Nucleosynthesis in Universe and Stars

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Abstract

In this set of lectures we discuss the formation of various elements in the Universe. We first review some of the basics of nuclear physics which are required to understand the synthesis of nuclei in the Universe. We then show that there are basically three distinct phases or scnarios during which the synthesis of nuclear elements takes place in the Universe. We shall see that the light nuclei, such as ^4He, are formed during the very early in the expanding Universe. The nuclei between helium and iron are formed during the long life of normal stars. Finally, the nuclei beyond iron are formed during a short time duringthe gravitational collapse of a star before it becomes a black hole or a neutron star.

1 Preamble

In nature we find a variety of elements starting from the hydrogen, the lightest element, having one electron and one proton to the heaviest naturally occurring element of uranium which has ninety two electrons and protons and about hundred and fourty two neutrons. As you know each of these elements is identified by its atomic number which gives the number of electrons in the atom of that element. The atomic number of hydrogen is 1 and that of uranium is 92. We also know that the atom of any element consists of a massive nucleus and a number of electrons going around it. Further, the nucleus of an element is composed of a number of protons and neutrons. Protons are charged and neutrons are neutral. The number of protons in a nucleus is equal to the number of electrons or the atomic number. Thus, the atom is charge neutral. We know that the chemical properties of an element are determined by its atomic number but its mass depends on the number of protons and neutrons in the nucleus. We find that although there are only 92 naturally occurring elements, the number of different of nuclides, nuclei having different number of neutrons and protons, occurring in nature is very large. For example, hydrogen comes with zero, one and two neutrons but has only one proton or we have three types of nuclides (or isotopes) having same number of protons and therefore same chemical properties. We find that some nuclei are abundantly found in nature where as some others are rare. For example, hydrogen is the most abundant element and in the universe, almost 75% of nuclei by weight consist of hydrogen having a nucleus consisting of a single proton. Other isotopes of hydrogen are rare. Next in abundance is helium, consisting of two protons and two neutrons, which is 25% in weight. The rest of the elements are comparatively rarer. All the rest of the elements together constitute less than 1%

of the baryon content by weight. Abundance of different elements generally decrease with the increase in their mass number. However it is not a monotonically decreasing function. Fig(1) shows the observed abundances of different elements in Earth's crust relative to silicon. Notice that some elements like oxygen, carbon, silicon, calcium, lead etc are more abundant than the other elements having mass numbers close to these. In Earth's crust abundance of hydrogen and helium is much smaller than what one finds in the universe. This is so because helium is a gas and hence absent. Hydrogen is in form of water and other organic compounds but a lot of hydrogen that was present initially when Earth was formed has escaped from Earth before water was formed.

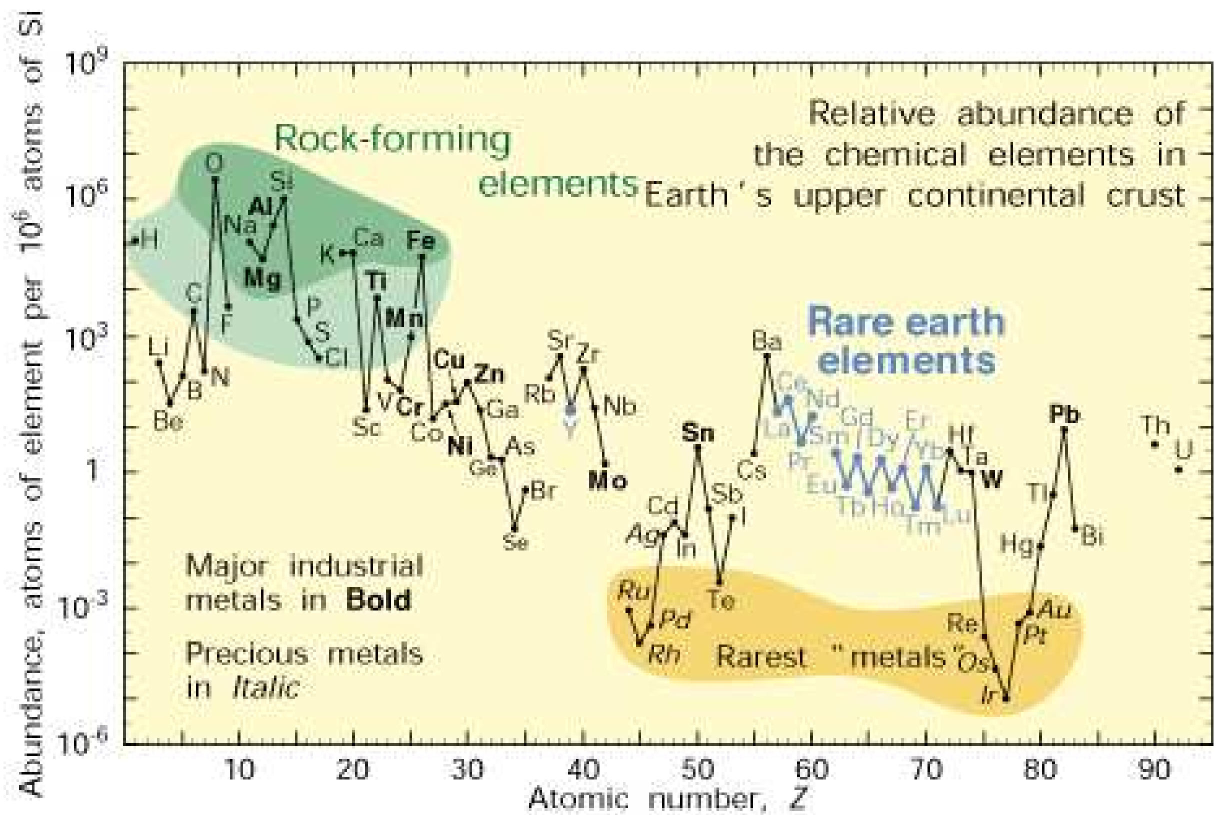

Figure 1: The abundance of various elements in Earth's crust. The abundance of hydrogen and helium is much smaller because helium is gas and hydrogen is in form of water. Notice that some elements like oxygen, calcium, lead etc are more abundant than the neighboring elements. This can be attributed to the shell structure of nuclei.

A natural question that one can ask is, is there a systematic way of understanding the natural abundances of nuclei qualitatively as well as quantitatively? And do we understand the cosmological and astrophysical processes which produce all these elements in nature? Is the current understanding of nuclear physics able to explain these observed facts? And does nuclear physics hellp us in a better understanding of cosmology and astrophysics? We are going to address these questions in a few lectures here. The answer to these questions is, yes, indeed nuclear physics is capable of explaining why we find these elements with the abundances that exist in nature. Not only that, from measured values of the abundances and the knowledge of nuclear physics, one is able to put stringent restrictions on cosmological and astronomical theories. In the following lectures we shall be investigating the formation of the whole zoo of nuclei during different stages of the evolution of the universe. We shall find that there are three main stages or phases during which different species of nuclei get formed. Particularly we shall see that very light elements, namely the deuteron, helium and lithium are formed during the phase when the Universe was only few tens of minutes old in a very short span of time of few minutes. Then the elements up to iron are formed in the 'usual' stars as a by-product of energy production in stars. We shall find that the process of element formation in stars is extremely slow, taking billions of years. Finally, the elements heavier than iron are formed during the violent process of the gravitational collapse of massive stars and it takes a very short time span of few hundredth of a second for cook these heavy elements.

Before we go on to discuss the formation of elements in nature, we need to revise some basic ideas of nuclear physics which are required for the description of nucleosynthesis in Universe. We shall do that first. We also need to have some idea of how the universe evolves when it is young. Fortunately we do not need to discuss that in the lectures here because Prof. Srivastava has already discussed that in his lectures. We shall borrow the required results from his lectures. Finally the mechanism of formation and evolution of stars is also required for the understanding of nucleosynthesis in stars. We shall address to these issues at the appropriate place during the lectures.

1.1 Nuclear Mass Systematics

One of the crucial nuclear physics input required in nucleosynthesis calculations is the systematics of nuclear masses. The masses of nuclei have been measured by a variety of methods and are known to a good accuracy. To a good approximation the mass of a nucleus ${}_NM_Z$ with N neutrons and Z protons (so the mass number of $A = N + Z$) is

$$ {}_NM_Z \approx N \times M_n + Z \times M_p $$

where M_n and M_p are neutron and proton masses. However, the masses differ from this value by a percent or so and that difference is very important. It arises from the binding of nucleons in a nucleus. Thus the correct equation is

$$ {}_NM_Z = N \times M_n + Z \times M_p - B/c^2 \tag{1} $$

Here B is the binding energy defined as the energy released when N neutrons and Z protons come together to form the nucleus and c is the velocity of light. Here we have used mass-energy relation of Einstein. It may be noted in passing that this is the first historically directly measured evidence of Einstein's mass-energy relation[1]. If the nucleus is stable against decay into N neutrons and Z protons, B is positive. One usually looks at the binding energy per nucleon (B/A) for systematics. The plot of B/A as a function of A is displayed in Fig(1.1). From this figure we can deduce the gross behavior of B/A. We find that B/A fluctuates below $A \sim 40$ or so and beyond that it changes smoothly. It first increases with A, attains a broad peak at $A \sim 60$ and then gradually decreases. We can understand this behavior in terms of liquid drop model. That is, the nucleus can be considered as a droplet of a liquid. Therefore the bulk of the binding energy of the nucleus can be expressed as a sum of volume, surface, Coulomb and terms. There are, of course, deviations from this gross prediction of the liquid drop model, particularly noticeable for small values of A and these are called shell effects, arising from the fact that the nucleus is, after all, a quantum system. These can be understood by assuming that nucleons move in a central potential and therefore some nuclei having certain numbers of protons and neutrons are more strongly bound in comparison with nuclei having proton or neutron numbers close to it. This is similar to the fact that the electrons in inert gasses are strongly bound in comparison with other atoms. We shall not go into the details of how the liquid drop model gives rise to such systematic behavior of binding energies and how the shell effects lead to the corrections to the liquid drop mass formula because we shall then be diverting from our main topic. We shall simply say that the peak in the B/A curve arises from the competition of volume, surface and Coulomb terms. But this systematic behavior of binding energies play an important role in the description of the nucleosynthesis in stars.

An important thing that one can infer from the binding energy curve is that when when one fuses two light nuclei the mass of the fused nucleus is smaller than the sum of masses of the fusing nuclei. This means that the fusing of light nuclei is energetically favorable. Further, this mass difference is liberated in form of energy in the fusion process. Similarly when a nucleus with mass number of 240 or more breaks up into two intermediate mass nuclei, the mass of the pair of nuclei is smaller than the mass of breaking nucleus and again the difference in the energy is released in form of energy.

The reason why nuclei nuclei heavier than mass number of 240 or so do not exist is precisely this breaking up or fission process. Crudely speaking, the probability of

[1] Such modification of mass is always there when we have a bound state of particles. For example, when electrons and nucleus come together to form an atom or when atoms come together to form molecules there is binding energy and therefore a reduction in the mass of an atom or a molecule. But the mass equivalent of these chemical and electronic energies are extremely small in comparison with the total mass of the composite. Such small changes are not measurable as a change in the mass.

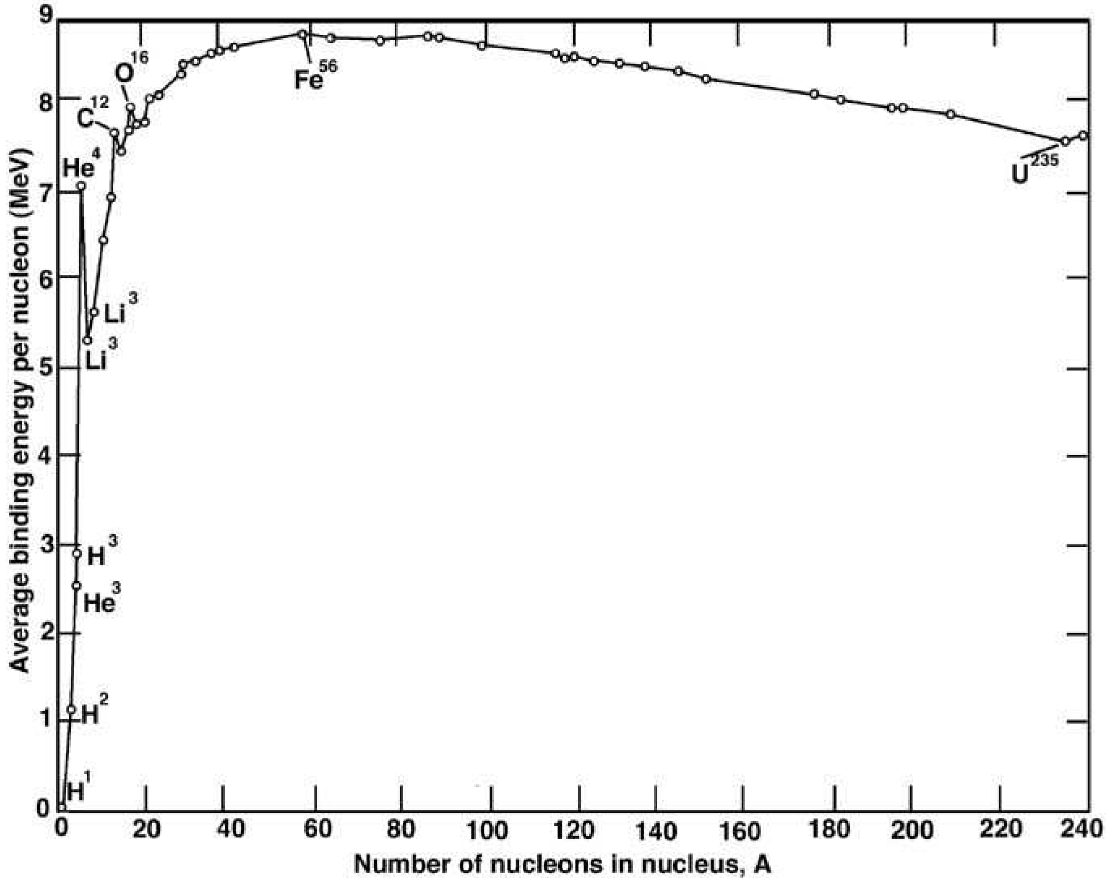

Figure 2: Plot of B/A as a function of A. Notice the prominent departures at small A. Also note that the curve first increases and then gradually decreases when the mass number increases.

the fission process depends on the energy liberated in fission which increases with the mass number. The decrease in B/A with the increase in A means that the energy released increases with A and therefore the decay probability decreases. Beyond a certain A, the decay probability becomes so small that these nuclei break up so quickly that they cannot be found in nature. That is, even if formed, they fission very quickly for us to observe them in the nature.

1.2 Nuclear Reactions

In the Universe the production of nuclei does not take place by bringing its constituent neutrons and protons together in one single step. For one thing, the probability of bringing a number of nucleons together at one time to form a composite nucleus decreases rapidly with the increase in the number of nucleons. So, this probability becomes prohibitively small even for three particles. This process is further inhibited by the Coulomb repulsion between charged protons. In the universe the nuclei are actually built up by adding neutrons and/or protons successively in a multi-step process. So, for nucleosynthesis, we come across the nuclear reactions of the form

$$A + a \rightarrow B + b + Q \tag{2}$$

where a and b are usually light nuclei or a neutron or a proton and A and B could be a light or heavy nucleus. Q is the energy difference between the masses of the nucleon the left and right side of the equation. If Q is positive, sum of the masses on the right is less than that on the left and therefore energy is released during the reaction. Such reactions are called exothermic reaction. When Q is negative, energy is needed for the reaction to proceed and such reactions are called endothermic reactions. By and large endothermic reactions are rare in nature as particles on left do not have sufficiently large kinetic energies for the reaction to occur.

The exothermic nuclear reactions are very important role in the energy production in stars. Because of the large amount of energies liberated in exothermic nuclear reactions, the stars are able generate energy which is eventually radiated light and heat. Most of the stars in the Universe which are in the early stage of evolution are going through fusion reactions converting hydrogen to (predominantly) helium.

Most of the time, even when a nuclear reaction is exothermic, it does not readily occur. This is because it is difficult to bring two nuclei in close contact for the nuclear reaction to take place. Nuclei are charged and at distances larger than sum of their radii, the only interaction between them is the repulsive Coulomb interaction. Thus, a large amount of energy is required to bring them in close contact for the nuclear reaction to proceed. The Coulomb energies required to bring a variety of nuclei together (so that they touch each other), for the nuclear reaction to proceed, are shown in Table(**??**). We find that these energies are enormous for medium weight and heavy nuclei. As a result, the matter needs to be heated to very high temperatures so that the kinetic energies of the nuclei may be sufficient to overcome the Coulomb barrier. This means that the nucleosynthesis predominantly proceeds by interaction of protons or neutrons or some light nuclei like helium with light or heavy nuclei.

1.3 Nuclear Shells

We have already noticed that the binding energy curve of Fig(1.1) departs from the smooth behavior for small A. We have argued that this is because of so-called shell effects. The model which explains these departures is the shell model which assumes that the nucleons in a nucleus move in an average central potential unmindful of rest of the nucleons. This is opposite of the liquid drop model which assumes that the nucleons are interacting strongly with each other. Although we can reconcile these

Table 1: Coulomb energy between two nuclei when they are just touching. The energies are in MeV and equivalent temperature is also shown.

nuclear combination	distance (fm)	energy (MeV)	temperature (^{o} K)
proton + proton	2.00	0.72	0.835×10^{10}
Helium + Helium	3.45	1.68	1.95×10^{10}
proton + oxygen	3.86	2.98	3.42×10^{10}
oxygen + oxygen	5.52	16.70	19.37×10^{10}
proton + calcium	4.86	5.93	6.88×10^{10}
calcium + calcium	7.52	76.60	84.26×10^{10}
proton + lead	7.61	15.52	18.0×10^{10}
lead + lead	13.02	743.67	862.7×10^{10}

apparently contradictory models, we shall not do that here. If we accept that the nucleons are moving in a central potential, one would have states of the potential and the nucleons would be occupying these states. Further, the nucleons are fermions so they will fill up these states starting from the lowest energy state. When a state is completely filled, the nucleons will have to fill the higher energy state, thus giving rise to less binding. The upshot is that there are certain nuclei which are more bound than the nuclei close to their mass number. Although the shell structure is more pronounced at small A, it exists throughout the periodic table. The nuclei which are more strongly bound are ^4He, ^{12}C, ^{16}O, $^{40,48}Ca$, ^{90}Zr and ^{208}Pb.

One of the consequence of relatively larger binding implies that the reactions producing these nuclei have larger Q value when compared with their neighbors. This means that these closed shell, strongly bound nuclei are produced with larger abundance. This detailed behavior is therefore very important in determining the abundances of different species of nuclei.

2 Nucleosynthesis in Early Universe

We now come to the nucleosynthesis in early Universe. We shall see below that in early Universe primarily light nuclei are are produced. We shall see that production of elements heavier than lithium does not occur during the early Universe phase. Before we go on further, let me first try to convince you that it is not possible to produce helium in stars during the later phase of the Universe. First, the measurements suggest that the amount of helium present in the Universe is about 25% by weight. Among different helium isotopes, practically all of it consists of the one having two neutrons and two protons. We shall argue that such large quantities of helium cannot be produced in stars. The argument goes as follows. The present luminosity of the milky way galaxy is about 0.2 ergs/gm/sec. This works out to the release of 0.2×10^{-18}

MeV/nucleon/sec. Assuming that the galactic luminosity has remained same during the whole life of the Universe ($\sim 10^{10} years$) then the energy liberated per nucleon is 0.063 MeV. Note that when four hydrogen nuclei are fused to form a helium nucleus[2], the energy liberated is about 24 MeV or 6 MeV per nucleon. If we further assume that all of this energy is emitted by conversion of hydrogen to helium, the amount of helium produced would be 0.01 or only 1% by weight. It is, of course possible that the luminocity of the galaxy was larger in the past. This is, however, unlikely because, for one thing, we know that, along with helium, heavier elements are also produced during the nucleosynthesis in stars. If 25% of helium is produced in stars, heavier elements would also be produced in larger abundance. This is not observed. Thus we can conclude that the helium observed in the Universe is not produced in stellar process and most of it is produced in the early Universe. We shall in fact assume that all of the observed helium is produced in early Universe.

2.1 The Standard Model of Early Universe

The standard model of the Universe assumes that the Universe began with a big bang at some time in past which can be considered as the origin of time. At the time of the big bang, the Universe was extremely hot and dense (in fact a singularity in space and time). After the big bang, it started expanding and cooled as time progressed. It is a fairly good approximation to assume that initially the contents of the Universe, all the elementary particles, were in thermal equilibrium because the strong, electromagnetic and weak reaction rates were much larger than the expansion rate of the Universe. Prof. Srivastava has described the evolution of early Universe in good details so I won't be repeating the that story again. We shall borrow some of his results required in our discussions. The subject of our interest is nucleosynthesis which occurs when the temperature of the Universe fell below the typical binding energies of nucleons in nuclei. In fact, we shall see that the nucleosynthesis began only after the temperature fell below 10^9 ^{o} K or about 0.1 MeV. Therefore here we are interested in the period when the temperature of the Universe fell below 10^{12} ^{o}K. This is because at higher temperature the constituents of the universe were quarks, gluons and leptons and at about this temperature, the Universe underwent the quark to hadron phase transition and the constituents after this were leptons and hadrons. The subject of quark-hadron phase transition is in itself very interesting but we will not be able to address to that here. Anyway, after this phase transition, the matter in the Universe consisted of electrons, photons, neutrinos and protons and neutrons and all these particles were in thermal equilibrium. Except for the proton and neutron densities, the densities of other particles were essentially determined by the Fermi-Dirac or Bose-Einstein distributions at the temperature of the Universe. The temperature being much larger

[2]Actually, the reaction involves weak interaction in which two protons are converted to neutrons. The reactions are, in brief, $p + p \to d + e^+ + \nu$ followed by $d + d \to^4 He$.

than the electron mass, electrons and positrons were in almost equal abundance.

The proton and neutron densities during this period is an important ingredient for deciding how the nucleosynthesis proceeds. In fact, what is important is these densities relative to the densities of other constituents. We can say several things about these densities. First, the difference between electron and positron densities should equal the proton density since the Universe is charge neutral. Secondly, one can argue that the photon (and neutrino) densities are essentially determined by the temperature of the Universe. This is because at this stage, the photons are in equilibrium with rest of the constituents. In fact, photon density follows a well-defined dependence on the age of the Universe through its temperature, which continues to fall due to expansion of the Universe. In fact, the photons present in the early Universe survive even today as the black body radiation having a temperature of 2.7 Kelvin. We can therefore determine the photon density during the period of nucleosynthesis from the present temperature of the all-pervading black body radiation. Also, the behavior of the sum of neutron and proton densities as a function of the age of the Universe can be determined from the expansion rate of the Universe. From observational cosmology, we have a good understanding of the ratio of matter density (which is basically the baryon density or the sum of proton and neutron densities) and radiation (or photon) density. It turns out that today this ratio is extremely small, $\sim 10^{-10}$. Important thing is, this ratio directly tells us about the neutron and proton densities or the ratio of baryon to photon density during the period of nucleosynthesis in early Universe. We will see that this ratio has a direct bearing on the amount and species of nuclei formed in the early Universe. In any case, the baryon density is negligibly small in comparison to the photon density at present. Further this ratio is small even when the Universe was very young. This means that the computation of nucleosynthesis can be divided into two independent processes, one of the evolution of the Universe and the second of nucleosynthesis. In other words, because of small baryon to photon density ratio one finds that the evolution of the Universe at the early time is essentially independent of what happens to baryons, that is how the nucleosynthesis takes place and how much matter goes into different nuclei.

We shall now consider the evolution of the Universe at the early stage. We shall start from the temperature of 10^{12} ^{o} K or about 100 MeV[3] when the muons have decayed and the Universe consists of photons, electrons, positrons and neutrinos. Their densities are given by Bose-Einstein or Fermi-Dirac distribution functions. Further, the chemical potentials of all these particles can be taken to be zero[4] Thus the energy density in the Universe, which controls the expansion of the Universe at this epoch,

[3] Boltzmann constant $k = 1.15 \times 10^{10}$ ^{0} K/MeV so approximately 10^{10} Kelvin of temperature can be expressed as 1 MeV

[4] Photon chemical potential is zero since photons can be emitted or absorbed in a reaction. Electron chemical potential is very close to zero because of small baryon to photon ratio. Similarly, neutrino chemical potential can also be taken to be zero.

is determined by the light particles, i.e. photons, electrons and neutrinos. This phase is also called as the radiation-dominated phase of the Universe because the particles which decide the expansion of the Universe in this phase are those whose rest masses are small as compared with the temperature. The energy density of the Universe is given by $\rho(T) = CT^4$ where the constant $C =$ is determined from the number of light particles, their degeneracies and whether they are Bosons of Fermions. The dynamics of the Universe is determined by solving the Einstein's field equations, which for the isotropic and homogeneous Universe is

$$\left(\frac{dR(t)}{dt}\right)^2 + k = \frac{8\pi G}{3}\rho(t)R^2(t) \tag{3}$$

and in addition, the energy conservation gives

$$\frac{dp(t)}{dt}R^3(t) = \frac{d}{dt}(R^3(t)(p(t) + \rho(t))) \tag{4}$$

Here $R(t)$ is the length scale, k is the curvature constant which is zero for flat Universe and 1 (-1) for Universe having positive (negative) curvature, $p(t)$ is the pressure and $\rho(t)$ is the energy density. For fluid having relativistic particles, $p(t) = \rho(t)/3$ and therefore the energy conservation equation becomes $\frac{d}{dt}(R^4(t)\rho(t)) = 0$ or $\rho(t) \propto R^{-4}(t)$. This also implies that $R(t) \propto T^{-1}(t)$. With this the Einstein's field equation becomes

$$\left(\frac{dR(t)}{dt}\right)^2 + k = CR^{-2}(t). \tag{5}$$

Further, one can show that $CR^{-2} >> 1$ and therefore we can neglect the curvature constant k on the right hand side and get the solution $R(t) = 2\sqrt{Ct}$. So, to summarize, the length scale of the Universe increases as square root of time and the temperature of the Universe drops as inverse of square root of time. The time and temperature relation as well as the neutron fraction in the Universe is given in Table(2.1).

2.2 Evolution of Neutron Fraction

We now come to the evolution of neutron fraction or the ratio of neutron to baryon density as a function of time. This quantity is important for nucleosynthesis because heavier nuclei are produced by fusing neutrons and protons. The neutron fraction evolves because of the weak interaction between baryons, electrons and neutrinos. The relevant reactions are

$$n + e^+ \leftrightarrow p + \overline{\nu} \tag{6}$$

$$p + e^- \leftrightarrow n + \nu \tag{7}$$

$$n \leftrightarrow p + e^- + \overline{\nu} \tag{8}$$

$$p \leftrightarrow n + e^+ + \nu \tag{9}$$

Table 2: The time dependence of temperature of the Universe and the neutron fraction. The time when the temperature is $10^{12}\ ^{o}$ K has been taken to be zero. Although we have predicted that the temperature decreases as inverse square root of time, there is a small departure from this when the temperature becomes comparable to electron mass. This is because at this stage electron-positron annihilation takes place which heats up the Universe somewhat and therefore cools more slowly.

Temperature(Kelvin)	Temperature (MeV)	time (sec)	neutron fraction X_n
10^{12}		0.0	0.496
3×10^{11}		0.00113	0.448
10^{11}		0.01078	0.462
3×10^{10}		0.1209	0.380
10^{10}		1.103	0.241
3×10^{9}		13.830	0.170
1.3×10^{9}		98.	0.150
1.2×10^{9}		119.	0.147
1.1×10^{9}		148.	0.143
10^{9}		182.	0.137
9×10^{8}		226.	0.131
8×10^{8}		290.	0.123
7×10^{8}		383.	0.112
10^{8}		2080.	0.021

At temperatures above 5×10^9 ^{o}K electrons and positrons are abundant and therefore these weak reaction rates are larger than the expansion rates. This means that the neutrons and protons are in thermal equilibrium and the ratio of neutron to proton densities is $e^{-\Delta m/T}$ where Δm is the neutron - proton mass difference and T is the temperature. So the neutron fraction is $e^{-\delta m/T}/(1 + e^{-\delta m/T} = 1/(1 + e^{\delta m/T})$. Note that $\Delta m = 1.29$ MeV and therefore for $T >> \delta m$, the neutron fraction is ~ 0.5. As the temperature drops, the neutron fraction decreases as $1/(1 + e^{\delta m/T})$. This can continue till the temperature drops to about 5×10^9 ^{o} K or 0.5 MeV. At this stage electrons and positrons annihilate and their densities drop to small values which are comparable to the baryon densities. Because of this the two-body reaction rates of eqs(6,7) become extremely small and the neutron fraction no more given by thermal equilibrium condition. From the Table(2.1) we find that this happens when the Universe is about 10 seconds old.

However, this does not mean that the neutron fraction remains frozen as the neutron β decay is possible. This means that the neutron fraction would continue to decrease and the decrease would be described by the formula $X_n(t) = X_n^0 e^{-\lambda t}$ where λ is the neutron decay constant (which is inverse of mean life) and X_n^0 is the neutron fraction when the neutron decay becomes the dominant mechanism in the evolution of the neutron fraction. To a good approximation X_n^0 is close to the value when the temperature drops to about 3×10^9 ^{0} K, that is, when the Universe is about 10 seconds old. Incidentally, the mean life of neutron is measured to be 886 sec.

For a detailed description of the neutron fraction one needs to do a dynamical calculation including weak reaction rates for all reactions in eqs(6-9), keeping track of electron-positron annihilation and computing the evolution of the temperature of the Universe as a function of time. The evolution of temperature and neutron fraction resulting from o such a calculation is shown in Table(2.1). These values of neutron fraction should now go into the nucleosynthesis calculations.

2.3 Nucleosynthesis

The question now is, when does the formation of nuclei begin and are the nuclei in thermal equilibrium with the contents of the Universe. This crucially depends on the baryon density. Because the baryon density is extremely small, nuclei are built up in two-body processes and not by bringing all the protons and neutrons together in one go. Thus, in order to build up heavier nuclei, deuteron must be formed in sufficiently large quantity so that heavier nuclei can be built up through binary reactions. That is the first step in nucleosynthesis is the reaction

$$p + n \leftrightarrow d + \gamma. \tag{10}$$

The crucial thing in determining the rate of deuteron formation is the deuteron binding energy, which is 2.2 MeV and the baryon to photon ratio. One would think that when the temperature of the Universe falls below the deuteron binding energy, the neutron capture by protons would dominate over deuteron break up by photons and the deuteron density would build up. But that does not happen because background photon density is large and deuterons that are formed are broken up by the high energy photons present in the background. This happens even when the temperature drops below the deuteron binding energy because of small baryon to photon ratio. That is, although the number density of high energy photons decreases exponentially, there are sufficiently large number of high energy photons, even at temperatures smaller than 2.2 MeV, to break up whatever deuterons are formed. Thus, the deuteron density can increase substantially only when the temperature drops below 0.2 MeV or $2 \times 10^0\ ^o$ K.

Once the deuteron density builds up, reactions like

$$\begin{aligned} d + p &\leftrightarrow {}^3He + \gamma && (11)\\ d + n &\leftrightarrow {}^3H + \gamma && (12)\\ d + d &\leftrightarrow {}^4He + \gamma && (13)\\ {}^3H + p &\leftrightarrow {}^4He + \gamma && (14)\\ {}^3He + n &\leftrightarrow {}^4He + \gamma && (15) \end{aligned}$$

begin and the nuclei on the right are rapidly produced. The build up of densities of these nuclei depend on the reaction rates which in turn depend on the binding energies of these nuclei through kinematic factors. The rates are larger if the final state nuclei are strongly bound. Therefore the reaction rates ensure that the densities of strongly bound nuclei built up faster. Now, among these nuclei, ${}^4He$ is the most strongly bound nucleus. The total binding energy of ${}^4He$ is 28 MeV where as that of ${}^3H$ and ${}^3He$ is about 7 MeV and deuteron binding energy is 2.2 MeV. This means that as soon as deuteron density starts building up, neutron and proton capture on deuterons, $3H$ and ${}^3He$ begins and very quickly ${}^4He$ density builds up.

A detailed dynamical calculation which includes the reaction rates of the reactions in eqs(11-15) shows that most of the neutrons which are forming deuterons very quickly end up in forming helium and very little of deuterons and other nuclei survive. However, a lot of protons also survive because the neutron fraction is less than 0.5. This means the amount of helium formed depends crucially on when the nucleosynthesis begins. Interesting thing is that the helium fraction formed during nucleosynthesis in early Universe can be estimated in almost model-independent way and only important input is the baryon to photon ratio. This ratio decides when the nucleosynthesis begins and whatever neutron fraction that exists at that time ends up into helium, thus fixing the helium abundance. For example, if we assume that the nucleosynthesis begins at $10^9\ ^o$ K, all the neutrons at this temperature would end up in helium. From Table(2.1) we find that X_n at this temperature is 0.137. Assuming that all these neutrons go into helium, the helium fraction would be $X_n/2$ or 0.0685. This corresponds to $4 \times 0.0685 = 0.274$ by weight since each helium nucleus has four

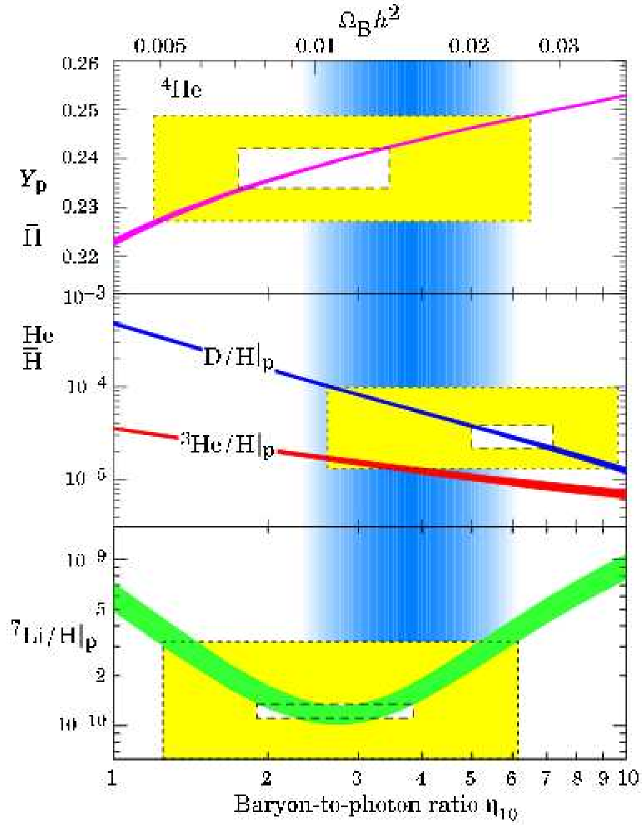

Figure 3: The computed values of nuclear abundance resulting from different ratios of baryon to photon number are shown. The observed abundances are marked by boxes. Note that the helium abundance is largest and deuteron, ^3H and ^7Li abundances are extremely small. The baryon to photon ratio of 3×10^{10} gives the best agreement with the observed values.

nucleons. This number is extremely close to the abundance found in the Universe. Fig(2.3) displays the abundances of light elements computed by assuming different baryon to photon density ratio. The agreement is excellent.

Can heavier elements be produced during the nucleosynthesis in early Universe? It seems the heavier elements are not produced. Question is why. The main difficulty is that the nuclei with mass number 5 and mass number 8 are unstable. That is we don't have ^5He, ^5Li and ^8Be in nature. Because ^4He density is large, the best way to produce heavier elements is by capture of neutron or proton on helium or by fusion of two helium nuclei. But that cannot happen. Only other possibility is the production of heavier elements by three body reactions. For example, ^{12}C can be formed by fusing three helium nuclei. Having three helium nuclei within nuclear volume at the same time is extremely improbable. Actually, ^8Be is a resonant state with somewhat small decay width so two helium nuclei may form this resonance and before it decays, third helium nucleus may interact, forming carbon nucleus. Indeed this happens in stars. But this is very rare in early Universe and the reason is very small baryon density. Thus, absence of stable nuclei with mass numbers 5 and 8 turns out to be the bottleneck for production of heavy nuclei. However, very small amount of ^7Li is formed in early Universe through a reaction $^4He +^3 H \rightarrow^7 Li + \gamma$. The abundance, however, is nine orders of magnitude smaller than that of helium.

2.4 Summary

We have seen that early Universe nucleosynthesis primarily produces light nuclei. We find that the main outcome of this is the large production of primordial helium. It turns out that the observed abundance of helium can be understood in terms of basic nuclear physics and dynamic evolution of the Universe following Einstein's field equations. The present baryon/photon densities put a strong constraint on helium abundance and observed number agrees very well with the theoretical computations. This is, indeed very satisfying.

We also find that heavier nuclei are not produced in early Universe nucleosynthesis. This again can be understood in terms of basic nuclear physics, namely the fact that nuclei with mass numbers 5 and 8 are not stable. We also find that very small amounts of other nuclei, such as deuteron, ^3He and ^7Li are produced and their abundances can also be computed by the same calculations and are in very good agreement with observations.

In summary, one can say that the explanation of helium abundance along with the omnipresent microwave background radiation s constitute a very strong evidence for big bang hypothesis. It is satisfying to see that basics of nuclear physics is able to explain the helium abundance.

3 Nucleosynthesis in Stars

Let us now consider the formation of elements in 'normal' stars. By normal we mean the stars which shine beacuse of nuclear fusion reactions and are quasi stable. That is, the fusion reactions in these stars continue at a relatively slow pace so that the size and temperature of the star remains constant for a large amount of time. We shall find that such stars go through a number of phases and strictly speaking, the size and temperature of the star remains constant while the star is in a particular phase. Generally, the life of the star in each of these phases is large, amounting to millions (if not billions) of years. We shall also see that the elements upto iron are produced by such stars. This is primarily because (as one finds from the binding energy curve of Fig(1.1) fusion of two elements lighter than iron liberates energy which is required for the stability of the star.

3.1 Birth of a Star

As we have seen in the previous lecture, the baryonic matter in the Universe at about 10 minutes after big bang predominantly consists of hydrogen and helium. By weight, about 75 % of the matter is hydrogen and the rest is helium (actually an isotope having two protons and two neutrons). As the Universe expands and cools, the temperature eventually drops below 13 eV, the energy required to ionize hydrogen atom. At that stage, electrons combine with protons and helium nuclei to form neutral atoms and the photons can no more interact with the atoms as they don't have sufficiently large energy to excite the atoms. At this stage, the Universe consists of mixture of hydrogen and helium gasses. The fluctuations in the gasses leads to their condensation and formation of galaxies. Further, due to gravitational attraction, the gasses condense into clouds which keep on shrinking. This process is essentially due to fluctuations which are always present. As a result of this condensation process, we have denser and rarer regions within a galaxy. The effect of gravity is to make make the denser regions more and more dense as time progresses. This increases collisions between atoms in the dense gas and as a result the gas gets heated up.

In principle, this condensation process may continue and after sufficiently large amount of time, we will have a hot and dense object which will continue shrinking and getting hotter. However, this doesn't continue indefinitely because as the object gets heated, there is outward pressure of the hot gas which balances the inward gravitational pull which is trying to compress the matter in the object. This may give a stability to the object. But this stability is not really permanent because this hot object will radiate photons in form of heat or light and loose energy. So the thermal or radiation pressure exerted by the constituents of the object only slows down the collapse. The object contracts, gets hotter, radiates energy even faster which leads to cooling of the object and as the temperature drops, it further shrinks and so on. So what is happening in this process is the gravitational energy that is released during shrinking is converted into thermal energy which is, in turn, radiated in form of photons. So the object keeps on getting hotter and denser as time progresses.

Eventually, when the object is hot enough, the kinetic energies of the atoms in the gas become large enough to overcome the repulsive Coulomb barriers between atoms and nuclear reactions start taking place. The nuclear reactions release large amounts of energy, much much larger than the gravitational energy. Therefore, if the nuclear reactions occur in a controlled fashion, the rate of energy loss by the object is balanced by the rate of nuclear energy production in the object. Then the object's temperature, size etc remain constant in time. Of course this is not really permanent because the fusion reactions would stop when all the light nuclei have fused. The binding energy systematics shows that the binding energy per nucleon is largest for mass number of about 60. That means, the fusion reactions will end up in producing nuclei of mass number of about 60. These are actually the iron nuclei. In other words, when the object has 'burnt' all its nuclear fuel, the light nuclei, it will consist of iron. After that the mighty gravitational force cannot be stopped and the object will continue shrinking[5]

We shall call the object described in the preceeding paragraph a star after the thermonuclear reactions begin and the object becomes quasi-stable. Not all the objects which condense from the clouds of gasses in a galaxy can develop into full-fledged stars. A minimum amount of mass is required for this purpose. For example, Jupiter is a massive object consisting primarily of gaseous hydrogen. But its mass is not large enough to collapse sufficiently and heat up so that hydrogen burning can start. The preceeding paragraph qualitatively describes the evolution of a star in nutshell. There are, ofcourse many more details to be filled up and that is what we shall be doing.

3.2 Hydrogen Burning

Hydrogen burning is the first nuclear reaction to in a star which is essential for controlling the collapse as well as for making stars as bright as they are. There are two reasons for this. One is that the bulk of the material of the proto star consists of hydeogen. So exothermic nuclear reactions involving hydrogen can only produce heat and pressure to stall the collapse. Secondly, as one can see from Table(1.2), the Coulomb barrier between two protons is smallest and therefore the nuclei of two hydrogen atoms can come within the range of nuclear forces at the lowest temperature. The Coulomb barrier between two protons is about 0.7 MeV and this corresponds to a temperature of 8.3×10^9 ^{o} K. That is a very high temperature. Fortunately the matter need not be heated to such high temperatures for the nuclear reactions to get started. Quantum mechanics comes to the rescue of the star. As we know, when the relative kinetic energy between two charged particles is below Coulomb barrier, particles cannot come in contact with each other if classical mechanics holds. But the barrier penetration is possible quantum mechanically. There is a small but nonzero quantum mechanical probability of these two particles penetrating the barrier and coming in contact with each other even when the relative kinetic energy of the two particles is below the Coulomb barrier. This probability decreases exponentially with the increase in the barrier height[6]. Thus, at very low temperatures, the penetration probability is exponentially small but it rapidly increases as the relative kinetic energy is increased. So, in principle, there is some probability of fusion reactions going on at small temperatures (which is same as small relative kinetic energy between two particles) but the rate of these reactions is negligible to have any effect on the

[5] Actually this is not completely correct. We shall discuss some situations when the object actually becomes permanently stable against gravitational collapse later in the lectures.

[6] Actually the difference between the barrier height and the relative kinetic energy of the two particles goes into the calculation of the penetration probability.

heating of the star. At some temperature, the reaction rate becomes sufficiently large to produce sufficient energy to balance the loss of energy due to radiaton. It turns out that this temperature can be three orders of magnitude smaller than the barrier height.

For heating up of the star which is required for maintaining the radiation pressure and stop or control the collapse, we require that the nuclear reactions should be exothermic. The amount of energy released in a reaction is an important criterion for deciding what temperature is maintained in the star. The third important criterion is the reaction rate of nuclear reactions is another crucial factor which decides how hot the star becomes. Here we don't want to go into the details of the energy production in stars but we shall describe the different processes qualitatively.

The first and most important reaction which starts the star on the path of nuclear reactions is the conversion of two protons into a deuteron. The reaction is

$$p + p \rightarrow d + e^{+} + \nu. \tag{16}$$

This reaction involves weak interaction in which a proton is converted into a neutron and is associated with the emission of a positron and a neutrino. Note that proton decay is not allowed by energy conservation as the proton is lighter than the neutron. However, this reaction is followed by the formation of a deuteron. Energy conservation is satisfied as the deuteron has a binding energy of 2.2 MeV which compensates the neutron-proton mass difference. So, the reaction which begins in the star is not a nuclear (strong) interaction but a weak interaction followed by deuteron formation. Because of the small strength of weak interactions, the reaction rate of this reaction is small and therefore this ensures that the hydrogen burning proceeds slowly. Among the by-products of this reaction, the positron quickly gets annihilated with the copious elecrons that are present in the star. The neutrino, however, escapes the star. Detection of these neutrinos provides a direct evidence of the fact that the source of energy production in stars is the nuclear reactions. In passing, we note that such neutrinos produced in Sun are observed on Earth. It has been found that the number of observed neutrinos are smaller by about a factor of two. For some time, it was not clear whether this descripancy is because we don't understand the details of nuclear reactions taking place in a star. It is now understood that this is due to the phenomenon called as neutrino oscillations and these oscillations imply that the neutrinos have a small but nonzero mass.

Once the deuteton concentration starts building up in a star, other nuclear reactions begin. As we have seen from the binding energy curve of Fig(1.1) helium isotope having two neutron has very large per nucleon binding energy and therefore first set of reactions will lead to conversion of four protons into a helium nucleus. Other nuclei will also be formed but these will be in a smaller quantities. The reactions which are involved in this process are

$$
\begin{aligned}
p+p &\rightarrow d+e^{+}+\nu && (17)\\
d+p &\rightarrow {}^{3}He+\gamma && (18)\\
{}^{3}He+^{3}He &\rightarrow {}^{4}He+p+p && (19)
\end{aligned}
$$

Each of these reactions is exothermic, liberating energy. Except for the first reaction, which emits neutrino which escapes the star carrying some energy with it, the energy produced is absorbed in the stellar atmosphere, leading to heating of the star. About 27 MeV of energy converted into heat when each helium nucleus is formed (rest being carried by the neutrino).

Note that the chain of reactions given above produce deuterons as well as ${}^{3}He$ while producing ${}^{4}He$. But because ${}^{4}He$ has largest binding energy per particle, production of ${}^{4}He$ is energetically favored. Because of this large binding energy, most of the deuterons and ${}^{3}He$ produced is converted to ${}^{4}He$ and the amount of deuteron and ${}^{3}He$ is rather small. Predominantly, this chain of reactions produce ${}^{4}He$ and therefore this process is correctly called hydrogen burning and the ashes of the burning are ${}^{4}He$ nuclei.

In addition to these set of reactions, another set of reactions are involved in the conversion of hydrogen to helium. These reactions require the presence of ${}^{1}2C$ nucleus, at least in trace amounts. In this set of reactions carbon nucleus acts as a catalyst and these series of reactions convert carbon to nitrogen to oxygen and back to carbon. Therefore this series of reactions is called CNO cycle and this series is very important in helium conversion at somewhat higher temperatures. These set of reactions are

$$
\begin{aligned}
{}^{12}C+p &\rightarrow {}^{13}N+\gamma && (20)\\
{}^{13}N &\rightarrow^{13} C+e^{+}+\nu && (21)\\
{}^{13}C+p &\rightarrow {}^{14}N+\gamma && (22)\\
{}^{14}N+p &\rightarrow {}^{15}O+\gamma && (23)\\
{}^{15}O &\rightarrow^{15} N+e^{+}+\nu && (24)\\
{}^{15}N+p &\rightarrow {}^{12}C+^{4} He && (25)
\end{aligned}
$$

$$
\begin{aligned}
{}^{15}N+p &\rightarrow {}^{16}O+\gamma && (26)\\
{}^{16}O+p &\rightarrow {}^{17}F+\gamma && (27)\\
{}^{17}F &\rightarrow^{17} O+e^{+}+\nu && (28)\\
{}^{17}O+p &\rightarrow 14C+^{4} He && (29)\\
& && (30)
\end{aligned}
$$

These reactions require higher energy protons when compared to the previous hydrogen burning chain. This is because of the larger charge of carbon, nitrogen and oxygen nuclei. Therefore temperatures of 15 million degrees or larger are required for these reactions to proceed in significant reaction rates. It turns out that the first set of reactions are dominant at temperatures of the order of 10 million degrees or less. The CNO cycle plays an important role at higher temperatures. Further, for the CNO cycle to be active, we require the presence of carbon. We have seen that carbon is not produced in the early Universe. So, the CNO cycle cannot be present in the first generation stars. If, on the other hand, if the heavier elements are produced in the first generation stars and if some of these stars explode, the later stars formed from the debris of these older stars may contain some heavier elements and would have CNO cycle.

The energy produced in hydrogen burning is quite large, 6 to 7 MeV per nucleon. This is slightly less that the binding energy of helium because some of the reactions in these chains are weak reactions in which neutrinos are emitted. These neutrinos escape the star and their energy does not contribute to the heating of the star. Anyway, because of the large energy production the star can maintain a temperature of tens of millions of degrees even when the reaction rates are slow. This also means that the hydrogen burning process continues for a very long time before much of hydrogen in the star is exhausted. This is very important. Had this rate been larger, much of the hydrogen will burn in a shorter time and the stars will die young. In fact, the hydrogen burning life of a star depends on its mass. Heavier stars contract faster and have higher temperatures. Higher temperatures means the hydrogen burning proceeds at a faster rate and these stars exhaust the hydrogen earlier and therefore they have a shorter life. The lighter stars, on the other hand last longer because they use up their fuel more slowly. They can get away by the slow rate of hydrogen burning because the inward gravitation pressure is smaller because of its smaller mass and this inward pressure can be balanced by lower kinetic pressure.

The hydrogen burning does not occur over the entire volume of the star at same pace. The reason is, the interior of the star experiences larger gravitational pressure and for static equilibrium, the interior has to be hotter to generate larger thermal pressure. As a result, the hydrogen burning proceeds at a faster pace in the interior of the star. The star, however, looses heat from the surface, which is cooler. Thus, there is a thermal gradient established from the interior of the star to the surface. And the energy produced in the interior is transported to the surface by conduction and convection. This is important to remember and is useful in understanding the later development of the star.

3.3 Production of Heavier Elements

When the interior of a star has exhausted the hydrogen, the thermonuclear reactions stop. So, as the heat in the interior of the star is conducted to the surface, the thermal pressure drops and the core starts collapsing under gravitational pressure. External shell, however, continues to burn hydrogen. The collapse of the core leads to increasing the density as well as temperature of the core. Beyond a certain stage, the temperature becomes high enough for the helium burning to start. We have already seen that there are no nuclei with 5 or 8 nucleons. So, two helium nuclei cannot form a stable nucleus having four protons and four neutrons. We had argued that this is a bottleneck in producing heavier elements in the early Universe. How is this bottleneck sidestepped in stars? In stars, the reactions proceeds through a quasi three-body reactions. Actually, ^8Be is not stable but there is a narrow resonance in $^4He -^4 He$ system. That is, the scattering cross section for helium-helium scattering has a narrow peak. Actually, a resonance is a short-lived bound system and the width of the sesonance is inversely proportional to the half life of the system. It has been found that the resonance in ^8Be has a half life of about 10^{-16} seconds. So what happens is, two helium nuclei collide to form a shortlived ^8Be nucleus and before this nucleus decays into two helium nuclei, a third helium nucleus collides with it and forms a ^{12}C nucleus. These reactions are

$$
\begin{aligned}
^4He +^4 He &\rightarrow \ ^8Be \qquad (31)\\
^8Be +^4 He &\rightarrow \ ^{12}C + \gamma \qquad (32)
\end{aligned}
$$

These reactions require that the two helium nuclei overcome the larger (compared to two hydrogen nuclei) Coulomb barrier and therefore one requires higher temperature for these reactions to proceed. The required temperature to have sufficiently large reaction rates is about 10^8 ^{o}K where as hydrogen burning requires temperatures of about 3×10^7 ^{o}K. At a slightly higher temperatures production of oxygen through the reaction

$$
^{12}C +^4 He \rightarrow^{16} O \qquad (33)
$$

begins. In this process, the content of carbon and oxygen in the core starts building and helium starts depleting. Although these reactions are exothermic, the amount of energy liberated is small in comparison to the energy liberated in hydrogen burning. As we find from Fig(1.1), the per nucleon binding energy of ^{12}C is about 7.6 MeV in comparison with that of helium being 7 MeV. So, the $0.6 \times 3 = 1.8$ MeV of energy is released when one ^{12}C nucleus is formed. Compare that with helium formation when 28 MeV is released. So, helium burning is not as efficient as hydrogen burning from the point of view of energy production. This means that the material in the star has to burn at a higher rate to maintain the higher temperature of the core. The result is that the helium in the core gets exhausted at a much faster rate.

At this stage, one may ask why the conversion of helium to carbon and oxygen does not occur in early Universe. The reason for this is that the baryon number density in the early Universe is small and the nucleosynthesis phase lasts for a very short time. For carbon nucleus to form, we should have a third helium nucleus reaching the short-lived berilium nucleus. The probability for that is very small because of the

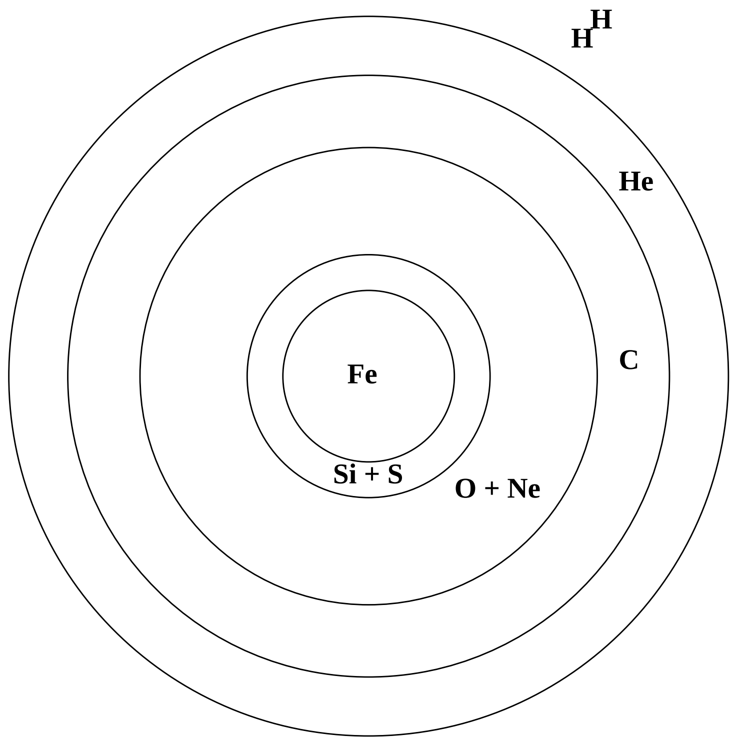

Figure 4: Onion-like structure of the star before the collapse starts.

low baryon density. The stars have advantage that the baryon densities in the core are large and the time available is billions of years and not few seconds.

When the helium gets depleted from the core, the story of collapse and reheating gets repeated again. However, this time, carbon and oxygen burning has to occur for sustaining the higher temperature of the core. Conversion of carbon and oxygen continues and this produces nuclei such as neon, silicone ans sulpher. This further leads to depletion of different fuels in the core. However, the outer layers of the star are going through earlier phases so the star has onion-like structure as shown in Fig(**??**). The final composition of the core is iron. Iron being most stable nucleus having largest per nucleon binding energy, further energy production cannot occur and therefore thermonuclear reactions stop. This means, the compression of the core due to gravitational pressure cannot be stalled by thermonuclear reactions. This also means that the elements that can be formed in the stars are restricted to mass number 60. Heavier elements cannot be formed by thermonuclear processes described in preceeding sections. We can summarize these results as follows.

1. The core of a star is hotter and the thermonuclear reactions proceed at faster rate in the core.

2. As a result, heavier elements are formed in the core of the star.

3. In older and heavier stars, an onion-like structure develops. In such stars different nuclear reactions are occuring in different layers of the star with heavier elements being formed in the inner layers.

4. The time required for burning heavier elements takes shorter time. This is because the energy released in thermonuclear reactions with heavier elements is smaller and these reactions occur at higher temperatures.

5. The thermonuclear reactions stop when iron core is formed. At this stage there is no mechanism for balancing the inward gravitational pressure and core starts collapsing.

6. Outer layers consist of lighter elements and there theromuclear reactions continue.

7. Till a star reaches the stage of the formation of iron core, it has produced elements upto mass number of 60 or so. Heavier elements are not produced until this time. Among the different elements formed, oxygen and carbon have largest fraction. Basically, these are present in the outer layers of the star.

8. Although these elements are formed in a star, there is no mechanism of getting them out of a star. This happens at a later stage in the evolution of a star.

4 'Death' of A Star

In the preceeding section we saw that the evolution of the star leads to the formation of iron core and the thermonuclear reactions stop. Actually, not all stars reach this stage because, for the formation of heavier elements the temperature of the core needs to rise above 10^8 degrees. This cannot happen in lighter stars. The stars having masses of the order of the solar mass and larger go through the processes described above. The later developments of a star takes different paths in evolution depending on their mass. We shall describe these paths. We shall see that in some cases, not only the heavier elements are formed but there is a mechanism for thorwing the outer layers of the star in the Universe. Actually this process is of interest to us here because we want to understand how the elements are formed. It is also important because, without this process we will not have heavy elements present in the Universe at large and on the Earth as well. In other words, without this process, we won't have life and intellegent human beings.

4.1 White Dwarfs

As discussed earlier, when the nuclear burning stops in the core, the core consists of iron. Since energy can no more be generated by thermonuclear reactions, the core cools and starts collapsing under the gravitational pressure. Can there be a mechanism for halting this collapse? It turns out that there is and that is the pressure of electrons. In this discussion so far we have not discussed the role of electrons. The compression as well as heating of the core results in ionisation of atoms. The electrons can not remain bound to nuclei and we have a system consisting of nuclei present in a bath of a gas of electrons. Because of the compression the electron density is very high and even at the temperatures of 10^9 degrees or more, the electrons form a degenerate Fermi gas with a chemical potential or Fermi energy much larger than the temperature of the core. As we know, in a degenerate Fermi gas, electrons occupy states upto Fermi energy and as a result the degenarate Fermi gas has nonzero internal pressure. This is essentially because of the Pauli principle. Because no two Fermions can occupy same state, there is a resistance to the compression of a Fermionic gas. The electrons also have large momenta and energies. There are two consequences of this fact. One is that the degenerate Fermi pressure now plays the same role as thermal pressure and could stop the star from collapsing. The theory of stars which are stabilised by electron degeneracy pressure was first developed by Chandrashekhar. This theory tells that the collapse can be halted by the Fermi degeneracy pressure if the mass of the star is less than about 1.4 solar masses even when the thermonuclear reactions are ceased. This limit is called the Chandrashekhar limit and such stars are called white dwarfs because these are quite small in comparison to the normal, hydrogen burning stars. These stars radiate all their energy, cool down but they cannot collapse further. Without going into the details of how a star evolves into a white dwarf, we will just state that before becoming a white dwarf, the star emits large amount of energy in a very short time. These are the novas or supernovas of type I. Another consequence of large electron density is the large electron Fermi energy. It turns out that this energy could be several MeV's and then it is energetically favourable to convert some of the protons in nuclei to neutrons by absorbing electrons which are at Fermi level. Therefore, such stars produce neutron-rich nuclei by absorbing electrons. This is one process by which nuclei heavier than iron are produced.

The white dwarf is stable for ever because the electron degeneracy pressure does not diminish with time so the collapse is halted for ever. As time progresses, the star radiates and cools and eventually becomes cold. The white dwarf is a compact object having a size of one hundredth of solar radius. One may consider this as the death of a star since it no more shines.

4.2 Neutron Stars, Supernovae and Nucleosynthesis

If a star is more massive than the Chandrashekhar limit, the electron degeneracy pressure is not sufficient to halt the collapse of the core. In such a situation many interesting things happen. We shall be describing these in this subsection. The actual calculations of the process of collapse are very complex and it is difficult to describe the details of the collapse. However, a vivid description of the collapse and subsequent explosion is given in the Scientific American article of Bethe and Brown. I shall be following that article here.

We had argued earlier that the time spent by the core in different phases of burning reduces rapidly. For example, the hydrogen burning takes tens of billion years for a star of the size of Sun and shorter by an order of magnitude for the stars ten times massive. The helium burning gets over in may be a million years or so. As heavier elements start burning, the time spent in burning decreases rapidly. Further, as argued in the preceeding section, the nuclear reaction rates are faster in the interior of the star and therefore the star develops an onion ring structure with the core consisting of inert iron and outer layers consisting of material having lighter elements undergoing thermonuclear reactions. Thus, the layer immediately outside iron core consists of burning silicon and sulphur and bulk of the material is in the outer layer where hydrogen burning is going on. When the mass of the iron core exceeds Chandrashekhar limit, it starts collapsing under gravitational pressure.The detailed dynamics of the collapse is determined by the total mass of the star. What we describe below is valid for stars having mass larger than five times solar mass.

The collapse of a star is a very fast process taking less than a second. During this time, there is not much of a change in the state of the outer layers. So the effect of the outer layers can be ignored during the collapse. These layers play a role later when the collapse has ended in the explosion and when the star ends up in a neutron star or a black hole. Below we shall describe the process of collapse.

1. Dynamically, the collapse process is homologous, that is the speed with which different layers of the core are collapsing is proportional to the radius of the layer. Beyond a certain layer, where the sound velocity in the medium exceeds the collapse velocity, the material is falling with velocity which decreases as radius increases. For our discussion we can consider the center-most layer. What we are describing occurs in the outer layers of the core as well. As the collapse continues, the material in the layer becomes denser and heats up. At this stage, the density of the material plays an important role. The increase in the density leads to an appropriate increase in the Fermi energy and Fermi momentum of the electron gas in the material. We have already seen that at these densities and temperatures electrons cannot remain bound to the nuclei.

2. As the electron Fermi energy becomes larger than neutron-proton mass difference, inverse beta decay begins as nuclei with large neutron numbers become

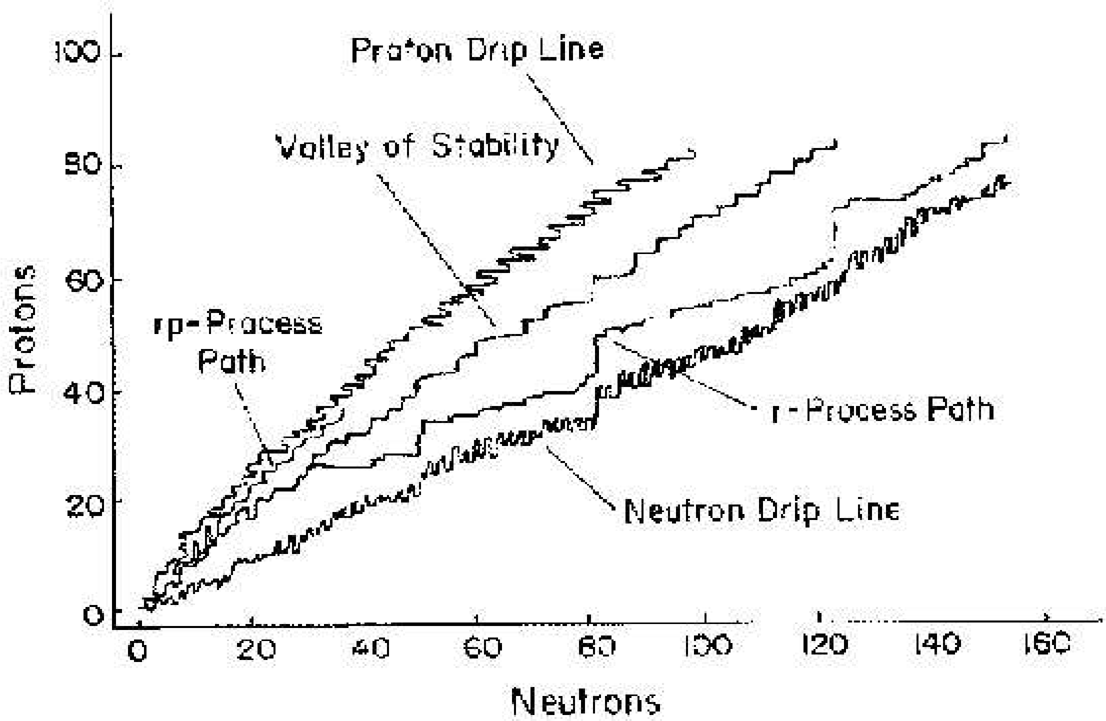

Figure 5: Part of the nuclear chart showing the valley of stability and neutron and proton drip lines. The figure also shows the path followed by r process during nucleosynthesis of heavy elements during the collapes of superheavy core.

stable against beta decay because of occupied electron states of large momenta. This process continues till neutrons can no more be bound in nuclei and a fluid of neutrons appears. These neutrons cannot decay to protons for the same reason. As the collapse continues, the neutron density builds up. But we still have nuclei separated from each other and in a bath of electrons and neutrons.

3. The presence of neutron bath leads to neutron capture as well as beta decay reactions going on at a rapid pace. Because of this, heavier nuclei are cooked in the core during the collapse. For this to happen, nuclei with large neutron excess have to be produced as these generally have small weak decay time. This process of neutron capture followed by beta decay is called rapid or r process. The path followed during the nucleosynthesis of heavy nuclei is shown in Fig(4.2).

4. The nuclear reactions during the r process are indeed rapid and therefore the abundances o nuclei beyond iron are close to what one would expect from the equilibrium distribution. Some of the features of the r process are one finds that

the nuclei near the proton magic numbers (such as Z=40) are produced with larger abundance. This is because for magic proton numbers a large number of isotopes with varying neutron numbers are stable and these nuclei are more strongly bound than other neighbouring nuclei.

5. As the collapse proceeds further, the increase in the matter density means the density of neutrons keeps building up and the nuclei also start coming closer. At some stage the nuclei start touching and at this stage the nuclei essentially fuse into a giant nucleus or nucler matter is formed. This happens when the matter density approaches the density observed in the center of nuclei.

6. When the nuclei fuse to form the nuclear matter, one thing that happens is that the compressibility of the matter increases by orders of magnitude. This is essentially because of the repulsive nucleon-nucleon interaction at short distances. This sudden increase in the compressibility when the nuclear matter is formed gives rise to an outward moving shock in the nuclear matter.

7. All this while the matter in outer layers of the core is falling inwards. When it meets the outward moving shock it is heated and expelled from the star. Thus the star explodes, throwing most of the matter in the outer layers of the core as well as the outer layers where thermonuclear reactions are going on. This is the birth of a supernova.

The matter thrown in supernova contains all the nuclei formed during the life of the star. It primarily consists of hydrogen from outermost layer but also contains helium and heavier nuclei (all the way up to Uranium and beyond). Besides the matter expelled from the star, the supernova is associated with emission of large amount of energy during a very short time. This energy essentially comes from the energy contained in the shock wave produced when nuclear matter is formed in the core. This energy is emitted in form of light. The intensity of the radiation is so strong that the star suddenly becomes bright by several orders of magnitude. Hence the name nova which means new star. The light intensity, however, rapidly decreases as the hot gas expelled from the star expands and cools.

In addition to the light energy, the supernova explosion is also associated with emission of large flux of neutrinos. These neutrinos are produced during the collapse of the cor when heavy elements are being cooked. The neutrinos are also produced during the cooling of the interior of the newly formed star in the center because, it turns out that the most efficient method of cooling a such high densities is by neutrino emission. It has been found that these neutrinos also supply additional energy to the shock and help the star in exploding.

The remnant of the star at the center consists of very dense nuclear matter. Actually, the matter predominantly consists of neutrons (about 90 percent) and the rest being protons. Hence it is called neutron star. The question at this stage is whether this configuration of the star is now stable against gravitational collapse or the collapse continues further. It turns out that, as in case of white dwarfs, degeneracy pressure helps in stalling the collapse of neutron stars as well. But here it is the neutron degeneracy pressure. The basic idea is same but there are two major differences in determining the stability of neutron stars. One is that these stars are so compact that the general relativity effects cannot be neglected in determining the stability. Second is that because nucleons are strongly interacting particles, the equation of state that goes in the calculation is uncertain because of the ambiguities in the nucleon-nucleon interactions. It turns out that, just like white dwarfs, neutron stars beyond certain mass cannot be supported by neutron degeneracy pressure and they continue the collapse. This limiting mass depends on the equation of state of nuclear matter but most of the reasonable equations of state give the limiting mass of two solar masses or smaller.

If the mass of the remnant exceeds the limiting mass, it continues the collapse and ends up into a black hole. So, the core of the supernova is a neutron star or a black hole.

5 Conclusions

In these series of lectures we have seen that the elements that we see in the Universe are formed in the various stages of the evolution of the Universe. We have seen that the result of the big bang is a Universe which has very little matter consisting of baryons and initially the baryons are in the form of neutrons and protons. Very early in the life of the Universe when the life of the Universe is of the order of few minutes, production of nuclei from neutrons and protons begins. The elements formed in this epoch are primarily ^4He with traces of deuterium, tritium and ^7Li. Heavier elements are not formed in this epoch. The amount of ^4He formed during this phase crucially depends on the ratio of baryon and photon densities.

Thermonuclear reactions in stars produce nuclei heavier than helium. 'Normal' stars, however produce nuclei upto mass number 60. Heavier nuclei cannot be produced in stars which generate energy by thermonuclear reactions. Production of these nuclei goes for bilions of year in stars.

Heavier nuclei are produced during the gravitational collapse of heavy stars. These nuclei are produced by (n, γ) reactions followed by beta decay. The collapse of a star is a very fast process, taking less than a second and all the nuclei beyond iron are produced in such a short time.

The collapse of a heavy star is followed by explosion of the star when the mantle of the star is blown out. By this process the elements beyond helium, which are formed in a star during its life, are blown into the galaxy.

6 Further Reading

1. S. Weinberg, 'First Three Minutes'. This book gives a vivid description of nucleosynthesis in early Universe.

2. 'Astrophysical Concepts'. This is one of the textbooks for study of astrophysics. It has one chapter where evolution of elements in stars is described.

3. H. A. Bethe and G. E. Brown, Scientific American, 252, 40(1985). This article gives a beautiful description of the collapse of a massive star and supernova production.

4. E. M. Burbridge, G. R. Burbridge, G. R. Fowler and F. Hoyle, Rev. Mod. Phys. 29, 547(1957). This article gives a detailed account of nucleosynthesis in stars.