QFT: Problem Set 4

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1. The Lagrangian for harmonic oscillator is

$$L = \frac{1}{2}m\Big(\frac{dx}{dt}\Big)^2 - \frac{1}{2}m\omega^2 x^2. \tag{1}$$

The propagator can be written as

$$U(x_2, t_2; x_1, t_1) = A(t_2 - t_1)e^{\frac{i}{\hbar}S_{cl}}, \tag{2}$$

where S_{cl} was calculated in the class.

Now using the fact that U satisfies time-dependent Schröedinger equation or using

$$U(x_2, t_2; x_1, t_1) = \int U(x_2, t_2; x', t')U(x', t'; x_1, t_1)dx', \tag{3}$$

show that

$$A(t_2 - t_1) = \sqrt{\frac{m\omega}{2\pi i\hbar \sin\omega(t_2 - t_1)}}. \tag{4}$$

2. The propagator satisfies

$$\psi(x', t') = \int U(x', t'; x, t)\psi(x, t)dx. \tag{5}$$

Verify this for harmonic oscillator with $\psi(x, t)$ as the ground state wave function

$$\psi(x, t) = \Big(\frac{m\omega}{\pi\hbar}\Big)^{\frac{1}{4}} \exp\Big(-\frac{m\omega}{2\hbar}x^2 - \frac{i}{2}\omega t\Big). \tag{6}$$