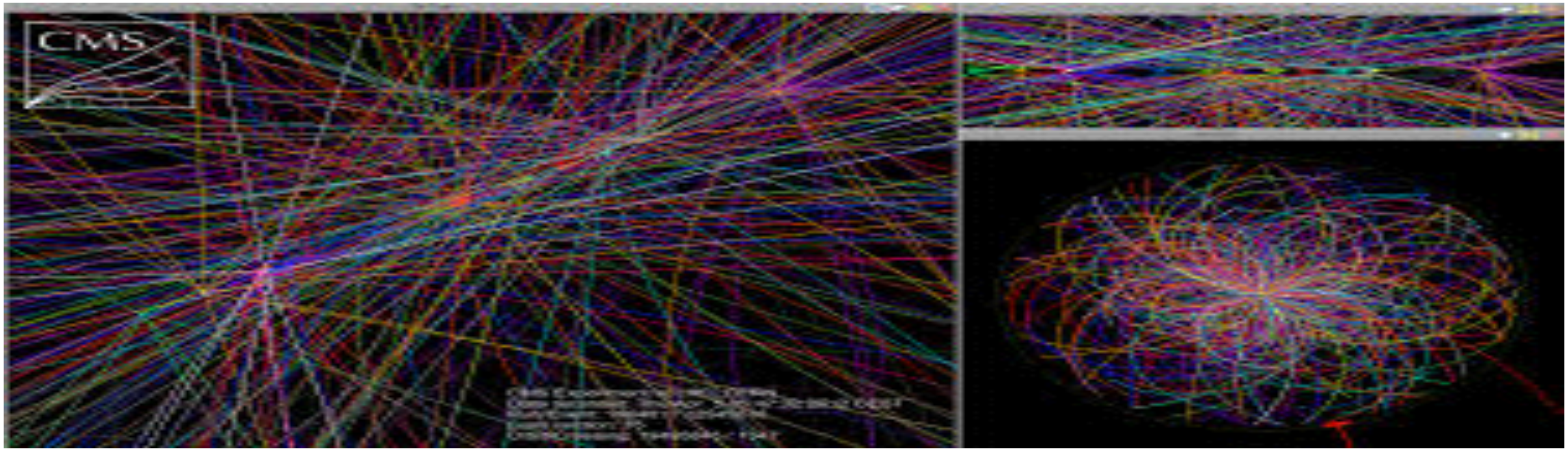
- Quantum Chromodynamics
- Electroweak Theory (SM)
- Theory of Gravity



Large Hadron Collider

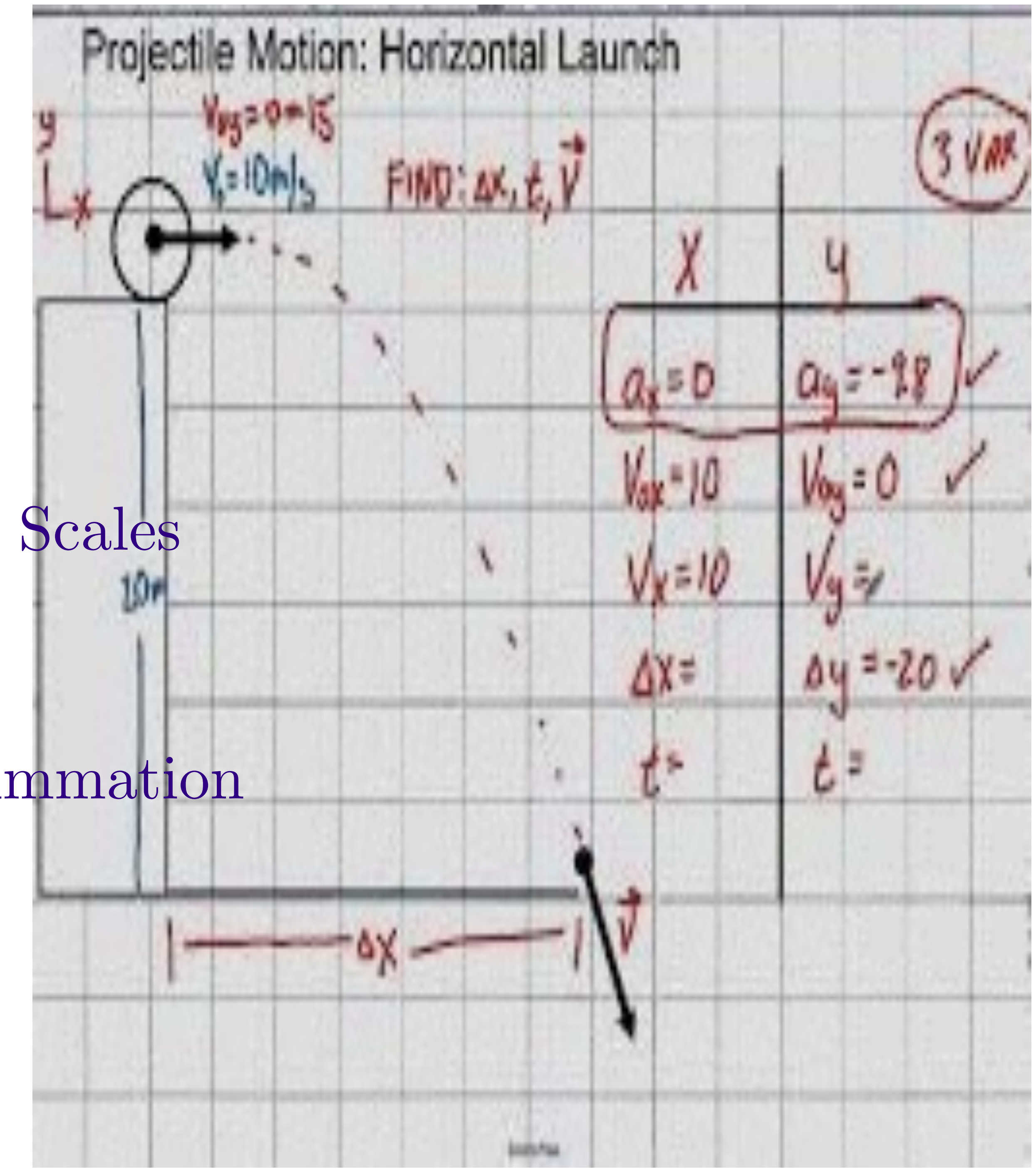
- **Large background**

- Large number of $\gamma, l^{\pm}, Z, W^{\pm}$
- Jets
- Large number of $t\bar{t}, b\bar{b}$



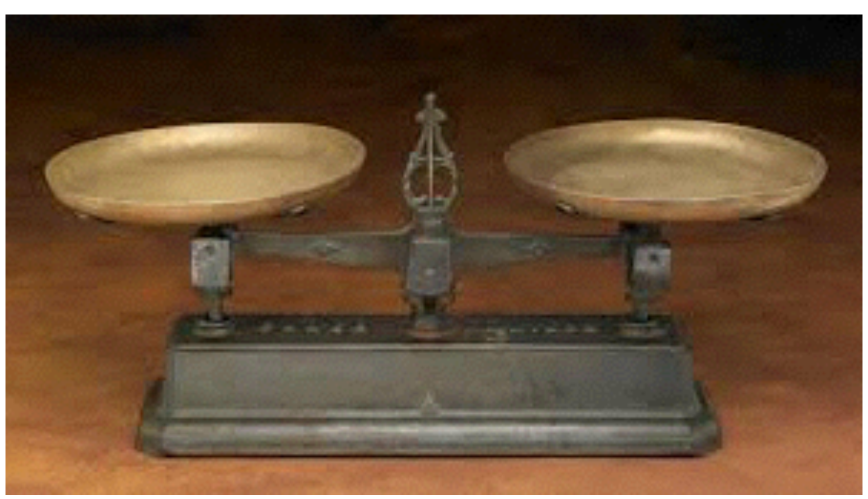
- Issues to be tackled

- Kinematics
- Normalisation
- Renormalisation and Factorisation Scales
- Parton distribution functions
- Phase space boundary effects, resummation



Why Precision ?

Precision Units Ltd

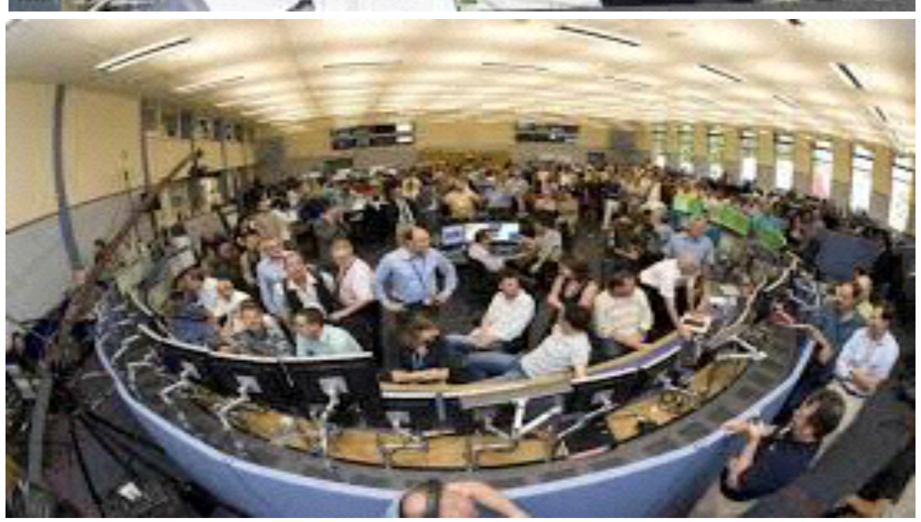


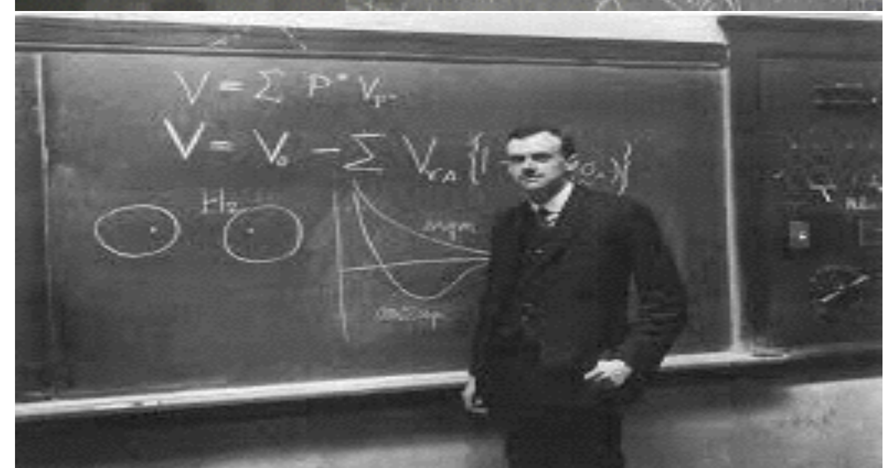
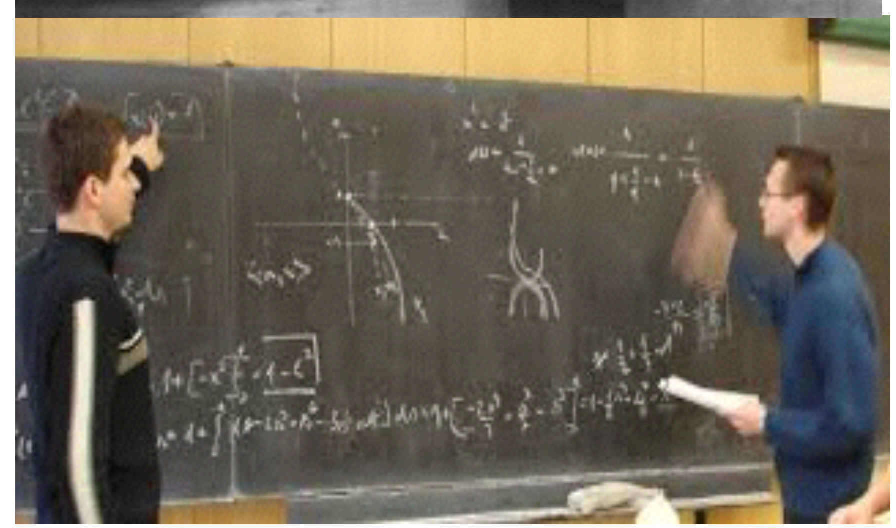
The truth is in the Details

Precision Physics

Experiment ⟷ Theory

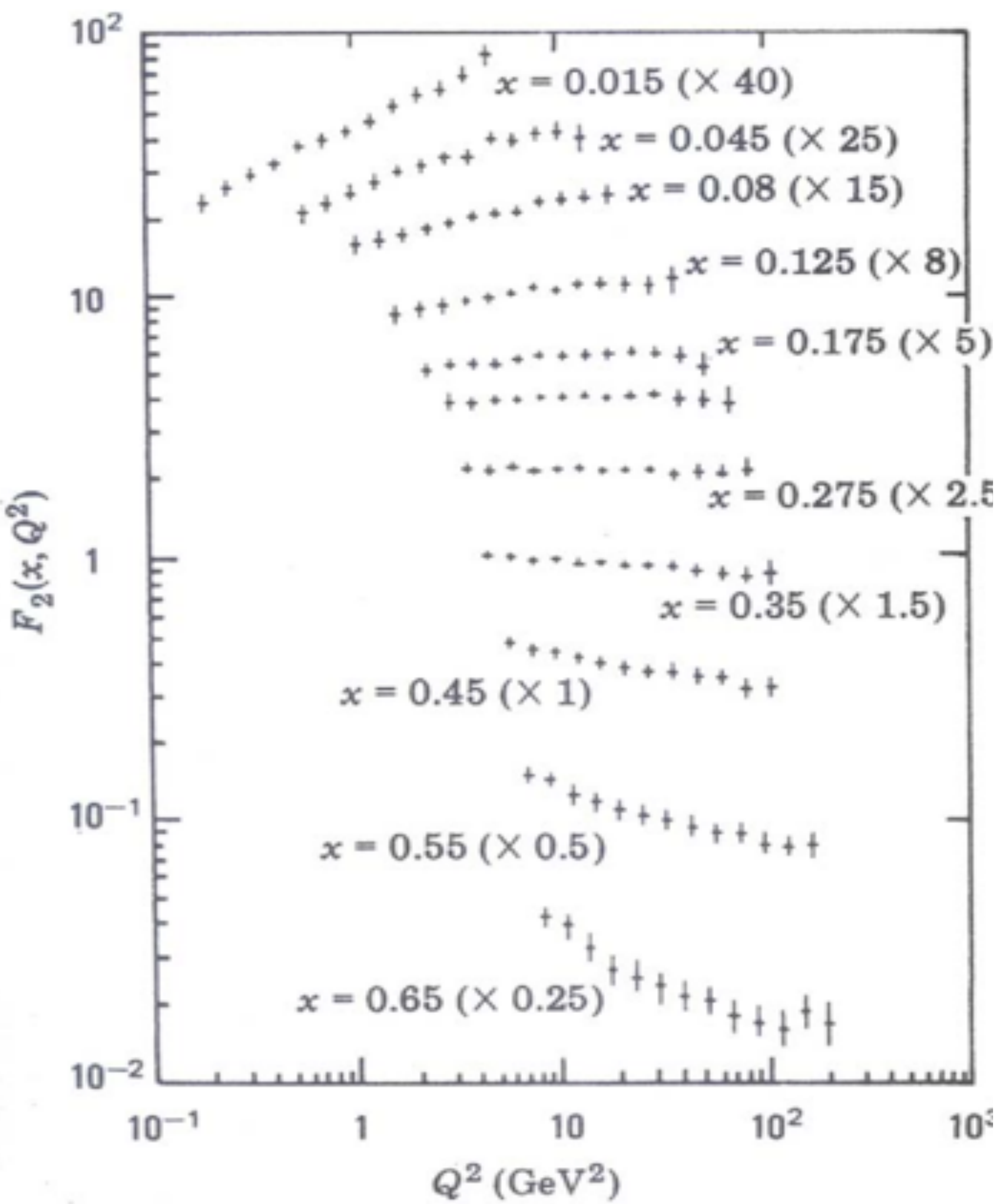




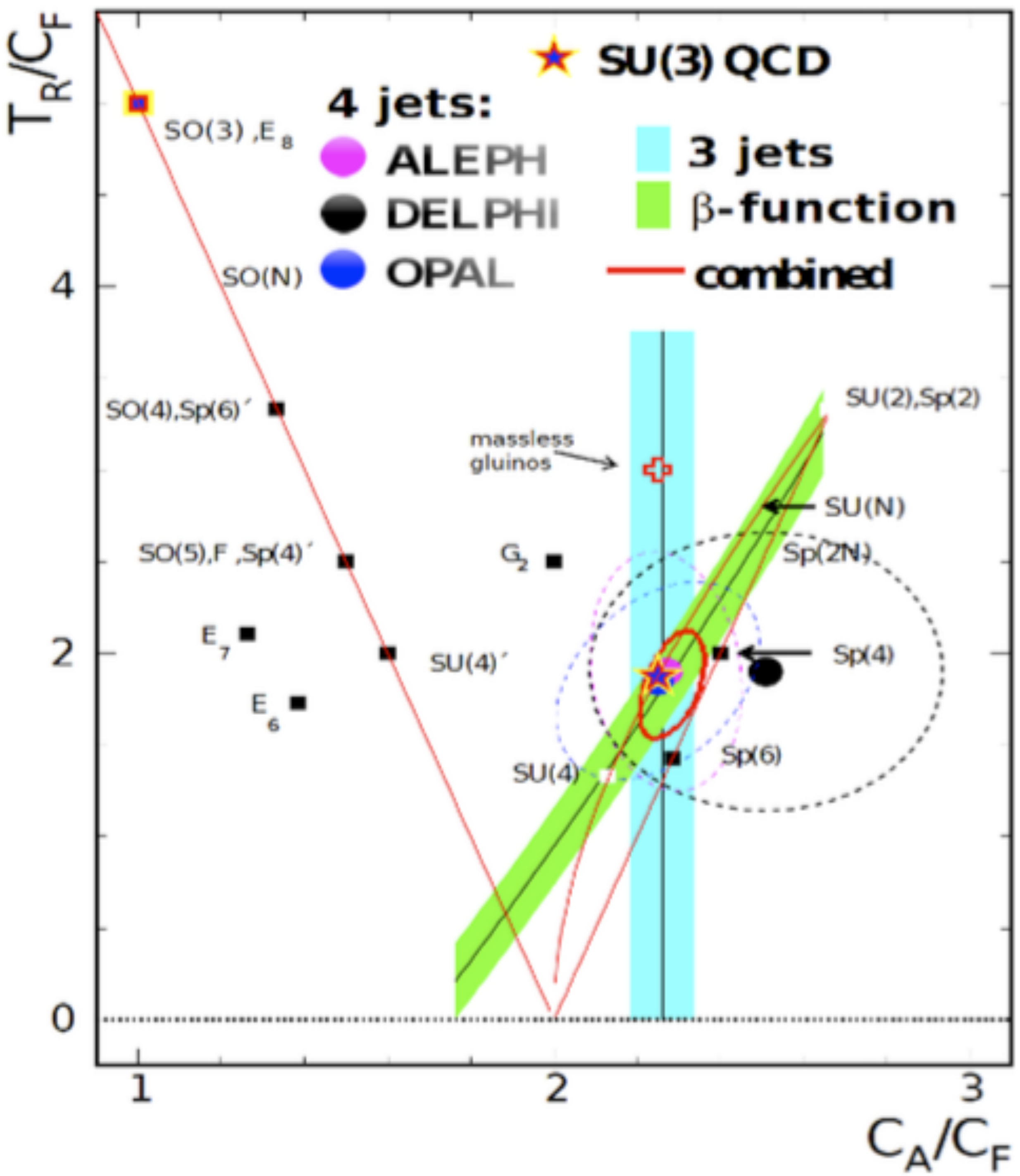



Tests of Quantum Chromodynamics

QCD RGE prediction for DIS



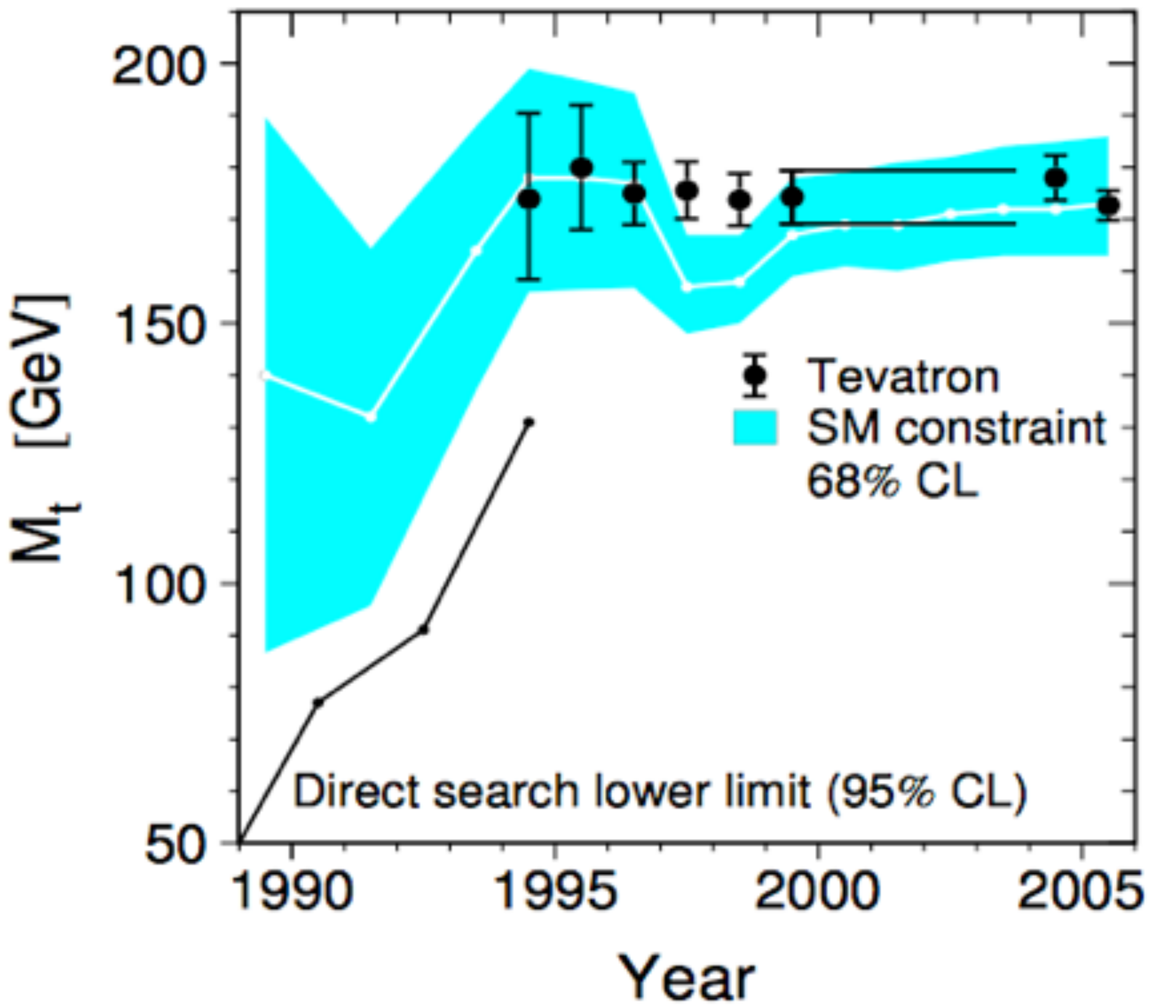
QCD Jets at LEP

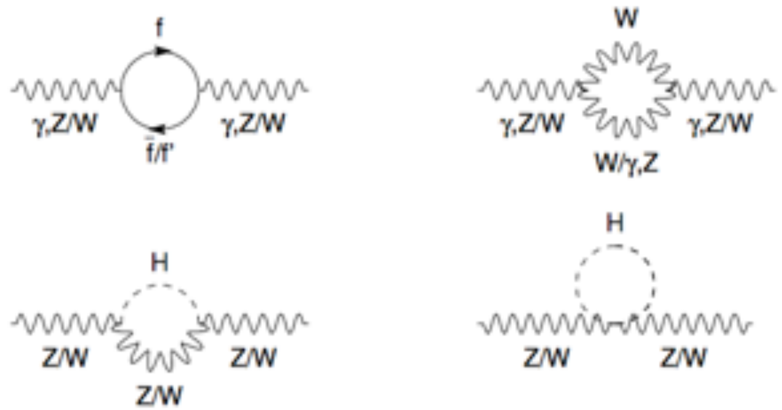


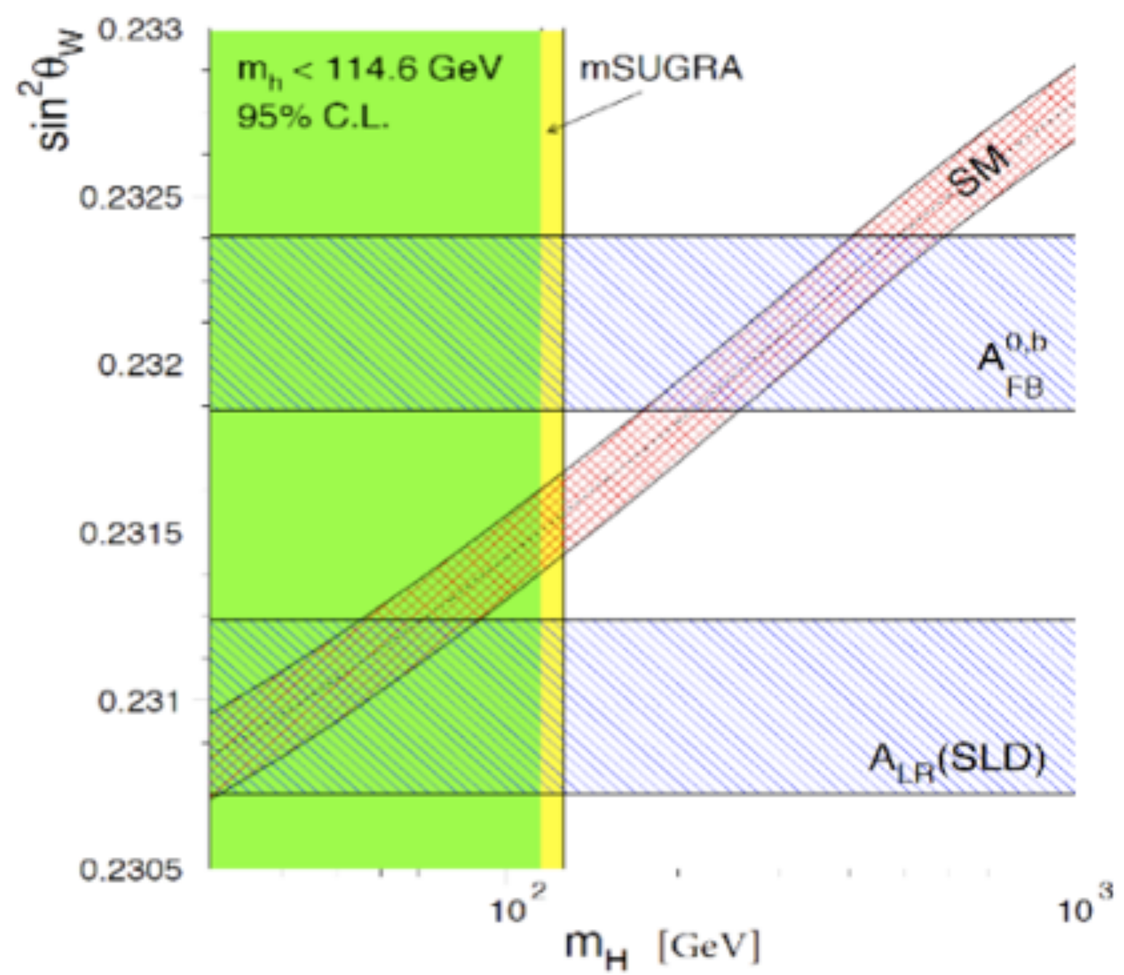
Hits from LEP for Top and Higgs

EW Radiative Corrections

$M_Z, M_H, m_t, \alpha_s(M_Z), \alpha(M_Z)$







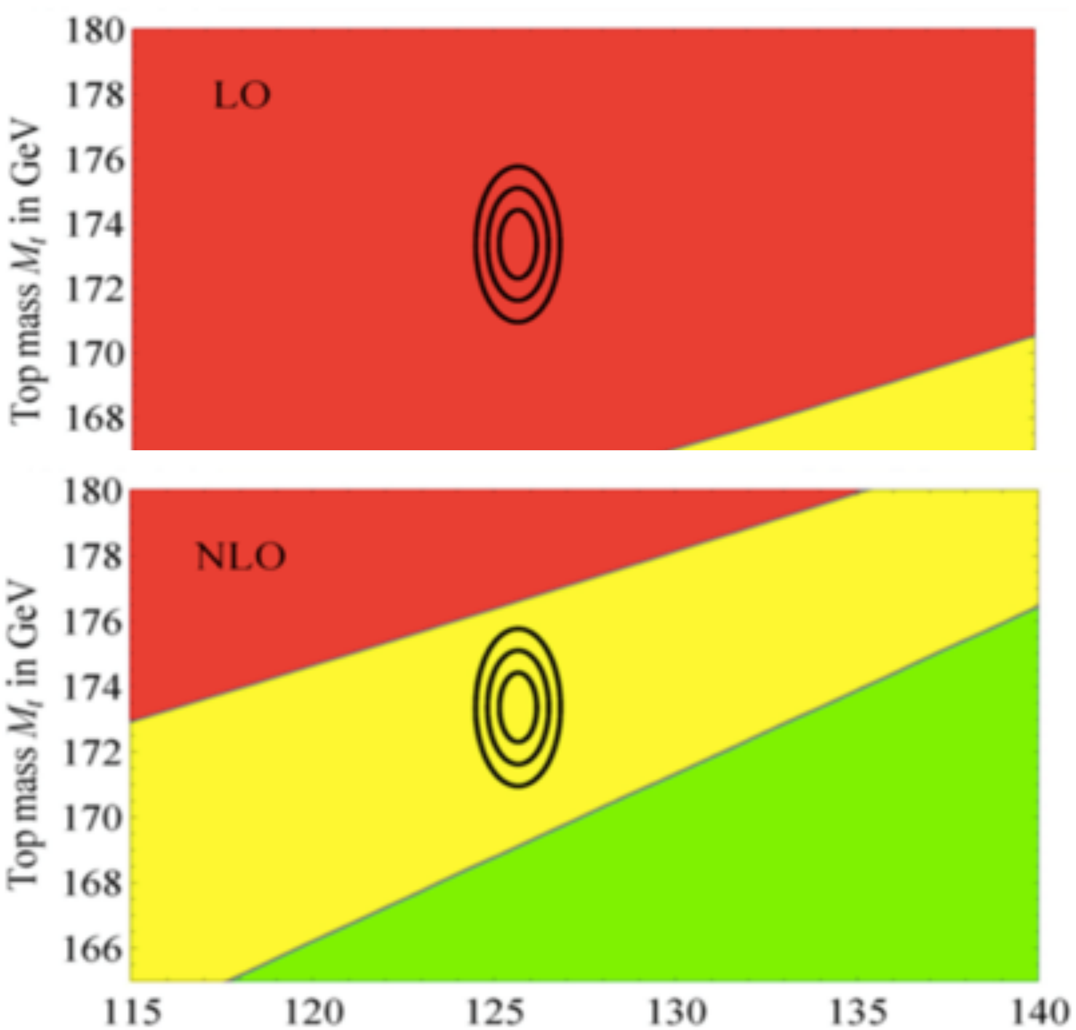
$$m_t = 178.5 \pm 3.9\ GeV$$

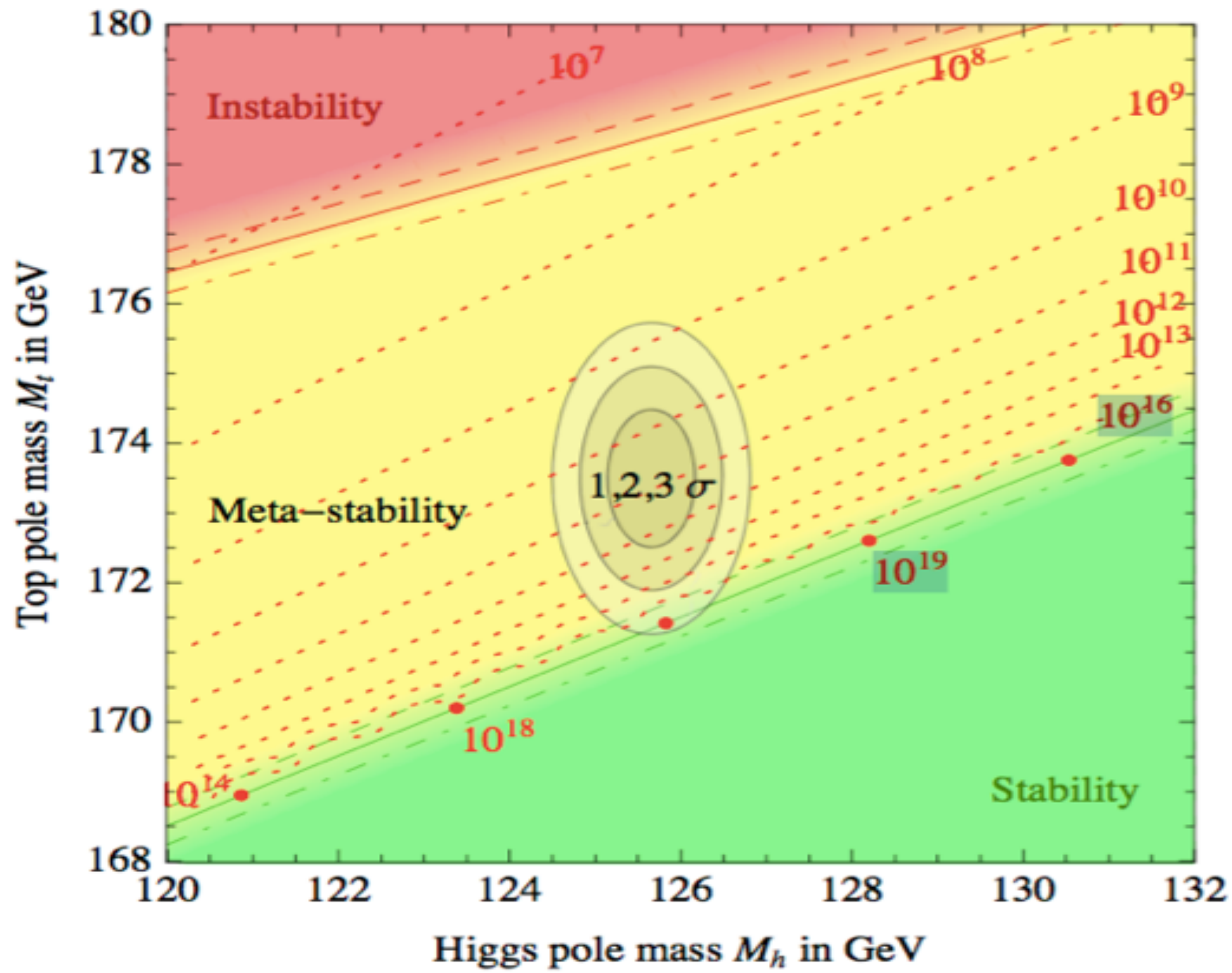
$$m_H = 129^{+74}_{-49}\ GeV$$

Stability of our Vacuum

NNLO Electroweak Correction :

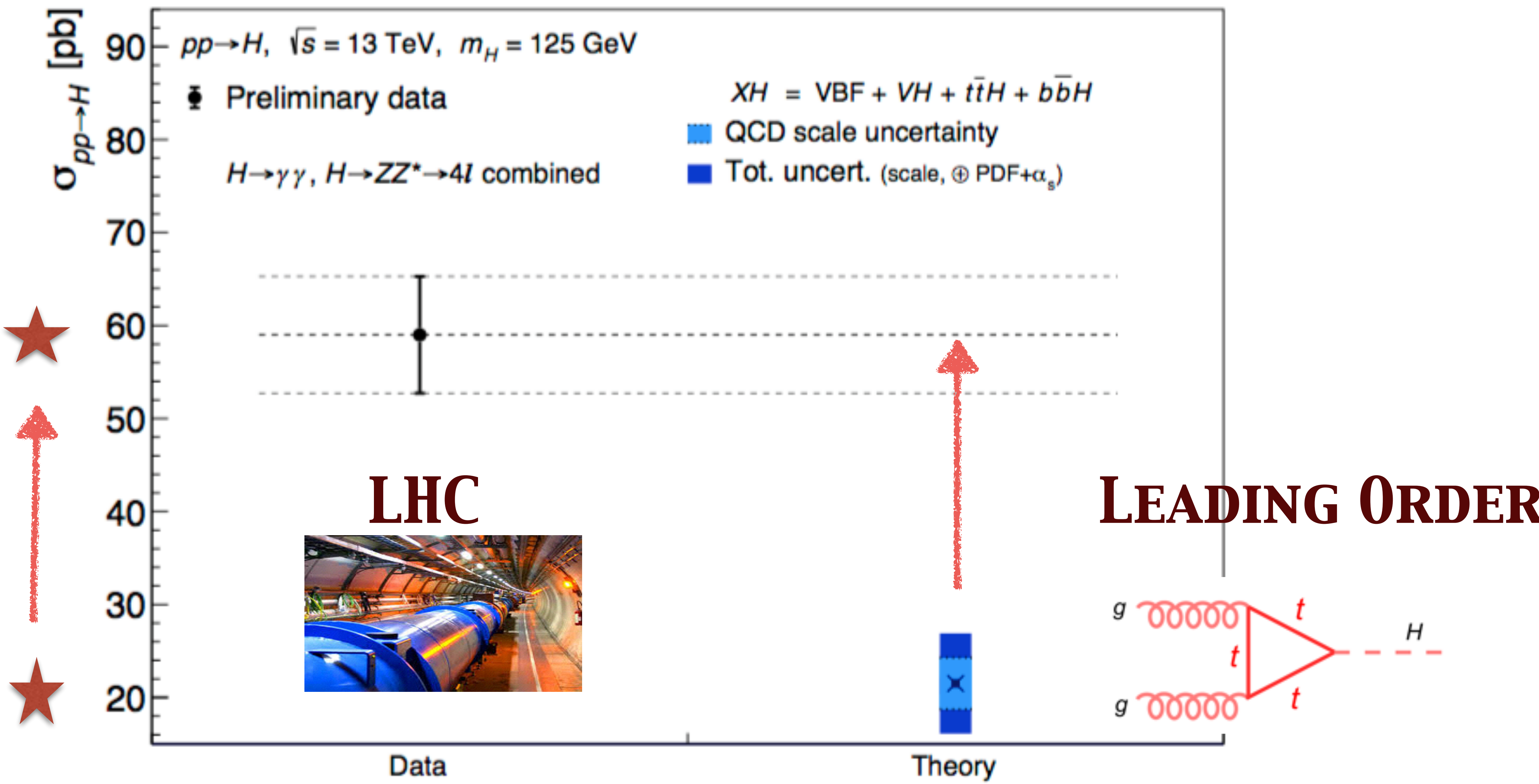
$$M_h[GeV] > 129.6 + 2.0\,[M_t(GeV) - 173.35] - 0.5\left[\frac{\alpha_s(M_z) - 0.1184}{0.0007}\right] \pm 0.3\,.$$



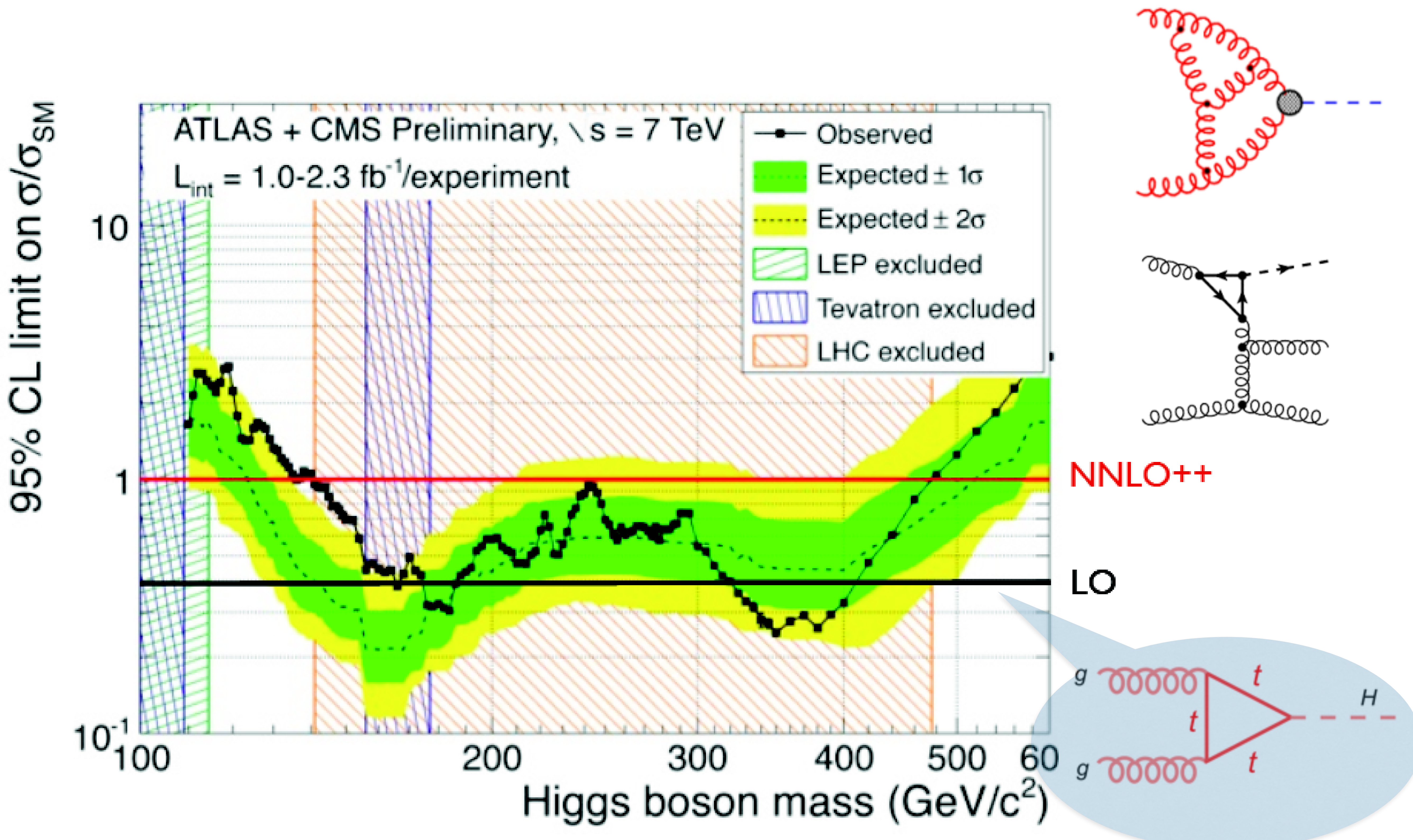


Fate of the universe depends on the mass of top

Leading order is often Crude in QCD

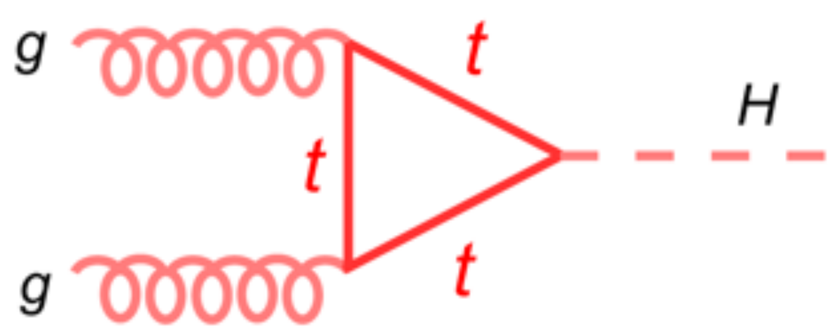


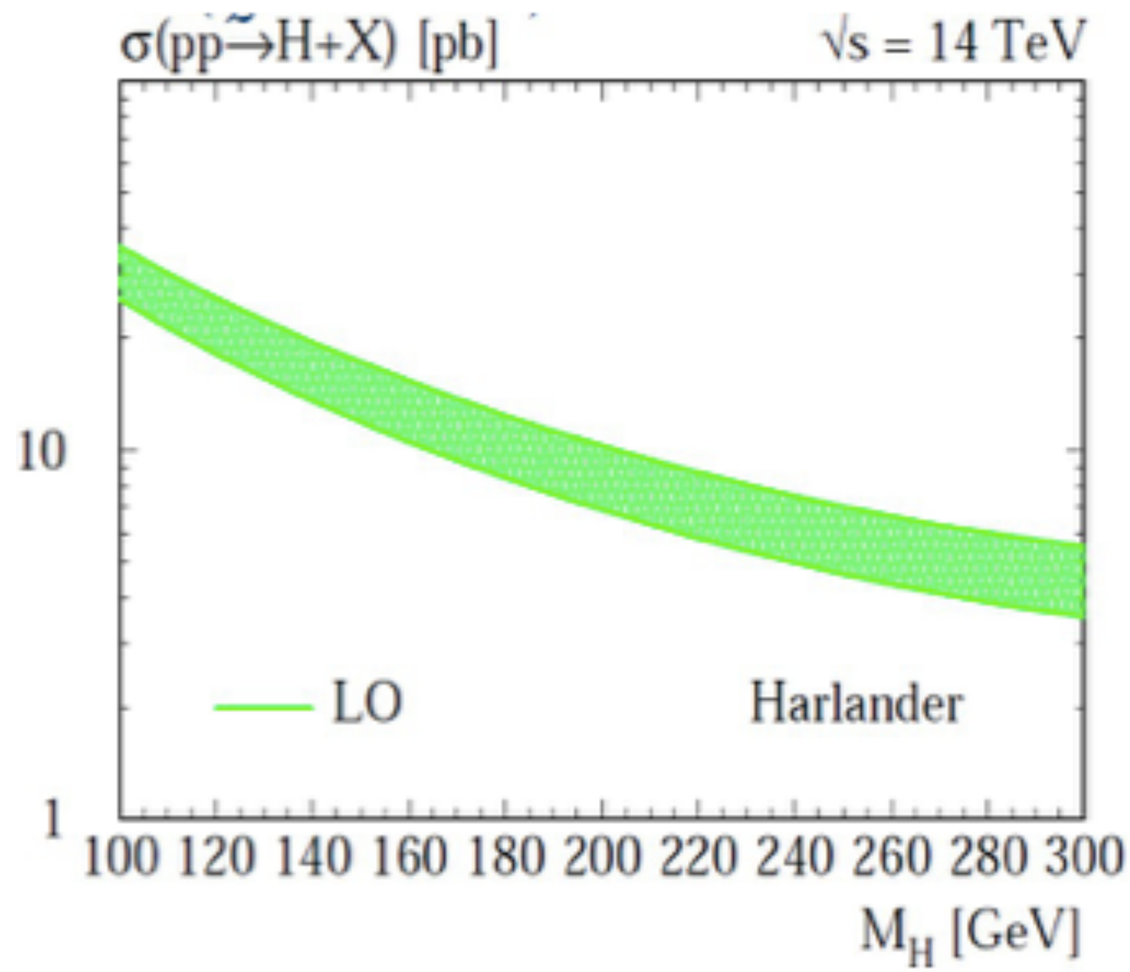
Exclusion Plot for Higgs mass



LO is often a crude estimate

$$2S\sigma^{H}(x,m_H) = \int_x^1 \frac{dz}{z}\Phi^{(0)}_{gg}(z,\mu_F)2\hat{s}\hat{\sigma}^{(0)}_{gg}\left(\frac{x}{z},m_H^2,\mu_R\right)+\cdots$$



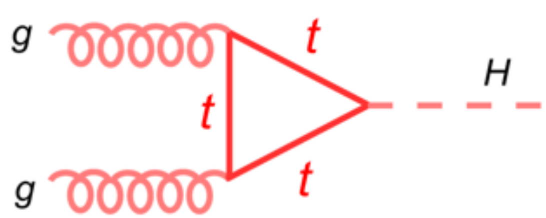


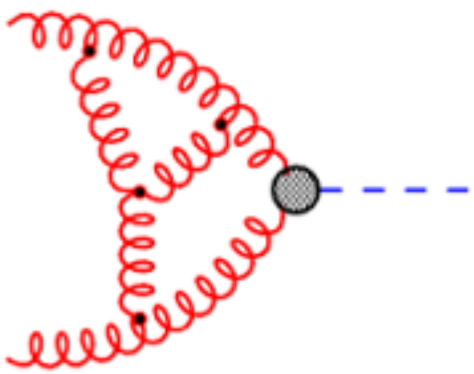
$$2\hat{s}\hat{\sigma}^{(0)}_{gg}\left(\frac{x}{z},m_H^2,\mu_R\right) = \alpha_s^2(\mu_R)G_F F(m_t,m_H)$$

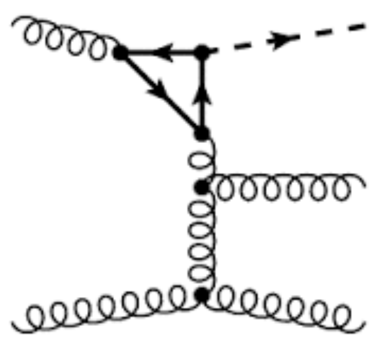
LO prediction is unreliable due to $100-200\%$ scale uncertainty

True Result for Higgs

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$

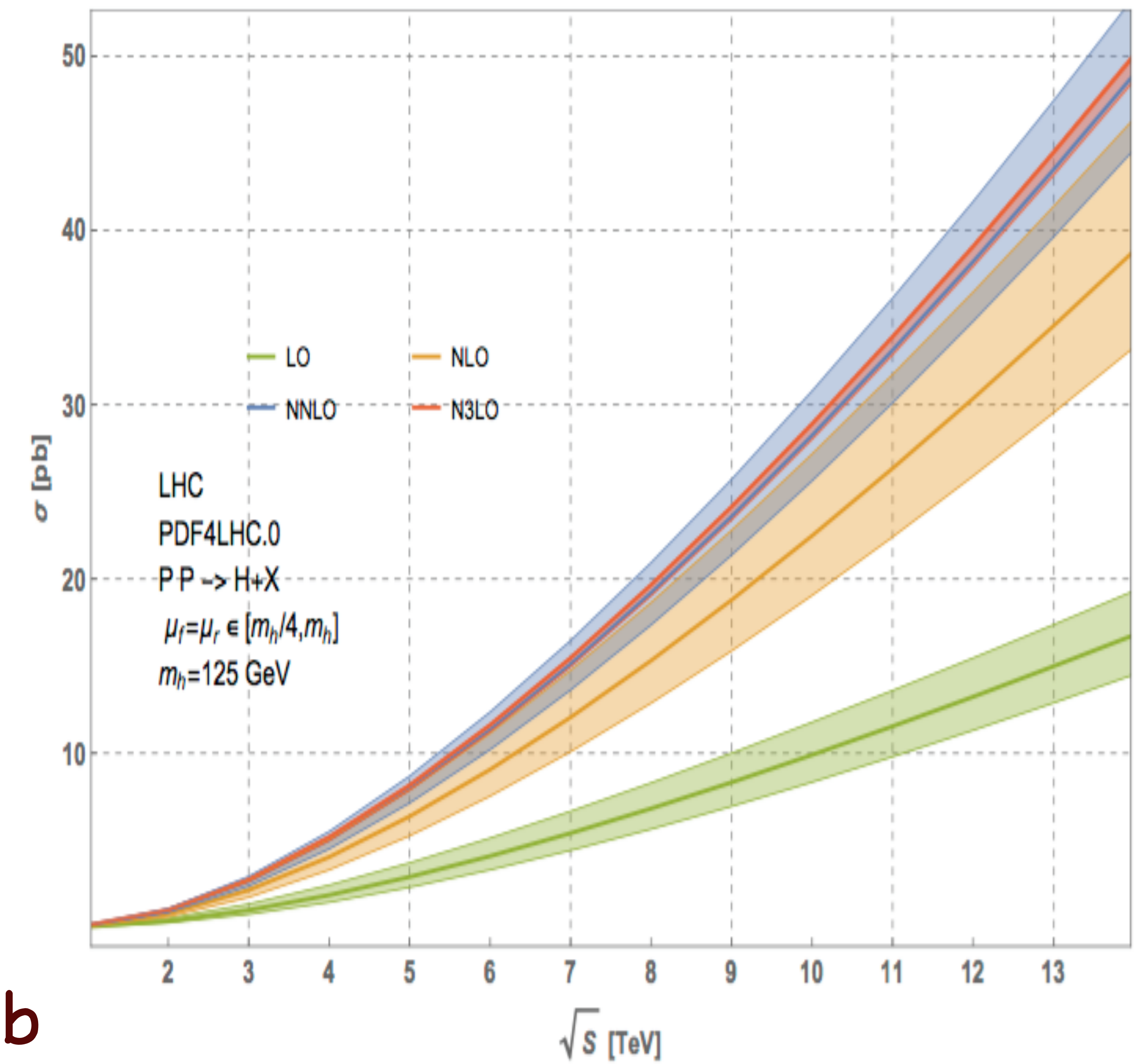






LO	$15.05 \pm 14.8\%$
NLO	$38.2 \pm 16.6\%$
NNLO	$45.1 \pm 8.8\%$
N3LO	$45.2 \pm 1.9\%$

pb



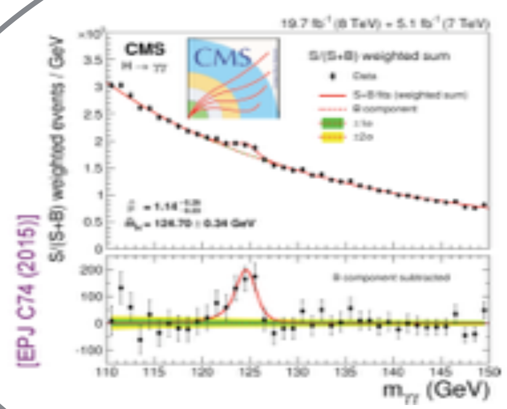
QCD + EW for Higgs

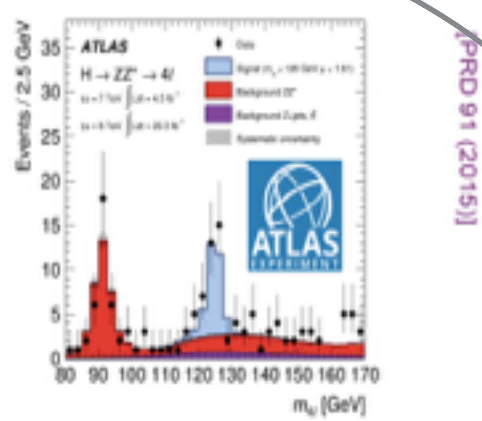
13 Tev LHC

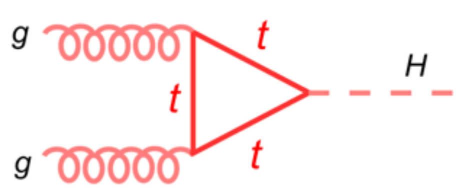
$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

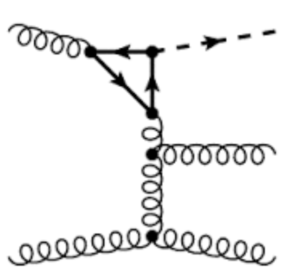
$$\begin{aligned}
48.58\,\text{pb} = {} & \ 16.00\,\text{pb} && (+32.9\%) && (\text{LO, rEFT}) \\
& +20.84\,\text{pb} && (+42.9\%) && (\text{NLO, rEFT}) \\
& -\ \ 2.05\,\text{pb} && (-4.2\%) && ((t,b,c),\ \text{exact NLO}) \\
& +\ \ 9.56\,\text{pb} && (+19.7\%) && (\text{NNLO, rEFT}) \\
& +\ \ 0.34\,\text{pb} && (+0.7\%) && (\text{NNLO},\ 1/m_t) \\
& +\ \ 2.40\,\text{pb} && (+4.9\%) && (\text{EW, QCD-EW}) \\
& +\ \ 1.49\,\text{pb} && (+3.1\%) && (\text{N}^3\text{LO, rEFT})
\end{aligned}$$

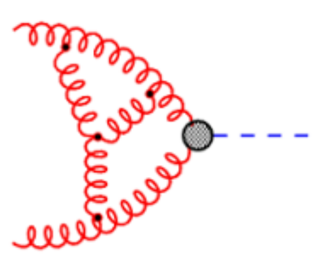
Theory Vs Experiment











Significance of excess:

$\gamma\gamma$: 5.6 σ (5.1 exp.)

ZZ: 6.6 σ (5.5 exp.)

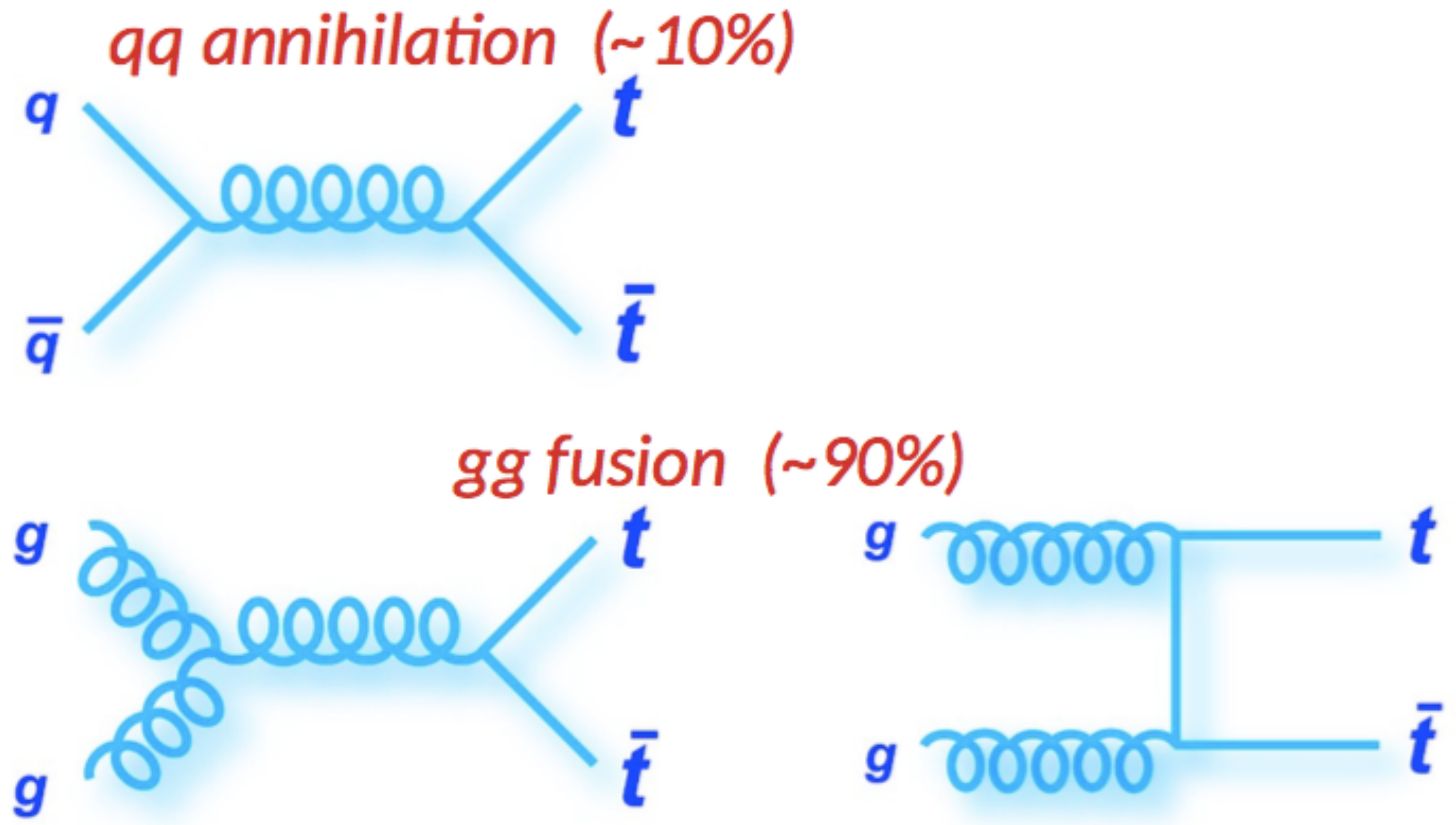
Signal strength $\mu = \sigma_{obs} / \sigma_{SM}$

$$\mu = 1.12^{+0.25}_{-0.23}$$

$$\mu = 1.51^{+0.39}_{-0.34}$$

Agreement with SM Higgs Boson

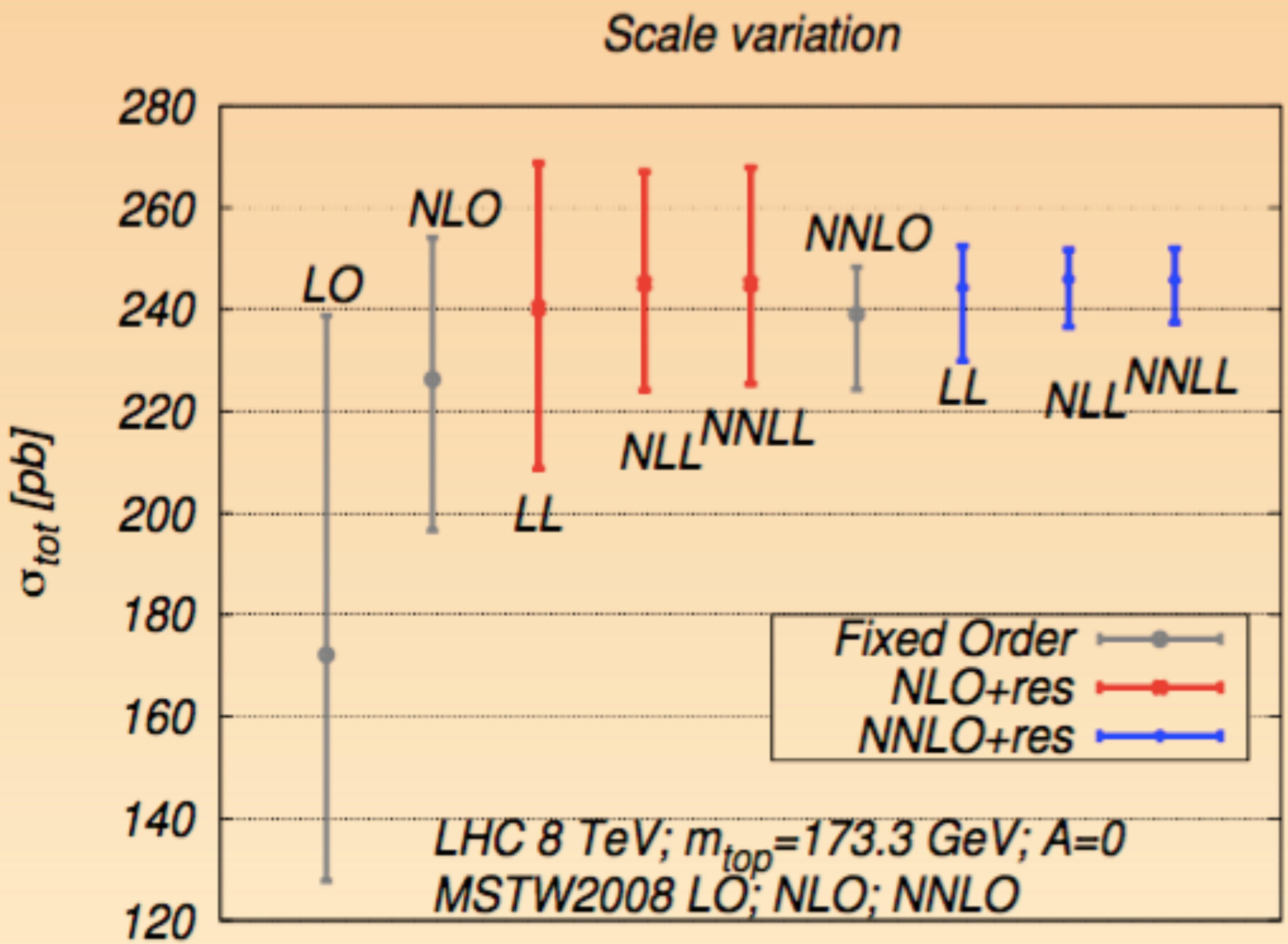
Top production



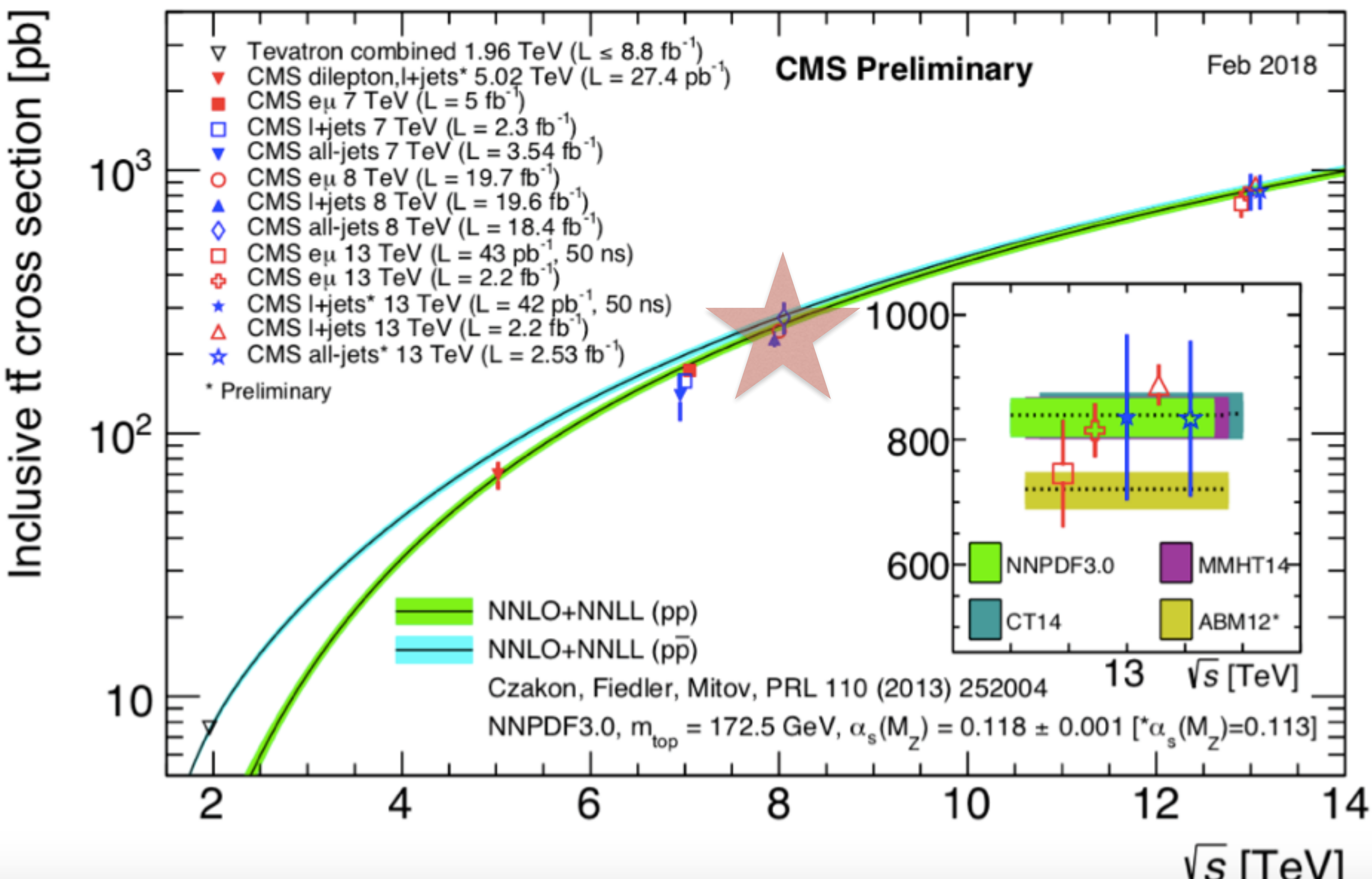
Large theory uncertainty

$$\alpha_s(\mu_R^2) \qquad f_g(x, \mu_F^2)$$

Theory Prediction



Theory Vs Experiment



Parton Model in QCD

Hadronic Cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A\left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right),$$

Partonic cross section:

Precision Measurements

Precise theory

Discover/Test Physics

Inputs that can affect

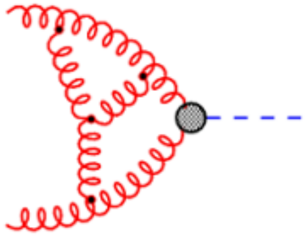
- UV Renormalisation Scale, Strong coupling

$$\alpha_s(\mu_R)$$

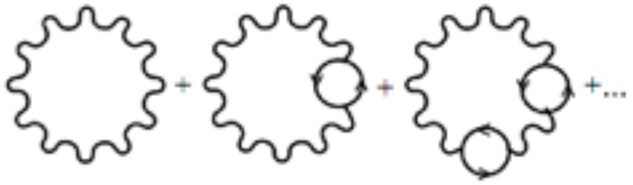
- Factorisation Scale and Parton Distribution Functions

$$f_a(x, \mu_F)$$

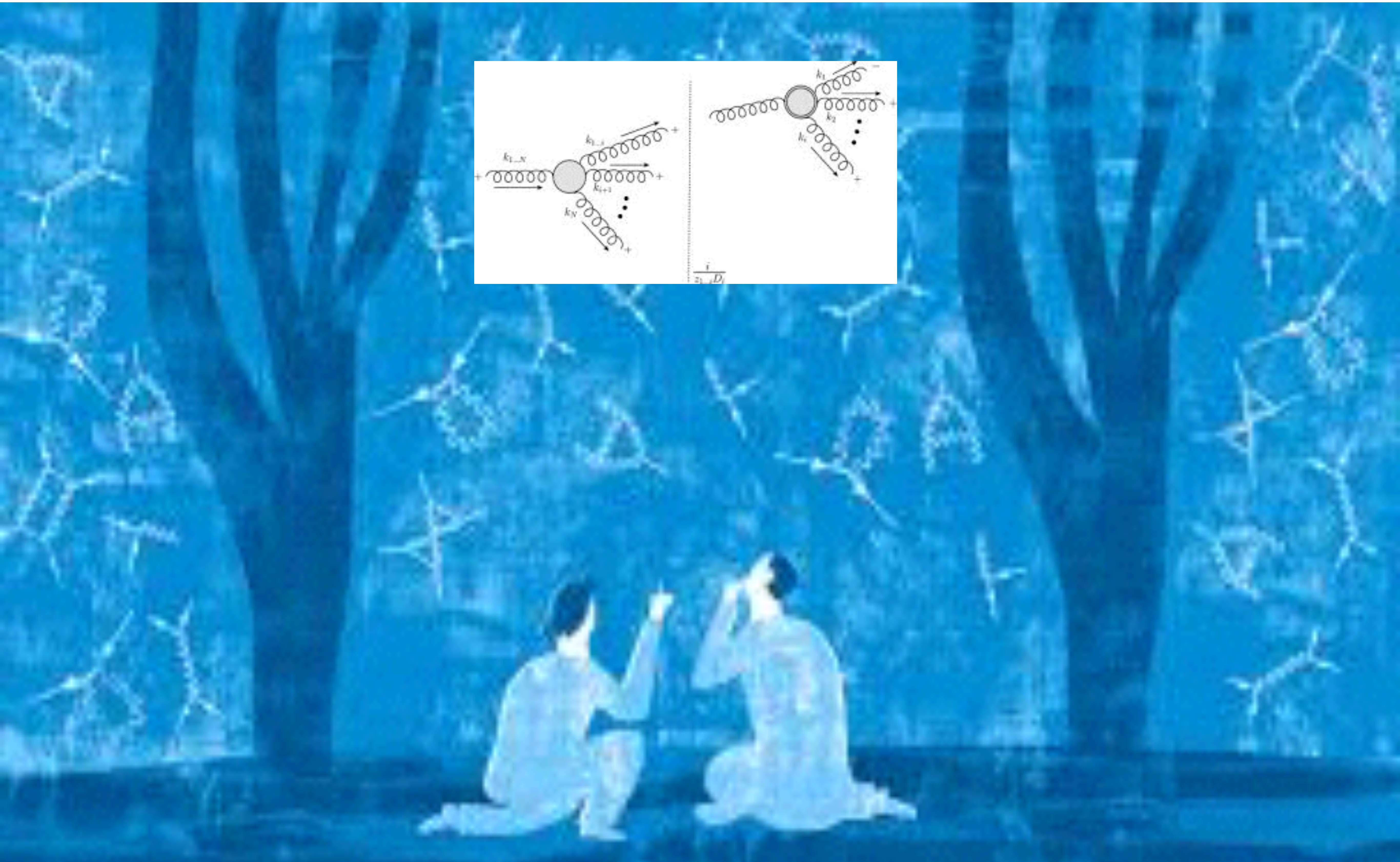
- Missing Higher Order corrections
- Stability of the perturbation theory



- Resummation Methods
- Hadronisation models



LO (Tree level)



No. of diagrams

For Jet / background to BSM

$$g + g \to g + g$$
$$g + g \to g + g + g$$
$$g + g \to g + g + g + g$$

$$g + g \to n\ g$$

n	no. of diagrams
2	4
3	25
4	220
5	2485
6	34300
7	559405
8	10525900

Polarisation Vectors

$$\epsilon^+(p,q) = \frac{\langle q|\,\gamma_\mu p]}{\sqrt{2}\,\langle qp\rangle}$$

$$\epsilon^-(p,q) = -\frac{[q|\gamma_\mu\,|p\rangle}{\sqrt{2}[qp]}$$

$$p_\mu \epsilon^\mu = 0.$$

$$(\varepsilon_\mu^+)^* = \varepsilon_\mu^-$$

★ Gauge Choice

$$\epsilon_\mu^-(\tilde{q}) - \epsilon_\mu^-(q) = \frac{[\tilde{q}\gamma^\mu\,|p\rangle\,[qp] - [q\gamma^\mu\,|p\rangle\,[\tilde{q}p]}{\sqrt{2}[qp][\tilde{q}p]}$$

$$= \frac{\sqrt{2}[\tilde{q}q]}{[qp][\tilde{q}p]}p^\mu.$$

★ Polarisation Sum

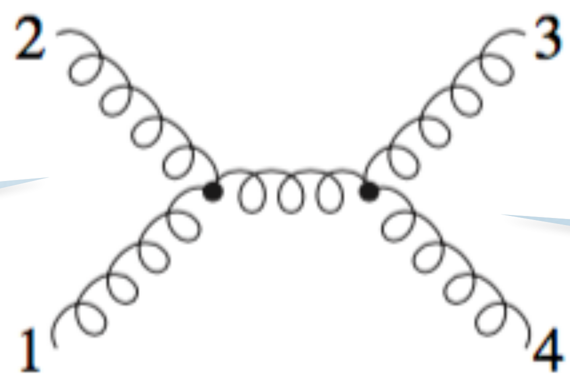
$$\sum_\lambda \epsilon_\mu^\lambda(p,q)(\epsilon_\nu^\lambda(p,q))^* = \frac{q_\mu p_\nu + q_\nu p_\mu}{p.q} - \eta_{\mu\nu}$$

$$\epsilon_i^\pm(q).q = 0$$

$$\epsilon_i^\pm(q).\epsilon_j^\pm(q) = 0$$

$$\epsilon_i^\pm(p_j).\epsilon_j^\mp(q) = 0$$

SU(N) color algebra



f^{abc}

f^{abc}

$$[T^a, T^b] = if^{abc}T^c \quad \longrightarrow \quad if^{abc} = 2\left[\mathrm{Tr}\left(T^aT^bT^c\right) - \mathrm{Tr}\left(T^bT^aT^c\right)\right]$$

$$\begin{aligned} if^{a_1a_2b}\,if^{ba_3a_4} &= 4\left[\mathrm{Tr}\left(T^{a_1}T^{a_2}T^b\right) - \mathrm{Tr}\left(T^{a_2}T^{a_1}T^b\right)\right]\left[\mathrm{Tr}\left(T^{a_3}T^{a_4}T^b\right) - \mathrm{Tr}\left(T^{a_4}T^{a_3}T^b\right)\right] \\ &= 2\,\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}) - 2\,\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_4}T^{a_3}) - 2\,\mathrm{Tr}(T^{a_2}T^{a_1}T^{a_3}T^{a_4}) \\ &\quad + 2\,\mathrm{Tr}(T^{a_2}T^{a_1}T^{a_4}T^{a_3}) \end{aligned}$$

QCD improved Parton Model

$$\mathcal{A}_n^{(0)}(g_1, g_2, \ldots, g_n) = g^{n-2} \sum_{\sigma \in S_n/Z_n} 2\,\mathrm{Tr}\left(T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(n)}}\right) A_n^{(0)}\left(g_{\sigma(1)}, \ldots, g_{\sigma(n)}\right)$$

Partial Amplitude

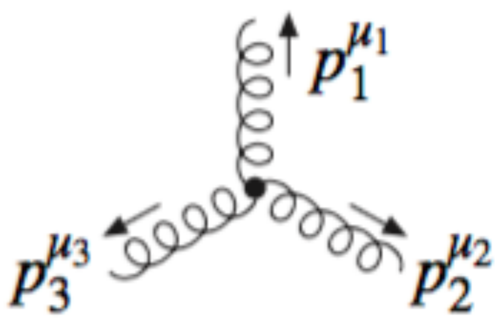
Partial Amplitudes
- No color information
- Gauge Invariant

n	4	5	6	7	8	9	10
unordered	4	25	220	2485	34300	559405	10525900
cyclic ordered	3	10	38	154	654	2871	12925

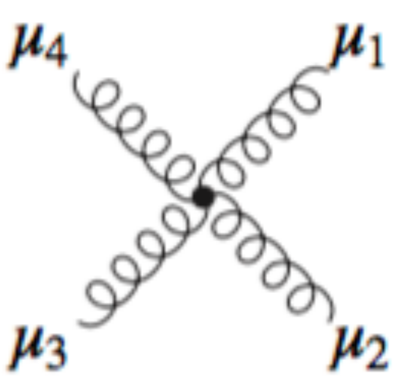
Cyclic Ordered Feynman Rules

No Color Factors !!!

$$= \frac{-ig^{\mu\nu}}{p^2},$$



$$= i\left[g^{\mu_1\mu_2}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)+g^{\mu_2\mu_3}\left(p_2^{\mu_1}-p_3^{\mu_1}\right)+g^{\mu_3\mu_1}\left(p_3^{\mu_2}-p_1^{\mu_2}\right)\right]$$

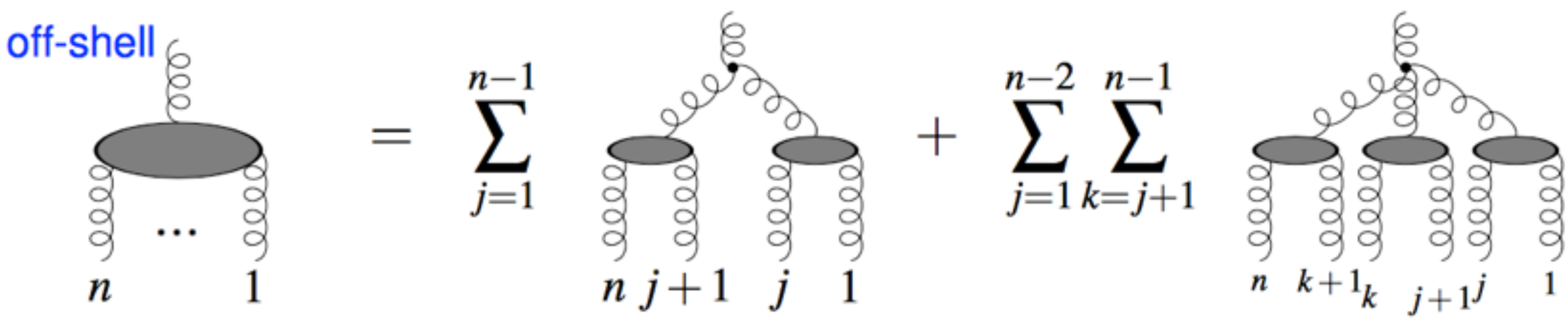


$$= i\left[2g^{\mu_1\mu_3}g^{\mu_2\mu_4}-g^{\mu_1\mu_2}g^{\mu_3\mu_4}-g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right].$$

Berends-Giele Recursion

- Off-Shell currents

- No Feynman diagrams

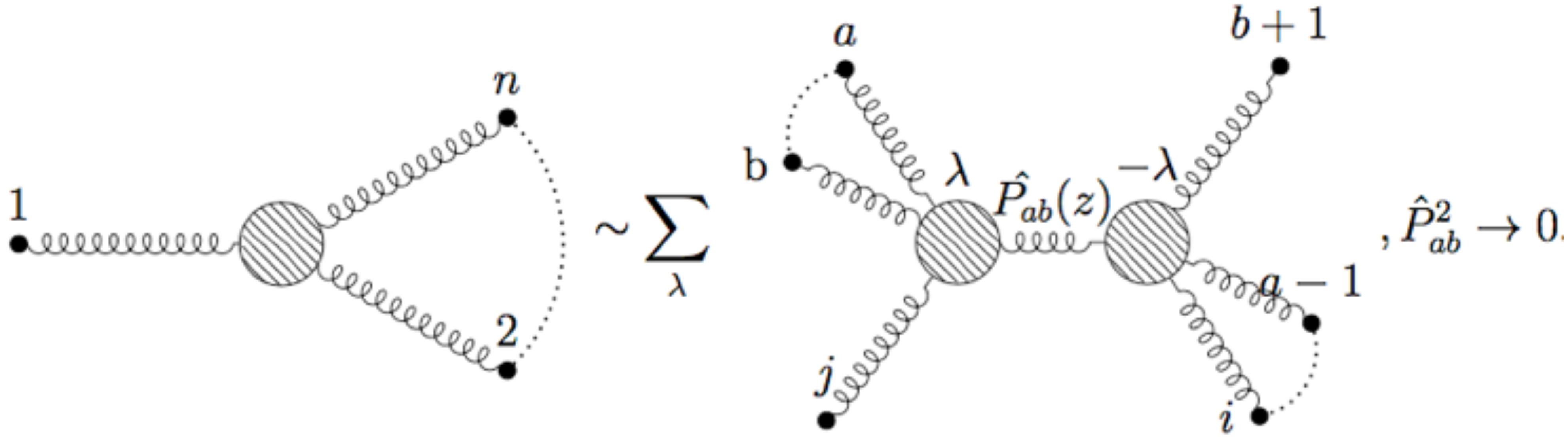


BCFW relation

$$0 = \frac{1}{2\pi i}\oint_C \mathrm{d}z \frac{A(z)}{z} = A(0) + \sum_{poles(z_\alpha \neq 0)} \mathrm{Res}\left(\frac{A(z)}{z}, z_\alpha\right)$$

$$\hat{P}_{ab}(z) = \sum_{k=a}^{b} |k\rangle [k| - z\,|i\rangle [j|$$

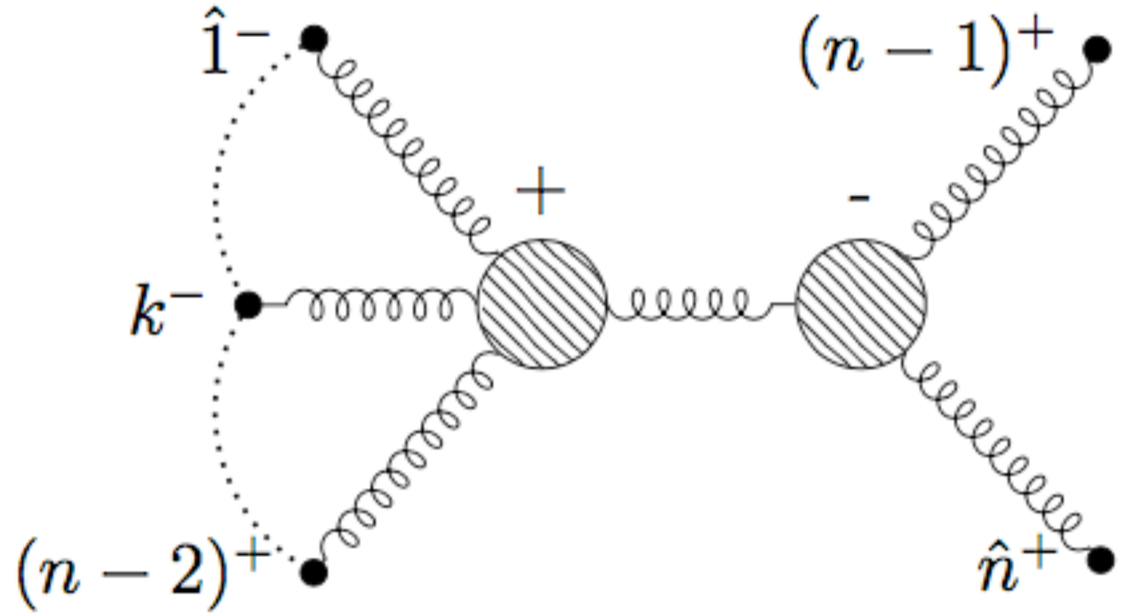
$$A(1,\cdots,n) \sim \sum_\lambda A_L(a,\cdots,b,-\hat{P}_{ab}^{\lambda}) \frac{1}{\hat{P}_{ab}^2} A_R(\hat{P}_{ab}^{-\lambda}, b+1,\cdots,a-1)$$



Park-Taylor Amplitude

MHV n - gluon Amplitudes

$$A_n(1^-, 2^+, \cdots, k^-, \cdots, n^+) =$$



$$A_n(1^-, 2^+, \cdots, k^-, \cdots, n^+)$$

$$= \frac{\langle 1k \rangle^4}{\langle 12 \rangle \cdots \langle n-3|n-2 \rangle \langle n-2|n-1 \rangle \langle n-1|n \rangle \langle n1 \rangle}.$$

Twister space

Momenta in bi-spinor $p^\mu \rightarrow \lambda_a \lambda_{\dot{a}}$

Scaling

$$\lambda_a \rightarrow z\lambda_a$$

$$\lambda_{\dot{a}} \rightarrow \frac{1}{z}\lambda_{\dot{a}}$$

Transform

$$\lambda_{\dot{a}} \rightarrow i\frac{\partial}{\partial \lambda^{\dot{a}}}$$

$$-i\frac{\partial}{\partial \lambda^{\dot{a}}} \rightarrow \lambda_{\dot{a}}$$

Fourier Transform

$$f(\overline{\lambda}^{\dot{a}}) = \int \frac{d^2p}{(2\pi)^2}\ exp(i\overline{\lambda}^{\dot{a}}\lambda_{\dot{a}})\ \ f(\lambda_{\dot{a}})$$

Weinzierl's comparison

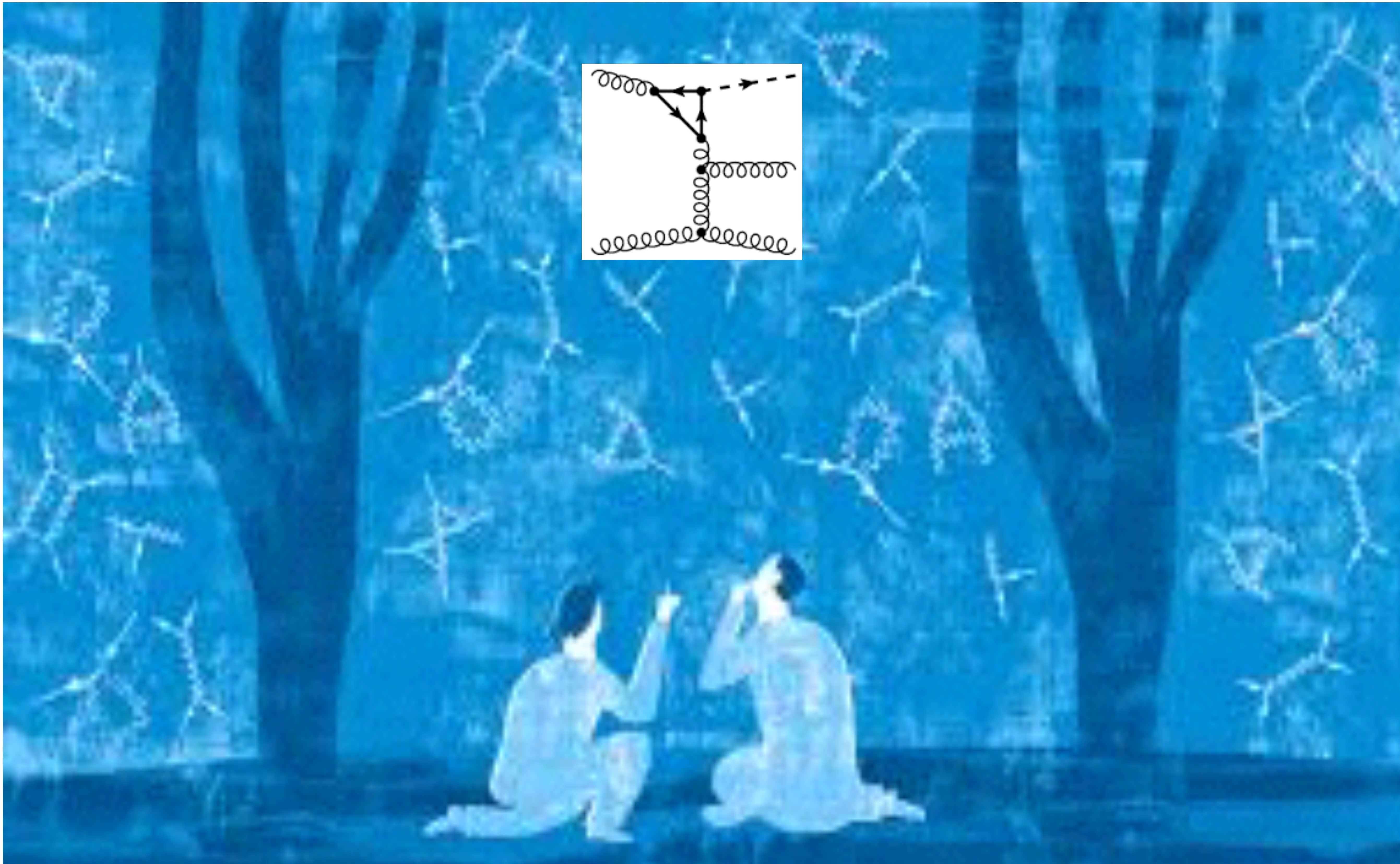
Compare algorithms based on different methods for the numerical computation of the Born gluon amplitude:

n	4	5	6	7	8	9	10	11	12
Berends-Giele	0.00011	0.00043	0.0015	0.005	0.016	0.047	0.13	0.37	1
Scalar	0.00014	0.00083	0.0033	0.011	0.033	0.097	0.26	0.7	1.8
MHV	0.00001	0.00053	0.0056	0.073	0.62	3.67	29	217	—
BCF	0.00002	0.00007	0.0004	0.003	0.017	0.083	0.47	2.5	14.5

CPU time in seconds for the computation of the n gluon amplitude on a standard PC (Pentium IV with 2 GHz), summed over all helicities.

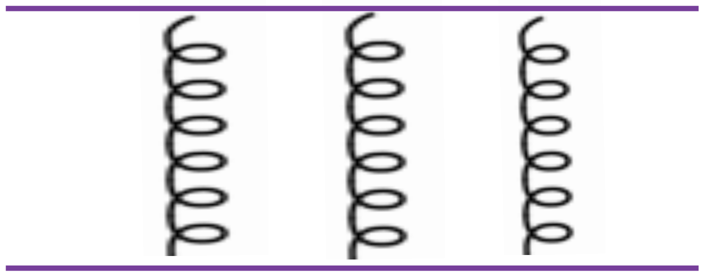
All methods give identical results within an accuracy of 10^{-12}.

NLO



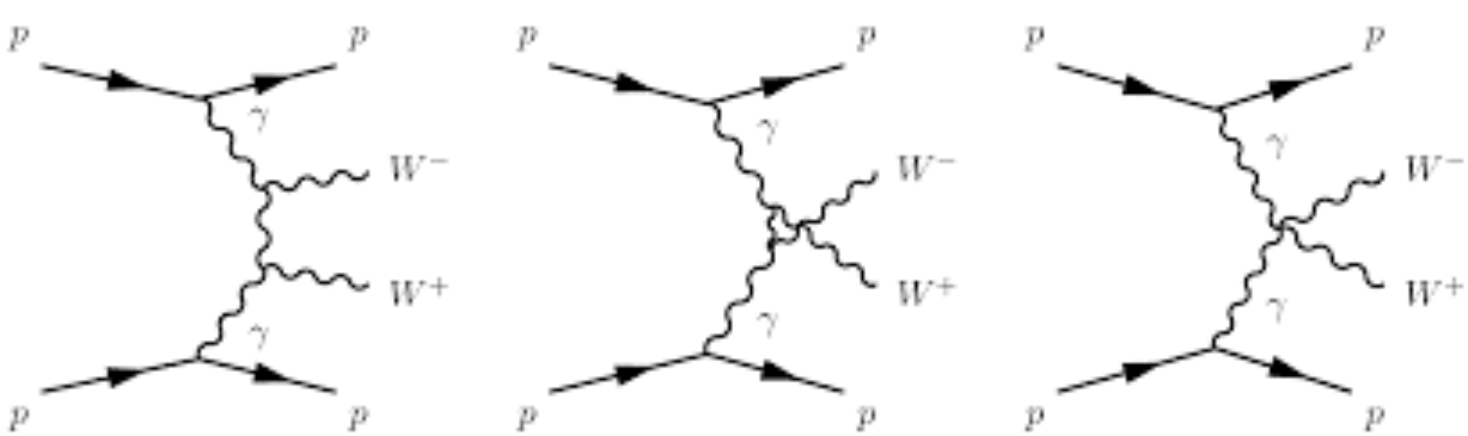
Beyond LO

Loop integral



$$\mathbf{I} = \prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{N(\{k_l\},\{p_m\})}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_d}}}$$

Phase space integrals



$$d\Phi_N = \prod_{i=1}^{N} \left(\int \frac{d^n p_i}{(2\pi)^n} \delta(p_i^2 - m_i^2) \right) (2\pi)^n \delta^n (q - \sum_i p_i)$$

Loop Integrals

Numerator: Reducible

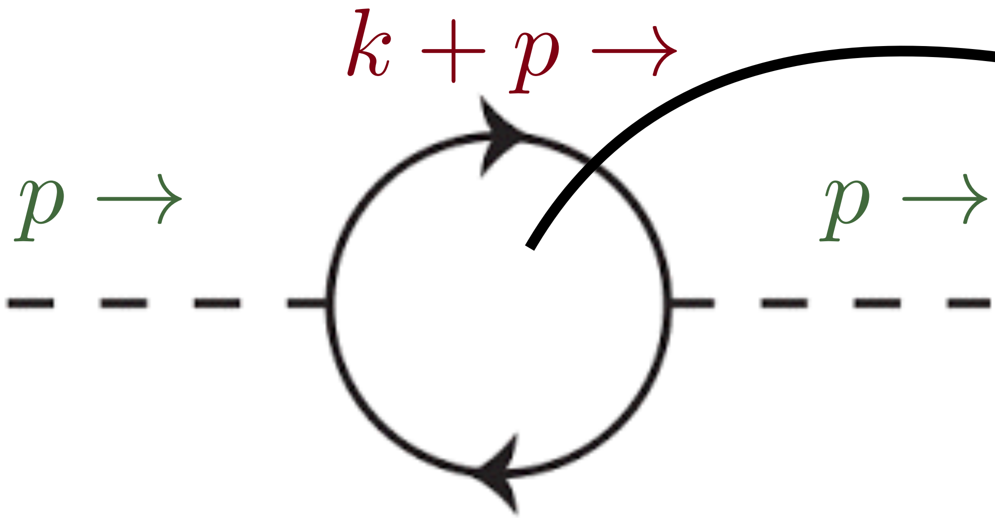
Irreducible

Reducible if

$$\left.\begin{array}{l}(p_i \cdot k_j)^{a_k} \\ \\ (k_i \cdot k_j)^{b_k}\end{array}\right\}$$ are expressible in terms of Denominators

$$\frac{(p \cdot k)_j}{D_j} = \frac{1}{C_j}\left(1 - \frac{D_j - C_j\,(p \cdot k)_j}{D_j}\right) , \qquad j = 1, \dots , N_d \ ,$$

Loop Integrals



$$\int \frac{d^n k}{(2\pi)^n} \frac{(k \cdot p)^a}{D_1 D_2}$$

$$D_1 = k^2 + i\epsilon$$

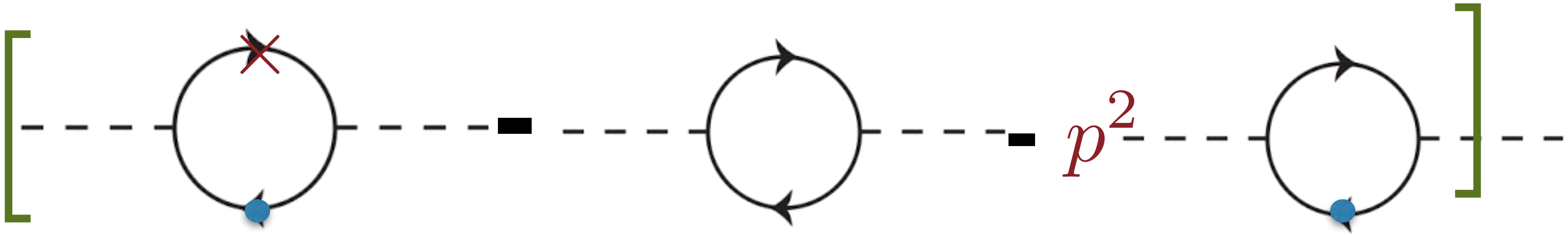
$$D_2 = (k+p)^2 + i\epsilon, \qquad p^2 < 0$$

$$\frac{k \cdot p}{D_1} = \frac{1}{2}\left(\frac{D_2}{D_1} - 1 - \frac{p^2}{D_1}\right)$$

$$\frac{k \cdot p}{D_2} = \frac{1}{2}\left(1 - \frac{D_1}{D_2} - \frac{p^2}{D_2}\right)$$

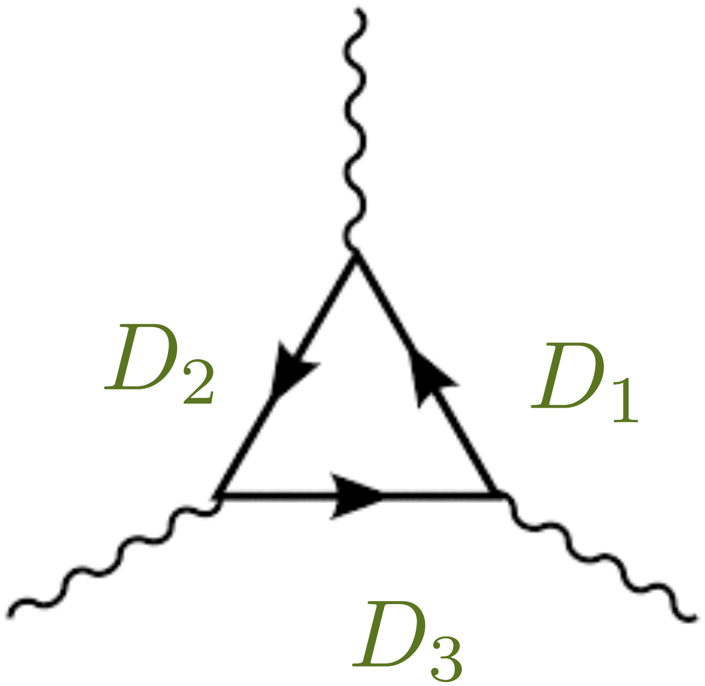
} Reducible

$$\int \frac{d^n k}{(2\pi)^n} \frac{k \cdot p}{D_1 D_2} = \frac{1}{2}\Big[$$



$\Big]$

Tensorial Reduction

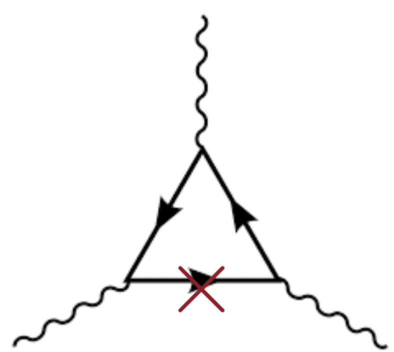
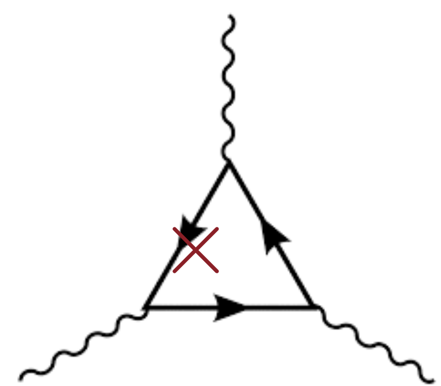
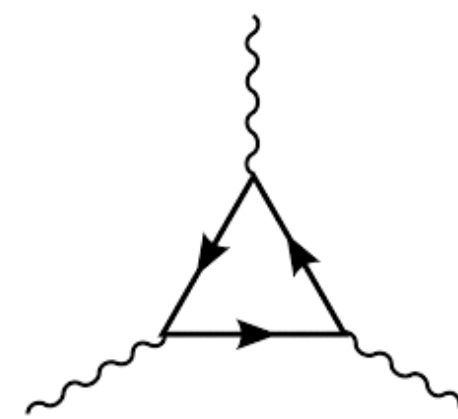


$$I_\mu = \int \frac{d^n k}{(2\pi)^n} \frac{k_\mu}{D_1 D_2 D_3}$$

$$I_\mu = A_1 \; p_{1\mu} + A_2 \; p_{2\mu}$$

$$I_1 = p_1 \cdot I = A_2 \; p_1 \cdot p_2$$

$$I_2 = p_2 \cdot I = A_1 \; p_2 \cdot p_1$$

$$A_1 = \frac{1}{2p_1 \cdot p_2}$$ [ -  + S ]

Ossola-Papadopoulos-Pittau (OPP method)

Integrand : $\dfrac{N(k,p_i)}{D_1 D_2 D_3 D_4}$

$$D_i = (k + \sum_j c_j p_j)^2 - m_i^2$$

$$\begin{aligned}
N(k,p_i) = & \sum_{i_1<i_2<i_3<i_4}^{m} \left[d(i_1 i_2 i_3 i_4) + \tilde{d}(k, i_1 i_2 i_3 i_4)\right] \prod_{i \neq i_1 i_2 i_3 i_4}^{m} D_i \\
& + \sum_{i_1<i_2<i_3}^{m} \left[c(i_1 i_2 i_3) + \tilde{c}(k, i_1 i_2 i_3)\right] \prod_{i \neq i_1 i_2 i_3}^{m} D_i \\
& + \sum_{i_1<i_2}^{m} \left[b(i_1 i_2) + \tilde{b}(k, i_1 i_2)\right] \prod_{i \neq i_1 i_2}^{m} D_i \\
& + \sum_{i_1}^{m} \left[b(i_1 i_2) + \tilde{b}(k, i_1)\right] \prod_{i \neq i_1}^{m} D_i + P(q) \prod_{i=1}^{m} D_i
\end{aligned}$$

Best suited for Numerical Methods

NLO QCD - Tool Kits

Analytical Tools

Faster generation of Feynman diagram
QGRAF
Symbolic Manupulation:
FORM,Mathematica
On-Shell Methods
BCFW
Recursion techniques
BG

Merging NLO with Showers

MC@NLO
POWEG
SHERPA
VINCIA
GENeVa
aMC@NLO
KRKMC

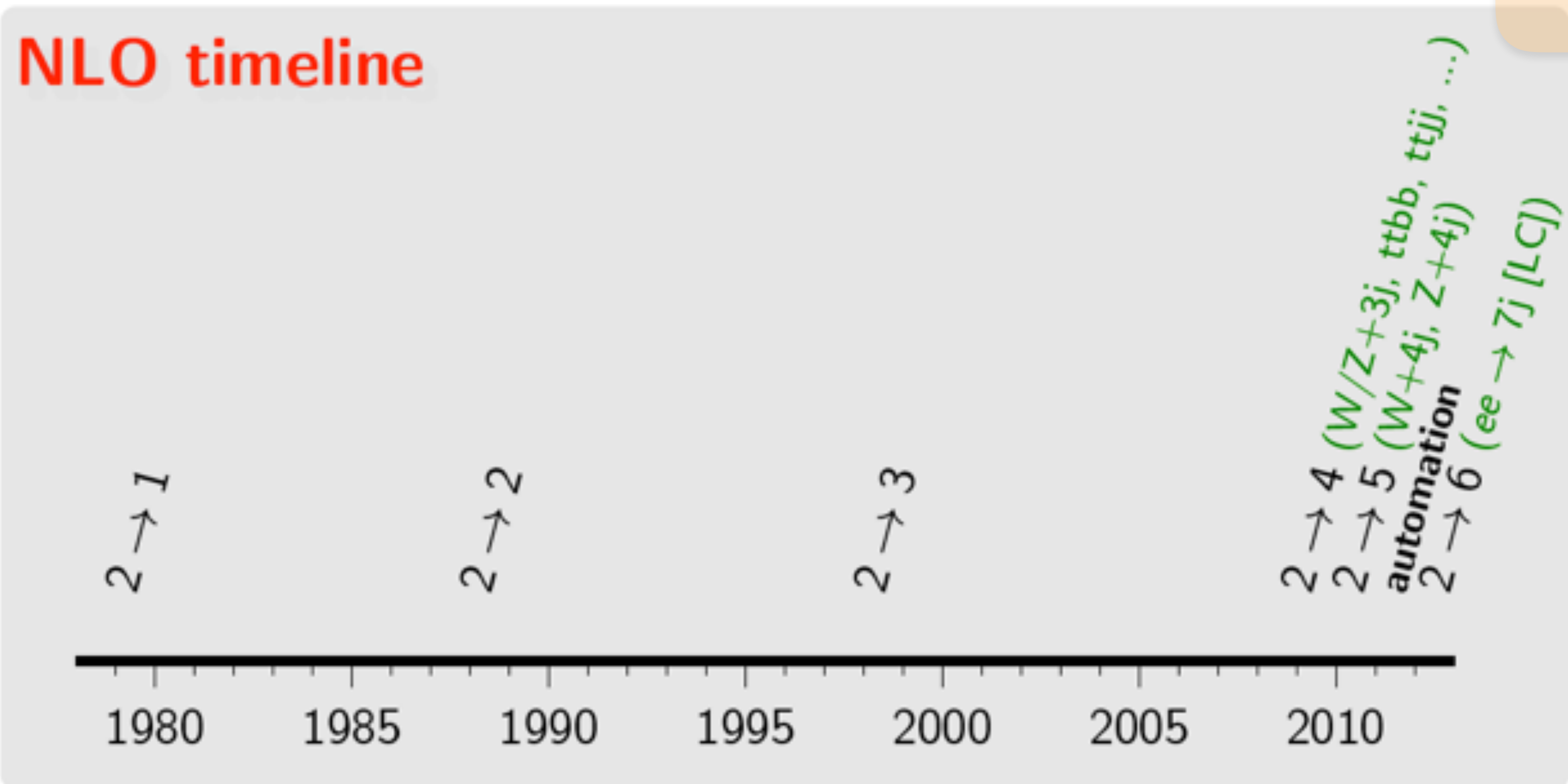
Semi-Numerical Methods

Helac-NLO
CutTools
BlackHat
Rocket
SAMURAI
MADLoop
GoSam
Ngluon

NLO revolution

1979: NLO Drell-Yan [Altarelli, Ellis & Martinelli]
1991: NLO $gg \to$ Higgs [Dawson; Djouadi, Spira & Zerwas]

1987: NLO high-p_t photoproduction [Aurenche et al]
1988: NLO $b\bar{b}$, $t\bar{t}$ [Nason et al]
1988: NLO dijets [Aversa et al]
1993: Vj [JETRAD, Giele, Glover & Kosower]

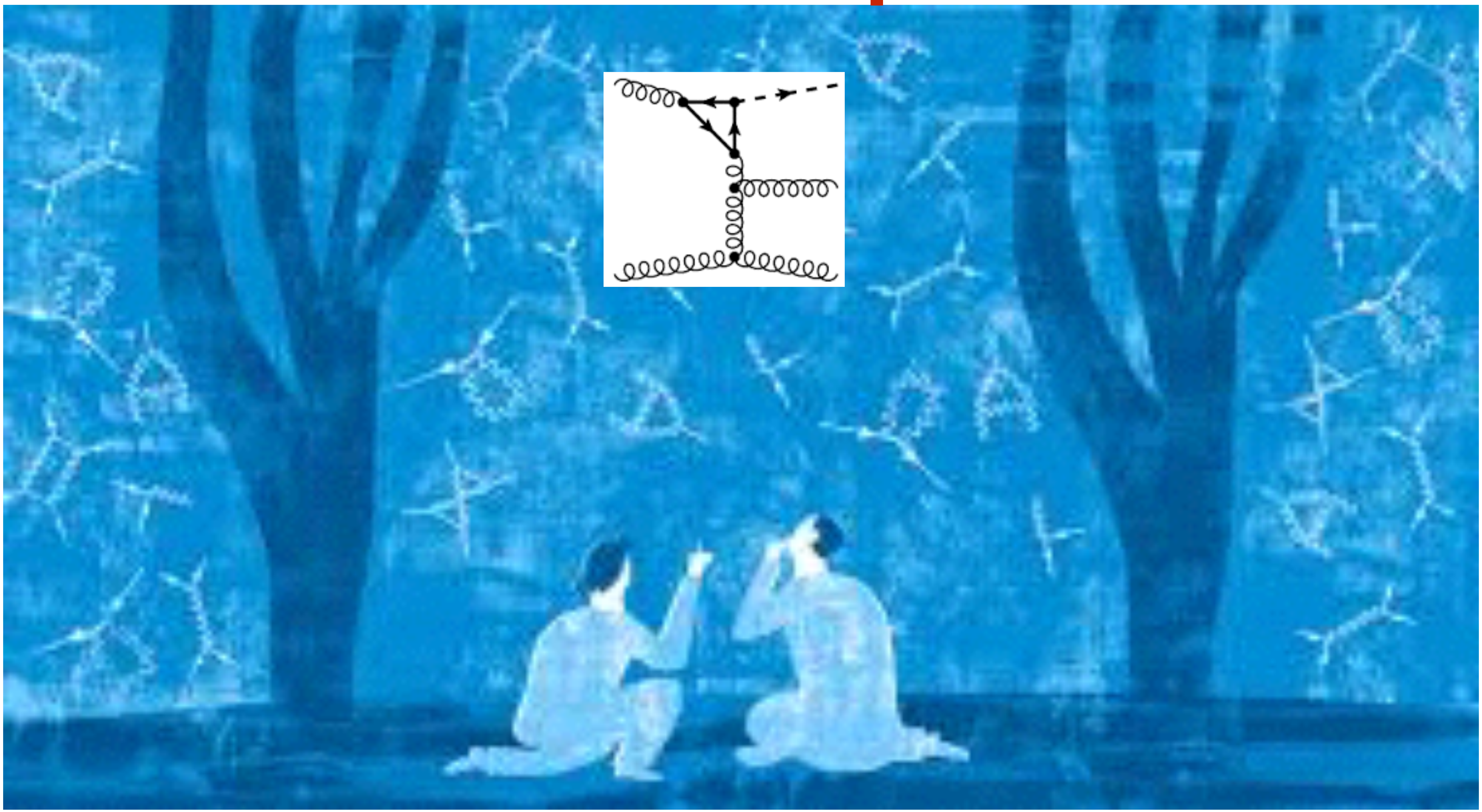


Golem, HELAC
BlackHat

1998: NLO $Wb\bar{b}$ [MCFM: Ellis & Veseli]
2000: NLO $Zb\bar{b}$ [MCFM: Campbell & Ellis]
2001: NLO $3j$ [NLOJet++: Nagy]
...
2007: NLO $t\bar{t}j$ [Dittmaier, Uwer & Weinzierl '07]
...

2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi]
2009: NLO $W+3j$ [BlackHat+Sherpa: Berger et al]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al]
2010: NLO $Z+3j$ [BlackHat+Sherpa: Berger et al]
...

2- loop



Integration By Parts (IBP)

$$D_i = q_i^2 + i\epsilon \qquad q_i = \sum_j k_l + \sum_l p_j$$

$$\prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{1}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_d}}}$$

n-Dimensional Gauss theorem

$$\int \prod_{i=1}^{N_k} \frac{d^n k_i}{(2\pi)^n} \frac{\partial}{\partial k_{j,\mu}} \left(\frac{k_{l,\mu}}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_e}}} \right) = 0 \qquad j, l = 1, \cdots N_k$$

$$\int \prod_{i=1}^{N_k} \frac{d^n k_i}{(2\pi)^n} \frac{\partial}{\partial k_{j,\mu}} \left(\frac{p_{l,\mu}}{D_1^{a_1} D_2^{a_2} \cdots D_{N_d}^{a_{N_e}}} \right) = 0 \qquad l = 1, \cdots N_e - 1$$

Integration By Parts (IBP) identities

Integration By Parts (IBP)

Number of IBP identities = $N_k(N_k + N_e - 1)$

IBP identities:

$$\sum_i C_i(s_{ij}, n)\; I_i(\{p_j\}, n, a_1, \cdots a_{N_e}) = 0$$

Integrals:

$$i = 1, \cdots N_I$$
$$s_{ij} = (p_i + p_j)^2$$

Solving IBP identities -> **Master Integrals:**

$$I_i(\{p_j\}, a_1, \cdots a_{N_e})$$
$$i = 1, \cdots N_{MI}$$

Good News $\quad N_{MI} << N_I$

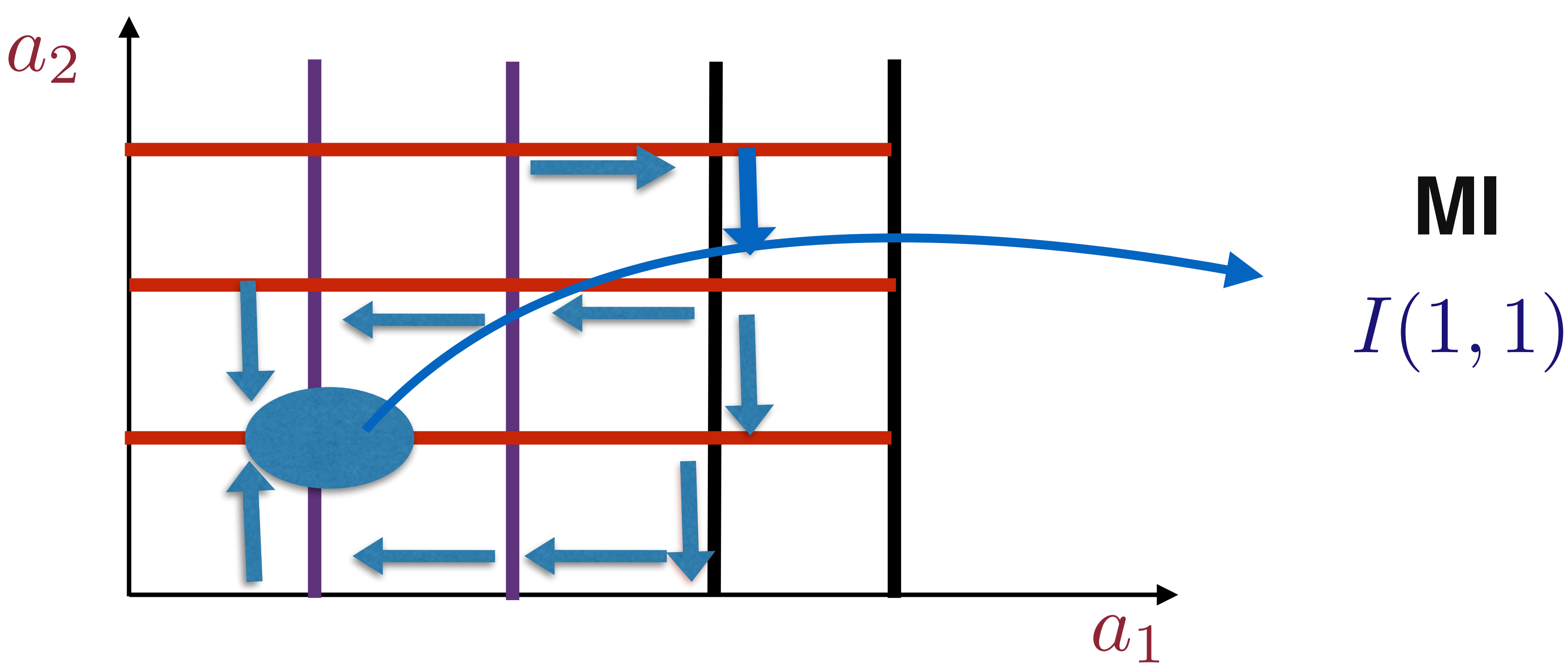
Integration By Parts (IBP)

$$I(a_1, a_2) = \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2)^{a_1}((k+p)^2)^{a_2}}$$

$$\int \frac{d^n k}{(2\pi)^n} \frac{\partial}{\partial k_\mu} \left(\frac{v_\mu}{(k^2)^{a_1}((k+p)^2)^{a_2}} \right) = 0$$

$$v = k, p$$

$$I(a_1, a_2) = \frac{a_1 + a_2 - n - 1}{p^2(a_2 - 1)} I(a_1, a_2 - 1) + \frac{1}{p^2} I(a_1 - 1, a_2)$$



Lorentz Invariant Identities

Integrals are Lorentz scalars

$$\omega^{\mu\nu} = -\omega^{\nu\mu}$$

$$p_i^\mu \to p_i^\mu + \delta p_i^\mu = p_i^\mu + \omega^{\mu\nu} p_\nu ,$$

$$I(p_i + \delta p_i) = I(p_i) + \omega^{\mu\nu} \sum_j p_{j,\nu} \frac{\partial}{\partial p_j^\mu} I(p_i) = I(p_i)$$

Anti-symmetry

$$\omega^{\mu\nu} = -\omega^{\nu\mu}$$

$$\sum_i p_{i,[\mu} \frac{\partial}{\partial p_i^{\nu]}} I(p_i) = 0$$

Anti-symmetry of $p_j^{[\mu} p_k^{\nu]}$

$$p_j^{[\mu} p_k^{\nu]} \sum_i p_{i,[\mu} \frac{\partial}{\partial p_i^{\nu]}} I(p_i) = 0$$

Solving Master Integrals

Consider a Master Integral

$$I(s_{ij}, n) = \prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} \frac{1}{D_1 D_2 \cdots D_{N_f}}$$

Define $s_{12} = s$

$$D_i = q_i^2 + i\epsilon \qquad q_i = \sum_j k_l + \sum_l p_j$$

$$s = (p_1 + p_2)^2$$

Differential w.r.t s

$$s\frac{\partial}{\partial s} I(s_{ij}, n) = \prod_{i=1}^{N_k} \int \frac{d^n k_i}{(2\pi)^n} s\frac{\partial}{\partial s}\left(\frac{1}{D_1 D_2 \cdots D_{N_f}}\right)$$

Master Integrals

Generalization with set of MIs

$$\vec{I} = \left(I_1, I_2, \cdot, \cdot, \cdot, I_N\right)$$

$\{I_i(\vec{x})\}$ depend on Scaling variables

$$x_i = f_i\left(\frac{s_{ij}}{Q^2}\right)$$

$$\vec{x} = \left(x_1, x_2, \cdot, \cdot, \cdot, x_M\right)$$

Differential equation:

$$d\vec{I} = \sum_{i=1}^{M} \mathbf{A}_i dx_i\ \vec{I}$$

$$\frac{\partial}{\partial x_i} \begin{bmatrix} I_1 \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{A_{11}} & \cdots & \mathbf{A_{1N}} \\ \cdot & \cdots & \cdot \\ \mathbf{A_{N1}} & \cdots & \mathbf{A_{NN}} \end{bmatrix} \begin{bmatrix} I_1 \\ \cdot \\ I_N \end{bmatrix}$$

$$i = 1, 2, \cdot, \cdot, \cdot M$$

Canonical/Henn's Basis

Consider Diff equation:

$$d\vec{I}(\vec{x},n) = \sum_i \mathbf{A}_i(\vec{x},n)dx_i \; \vec{I}(\vec{x},n)$$

Choose U Transformation such that

$$U^{-1}\mathbf{A}(\vec{x},n)U - U^{-1}dU = (n-4)\;\overline{\mathbf{A}}(\vec{x})$$

Diff equation contains ,n' independent A

$$d\vec{\overline{I}}(\vec{x},n) = (n-4)\sum_i \overline{\mathbf{A}}_i(\vec{x})dx_i \; \vec{\overline{I}}(\vec{x},n)$$

Solution

$$\vec{\overline{I}}(\vec{x},n) = \vec{\overline{I}}(\vec{x}_0,n)\;\mathbf{P}\exp\left((n-4)\int\frac{d\lambda}{\lambda}\overline{\mathbf{A}}(\lambda)\right)$$

$\mathbf{P}$ - Path Ordered exponential

Canonical/Henn's Basis

Start with Henn's Diff equation:

$$s\frac{\partial}{\partial s}\overline{I}(s,n) = (n-4)\overline{A}(s)\ \overline{I}(s,n)$$

If $\overline{A}$ contains poses at s_i

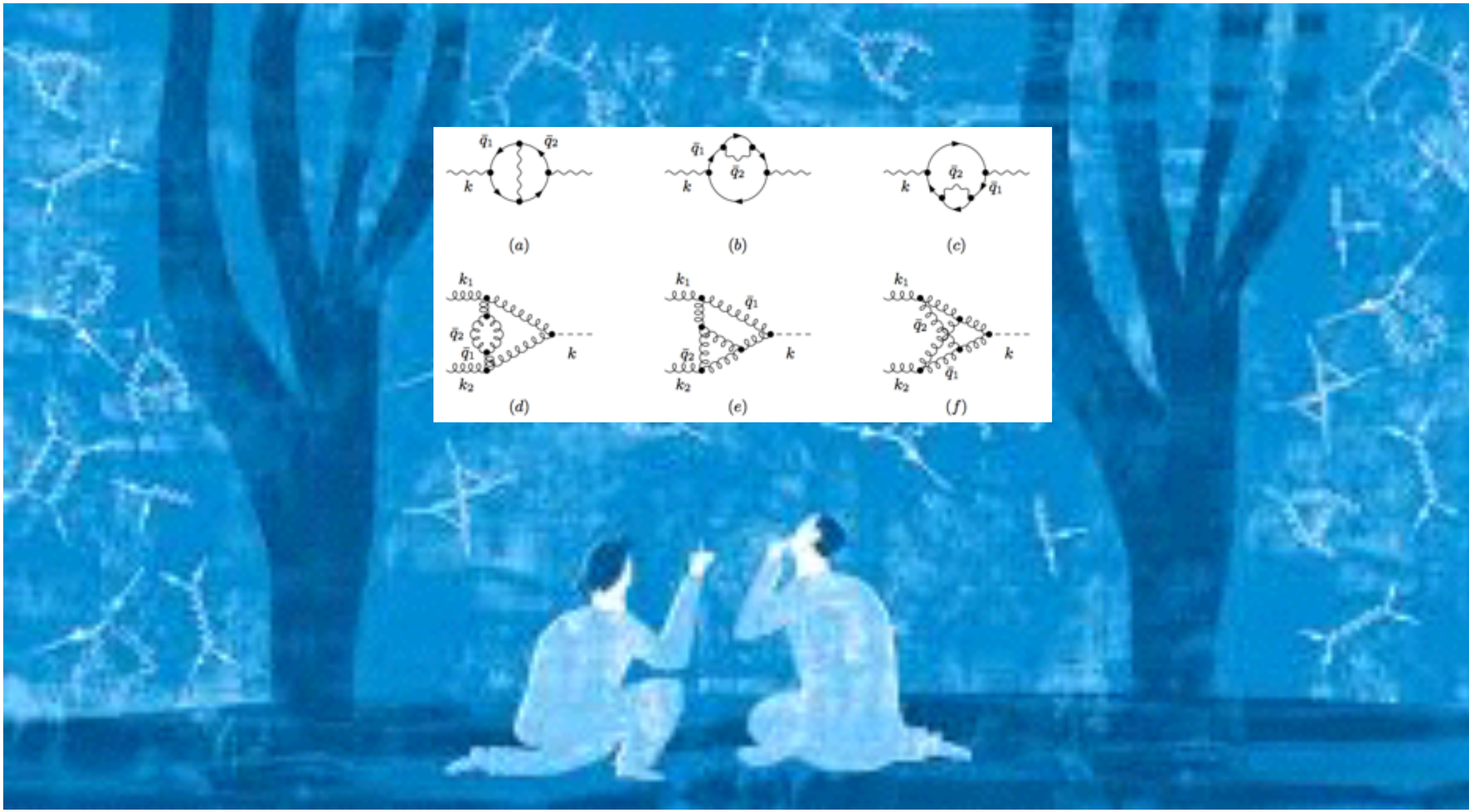
$$\overline{A}(s) = \sum_i \frac{\tilde{A}_i(s_i)}{s-s_i}$$

$$\overline{I}(s,n) = \overline{I}^{(0)}(s_0) + (n-4)\sum_i \tilde{A}_i(s_i)\log\left(\frac{s-s_i}{s_0-s_i}\right)$$

$$+(n-4)^2\sum_i \tilde{\tilde{A}}(s_i)\mathcal{L}_i(s_0,s_i) + \cdots$$

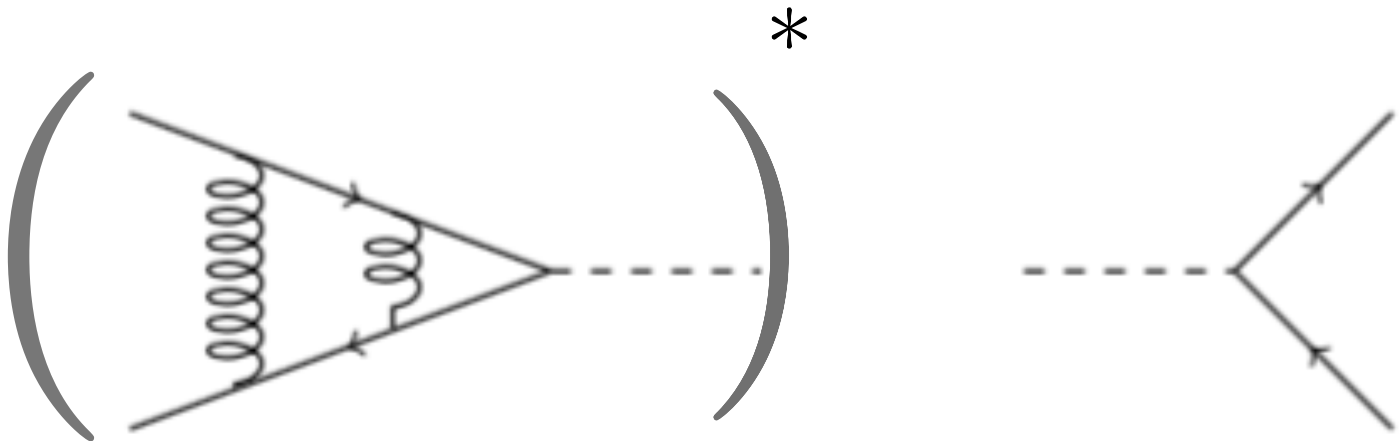
Polylogarithms - Uniform transcendental terms

NNLO



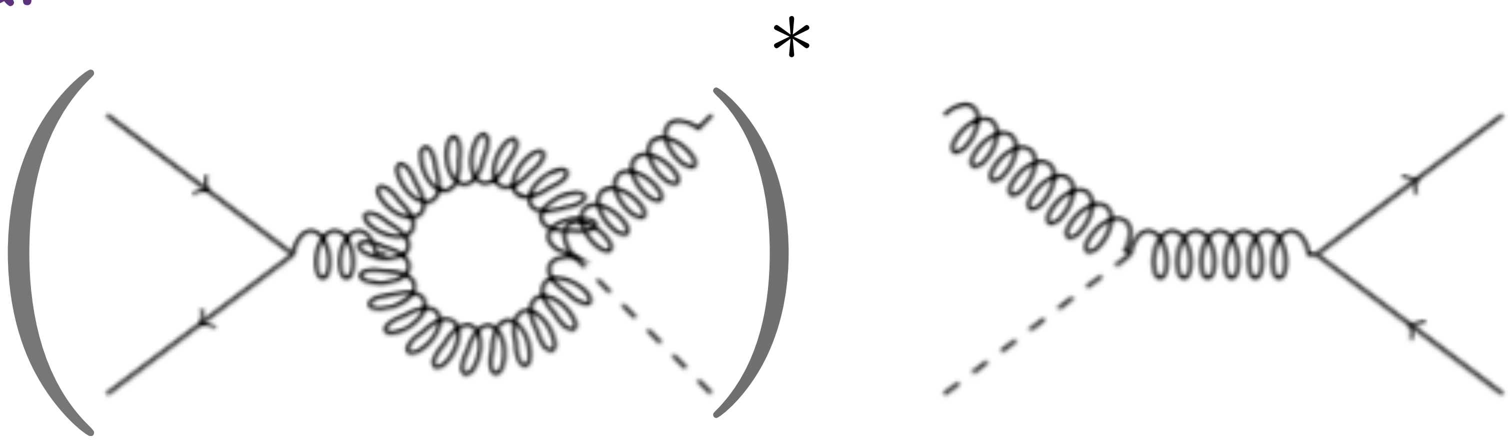
NNLO corrections:

Pure Virtual



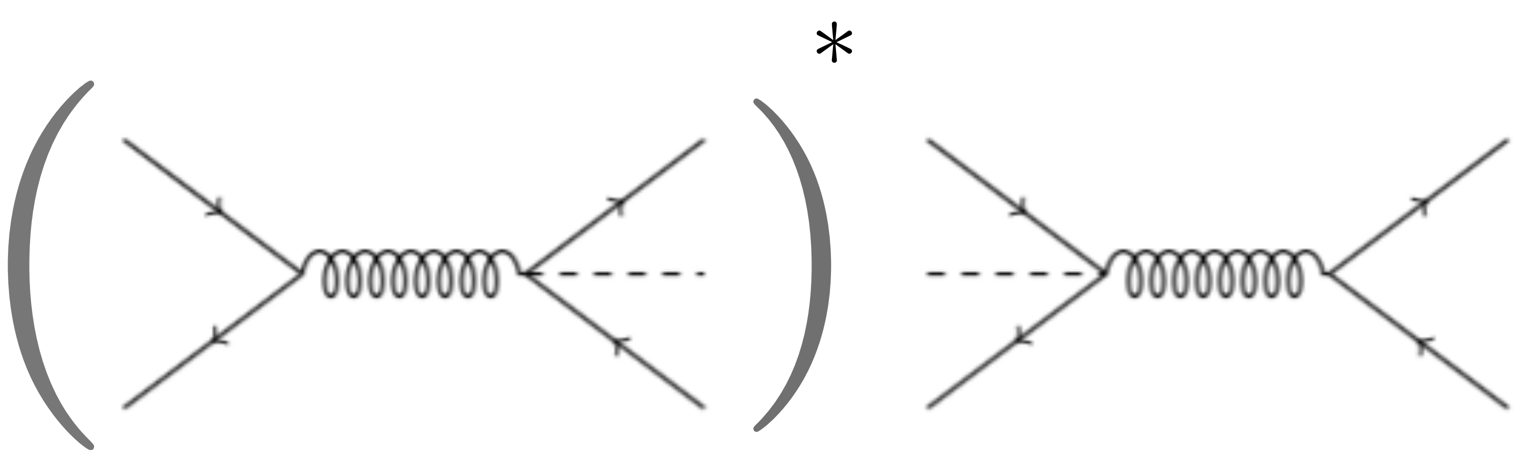
$+\,53$ terms.

Virtual - Real



$+\,171$ terms.

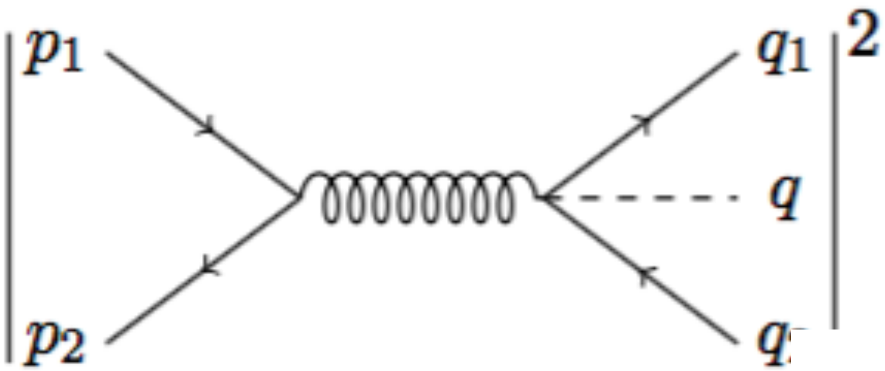
Real-Real



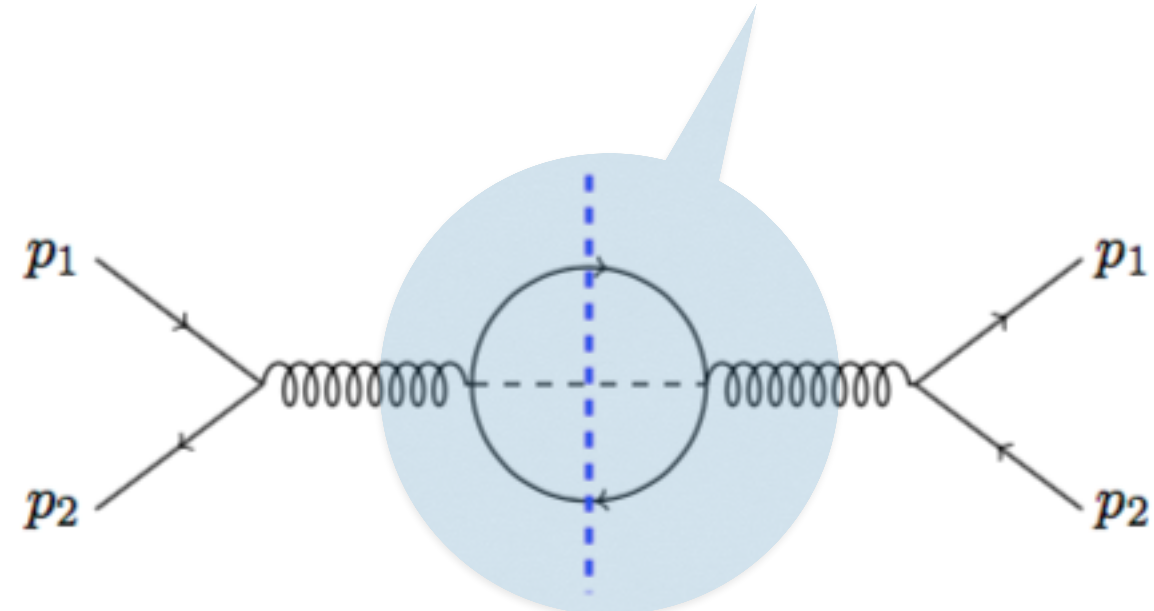
$+\,293$ terms.

Reverse Unitarity

Real-Real



$$\propto \int \frac{d^n q_1}{(2\pi)^{n-1}} \frac{d^n q_2}{(2\pi)^{n-1}} \delta_+(q_1^2)\delta_+(q_2^2)\delta_+(q^2 - m_h^2)[\cdots]$$



Reverse Unitarity

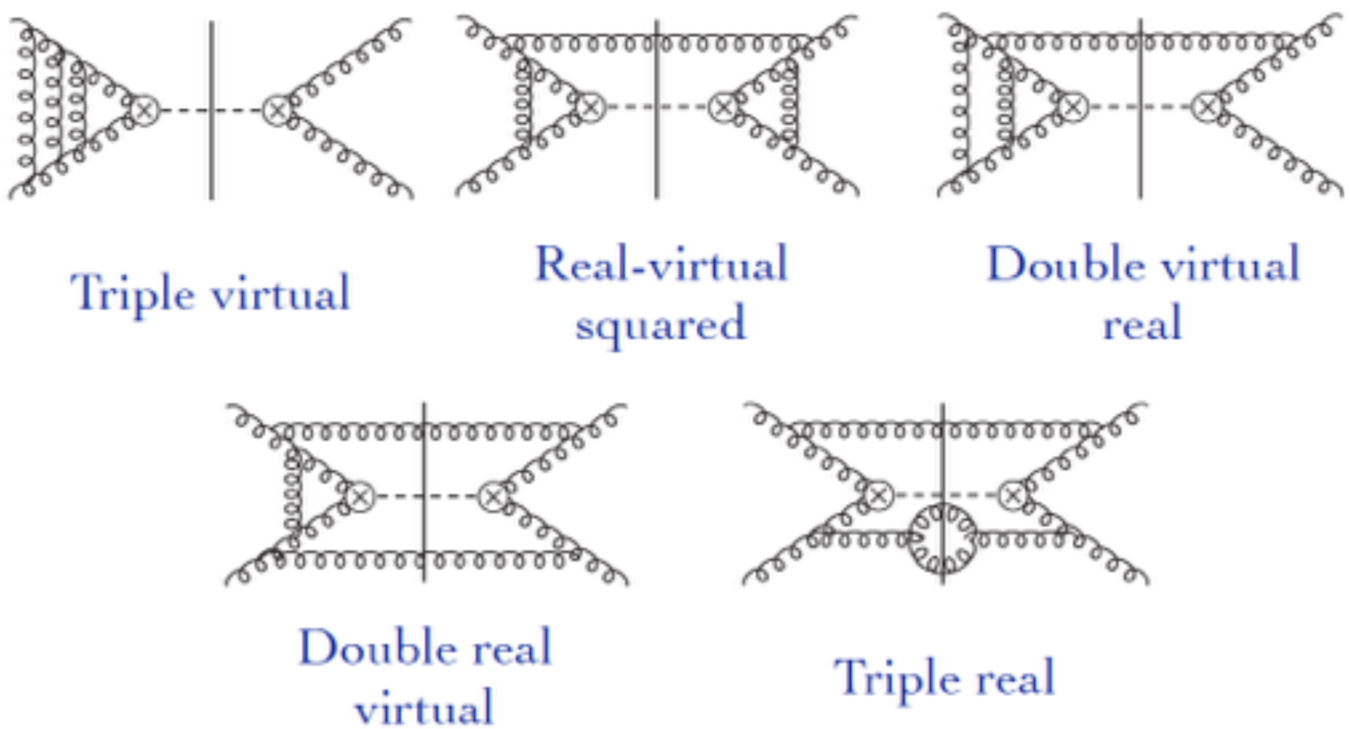
$$\delta_+(q^2 - m^2) \sim \frac{1}{q^2 - m^2 + i\varepsilon} - \frac{1}{q^2 - m^2 - i\varepsilon}$$

→ Loop Integrals

Higgs production to N^3LO in QCD at the LHC

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

100 000 diagrams



Integration By Parts

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \int \frac{d^d k_3}{(2\pi)^d} \frac{\partial}{\partial k_i} \cdot \left(v_j \frac{1}{\prod_l D_l^{n_l}} \right) = 0$$

Lorentz Invariance

$$p_i^\mu p_j^\nu \left(\sum_k p_{k[\nu} \frac{\partial}{\partial p_k^{\mu]}} \right) J(\vec{n}) = 0.$$

↓

Master Integrals

	Integrals	Master Integrals
NNLO	50 000	27
N3LO	517 531 178	1028

NNLO n-Jettiness:

$$\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}.$$

$$\mathcal{T}_N < \mathcal{T}_N^{cut}$$

$$\sigma(\mathcal{T}_N < \mathcal{T}_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n \right] + \cdots .$$

NNLO - analytical

- H- pure virtual
- B - Initial state beam fns.
- S - Soft distribution fns.
- J - Final state jet fns.

$$\mathcal{T}_N \geq \mathcal{T}_N^{cut}$$

$$\sigma\left(\mathcal{T}_N \geq \mathcal{T}_N^{cut}\right)$$

NLO with two jets

finite- numerically

NNLO n-Jettiness:

$\mathcal{T}_N < \mathcal{T}_N^{cut}$ and $\mathcal{T}_N \geq \mathcal{T}_N^{cut}$

$$\mathcal{T}_N^{\rm cut} \to \mathcal{T}_\delta = \delta_{\rm IR} Q,$$

$$\sigma(X) = \int_0^{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} + \int_{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$
$$= \sigma^{\rm sing}(X, \mathcal{T}_\delta) + \int_{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} + \mathcal{O}(\delta_{\rm IR}) \, .$$

$\sigma^{\rm nons}(X, \mathcal{T}_\delta)$ is of $\mathcal{O}(\mathcal{T}_\delta/Q) = \mathcal{O}(\delta_{\rm IR})$.

qT subtraction at N3LO:

$$d\sigma^{F}_{(N)NLO} = \mathcal{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

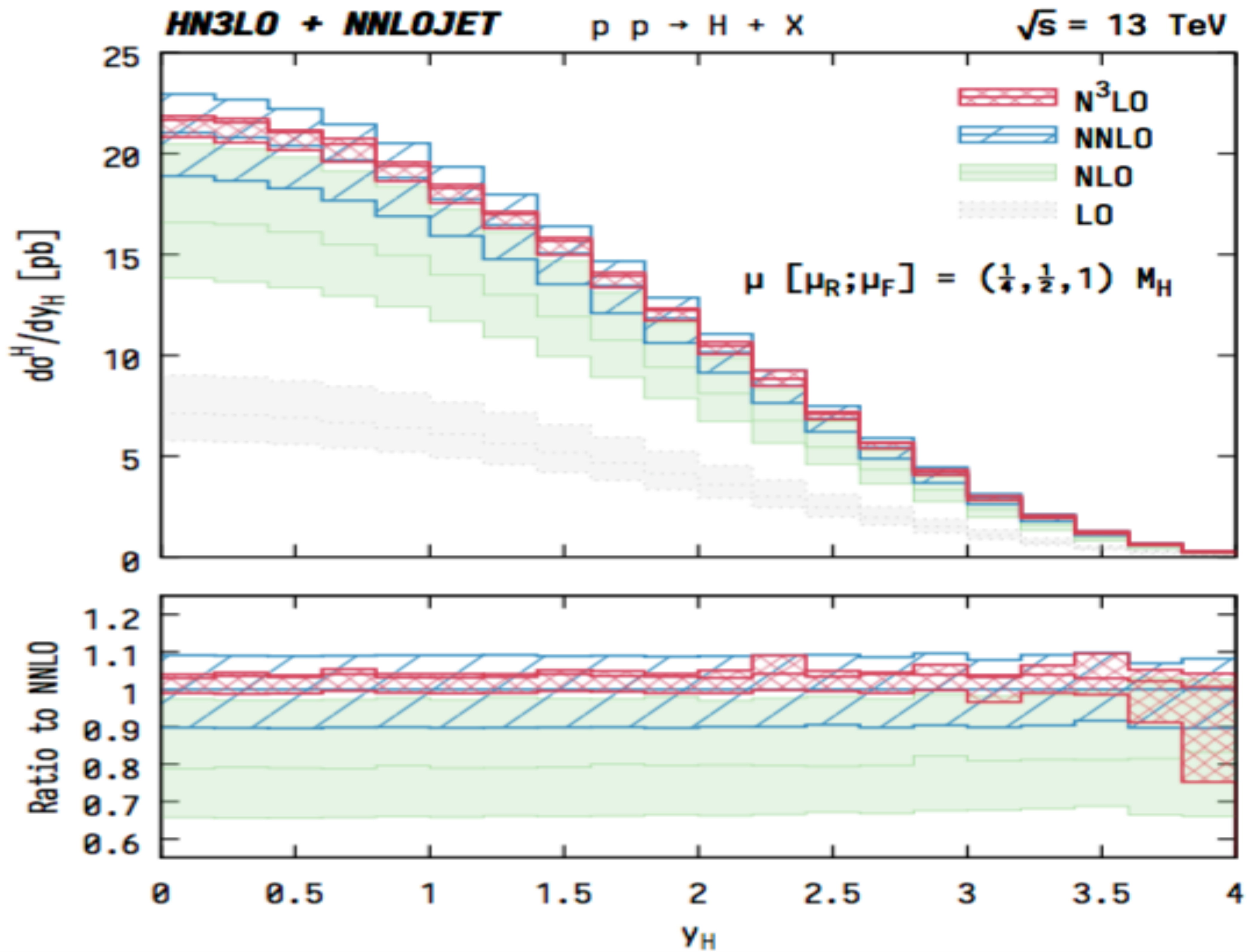
$$d\sigma^{CT} = d\sigma^{F}_{LO} \otimes \Sigma^{F}(q_T/Q)\, d^2\mathbf{q}_T .$$

Note that
$$d\sigma^{F}_{(N)NLO}\big|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

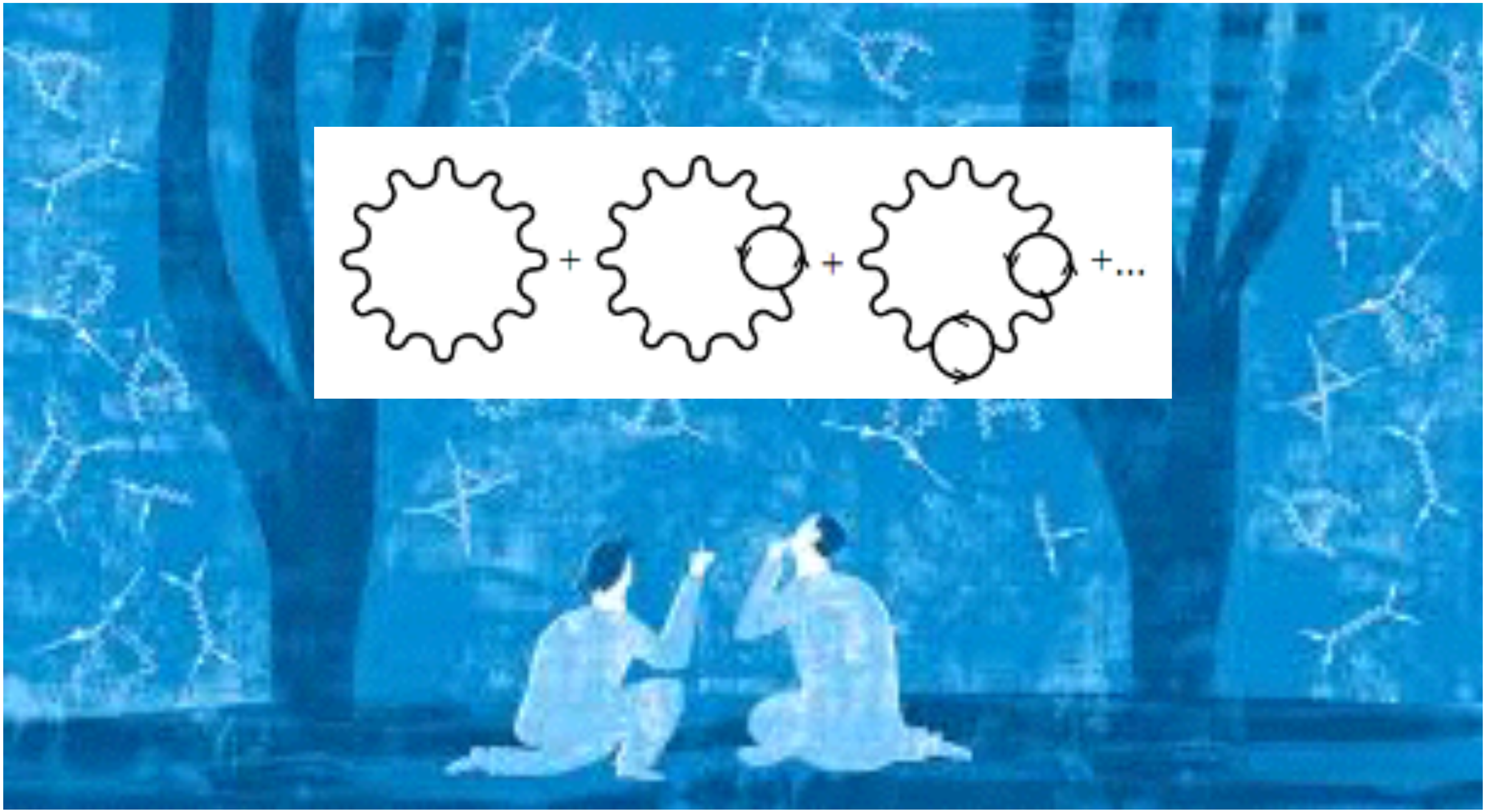
qT resummation gives

$$\Sigma^{F}(q_T/Q) \xrightarrow[q_T \to 0]{} \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n} \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2} \quad .$$

qT subtraction at N3LO:

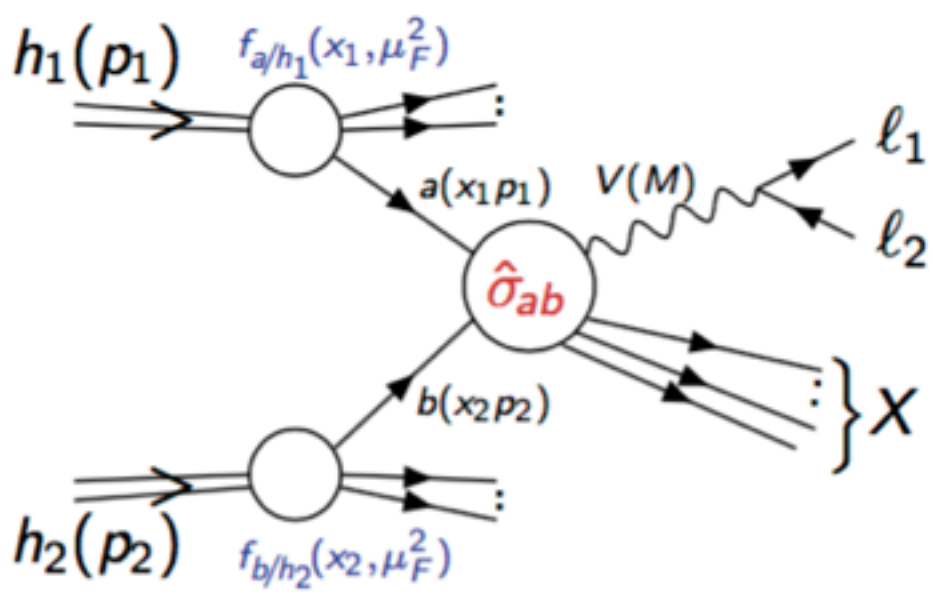


Resummation



Small qT Resummation for DY

$$\frac{d\sigma}{dq_T^2}(q_T,M,s)=\sum_{a,b}\int_0^1 dx_1\int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\,\frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2)$$



For $q_T \to 0,\ \alpha_S^n \log^m(M^2/q_T^2) \gg 1$

$$\frac{d\hat{\sigma}}{dq_T^2}=\frac{d\hat{\sigma}^{(res)}}{dq_T^2}+\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}$$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2}=\frac{M^2}{\hat{s}}\int_0^\infty db\,\frac{b}{2}J_0(bq_T)\,\mathcal{W}(b,M)$$

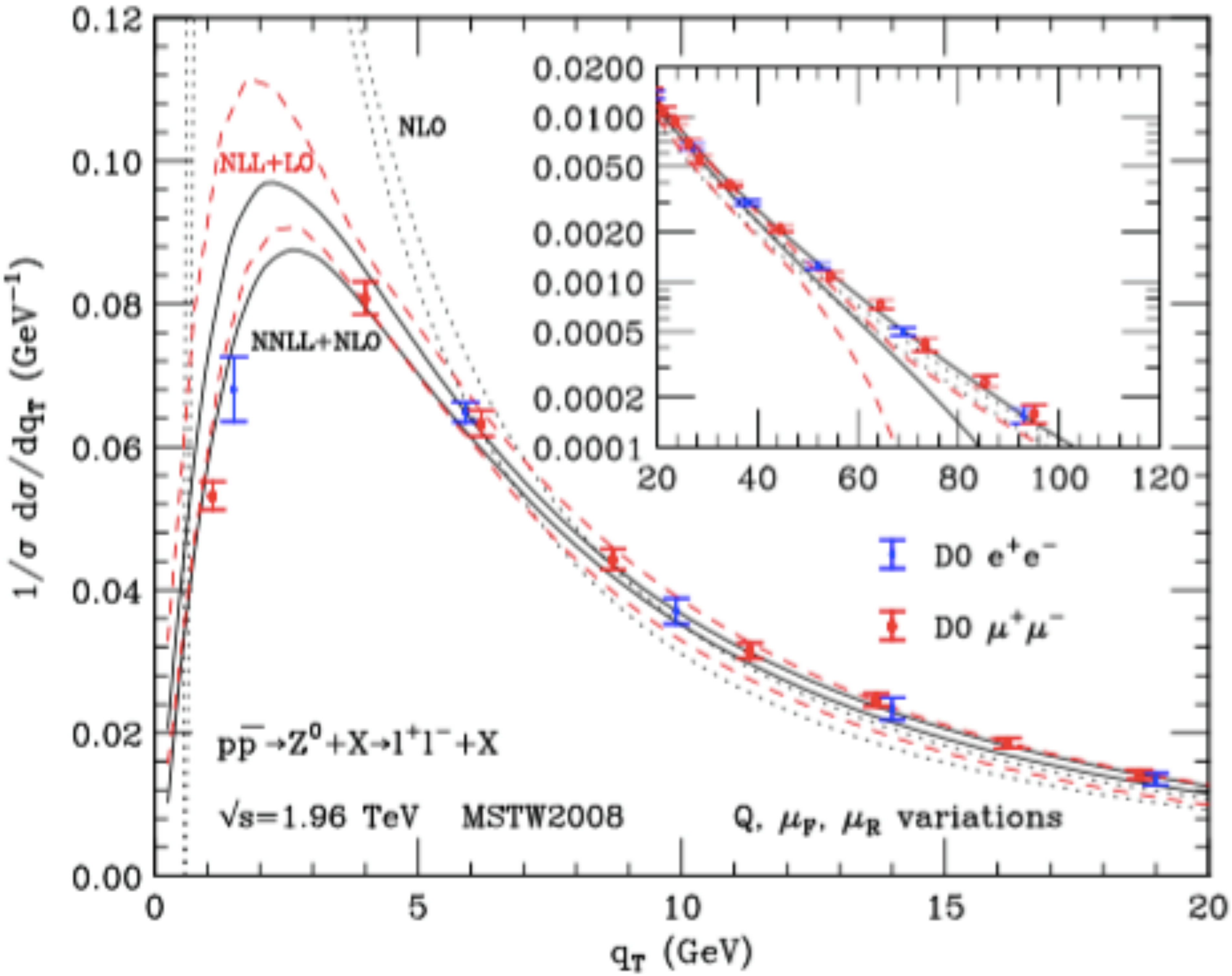
$$\mathcal{W}_N(b,M)=\mathcal{H}_N(\alpha_S)\times\exp\{\mathcal{G}_N(\alpha_S,L)\}$$

$$L\equiv\log(M^2b^2)$$

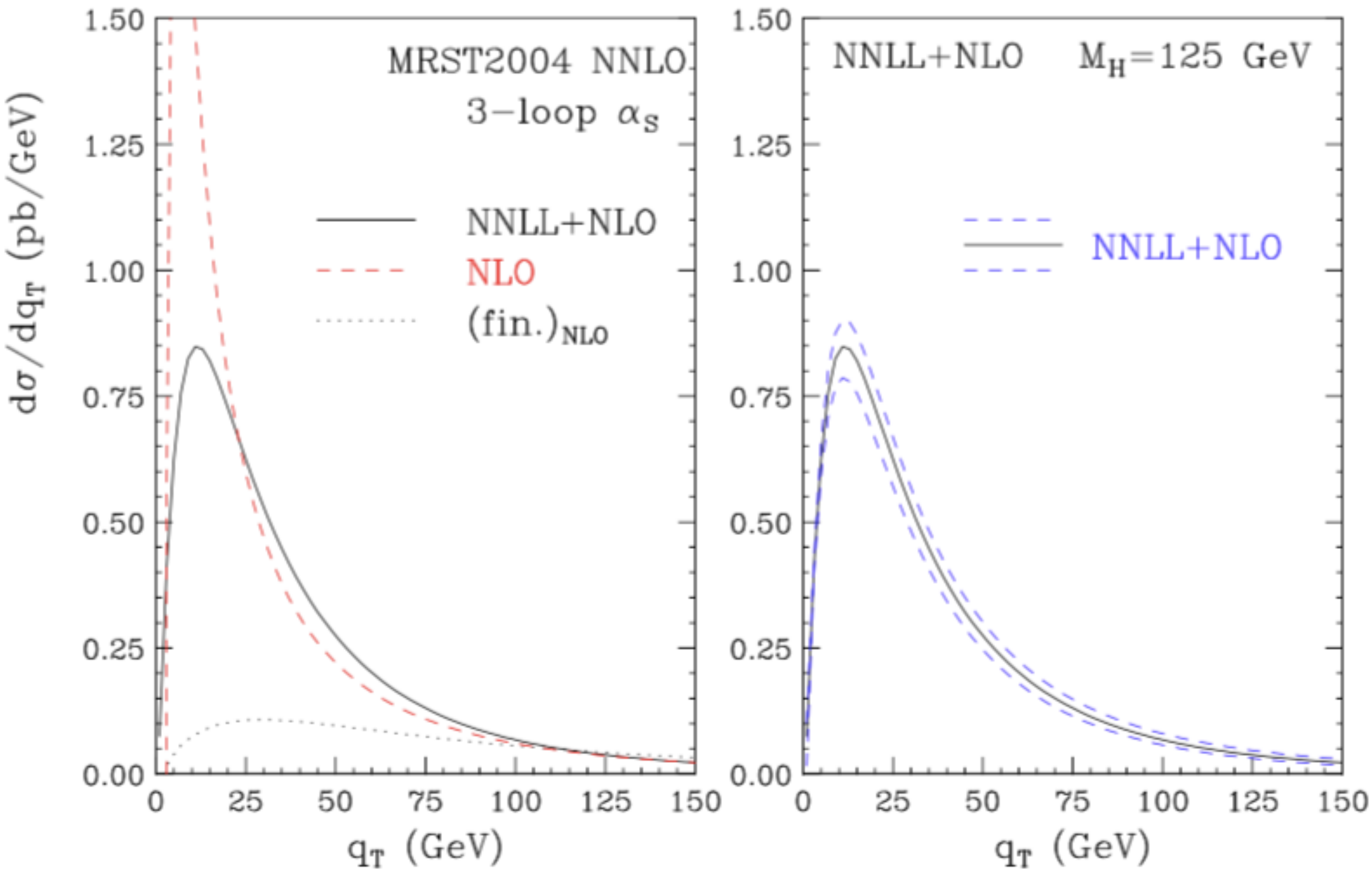
$$\mathcal{G}_N(\alpha_S,L)=L\,g^{(1)}(\alpha_S L)+g_N^{(2)}(\alpha_S L)+\frac{\alpha_S}{\pi}g_N^{(3)}(\alpha_S L)+\cdots$$

Small qT Resummation for DY

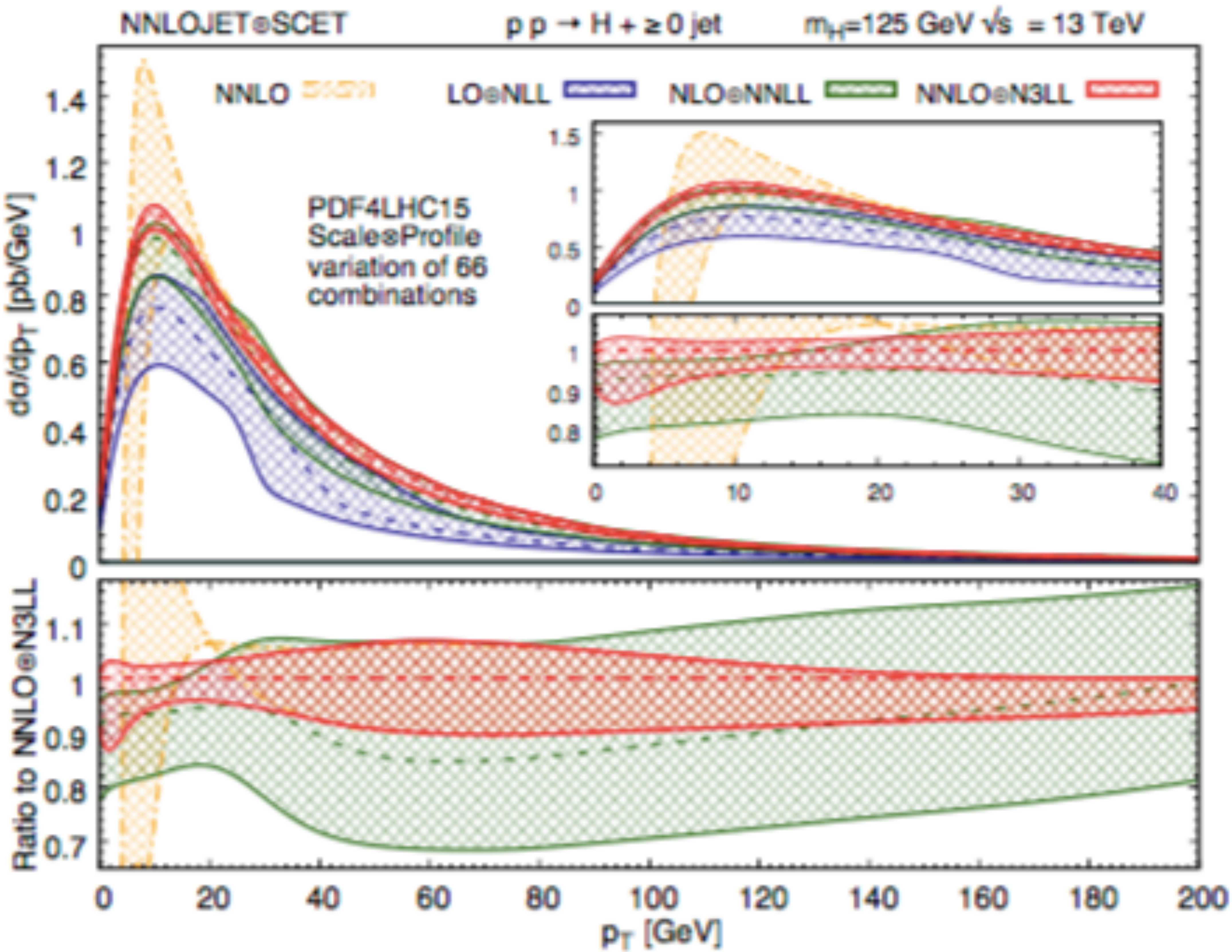
D0 data for the Z q_T spectrum compared with perturbative results.



Small qT Resummation for Higgs



Small qT Resummation for Higgs



Rapidity Distribution

Rapidity Distribution of any colorless particle:

$$\frac{d\sigma^I}{dy} = \hat{\sigma}^I_B \sum_{ab=q,\bar{q},g} \int_{x_1^0}^1 \frac{dz_1}{z_1} \int_{x_2^0}^1 \frac{dz_2}{z_2} \hat{\mathcal{H}}^I_{ab}\left(\frac{x_1^0}{z_1}, \frac{x_2^0}{z_2}, \mu^2\right) \hat{\Delta}^I_{d,ab}\left(z_1, z_2, q^2, \mu^2\right)$$

DY production of lepton pairs $\sigma^I = \frac{d\sigma^q\left(\tau, q^2, y\right)}{dq^2}$.

Higgs through gluon (bottom anti-bottom), $\sigma^I = \sigma^{g(b)}\left(\tau, q^2, y\right)$.

Rapidity: $y = \frac{1}{2}\ln\left(\frac{p_2 \cdot q}{p_1 \cdot q}\right) = \ln\left(\frac{x_1^0}{x_2^0}\right)$, $\tau = x_1^0 x_2^0$

Partonic Scaling variables:

$$z_1 = \frac{x_1^0}{x_1}, \qquad z_1 = \frac{x_2^0}{x_2}$$

Soft and Virtual terms

$$\Delta_d^I = \delta(1-z_1)\delta(1-z_2) + a_s\Bigg\{ c_1^{(1)}\delta(1-z_1)\delta(1-z_2)$$

$$+c_2^{(1)}\left(\frac{\ln(1-z_1)}{1-z_1}\right)_+ + R^{(1)}(z_1,z_2) + z_1 \leftrightarrow z_2 \Bigg\} + \mathcal{O}(a_s^2)$$

$$\Delta_d^I(z_1,z_2) = \Delta_d^{I,SV}(z_1.z_2) + \Delta_d^{I,hard}(z_1,z_2)$$

Virtual , Soft $\delta(1-z_i)$ $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$

$$\tilde{\Delta}_d^{I,\mathrm{SV}}(\omega) = \int_0^1 dz_1 z_1^{N_1-1}\int_0^1 dz_2 z_2^{N_2-1}\Delta_d^{I,\mathrm{SV}}(z_1,z_2)$$

Resummation to NNLL

Logarithms that are resummed in g_d^I

	LL	NLL	NNLL
$\mathcal{O}(a_s)$	$\ln^2(\bar{N}_1\bar{N}_2)$	$\ln(\bar{N}_1\bar{N}_2)$	
$\mathcal{O}(a_s^2)$	$\ln^3(\bar{N}_1\bar{N}_2)$	$\ln^2(\bar{N}_1\bar{N}_2)$	$\ln(\bar{N}_1\bar{N}_2)$
$\mathcal{O}(a_s^3)$	$\ln^4(\bar{N}_1\bar{N}_2)$	$\ln^3(\bar{N}_1\bar{N}_2)$	$\ln^2(\bar{N}_1\bar{N}_2)$
Resummed terms:	$a_s^m \ln^{m+1}(\bar{N}_1\bar{N}_2)$	$a_s^m \ln^m(\bar{N}_1\bar{N}_2)$	$a_s^{m+1} \ln^m(\bar{N}_1\bar{N}_2)$
Function that resums :	$g_{d,1}^I \ln(\bar{N}_1\bar{N}_2)$	$g_{d,2}^I$	$a_s g_{d,3}^I$

Soft Gluon Resummation

Double Mellin Transformation:

$$\tilde{\Delta}_d^{I,\mathrm{SV}}(\omega) = \int_0^1 dz_1 z_1^{N_1-1} \int_0^1 dz_2 z_2^{N_2-1} \Delta_d^{I,\mathrm{SV}}(z_1, z_2)$$

Resumed Rapidity distribution:

Ni dependent

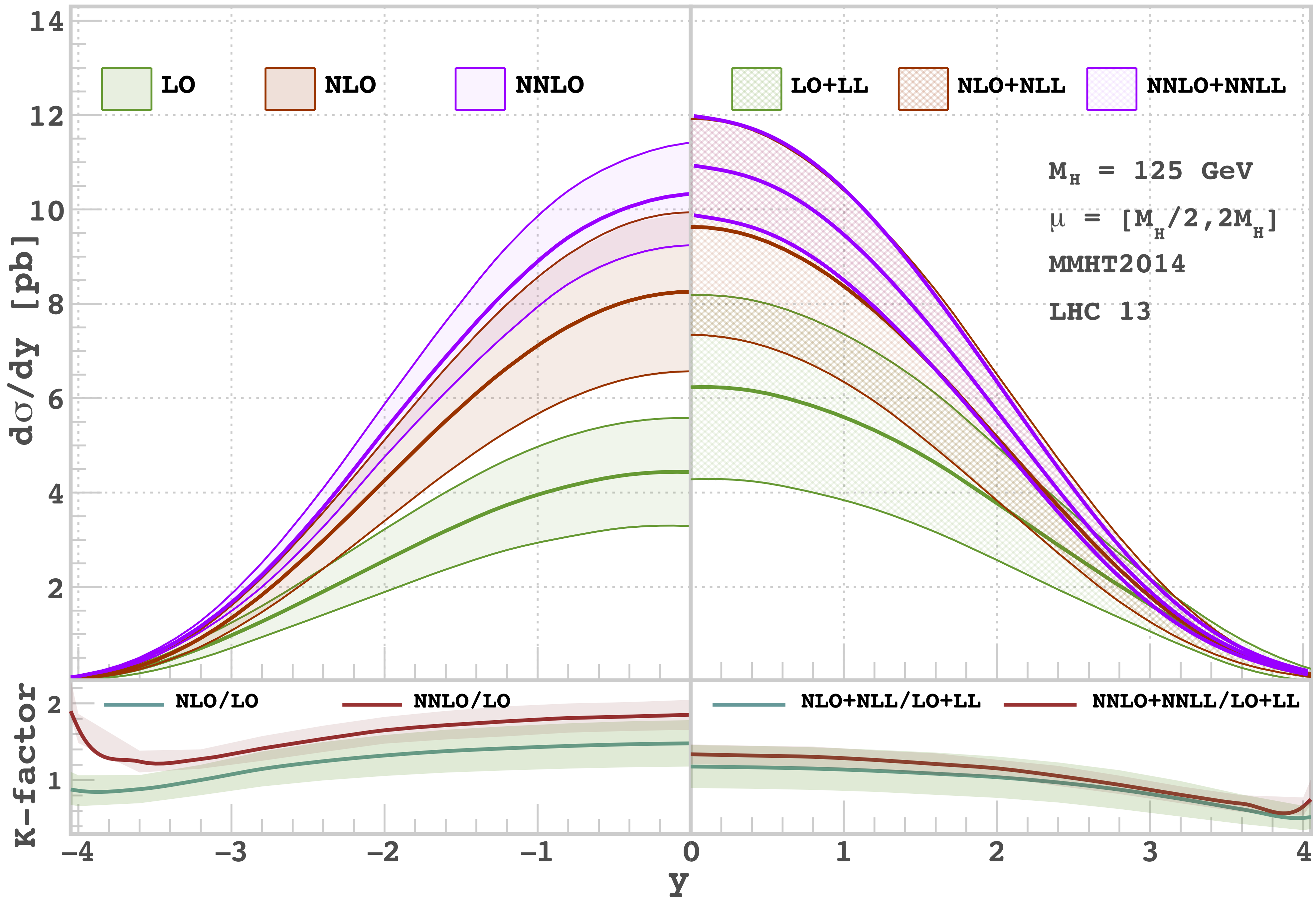
$$\tilde{\Delta}_d^{\mathrm{SV},I}(\omega) = \tilde{g}_{d,0}^I(a_s) \exp\left(g_d^I(a_s,\omega)\right)$$

Ni independent

$$\omega = a_s \beta_0 \ln\left(\overline{N}_1 \overline{N}_2\right)$$

$$\overline{N}_i = e^{\gamma_E} N_i$$

Rapidity of Higgs at NNLO + NNLL



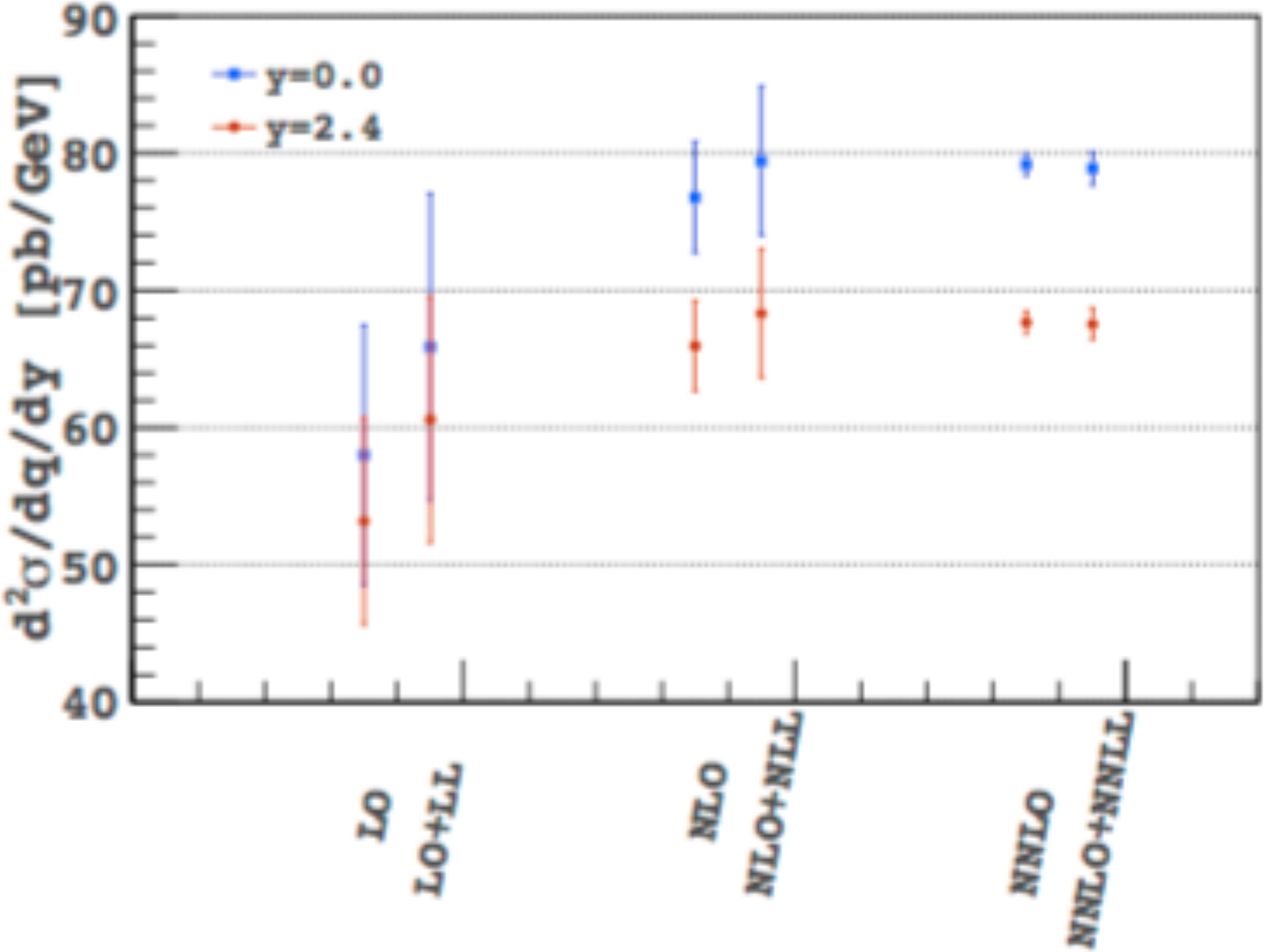
Fixed order CS

$$M_H/2 \leq \mu_{R,F} \leq 2M_H \qquad \Longrightarrow$$

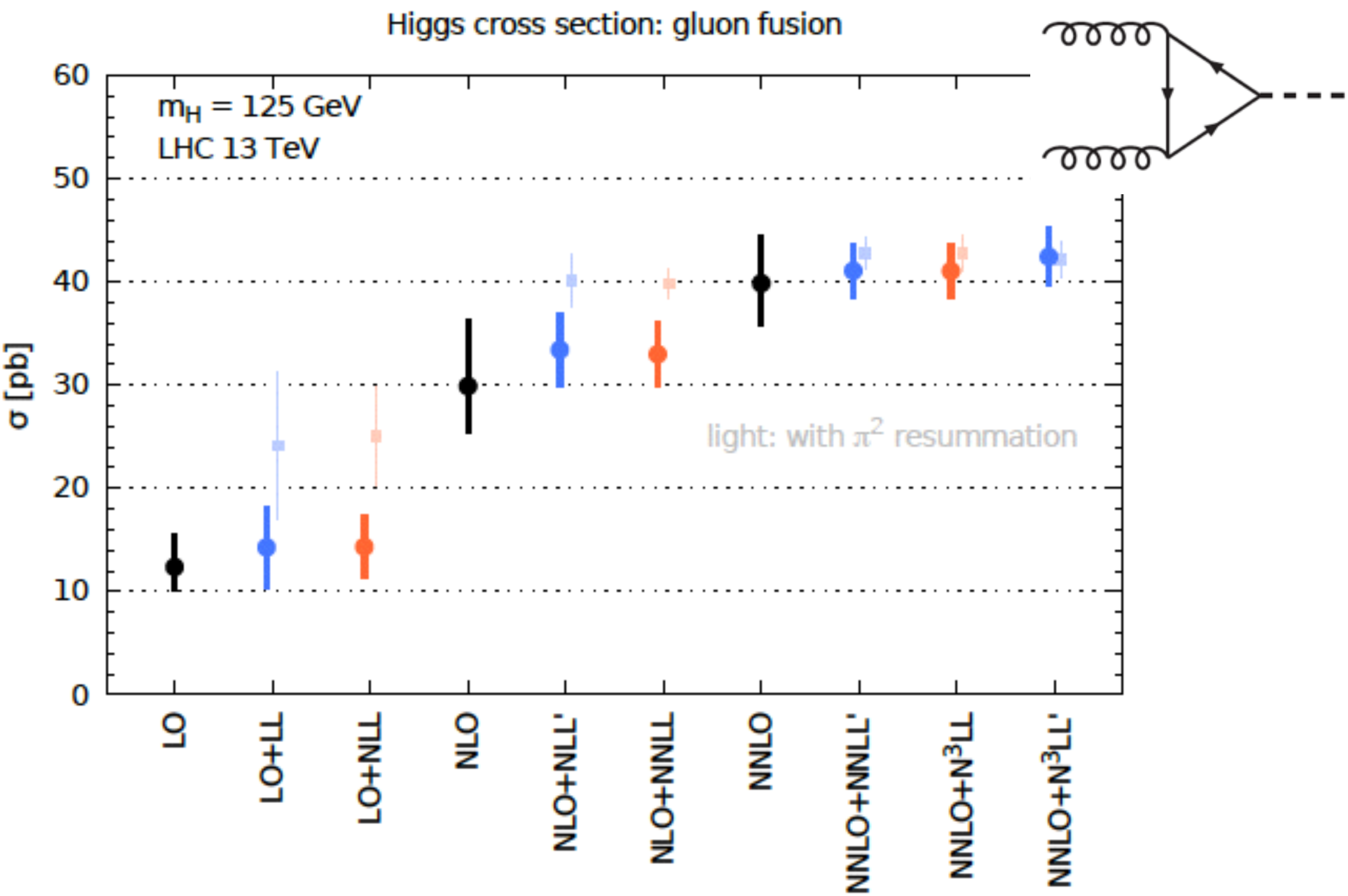
Resummed CS

At NNLO+NNLL result stabilises convergence of perturbation series !

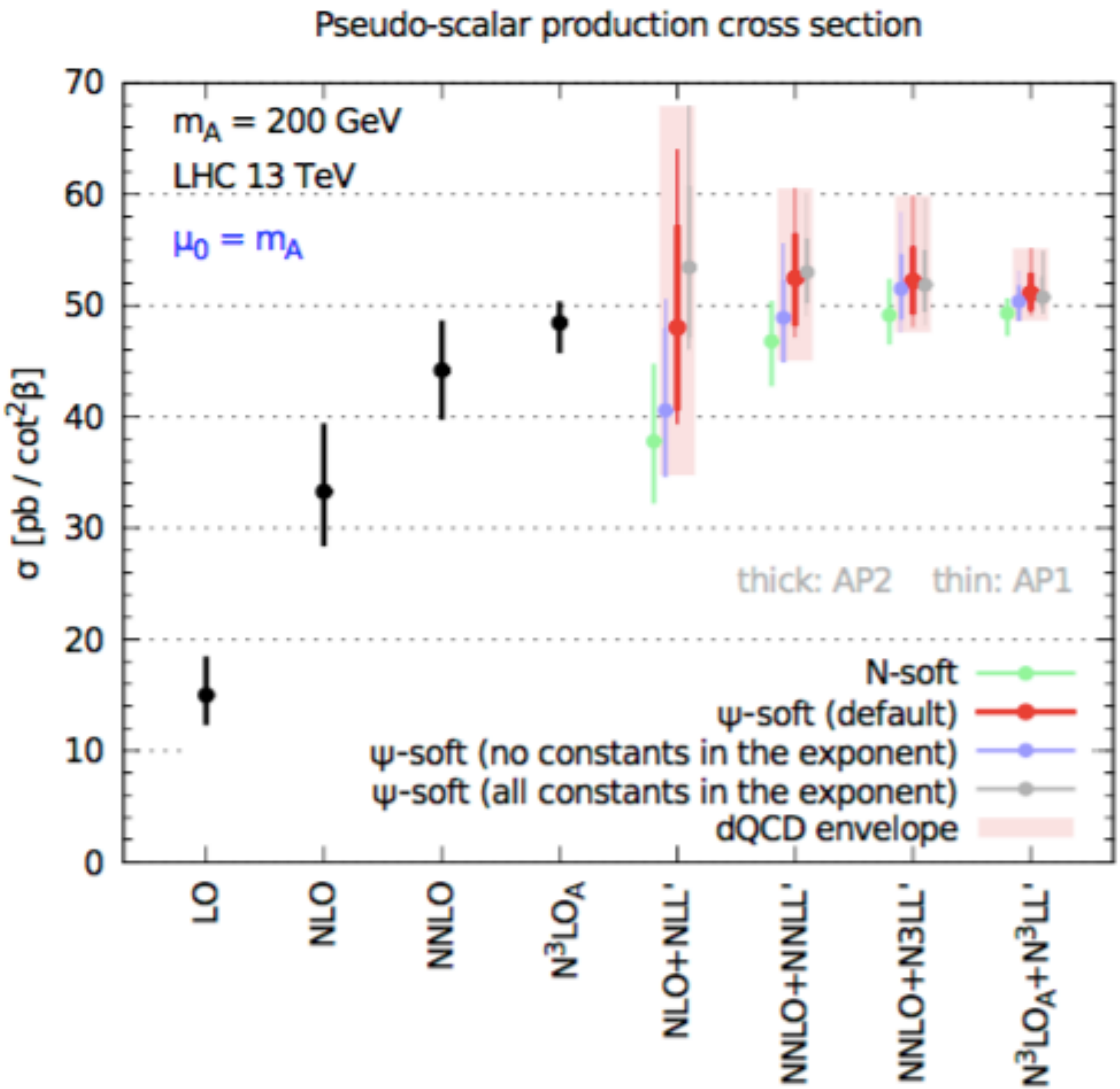
Rapidity of DY at NNLO + NNLL



Resummation at N3LL for Higgs



Pseudoscalar Higgs at N3LO(A) + N3LL



Conclusion

The truth is in the Details