QFT: Problem Set 1

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1. *Estimation*

A particle is in one dimensional potential given by

$$V = V_0 e^{\frac{ax^2}{2}}, \tag{1}$$

where V_0 is a constant.

Estimate the energy of the two lowest eigen states. What are the parities of these states?

2. *Love thy neighbour!*

In class, we have seen that for a linear one dimensional chain with nearest neighbour interaction, the dispersion relation in the small $|\mathbf{k}|$ limit goes as

$$\omega \sim |\mathbf{k}|. \tag{2}$$

We would like to see that this behaviour remains unchanged as long as interactions have finite range.

Consider the Lagrangian (following class notations):

$$L = \sum_{I=1}^{N} \Big[\frac{1}{2} M \dot{R}_I^2 - \sum_{m=1}^{r} \frac{1}{2} K_m (R_{I+m} - R_I)^2\Big]. \tag{3}$$

(a) Show that the dispersion relation is now given by

$$\omega = 2\sqrt{\sum_{m=1}^{r} K_m \frac{\sin^2(mka/2)}{M}} \tag{4}$$

(b) Show that in the long-wavelength limit the dispersion relation reduces to

$$\omega = a\Big(\sum_{m=1}^{r} m^2 K_m / M\Big)^{\frac{1}{2}} |\mathbf{k}|. \tag{5}$$

(c) Comment on the case where $N, r \to \infty$ and $K_m = 1/m^p$ (for $1 < p < 3$).