Story of metal, semiconductor and insulator

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1 Introduction

In this pedagogical article I try to explain one of the simple yet profound property of metal, insulator and semiconductor. Among many properties that differentiate a metal from semiconductor or insulator electrical conductivity is one. Metals are known for a very high electrical conductivity compared to insulator and semiconductor. The motivation to take this simple phenomena is many-fold. Firstly the article would show an example which a material science or a condensed matter researcher is interested in. Secondly, through this example we would explain some basic but profound advancements in modern day physics such as application of quantum mechanics and statistical mechanics. No prior exposure to the above subjects are assumed. The importance of understanding how metal, semiconductor and insulator behaves is immense for numerous technological applications. For example the metal-semiconductor junction device paved the discovery of diode and transistors which revolutionized the electronics world. All modern day electronics gadgets have transistors as basic building block.

Guide to use this pedagogical article

- For basic definition of metal, insulator, semiconductor and pedagogical understanding of main criteria to determine this one can read only first part of Sec. 2 i.e 2.1. This is mainly aimed for up to higher secondary students.
- Those wish to move further and understand the basic mechanism of electric transport and which causes finite resistivity can go up to last part of Sec. 2 i.e 2.2.
- To know the finite temperature behavior and basic understanding of it one can refer to Sec. 3. This is aimed for little advanced student up to BSc first year or second year.

Effort has been made to write the whole article in such a way that any student in any discipline can understand it given a little time and patience.

2 Metal, Semiconductor and Insulator

Usually metals are known as good conductor of electricity which means that if a potential difference is applied between two ends of a metal object, current flows from positive to negative terminal. Copper, Aluminum, Iron are example of this. For an insulator the current almost

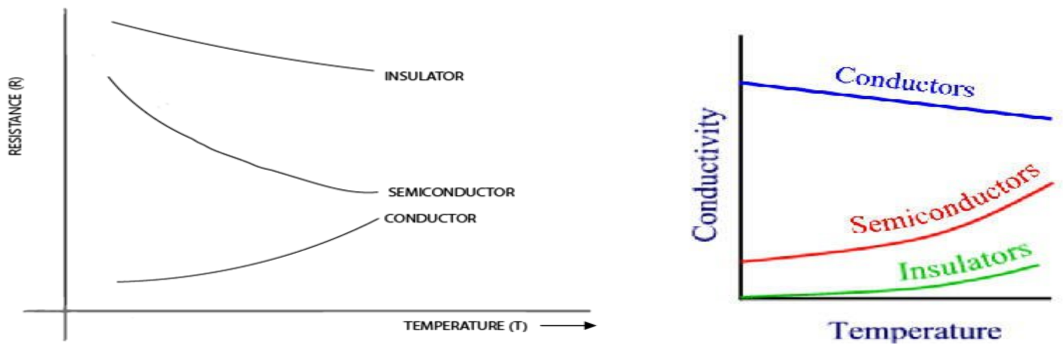

Figure 1: In the left panel plots of resistivity is shown for metal, insulator and semiconductor. In the right panel plots of conductivity(inverse of resistivity) is shown. As we see at low temperature resistivity of metal is minimum and for insulator it is maximum. While for metal resistivity increases as temperature increases, it decreases for semconductor and insulator.

does not flow. Semiconductor lies in between metal and insulator. Silicon, Germanium are known examples of semiconductor. Conductivity is a measure of ability to conduct electricity through a material and resistivity is inverse of conductivity i.e is a measure of resistance the material offers against conduction of electricity through it. In the SI system the unit of conductivity is $\Omega^{-1}m^{-1}$ where Ω is unit of resistance and called Ohm and 'm' is for meter. A comparative values of conductivity for metal, semiconductor and metal are given in Table 1. Apart from this quantitative description of metal, insulator and insulator what is truly interesting to note how the conductivity or resistivity (the inverse of conductivity) varies with temperature. In Fig. 2.2 we show how the conductivity of metal, insulator and semiconductor vary with temperature.

As we observe, the resistivity of a metal increases with temperature which implies that for a given voltage difference between the two ends of the bar (as shown in right panel of Fig. 2.2), current decreases as we increase the temperature. On the other hand for semiconductor and insulator the resistivity decreases with temperature. While at a given temperature the resistivity of semiconductor is much lower than that of insulator. How do we understand this different behavior of conductivity or resistivity of metal, insulator and semiconductor? We note two facts, first conductivity of metal is higher than semiconductor and insulator at low temperature. Secondly the temperature dependence of conductivity or resistivity of metal and semiconductor is different. For a material scientist this is one of the many interesting questions that they aim to solve for. In this article we will try to answer this without much technicality and exposure to formal science background. At the beginning we must know what is the basic microscopic property that determines whether a given material could behave like metal, insulator or semiconductor. We first answer this question. The answer might look very simple, there must be some mechanism in metal which allows an outside electron to enter inside the material and travel through it without much difficulties in comparison to insulator or semiconductor. However temperature dependence of the same mechanism works differently in metal in comparison to insulator and semiconductor. This offers a very prototype basic question that a material scientist wants to answer.

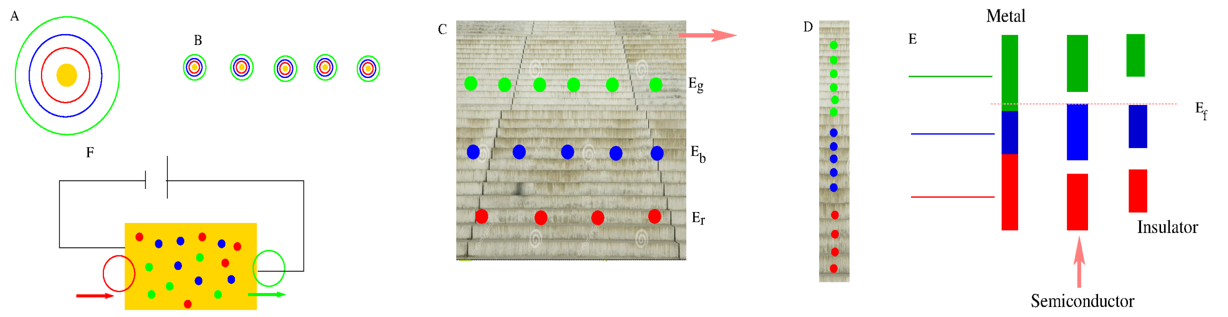

Figure 2: In panel A cartoon of an isolated atom is drawn. Green, blue and red circles schematically show electronic orbitals. The energies of electrons in each orbit are different. In panel B we show many such atoms at infinite distance. A crystal is made by bringing these atoms slowly very close to each other. In panel C we show that the energy of electrons of a given orbit in different atoms are identical and they are placed accordingly at a given step of the stair. Now we imagine that the width of the stairs are decreasing to mimic the formation of a crystal. In extreme limit the electrons would try to sit on each other and this is not allowed. Thus we need to place electrons on different steps vertically which yield each electron to have different energy and forms a band. In panel E, we show the bands of metal, insulator and semiconductor, as indicated. In metal the bands due to green, blue and red orbits overlap. In panel F, a cartoon of a electrical circuit is shown.

2.1 What allows an electron to enter into a metal so easily?

Example of Indian railway system

To understand it we take the example of long distance intercity Indian express train. In such express train there are in general three distinct classes. A sitting class where people can board the train with a ticket which costs a minimal fare. There is slipper class where someone needs a little high amount of money. And finally there are AC class where a substantial amount of money is needed to buy the ticket. As we see buying a sitting class ticket (and getting right to enter into it) is most affordable in comparison to other two classes. Sitting, Slipper and AC department can be thought of like metal, semiconductor and insulator. And money is equivalent to energy required. The microscopic energy distribution among the electrons is such that in a metal if one electron wants to enter into it the electron does not need any energy cost but for semiconductor and insulator there is finite cost. The energy barrier is called "Band gap". Now we will try to understand what the band gap means and how it is formed in a material pedagogically.

Let us first try to understand the process of forming a material from the perspective of many individual atom brought in together. In the beginning they are at large distance from each other. We try to bring them close to each other and form the final state of material where they are few Angstrom (10^{-8}cm) apart from each other. In Fig. 2 A, we show an isolated atom. In the center resides a heavy nucleus which is shown in yellow solid circle. Electrons move around this nucleus in many different orbits which are shown in green, blue and red circles for illustrative purpose. They need not be circles but here for simplicity we assume it as circles. The number of electrons in these orbits are governed by quantum mechanical rule which can be found in modern text book. The energy of the electrons in different orbitals are different, say those are in red, blue and green orbits have energy E_r, E_b and E_g respectively. If such many identical atoms are far from each other, the electrons in each atoms are in identical situations. This means that all the electrons who are in green orbits are having same energy. Same is true for electrons in other orbits.

Now we try to understand what happens when many such atoms are brought in close to each other. To understand this we play a hypothetical game with respect to the Fig 2 C and D. The electrons of different orbits are kept in different steps of the stair as shown in Fig 2 C. The height of the steps denote the energy of the electrons. Initially electrons in a given orbit has same energy and thus kept at a particular step. We try to reduce the width of the stair. This will bring electrons of different atoms closure to each others. In one limit individual orbits tend to overlap with each other. What do we do then? This electrons are needed to be spread over many steps as shown in Fig 2 D. While we spread the electrons vertically electrons are attributed to different energies than their original energy. For example if there are initially $\mathcal{N}_g$ electrons having identical energy E_g, now they are spread into a band where the energy of each green electrons are different such as $E_{g,i}$ and $E_{g,i} \neq E_{g,j}$. The identical thing happen for all the blue and red electrons. This spreading of energy among the electrons is called formation of band. Three quantum mechanical principle works in tandem to determine the individual energy of electrons in a given band and they are listed below.

Pauli exclusion principle, Bloch theorem and Coulomb repulsion

- Pauli exclusion principle: No two electrons can be in identical quantum state. In this naive picture when we try to bring two green electrons closure to each other, their quantum state tends to be identical as they are identical particle. The Pauli exclusion principle says that their energy now be different, i.e they needed to be placed in different steps.
- Coulomb energy: When we bring in electrons closure to each other extra work is needed to overcome the Coulomb energy which is $V(r_1, r_2) = \frac{e^2}{|r_1 - r_2|}$.
- Bloch theorem: Finally the atoms do form a regular periodic arrangement as shown in Fig 2, B. Electrons now roam inside this periodic potential. These three ingredient determine the individual energies of the electrons through quantum mechanical equation.

Now we are in a position to describe what is the basic difference in band structure in metal, semiconductor and insulator.

Table 1: Comparison of conductivity of metal, semiconductor and insulator

A	Typical conductivity $\Omega^{-1}cm^{-1}$	Band gap in eV
Conductor or metal	10^8 to 10^3	$E_g \approx 0$
Semiconductor	10^3 to 10^{-6}	$E_g = 1.1(Si), 0.67(Ge).1.47(GaAs)$
Insulator	10^{-7} to 10^{-24}	$E_g > 4$

Metal, Insulator and Semiconductor

- **Metal :**The band structure for metals are shown in Fig 2, D. The spreading of discrete energy levels such as blue, green and red are such that they overlap and there are continuous energy levels available as shown. This implies that given an energy level E_i, the next energy level available is $E_i+\delta$ where $\delta \to 0$.
- **Semiconductor :** The bands formed due to spreading of discrete atomic energy levels given in green, blue and red color are such that there is a separation between the bands. However this separation is small and of the order of KT where K is Boltzmann constant and T is temperature. The gap between two bands is called Band gap.
- **Insulator :**For this material the band gap is much higher than the semiconductor. A typical estimation of band gap in different materials are shown in Table 1.

Why the above band formation is important for electrical conductivity

We refer Fig. 2 F where potential differences has been applied across a material. Under the effect of electric field electrons move opposite to the direction of field. The electrons are ejected from the right edge and are ready to enter from the left edge. This process of electrons coming out of the material and coming into the material is a non-equilibrium process. At any instantaneous time the number of electron coming out and coming in is not equal. If an electron tries to enter into the material, the nature of band inside plays a vital role in allowing (or not allowing) the electron inside. First criteria an electron needs to satisfy is that its energy must be such that it matches to those allowed by the band inside and deciding factor is how easily the electron can do that. For metal up to some energy level(called Fermi energy and denoted by E_f) the states are taken by some electron already present inside the material. The minimum energy needed for an electron is E_f. This is usually not a problem at all. Because the system allows an electron having energy $E_f+\delta$ as it already allowed an electron with energy E_f. This $\delta \to 0$ can be supplied by a small potential difference. Now consider an insulator. If an electron tries to enter into an insulator, minimum energy it needs $E_f+\Delta$ and Δ is 4-5 electron volts. This additional energy can not be supplied by the temperature alone. The electron must get it from the battery. But it is not convenient because it amounts to large power consumption. Thus the basic difference between metal and insulator is that under the influence of very small electric field, an electron can easily enter into the material and conduct electricity but for insulator it is not so. The semiconductor is in between metal and insulator.

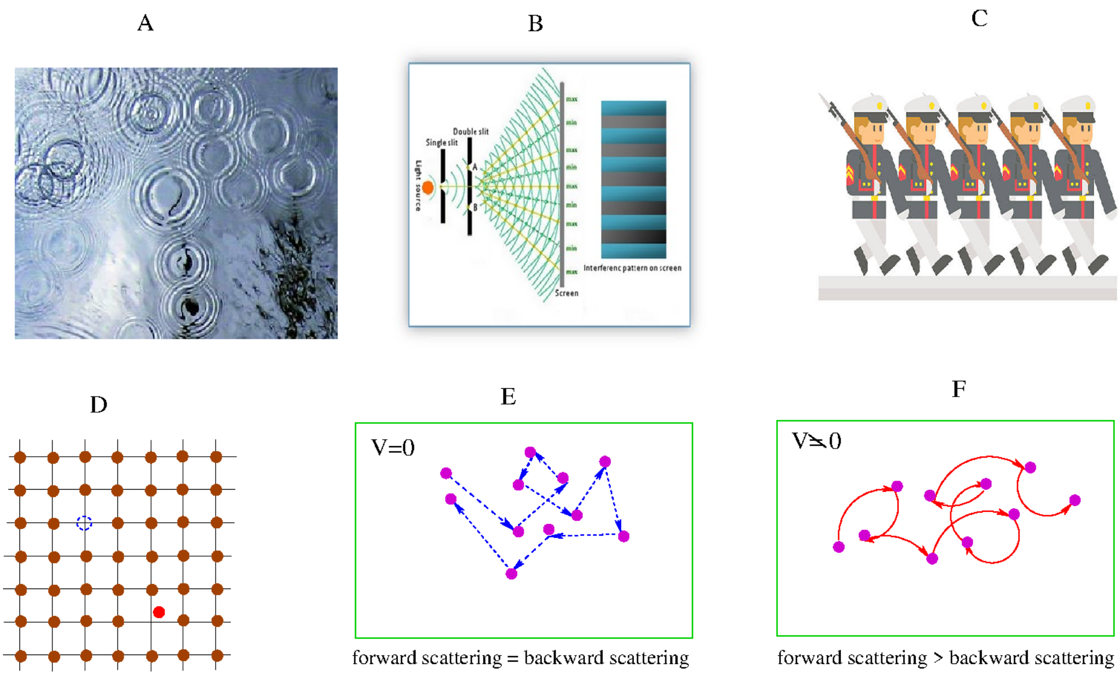

Figure 3: In panel A, interference pattern due to water waves are shown. Whenever two circular water waves meet, a pattern of crest and fall happens. In Panel B interference pattern due to electron wave is shown. Being a quantum mechanical object electron has dual identity such as particle and wave. An alternate pattern of bright and darkness (high density and low density) is made in space. In panel C, a cartoon of disciplined soldiers are shown to mimic the propagation of this interference pattern through the lattice without any collision. In panel D, a lattice with point defect is shown where some atom is either missing or displaced from its designated position. In panel D, we elucidate that in the absence of any electric field or potential difference, electrons undergo forward and back scattering with same probabilities and hence do not move forward. In panel E, we show pictorially how electrons move forward when certain potential difference is applied.

2.2 Basic mechanism of electrical transport

We are almost very close in explaining why the conductivity is very high in metal in comparison to semiconductor and metal. The role of band gap explains how easily an electron can get into the metal. Once they enter into it through one(say negative terminal) end of the material many things can happen before they exist from other end (the positive terminal). First of all, electrons behave like a wave, an extended object, not a tiny particle but like an extended cloud. The size of this cloud is actually bigger in metal than in insulator or semiconductor. On their course of travel from negative terminal to positive terminal, there are possibilities of collision with other electrons and also positive nucleus which are refereed as ions. This gives rise to resistance. In a metal due to the more extended nature of wave function there are more collision but still the conductivity is much higher due to two facts. First the easy access of an electron to get inside a metal. Second the valence electrons (the electrons in the outermost orbit, in Fig 1 A they are in green orbital) which are already there inside a metal are almost free (they come out of their parent atom and roam inside the material freely) and conduct electricity. The total number of free electrons in metal is much higher than semiconductor or insulator. In insulator or semiconductor the electrons are not so free and a bit tightly attached to their original parent nucleus. Due this large number of free electrons in metal conductivity is very high. Let us look at the final formula for conductivity for a metal or semiconductor or insulator in general.

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \tag{1}$$

In the above n is the density of free electron. m is the mass of electron, e is charge of the electron and τ is called mean free time between two successive collision of electrons with ions or other electron. We request the reader to imagine a little why the resistivity is proportional or inversely proportional to a particular parameter that appears in Eq. 1.

A	Number density n	Mean free time τ
Conductor	$2.6 \times 10^{26} m^{-3}$	$300 Angstrom$
Semiconductor	$10 \times 10^{18} m^{-3}$ to $10 \times 10^{23} m^{-3}$	as above

A metal at zero temperature should have zero resistance

We have mentioned before that an electron behaves like an extended object which is called wave. There are many interesting consequences of this. We know that light shows interference and diffraction pattern when suitable obstacles or apertures are kept satisfying certain conditions. The same is true for electrons as well. The interference pattern formed by light is shown in Fig 3, B. A monochromatic light source is kept in front of a single slit barrier. Then another double slit arrangement is kept in front. The monochromatic wave emanated from the single slit is splitted into two different sources of waves. These are called secondary waves. These two secondary waves now superpose in such a way that the amplitude of resultant wave has maxima and minima depending on the location. In the screen the amplitude of wave function is shown. The amplitude has alternate pattern of maxima and minima. This kind of interference pattern is easily observed in a water body if we throw two stones at certain distance. The wave originated from the two stones combine and produce crest and fall as shown in Fig 3 A. Identical things happen here. The regular arrangement of atoms in a material acts as source of secondary waves and these secondary waves interfere and produce a pattern of maxima and minima through out the space. The maxima can be thought of as a new kind of particle and the minima absence of it. These pattern now moves through the lattice without any resistance. To understand this we can think of marching soldiers which are highly trained to maintain a certain distance and regular pattern in their movements and the position of ions cause no barrier to them. Thus in a perfect metal or conductor there should be no resistance at zero temperature. However the experiment does suggest all known metals show a finite resistance at very low temperature until superconductivity sets in. The origin of this resistance is that no material exists in perfect lattice arrangement, there are defect in built. In Fig. 3 D, we have shown such two defects (which are most simple one). In the position "A", the atom is missing and in place "B", the atom is slightly displaced from its original position. These are called point defects and they are the simplest one. There are other much more interesting defects and it is a subject itself. Whenever such defect sites are present, constructive interference pattern does not result in and electrons continue to get scattered incoherently. This results in finite resistance at zero temperature.

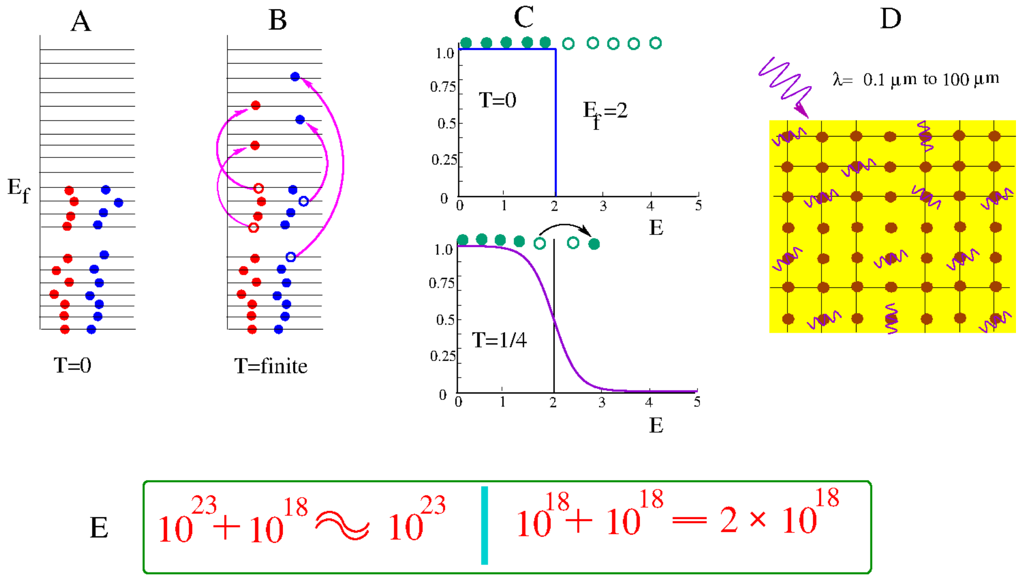

Figure 4: Various key aspects of increase of temperature in a material are shown pictorially. In A we show that at zero temperature all the energy levels up to a maximum energy called E_f are occupied by a pair of electrons. The red and blue filled circles denote up spin and down spin respectively. In B, we show that at finite temperature some of the electrons jump to higher energy levels by absorbing thermal energy. The empty circles denote available energy levels for external electrons to occupy. This transition to higher energy levels follow Fermi distribution $f(E) = 1/(1+e^{\frac{E-E_f}{KT}})$. In C, we have plotted $f(E)$ for T=0 and $KT = 1/4$ in the upper and lower panel respectively. The solid green circles and empty green circles denote occupied and unoccupied energy levels respectively. In D we have shown a cartoon picture depicting the atomic vibrations by absorbing thermal energy. This vibrations are random and incoherent and give rise to resistivity. In E, in the left we have shown that increase in carrier concentration in metal is marginal and does not have any advantage as adding 10^{18} to 10^{23} is still about 10^{23}. On the other hand for semiconductor this increment shows tangible increase in carrier concentration.

3 Temperature dependence of conductivity

Now we discuss the role of temperature on the conductivity. We have already explained why the conductivity is larger in metal than in semiconductor in terms of vanishing band gap and more free electrons or carrier concentration available. But as we increase the temperature the conductivity in a metal decreases and in semiconductor it increases and at large temperature a semiconductor behaves like metal. This intriguing phenomena we now try to explain in a simple way. The first question is when we increase the temperature of the material what happens? And first of all what actually temperature or heat energy means? When we increase the temperature of a given system, we supply heat energy which is a kind of electromagnetic wave such as light or microwave etc. The wavelength of light is around $4 - 7 \times 10^{-7}$ meter. Where as the heat wave has wavelength $0.1 - 100 \times 10^{-3}$ meter.

Light and heat both (and other electromagnetic waves of appropriate energy) interact with electrons and atoms inside a meterial. How this interaction happens is a quantum mechanical process and one of the mystries of nature. Here we discuss effect of the heat energy. When the temperature of the material is increased say by heating or placing it on sunlight thermal energy is supplied to the electrons and ions as well. This thermal energy is expressed as quantum of photon, like tiny energy pills which are absorbed by electrons and ions. Assume that an electron has energy E_i initially. When it absorbs a quantum of energy ϵ_0 it does not immediately jump to the energy level situated at $E_i + \epsilon_0$. If initially there are N electrons at an energy level E_i, only few of them say n_i jumps to the higher energy level by absorbing the thermal energy. Thus there is only a probability to go at higher energy level and as the energy of that level is higher this probability decreases. This fractional probability $f(E) = n_i/N$ is always less than 1 and decrees as E increases. This is shown in Fig 4, A where at zero temperature electrons occupy energy levels up to some energy E_f. At finite temperature some of the electrons try to jump at higher energy level. This process creates some vacant states available below E_f and more electron can enter into the system easily to conduct electricity. The concept of probability is shown in Fig 4 B. In the upper level all the energy states are filled up to E_f. This has been shown by filled green dots. The empty dots represent unoccupied levels and probability of occupying them is zero at zero temperature. In finite temperature some electrons jump to the higher energy level by absorbing thermal energy. The empty states below Fermi energy facilitates more electron to enter into the system and conduct electricity.

But for metal the increment in carrier concentration due to temperature is not that significant as it is in semiconductor. Moreover Pauli exclusion principle also plays a role. In metal there are many electrons and many of them will try jump to higher energy levels however if some of the higher energy levels are already occupied by some electrons, other electrons can not occupy it. Thus though temperature tries to excite more electrons to occupy higher energy level, Pauli exclusion principle prohibits it. For semiconductor the role of Pauli exclusion principle is not that effective and as a result number of free electrons increases in semiconductor. This situation is depicted in Fig 4, C. If we add 10^{18} electrons to an already existing 10^{23} it is still about 10^{23} electrons, this is the case of metal. But for semiconductor it doubles the number of free electrons. The exact form of carrier concentration in semiconductor as a function of temperature is given below.

$$\boxed{n = A_0^{1/2} T^{3/2} e^{-E_{g0}/2KT}} \tag{2}$$

Now there is one more effect of temperature to consider. As the temperature is increased the electrons collide with themselves more frequently. Not only that, ions also vibrate due to thermal energy (as shown in Fig 4 D) and this also leads to more collison or resistance to electrons. This vibrations leads to virtual defect in lattice which works against constructive interference of waves. This fact reduces the mean free path of the electron τ. For metal or semiconductor the temperature dependence of mean free time due to scattering from vibrating ions are $\tau = \frac{\tau_0}{T}$ and $\tau = \tau_0 T^{-1/2}$ respectively. Combining everything the final formula for conductivity for metal and semiconductor is obtained as given below.

$$\boxed{\sigma_m = \frac{ne^2\tau_0}{mT}, \quad \sigma_{sc} = \frac{e^2 A_0^{1/2}}{m} T e^{-E_{g0}/2KT}} \tag{3}$$

In the above σ_m and σ_{sc} refer the conductivity of metal and semiconductor respectively. As we see the conductivity of metal is inversely proportional to square root of temperature implying if temperature increases, conductivity decreases. For semiconductor it is now obvious that conductivity increases with temperature. The reader may wish to plot the functional dependence as shown in Eq. 3. However we note that the above temperature dependence of resistivity is a very simplified one and depending on different scattering mechanism and temperature range we are interested in the final formula would be much more complex.

4 Discussion

In this pedagogical article we have tried to explain one of the many interesting phenomena that basic science generally try to answer. The metal is known for their ability to conduct electricity where as insulators do not conduct electricity. The semiconductor can be defined as bad insulator or bad metal, they are in between metal and insulator. The ability to carry current is known as conductivity. Where as at zero temperature the conductivity of metal is much greater that insulator and semiconductor, the interesting fact is that with temperature the behavior of conductivity of metal is opposite to that of insulator and semiconductor. We have explained in detail why this happens. Along this journey we have also touched upon many fundamental physical concept and phenomena that are relevant such as wave nature of electrons, Pauli exclusion principle, effect of periodic potential, role of interference at zero temperature and Fermi distribution function. All these facts elucidate how complex a physical phenomena such as metalicity is and how much effort and time is needed to decode all the mysteries.

A pertinent question that is always asked what is the usefulness of this study. Metals, insulators and semiconductors are found in all technological applications and it is very important to know the temperature dependence of one of their basic property such as electrical conductivity. Only then it can help optimization of the particular technology. Here we discuss one such applications on which the whole modern electronics and communication technology is dependent. Once metal, insulator and semiconductors are discovered and identified with there different properties with regard to electrical conductivity, the basic science curiosity leads to the question what happens to the junction of a metal and semiconductor which paved the discoveries of diode. Later a semiconductor-metal-semiconductor junction was proposed and this lead the discovery of transistors which revolutionized the electrical and electronics technology. All modern day computers and electronics gadgets have transistors in varying numbers in them.

Study and research on various aspect of metal, insulator and semiconductor are very active today with many fundamental questions as well as many technological applications. This article has no scope at all to discuss all those. However we would like to mention that the curiosity to investigate what happens to resistivity of metal with temperature led to one of the most fascinating phenomena in condensed matter system and that is superconductivity. If one looks at Fig. 1 left panel, the resistivity of metal at low temperature seems to saturate at finite value at $T = 0$. However to their surprise, Prof H. Kamerlingh-Onnes and his students found that resistivity of mercury drops to very low values $R < 10^{-5}\Omega$ at $T = 4.2K$ and a persistent current flows without attenuation. Later it was realized that at very low temperature most metals undergo a phase transition to a new phase called superconductivity. The discovery of superconductivity further led to many insights into fundamental theory and understanding of condensed matter system. Superconductivity today is a very active field of research in theoretical as well as technological application. Probably most important application of superconductivity is in MRI(magnetic resonance imaging) to produce high magnetic field. We have discussed that the transistors are the active component of modern day computer and they are a device made of semiconductor-metal-semiconductor junctions at the basic level. Similar junctions are realized for superconductors which are known as Josephson junction and are superconductor-insulator-superconductor junction. This Josephson junction is at the heart of a quantum circuits which are being actively considered to build quantum computers which if successful may increase the efficiency of computer many many times. This would enable us to solve many challenging problems which can not be solved by most powerful computers available today within reasonable time.

Guide to references:

For introductions to basic band theory and classifications of metal, insulator and semiconductor one may look at [1]. For definition of conductivity see [1, 2]. For temperature dependence of resistivity (or conductivity) for metal refer [3]. For temperature dependence of resistivity of semiconductor see [4]. For introduction to quantum mechanics have a look at [5, 6]. For an introduction to interference effect of light see [7]. For introduction to transistors consult [8, 9]. To have a general idea about superconductivity and its history see [10, 11]. To know how to use superconductivity as a quantum bit refer [13]. To know more about the application of superconductivity in medical science read [14]. The above references are not exhaustive but merely a reflection of what I have gone through at some point of time and found them very useful.

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