QFT: Problem Set 3

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1. Consider scalar field Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2. \tag{1}$$

Keeping Lorentz invariance in mind we could have also chosen a Lagrangian density

$$\mathcal{L} = a\partial_\mu\phi\partial^\mu\phi + b\partial_\mu\partial^\mu\phi + c\phi\partial^\mu\partial_\mu\phi + d\phi^2 + e\phi, \tag{2}$$

where a, b, c, d, e are all arbitrary constants. Assuming m^2 and d are not equal to zero, show that both the Lagrangian densities are equivalent to each other.

2. Lagrangian density for Maxwell field is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu. \tag{3}$$

Via equations of motion show that $\partial_\mu j^\mu = 0$. Further show that the current here is conserved since we insist invariance of the action under gauge transformation.

3. Assume the Lagrangian density of an interacting scalar field is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \lambda\phi^4. \tag{4}$$

The action then has a symmetry under

$$x \to bx, \phi \to b^{-1}\phi, \tag{5}$$

where b is a constant. Find the conserved current.

4. Suppose the action of certain field theory of field Φ is invariant under space-time translation as

well as dilatation $x \to bx, \Phi \to \Phi$. Show that the energy-momentum tensor in this case satisfies $T^{\mu}_{\mu} = 0$.

5. Consider complex scalar field with Lagrangian density

$$\mathcal{L} = \partial_\mu \phi \partial \phi^\dagger - m^2 \phi^\dagger \phi. \tag{6}$$

Under space-time translation $x^\mu \to x^\mu + a^\mu$ with $\phi \to \phi$, the Lagrangian density changes by a total derivative. Find the conserved currents.