LECTURE - 8 16:30 - 18:00

QUANTUM CORRELATIONS

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1935 :- Einstein, Podolski, Rosen (EPR Paradox)

1952 :- EPR Interms of Bohm's Spin $\frac{1}{2}$ Representation $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$

1957 :- Gleason's Theorem

1964 :- Bell's Paper $\longrightarrow$ Local HVT's incompatible with Quantum Mechanics

1968 : Kochen Specker $\rightarrow \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{Bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{Bmatrix}$ Now assume $A_k = B_\ell$ for some k & ℓ

$\downarrow$ (some observable common to both sets)

1993 RMP Mermin

$[A_k, A_l] = [B_k, B_l] = 0$

$\downarrow$

With 117 vectors $\rightarrow$ This will give you a contradiction.

$\downarrow$

Where's the rub? Value of A_k = Value of B_ℓ always (assumed)

Contextuality!

Moral :- Hidden-Variables must not be non-contextual.

Common to all these things $\Rightarrow$ Composite Quantum Systems

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

$|\psi\rangle_{ABC} = |\chi\rangle_A \otimes |\xi\rangle_B \otimes |\eta\rangle_C$ is the one possibility.

Now linearity implies $|\chi_1\rangle \otimes |\xi_1\rangle \otimes |\eta_1\rangle + |\chi_2\rangle \otimes |\xi_2\rangle \otimes |\eta_2\rangle$ is also legit state

But this is $\leftarrow$ Entangled in general

ENTANGLEMENT

If $\rho_{ABC} = \sum_i \omega_i \rho_i^A \otimes \rho_i^B \otimes \rho_i^C$ → Then Separable ; Otherwise Entangled
$(0 \leq \omega_i \leq 1 \;\&\; \sum_i \omega_i = 1)$

Physical Realization → Separable States canbe prepared locally

FOR BIPARTITE PURE STATES

$|\psi\rangle_{AB} = \sum_{i,j} \alpha_{ij} |i\rangle_A \otimes |j\rangle_B$ → from Singular Value Decomposition Theorem one can always write this as $= \sum_{K=1}^{N} |k_A\rangle \otimes |k_B\rangle$ in some other basis → (Schmidt Decomposition)

If $N=1$ → Separable

But for multipartite states → no unique Schmidt Decomposition → Can't be done

Bipartite Systems:- Q> Is it Entangled ?
Q> If yes - How much Entanglement ?

Entanglement Quantification interms of Teleportation Protocol

Compare States in terms of their Entanglement ... → Take Singlet (Original Benett Protocol) → 100% Exact Teleportation ; But take some other initial state → inexact teleportation

In terms of fidelity wrt target

Suppose we have a state like $a|00\rangle + b|11\rangle$ ..... allowed operation = LOCC

Under LOCC → $\underbrace{\rho^{\otimes n}}_{\text{Less Entangled}} \longrightarrow \underbrace{\sigma^{\otimes m}}_{\text{highly entangled}}$ $[m < n]$ Entanglement concentration $m = n\,(S(\rho_A))$ → Benett et al ...

Schumacher Noiseless Data Compression Theorem → Allows the reverse process

$\Rightarrow S(\rho_A) =$ Entanglement of a pure bipartite state → Easily Calculable

What about mixed Bipartite States ?

Two Defns → 1) Distillable Entanglement
2) Entanglement of Formation

$\}$ $\neq$ for mixed states

$EOF(\rho_{AB}) = \inf \sum_i p_i E(|\psi_i\rangle_{AB})$ over all pure state decompositions $\rho_{AB} = \sum_i p_i |\psi^i_{AB}\rangle\langle\psi^i_{AB}|$

Distillable Entanglement $= \lim_{n\to\infty} \frac{m}{n}$

But these optimizations are hard to do ☹

PROPERTIES A GOOD MEASURE OF ENTANGLEMENT MUST SATISFY

- Vanishes for Separable State
- LU-invariant
- Monotone decreasing under LOCC
- Additivity, Convexity, Continuity → NOT Necessary but desirable (like Sixpack abs ☺)

Hastings → EoF is not additive (Recent Result)

One good candidate with all those properties → Squashed Entanglement (Winter)
↓
But notoriously hard to calculate (Winter ☺)

LOG-NEGATIVITY / CONCURRENCE (2x2) → Easy to Calculate

LOG-NEGATIVITY ↓ Easy to Calculate
$\sim \log|N|$
N = Negativity (under Partial Transpose)
↓
Additive but not convex (Plenio)

CONCURRENCE (2x2) ↓ EoF = Simple fn of Concurrence

Squashed Entanglement → Depends on Quantum Mutual Information ... So Good for Chann-els

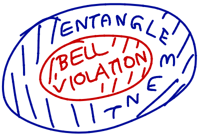
DETECTION OF ENTANGLEMENT

Prob:- Given ρ_{AB} is it entangled / separable ?

Initially people thought → Bell Violation ≡ Entanglement.

But for Werner States $p|\varphi^-\rangle\langle\varphi^-| + \frac{(1-p)}{4}\mathbb{1}_4$ → if $p > \frac{1}{3}$ → Entangled

Possible to have no Bell Violation even with Entanglement — But $p > 0.707 \ldots$ → Bell Violation



PARTIAL TRANSPOSE

If you have an operator which is +ve but not CP → Will Serve as a detector
(e.g. Transpose Operator) (Hahn-Banach)

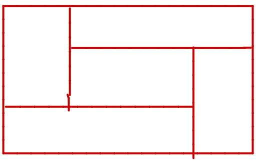
$\breve{\rho}_A = (\rho_{AB})^{T_B} \equiv$ Partial Transpose on B
$\breve{\rho}_B = (\rho_{AB})^{T_A} =$ Partial Transpose on A

If all eigenvalues of $\breve{\rho}_A$ are ≥ 0 → PPT states
If at least one -ve eigenvalue → NPT states → Entangled for sure

But does PPT ⇒ Separable ?

2×2, 2×3 → true
for higher dimensional states → False → ∃ Bound Entangled State (Entangled but not distillable)

Example of a PPT Entangled State in 3×3 systems



$|0\rangle\otimes\frac{|1\rangle\pm|2\rangle}{\sqrt{2}}$ $\quad\frac{|0\rangle\pm|2\rangle}{\sqrt{2}}\otimes|2\rangle$

$|2\rangle\otimes\frac{|0\rangle\pm|1\rangle}{\sqrt{2}}$ $\quad|1\rangle\otimes|1\rangle$

$\frac{|0\rangle\pm|1\rangle}{\sqrt{2}}\otimes|0\rangle$

→ Unextendible Product Basis ⇓ Implies Existence of Bound Entangled States

Here $\rho = [\mathbb{1} - \sum_{i=1}^{9}|\varphi_i\rangle\langle\varphi_i|]$ ⋯→ Bound Entanglement

EoF > 0 But Distillable Entanglement = 0

Reduction Criteria Violation ⟹ Distillable

$U\otimes U^*$ invariant states $\equiv \mathbb{1} + \beta P^+$ [Isotropic States] where $P^+_{d-1} = \frac{\sum_{i=0}^{d-1}|ii\rangle}{\sqrt{d}}$

Either Separable / Distillable

Definite form of Entanglement available

$U\otimes U$ invariant states $\equiv \mathbb{1} + \beta V$ [Werner States] → Satisfies Reduction Criteria → Can be Bound Entangled

$\rho_{AB} \rightarrow$ if you find $|\psi\rangle$ of rank=2 and $\langle\psi|\underbrace{\rho_{AB}^{T_A}}_{\text{(Partial Transpose)}}|\psi\rangle < 0 \rightarrow$ Distillable

$> 0 \rightarrow$ One copy undistillable

⋮

Similarly n-copy undistillability

Tomorrow :- Multipartite Entanglement, Non Classical Correlations Beyond Entanglement