QFT: Problem Set 5

Instructor: Sudipta Mukherji

1. Starting from

$$Z[F] \sim \int dq\ e^{\frac{i}{\hbar}\int dt L},\ \text{with } L = \Big(\frac{dq}{dt}\Big)^2 - \frac{1}{2}\omega^2 q^2 - F(t)q, \tag{1}$$

show that

$$Z[F] \sim Z[F=0] e^{\frac{i}{2\hbar}\int dt dt' F(t) D(t-t') F(t')}. \tag{2}$$

Work out explicit form of $D(t-t')$.

2. Consider a real symmetric 2×2 matrix A_{ij} and x as a vector $x_i, (i, j = 1, 2)$.

Show that

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2\ e^{-\frac{1}{2}x.A.x + J.x} = \sqrt{\frac{(2\pi)^2}{\det\ A}} e^{J.A^{-1}.J}. \tag{3}$$

Now argue that for real symmetric $N \times N$ matrix A_{ij} the above result generalises to

$$\int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} dx_1 ... dx_N\ e^{-\frac{1}{2}x.A.x + J.x} = \sqrt{\frac{(2\pi)^N}{\det\ A}} e^{J.A^{-1}.J}. \tag{4}$$

3. Consider Euclidean path integral in one-dimension

$$Z = \int \mathcal{D}\phi\ e^{-S} \text{ with } S = \int dx\ [\frac{1}{2}\partial_x\phi\partial_x\phi + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4] \tag{5}$$

Now consider discretising x direction into a lattice with unit lattice spacing. Introduce two new constants α and γ such that

$$\gamma = \frac{\lambda\alpha^2}{4},\ \alpha = \frac{2-4\gamma}{m+2}. \tag{6}$$

Show that the path integral can be written, upto an overall normalisation constant, as

$$Z = \int \prod_x \mathcal{D}\phi_x e^{\alpha \sum_x \psi_{x+1}\psi_x + \sum_x (-\psi_x^2 - \gamma(\psi_x^2 - 1)^2)}, \tag{7}$$

where $\psi_x = \alpha^{-\frac{1}{2}}\phi_x$. Now, plot

$$e^{(-\psi_x^2 - \gamma(\psi_x^2 - 1)^2)} \tag{8}$$

as a function of ψ_x for different γ and argue that in the large γ limit, the path integral is dominated by $\psi_x = \pm 1$ (Ising model in one dimension with neraest neighbour coupling $\sim \alpha$!)