Trade-off relation between generalized which-way information and fringe visibility

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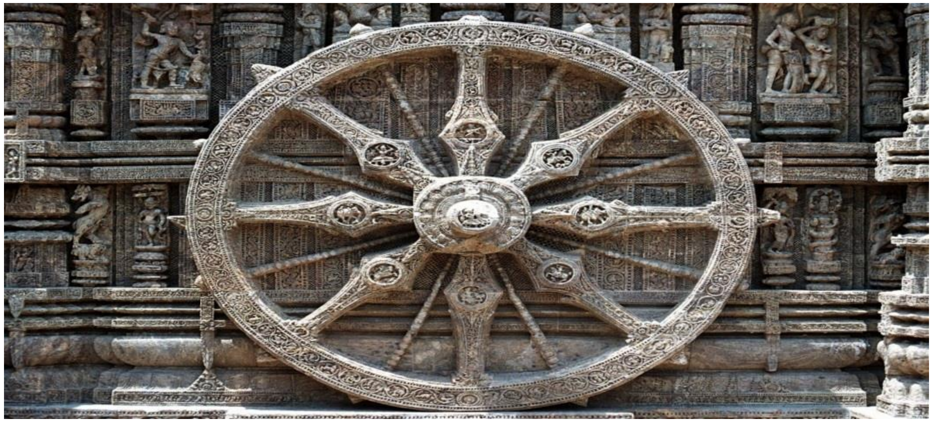
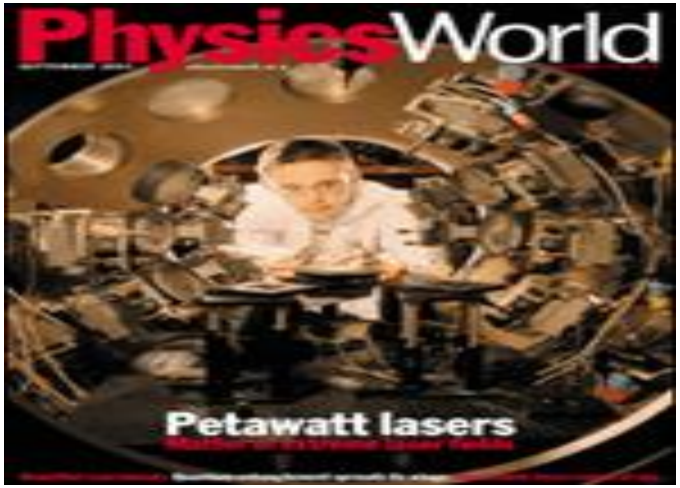


iscqi

IOP, Bhubaneswar
February 9-18, 2016

Einstein and Bohr debated over quantum theory for years, and never agreed. The debates represent one of the highest points of scientific research in the first half of the twentieth century because it called attention to quirky elements of quantum theory, complementarity, non-locality and entanglement, which are central to the modern quantum information science.



The most beautiful experiment

Sep 1, 2002

The most beautiful experiment in physics, according to a poll of *Physics World* readers, is the interference of single electrons in a Young's double slit.

Which is the most beautiful experiment in physics according to you?

This question was asked to Physics World readers - and more than 200 replied. Majority vote was for the classic experiments by Galileo, Millikan, Newton and Thomas Young. But uniquely among the top 10, Young's double-slit experiment applied to the interference of single electrons remained as one of the most beautiful experiments in physics.

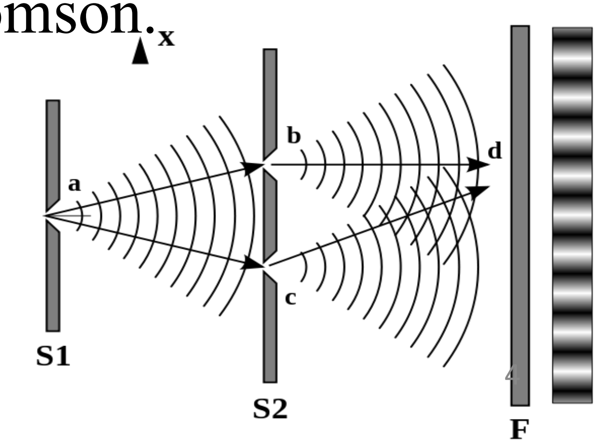
Wave or particle?

- First decade of 1800: Young – Double slit interference.
- 1909: Geoffrey Ingram (G I) Taylor – Interference with feeblest light (equiuivalent to "a candle burning at a distance slightly exceeding a mile“) leads to interference.

---- Dirac’s famous statement “each photon interferes with itself”

1927: Clinton Davisson and Lester Germer -- Diffraction of electrons from Nickel crystal – wave nature of particles (electrons) -- 1937 Nobel prize for the "discovery of the interference phenomena arising when crystals are exposed to electronic beams“ along with G. P. Thomson.

Thomas Young's sketch of two-slit interference based on observations of water waves.



Experiments on the Diffraction of Cathode Rays.

By G. P. Thomson, M.A., Fellow of Corpus Christi College and Professor of Natural Philosophy in the University of Aberdeen.

(Communicated by Sir Joseph Thomson, F.R.S.—Received November 4, 1927.)

[Plate 19.]

1. M. L. de Broglie has introduced a theory of mechanics according to which a moving particle behaves as a group of waves whose velocity and wave-length are governed by the speed and mass of the particle. In fact if m_0 is the mass for slow speed and v the speed of a freely moving particle, the wave-length is given by $\lambda = h\sqrt{1 - v^2/c^2}/m_0 v$, and the wave velocity V by $V = c^2/v$, the

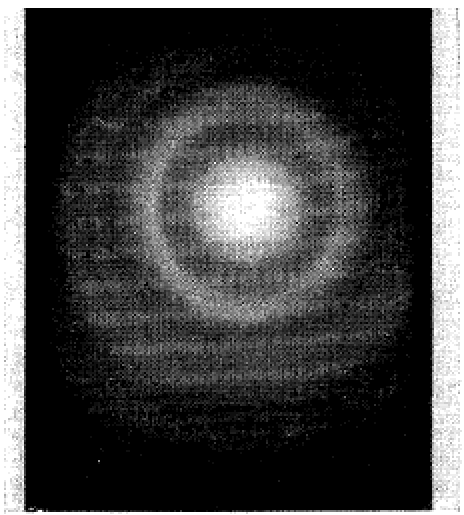


Fig. 4.—Gold.

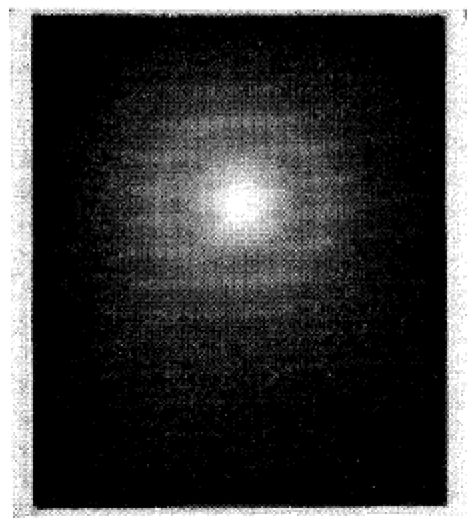
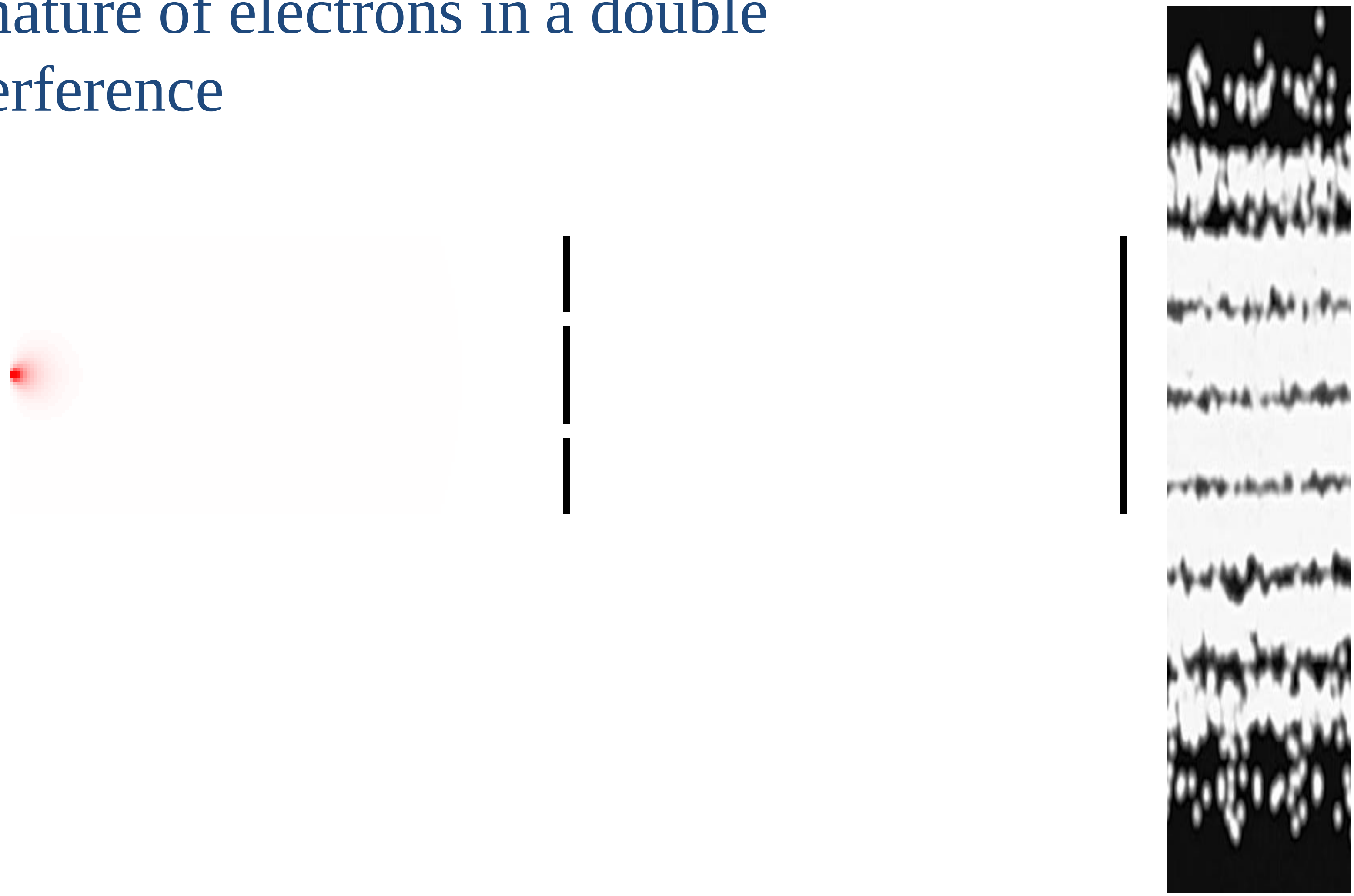


Fig. 5.—Celluloid.

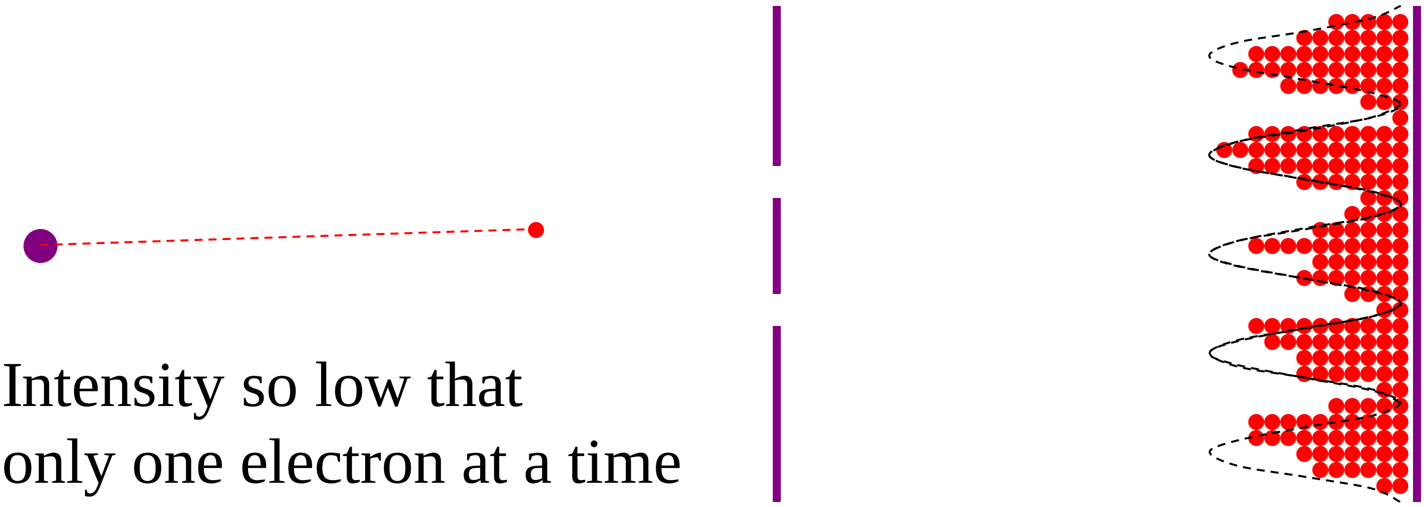
Wave nature of electrons in a double slit interference



4000 clicks

C. Jönsson , Tübingen, Germany, 1961

Single Particle at a time

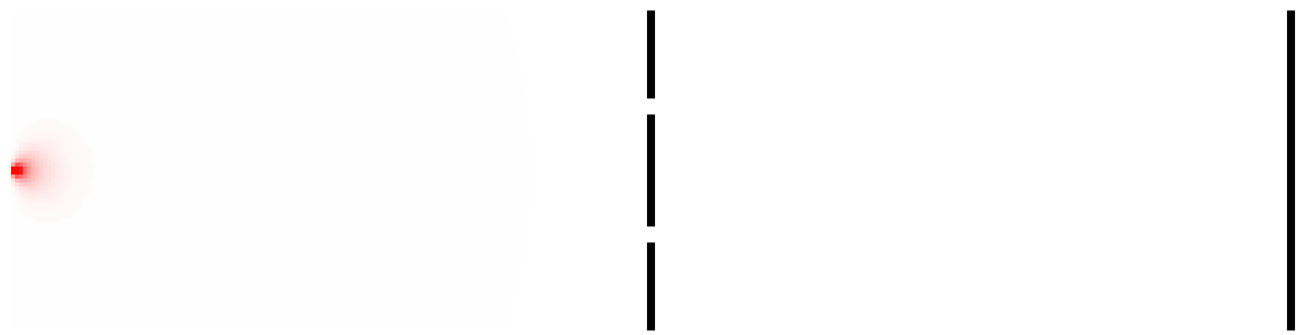


Intensity so low that
only one electron at a time

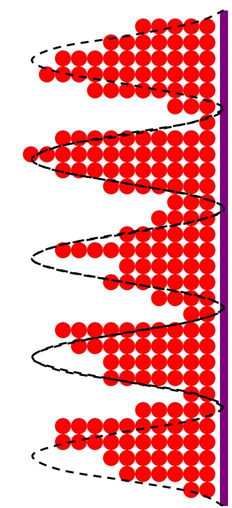
- Not a wave *of* particles
- Single particles interfere with themselves !!

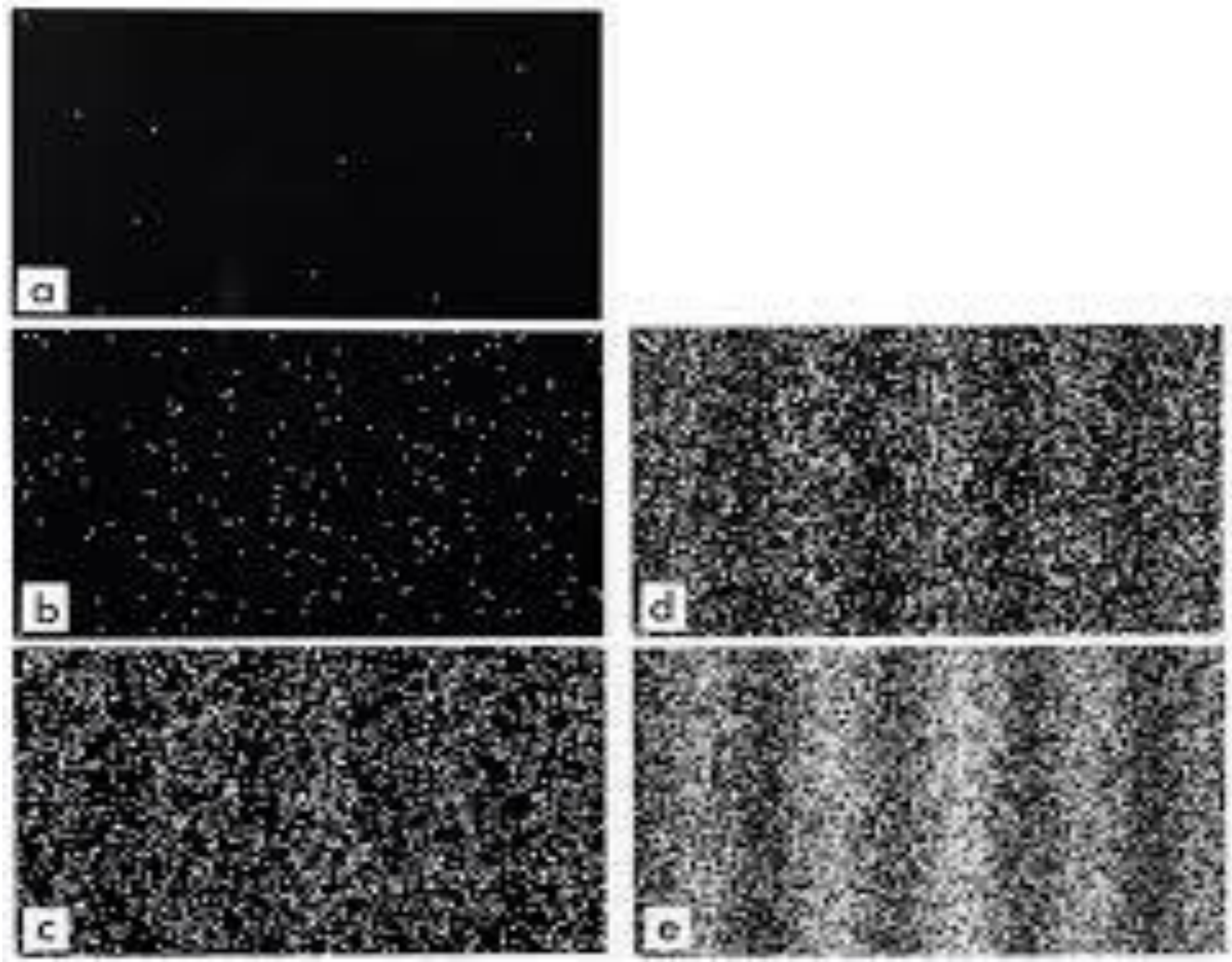
Akira Tonomura and co-workers, Hitachi, 1989

Single particle interference

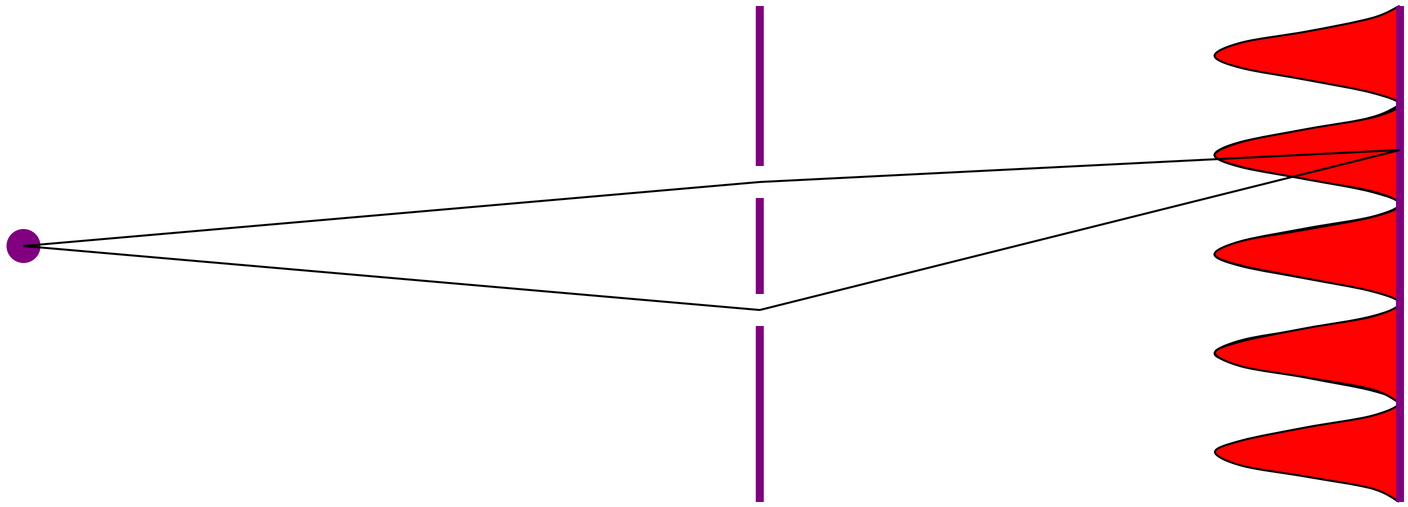


- Two-slit wave packet collapsing
- Eventually builds up pattern
- Particle interferes with itself !!



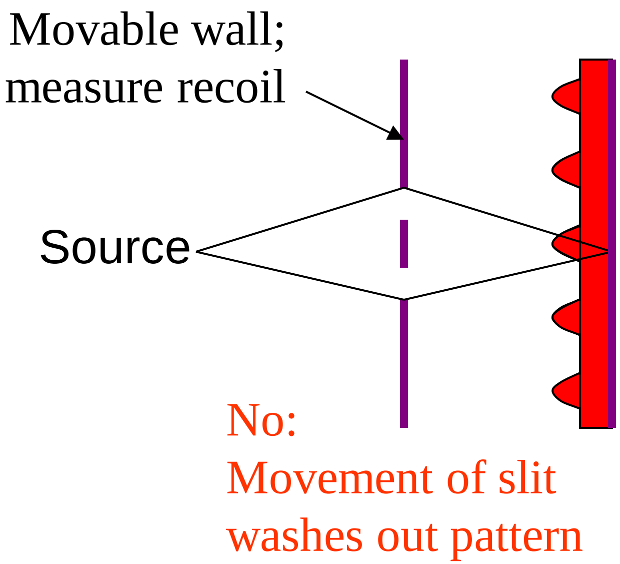

a
b
c
d
e

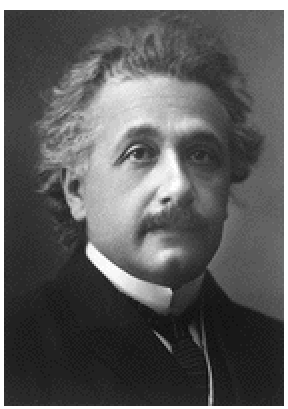
Which path ?



- A classical particle would follow some single path
- Can we say a quantum particle does, too?
- Can we measure it going through one slit or another?

Which path ?

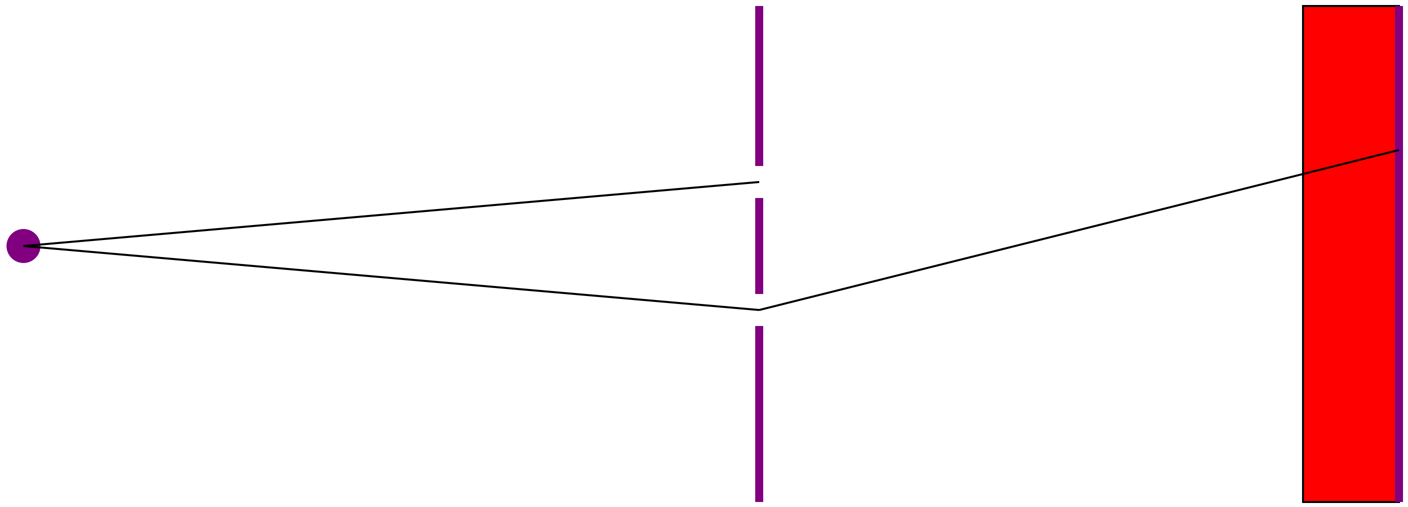



Albert Einstein

- Einstein proposed different ways to measure which slit the particle went through, without blocking it
- Each time, Bohr showed how that measurement would wash out the wave function.


Niels Bohr

Which path ?



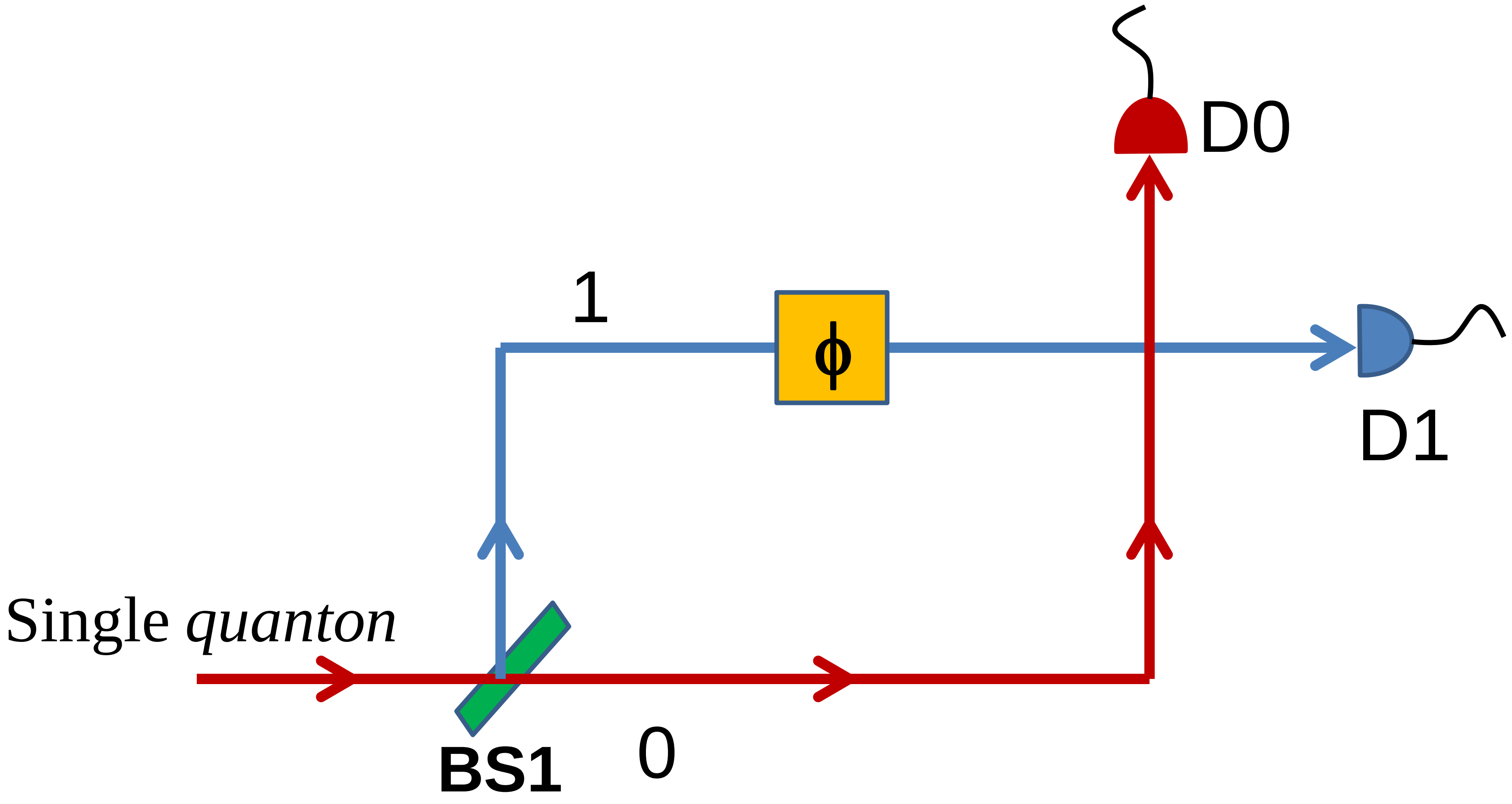
- Short answer: no, we can’t tell
- Anything that blocks one slit washes out the interference pattern

Bohr's Complementarity principle (1933)

Niels Bohr

- Wave and particle natures are complementary !!
- Depending on the experimental setup one obtains either wave nature or particle nature – not both at a time

Mach-Zehnder Interferometer -- Open Setup

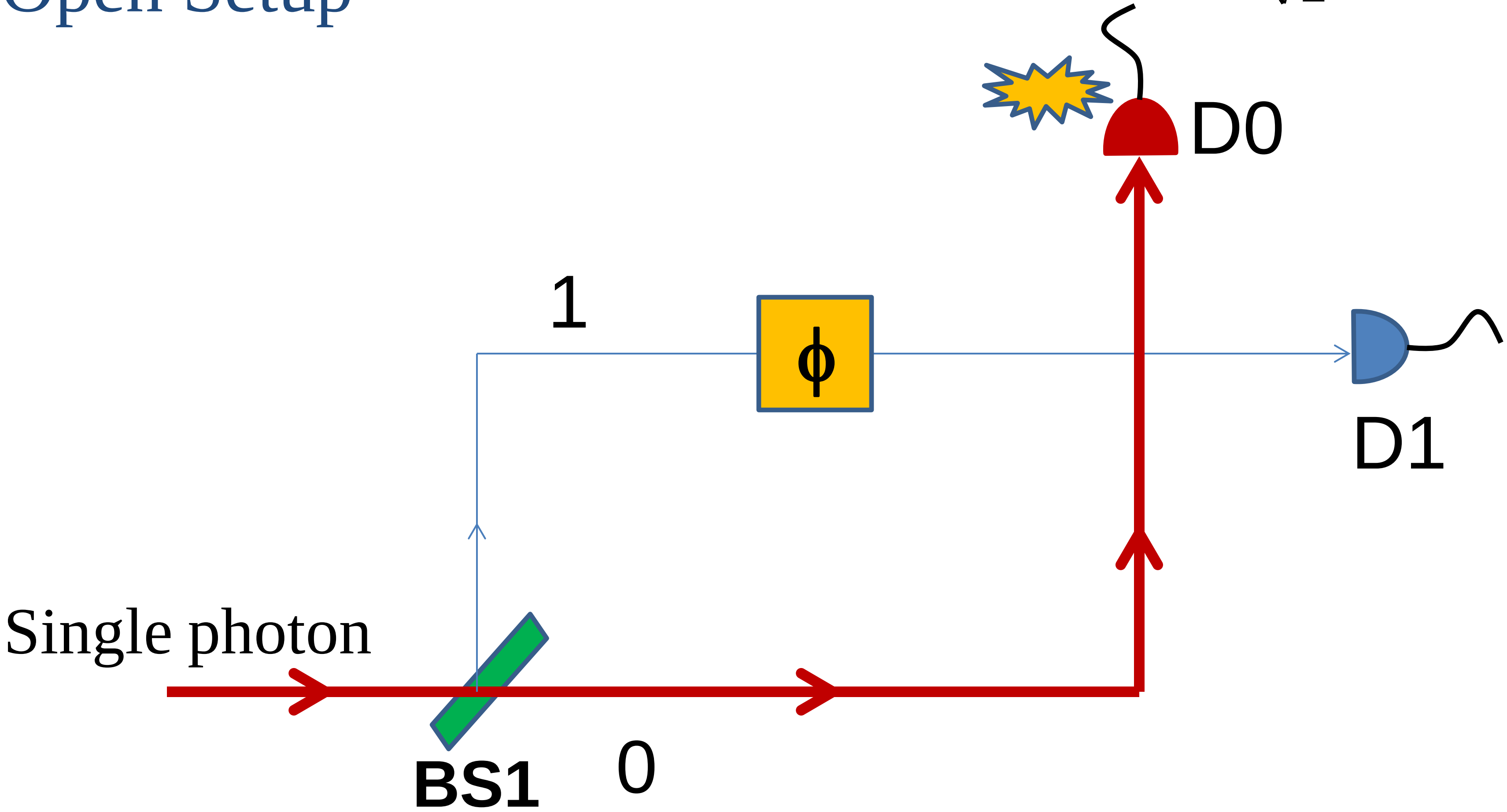


Only one detector clicks at a time

Mach-Zehnder Interferometer --

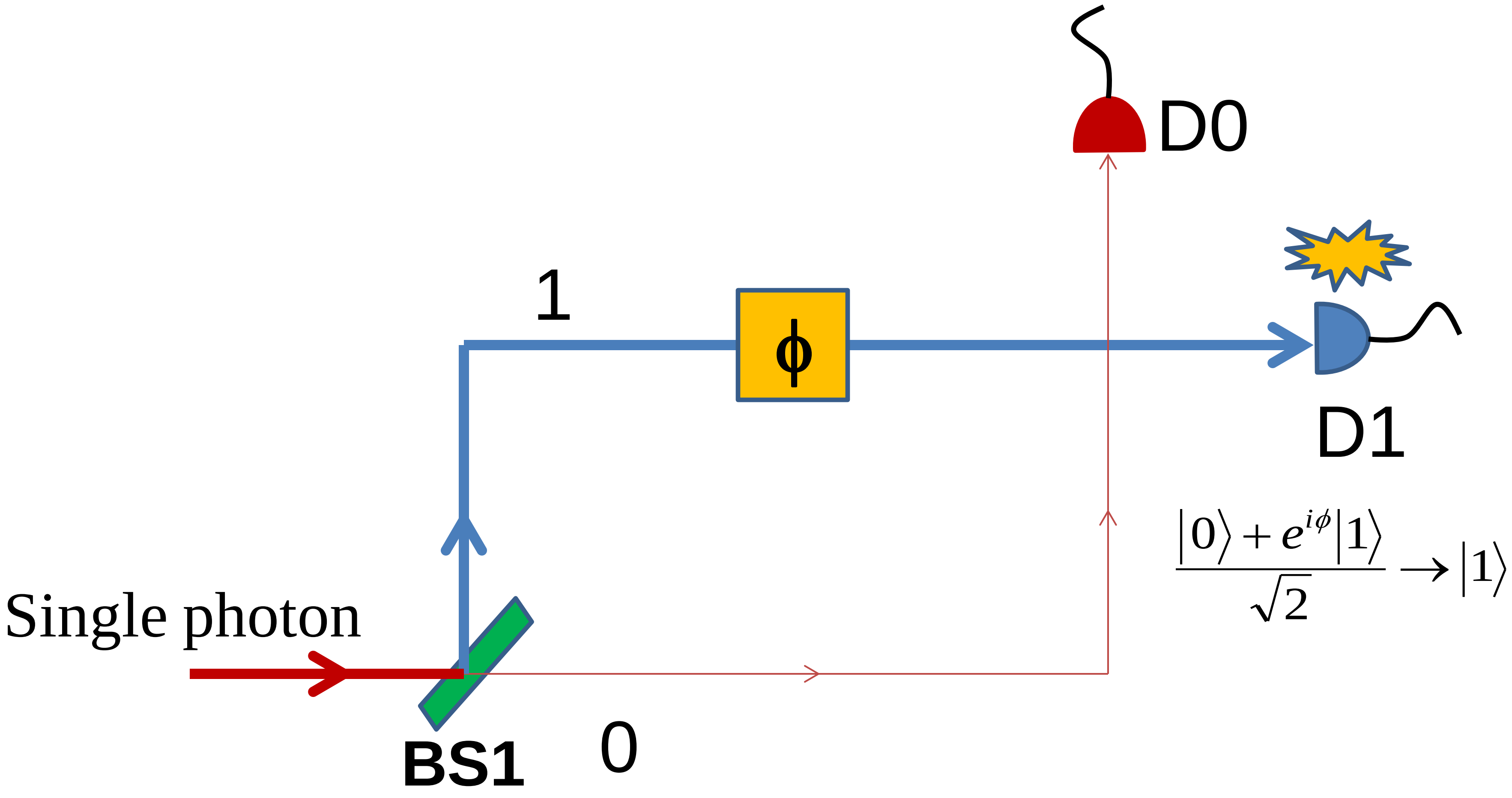
Open Setup

$$\frac{|0\rangle + e^{i\phi}|1\rangle}{\sqrt{2}} \rightarrow |0\rangle$$



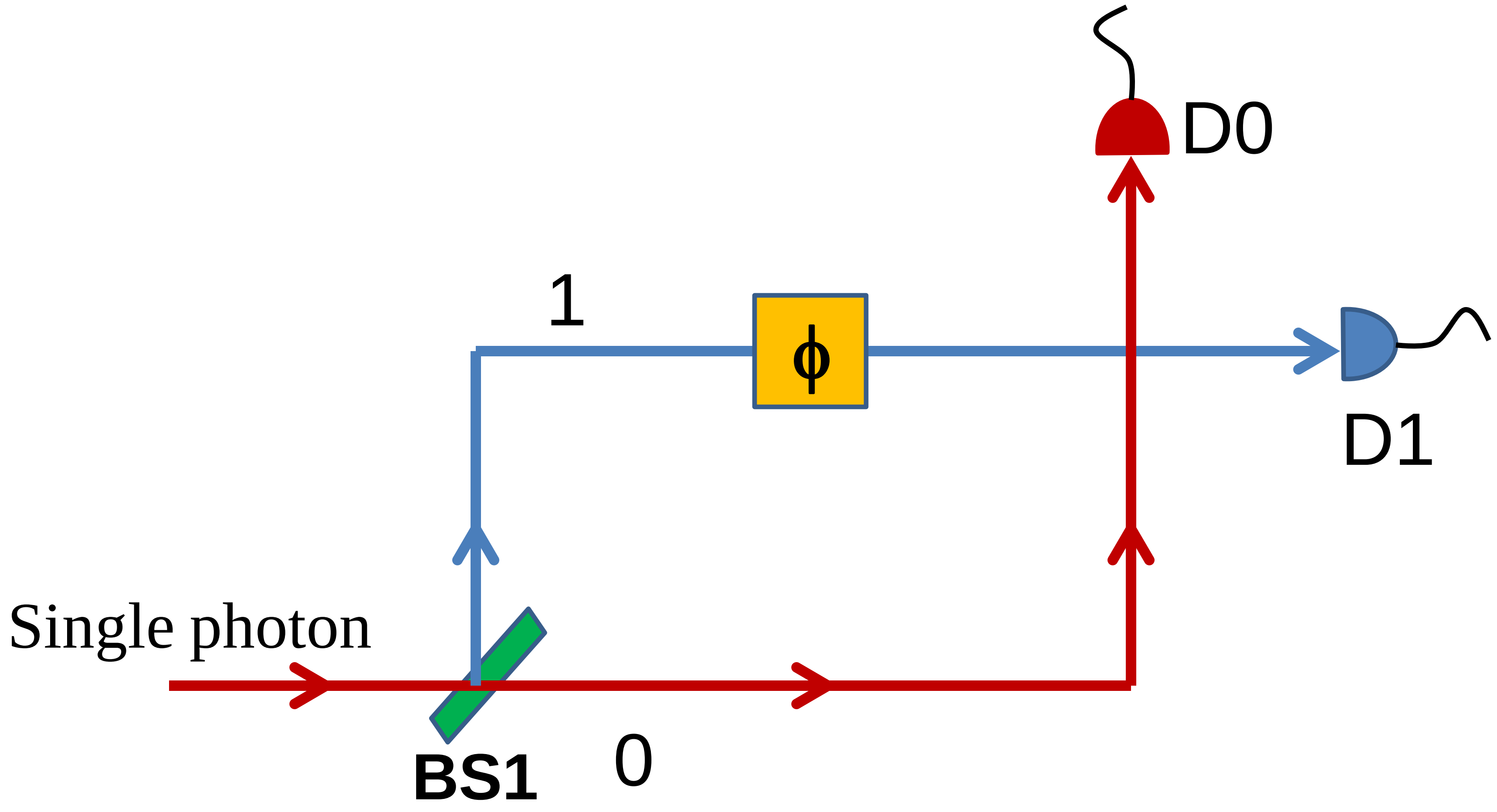
Trajectory can be assigned

Mach-Zehnder Interferometer -- Open Setup



Trajectory can be assigned

Mach-Zehnder Interferometer -- Open Setup

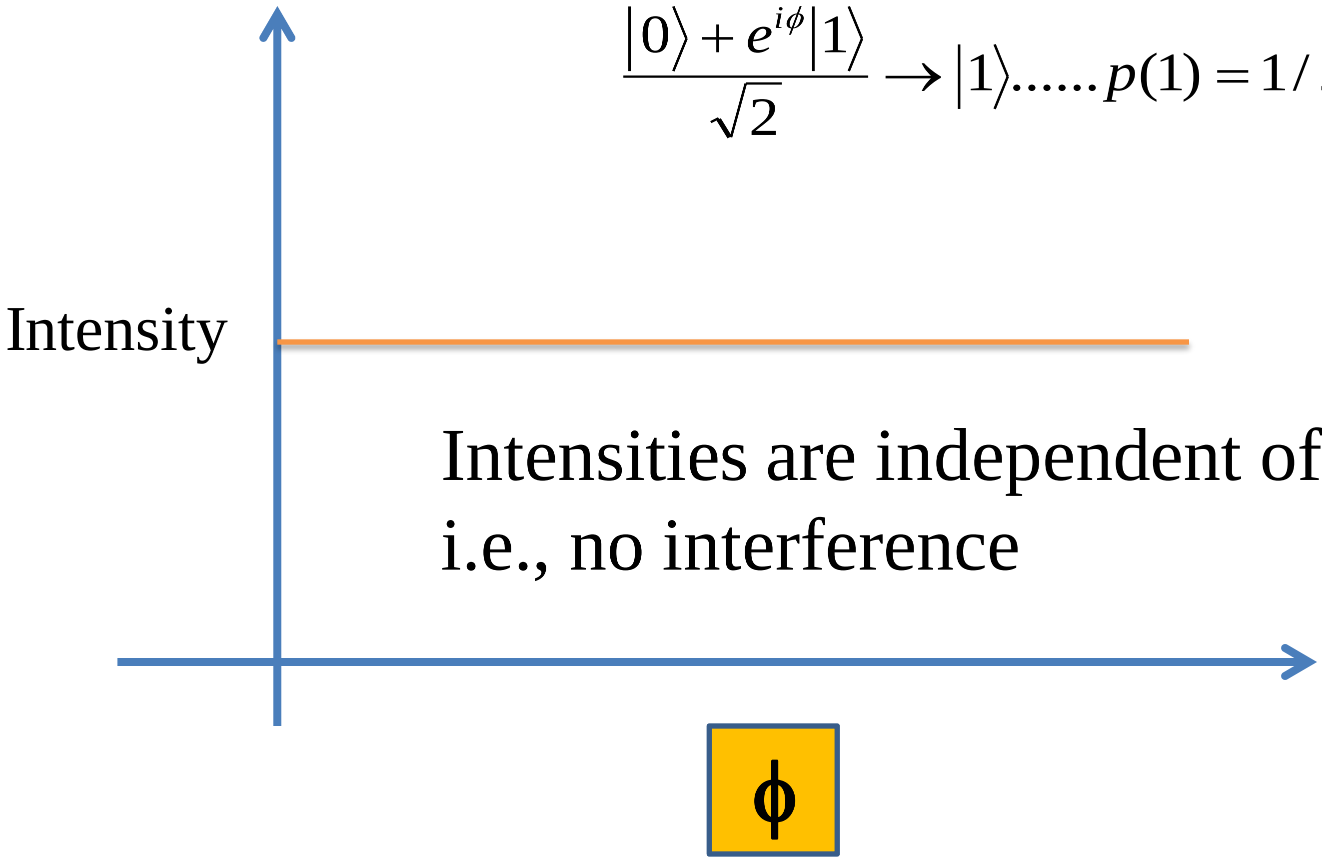


Trajectory can be assigned : Particle nature !!

Mach-Zehnder Interferometer -- Open Setup

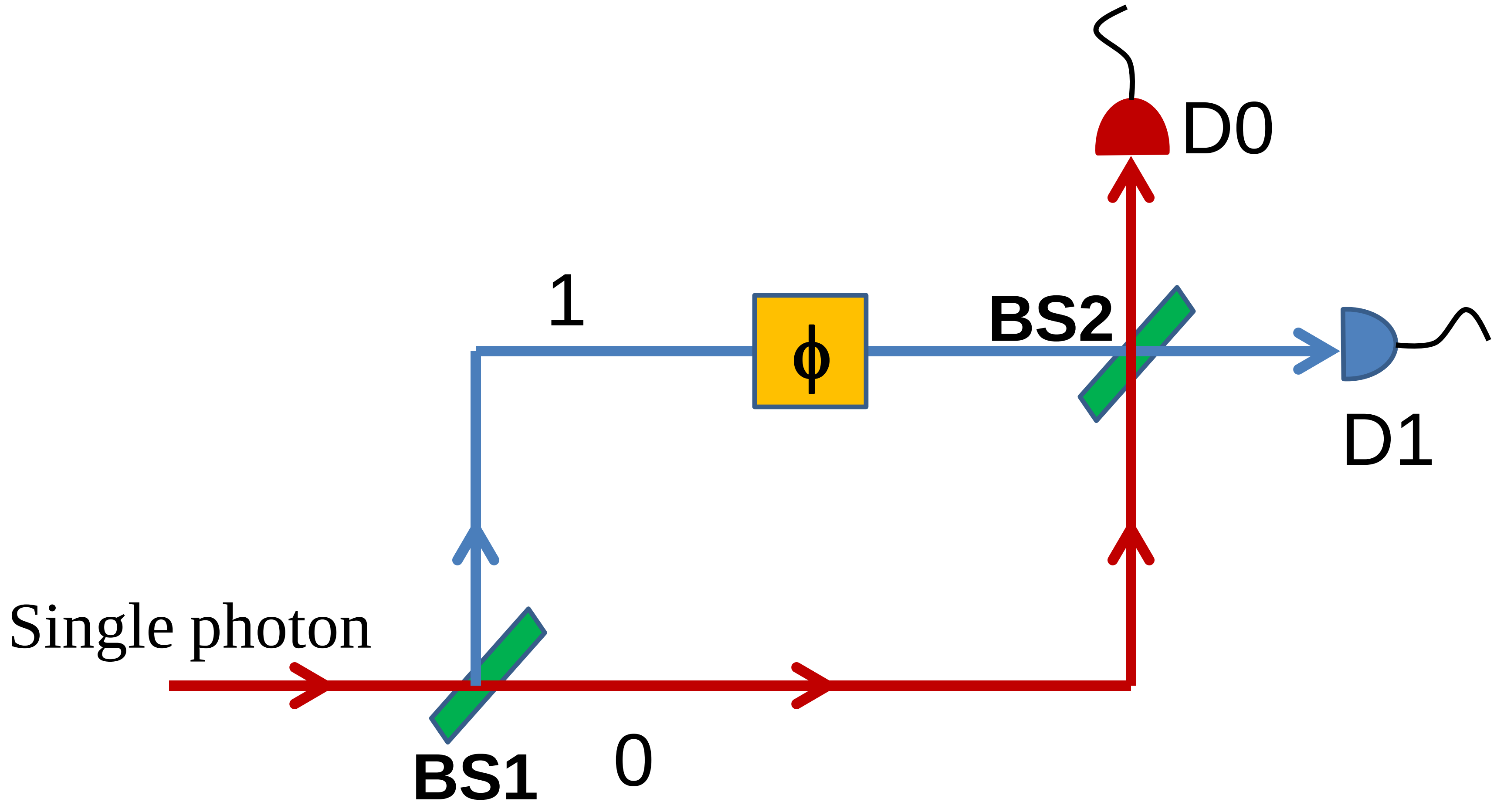
$$\frac{|0\rangle + e^{i\phi}|1\rangle}{\sqrt{2}} \rightarrow |0\rangle ......p(0) = 1/2$$

$$\frac{|0\rangle + e^{i\phi}|1\rangle}{\sqrt{2}} \rightarrow |1\rangle ......p(1) = 1/2$$



Intensities are independent of ϕ
i.e., no interference

Mach-Zehnder Interferometer -- Closed Setup



Again only one detector clicks at a time !!

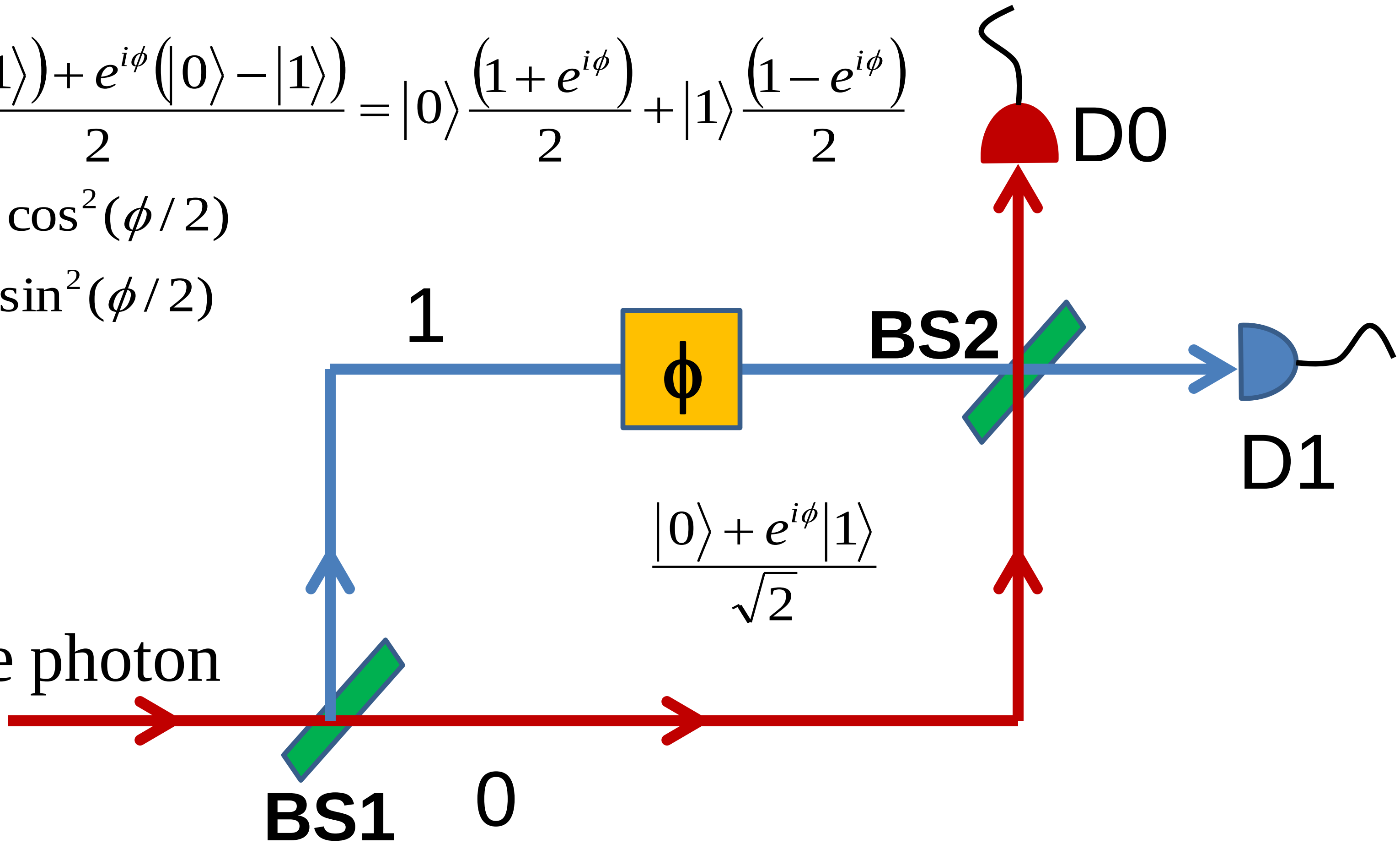
Mach-Zehnder Interferometer -- Closed Setup

$$\frac{\left(|0\rangle + |1\rangle\right) + e^{i\phi}\left(|0\rangle - |1\rangle\right)}{2} = |0\rangle \frac{\left(1 + e^{i\phi}\right)}{2} + |1\rangle \frac{\left(1 - e^{i\phi}\right)}{2}$$

$$p(0) = \cos^2(\phi/2)$$

$$p(1) = \sin^2(\phi/2)$$

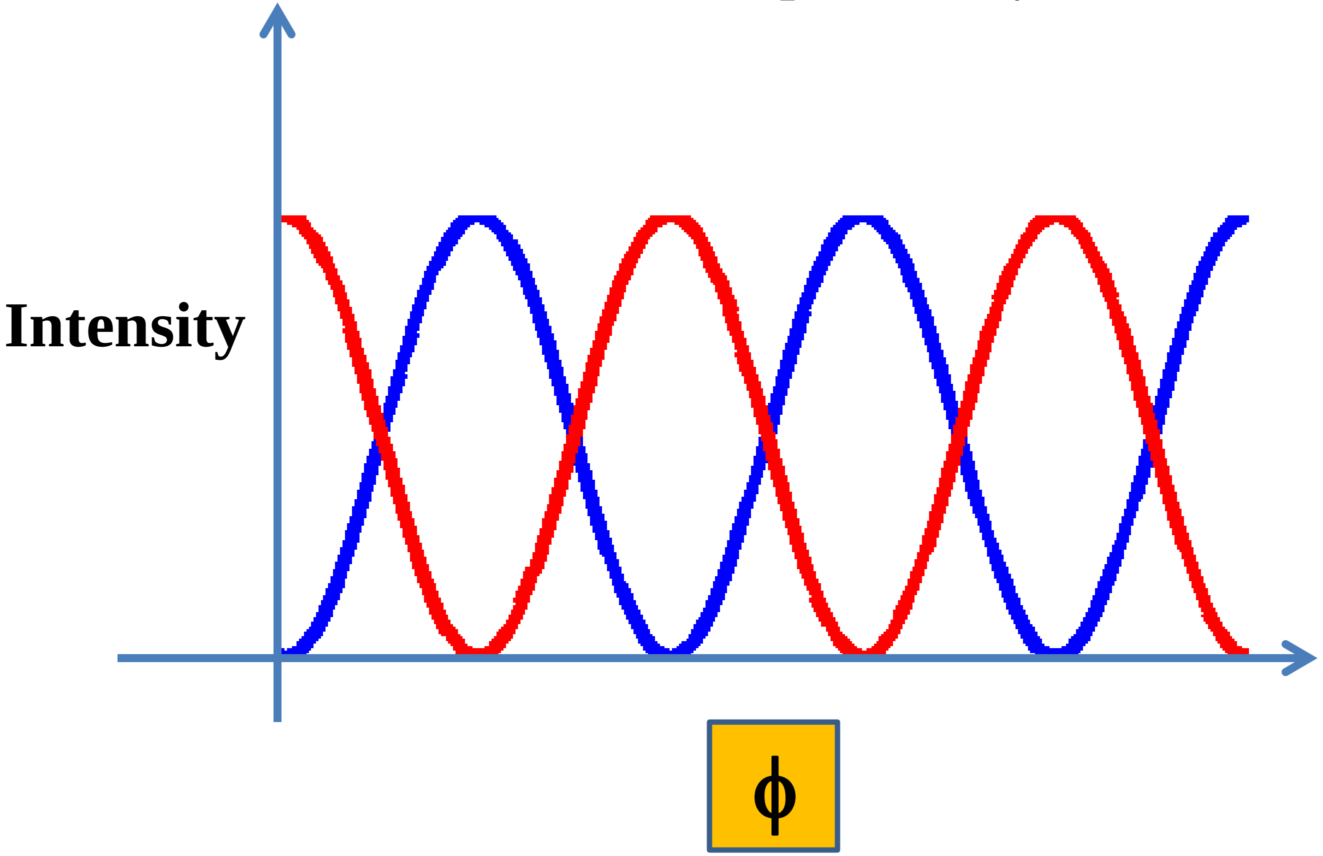
Single photon



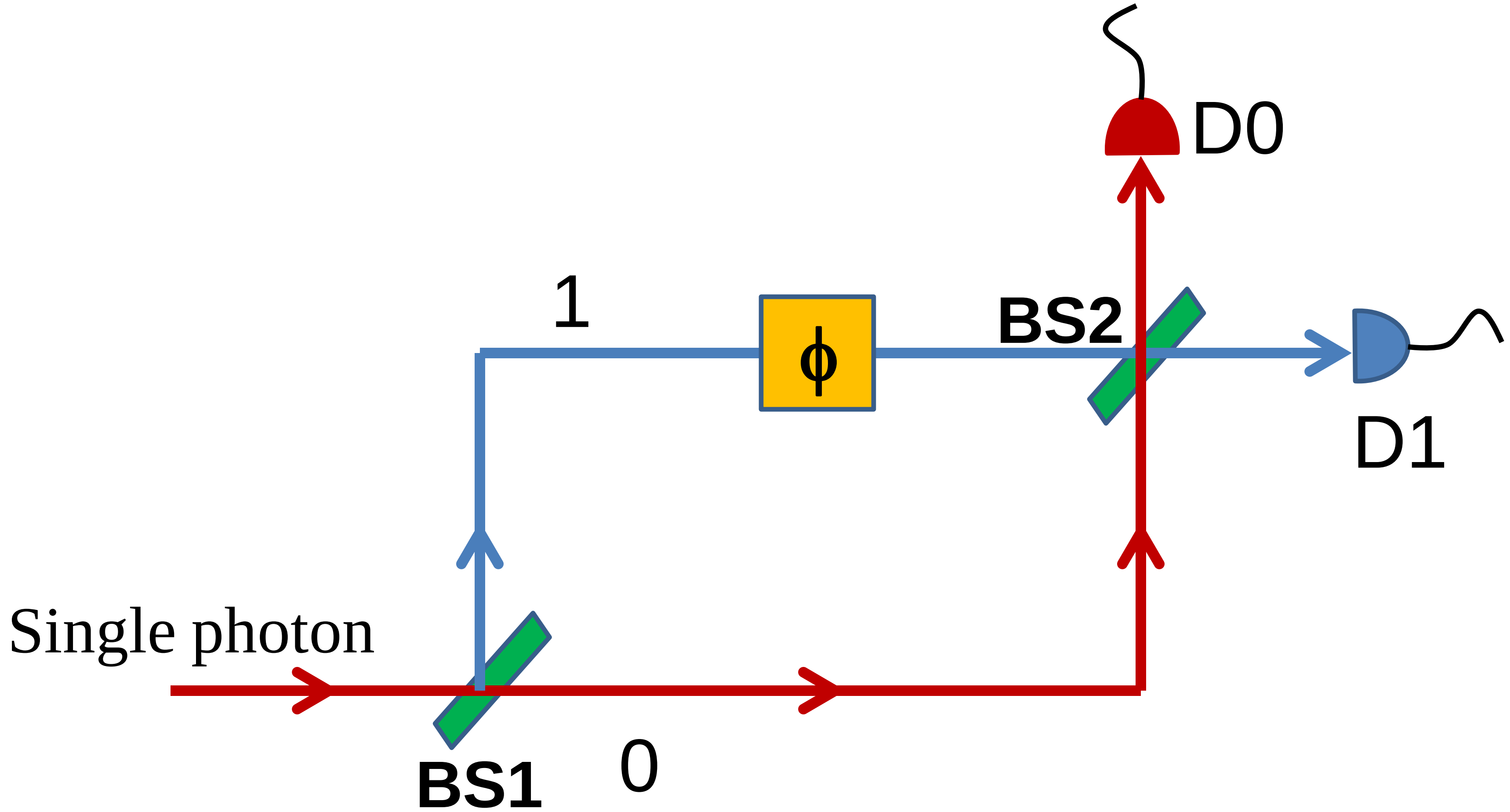
Again only one detector clicks at a time !!

Mach-Zehnder Interferometer -- Closed Setup

Intensities depend on ϕ : Interference!!



Mach-Zehnder Interferometer -- Closed Setup



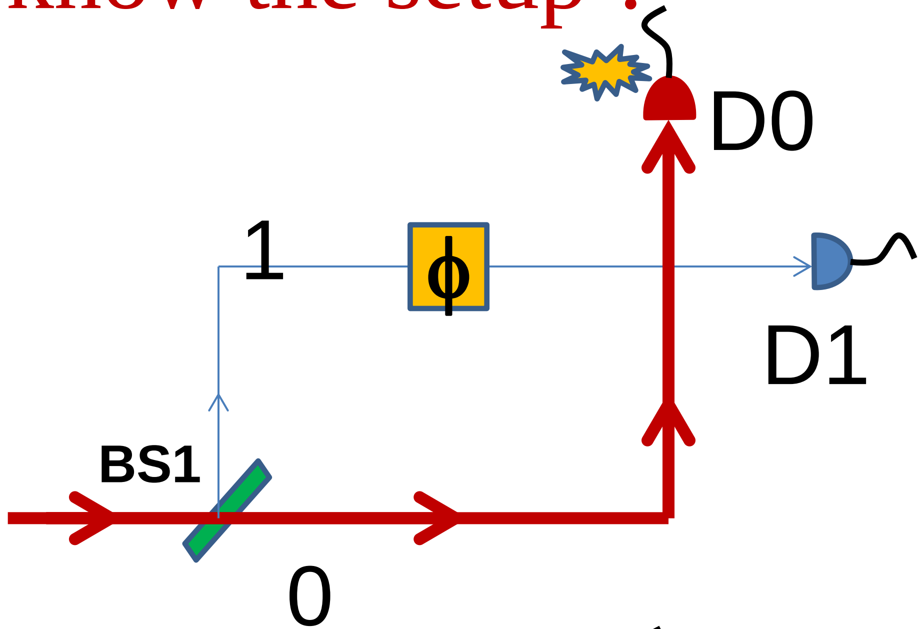
BS2 removes 'which path' information Trajectory

can not be assigned : Wave nature !!

Does ***quanton*** know the setup ?

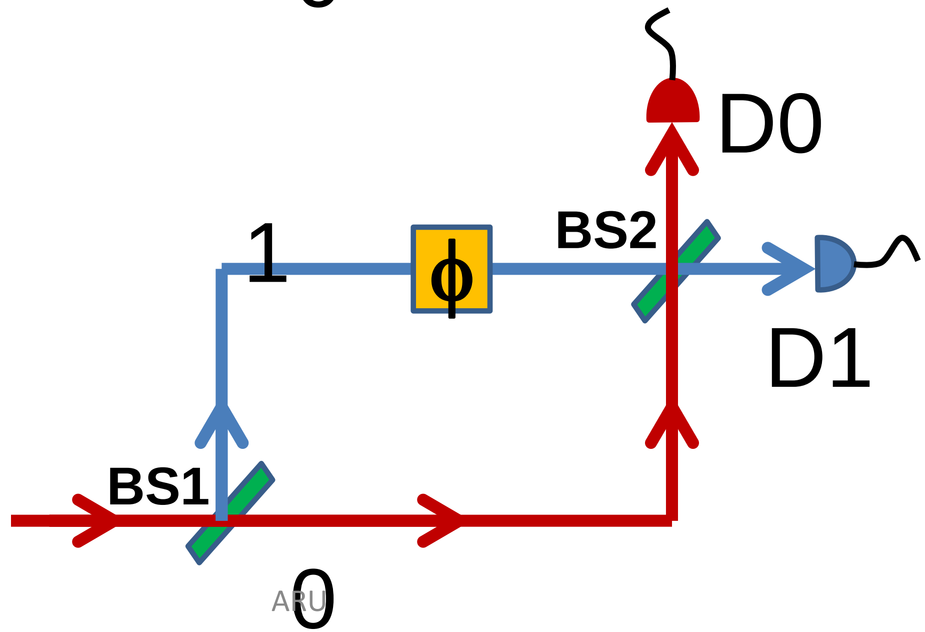
Open Setup

Particle behavior



Closed Setup

Wave behavior



Two schools of thought

Bohr, Pauli, Dirac, ….

- Intrinsic wave-particle duality
- Reality depends on observation
- Complementarity principle

Einstein, Bohm, ….

- Apparent wave-particle duality
- Reality is independent of observation
- Hidden variable theory

Bohr's complementarity principle: Every quantum system has mutually incompatible properties which cannot be simultaneously measured.

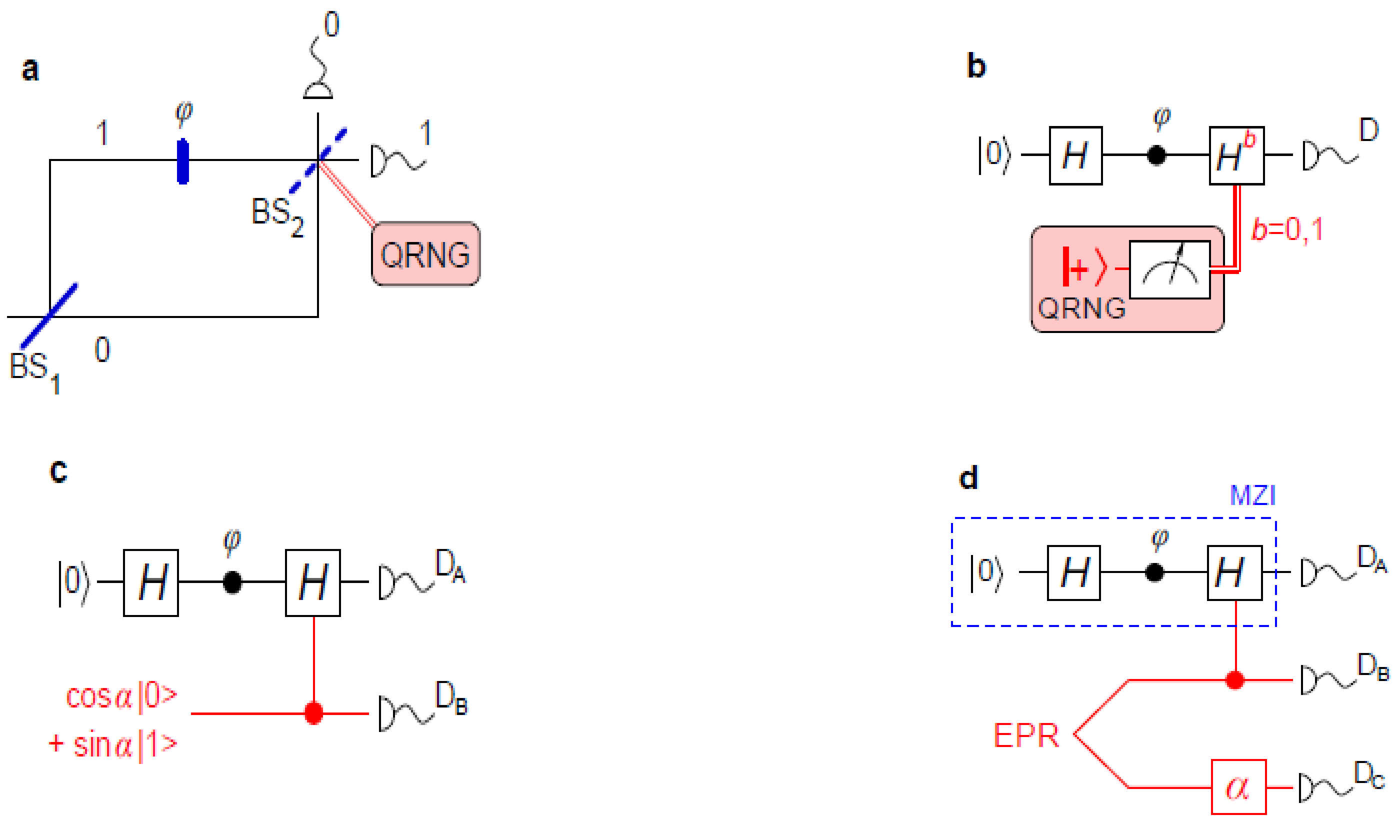
Delayed Choice Experiments

An idea introduced by John A Wheeler of the University of Texas at Austin in 1978

Suppose that the path lengths of a Mach-Zehnder interferometer have been tuned to make the quanton come out of one port of the final beam splitter with probability 1. After the quanton has passed the first beam splitter so that it is fully inside the interferometer, and before it has reached the second beam splitter, you decide to whisk away that second beam splitter, preventing any interference between the quanton's two paths from taking place. Without interference, the quanton behaves like a particle and emerges with equal probability out of either of the two ports of the apparatus where the second beam splitter used to be.

J. A. Wheeler, Mathematical Foundations of Quantum Mechanics, edited by A.R. Marlow (Academic, New-York, 1978) pp. 9-48; *Quantum Theory and Measurement*, J. A. Wheeler, W. H. Zurek, Eds. (Princeton Univ. Press, Princeton, NJ, 1984), pp. 182–213.

The evolution of the delayed-choice experiment.



In Wheeler's classic delayed choice experiment, the second beam splitter is introduced or removed after the photon enters the interferometer; this prevents the photon from "changing its mind". The detectors observe either an interference pattern dependent on the phase φ (wave behaviour) or an equal distribution of hits (particle behaviour). A quantum random number generator (QNRG) determines whether the BS_2 is introduced or not.

B.-G. Englert, Fringe Visibility and Which-Way Information: An Inequality, Phys. Rev. Lett. 77, 2154 (1996).

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Fringe Visibility and Which-Way Information: An Inequality

Berthold-Georg Englert*
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany
(Received 21 May 1996)

An inequality is derived according to which the fringe visibility in a two-way interferometer sets an absolute upper bound on the amount of which-way information that is potentially stored in a which-way detector. In some sense, this inequality can be regarded as quantifying the notion of wave-particle duality. The derivation of the inequality does not make use of Heisenberg's uncertainty relation in any form. [S0031-9007(96)00950-7]

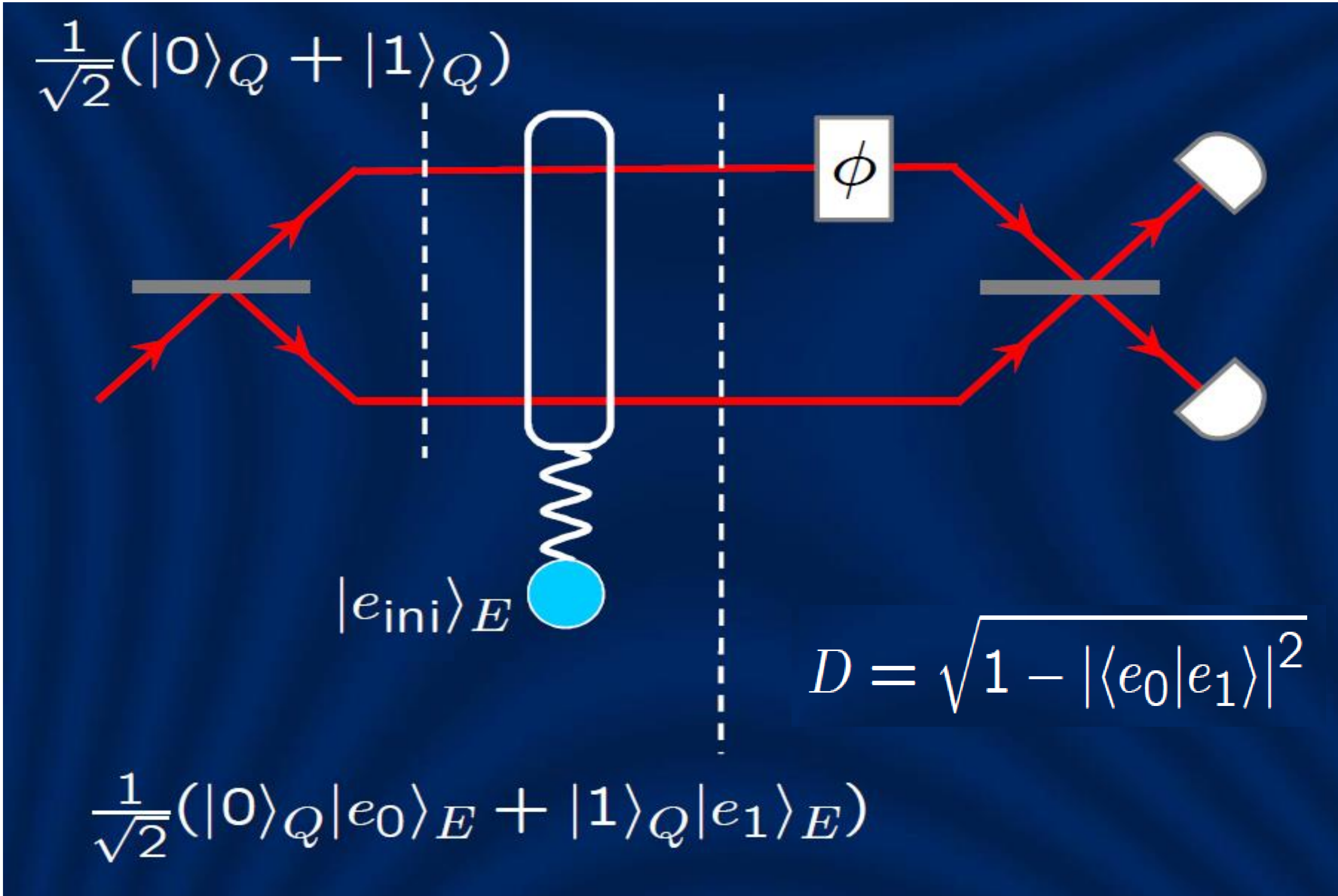
The trade-off between the amount of which-way information encoded in the detector system and the fringe visibility is captured in terms of a generalized complementarity relation

$$D^2 + |V|^2 \leq 1$$

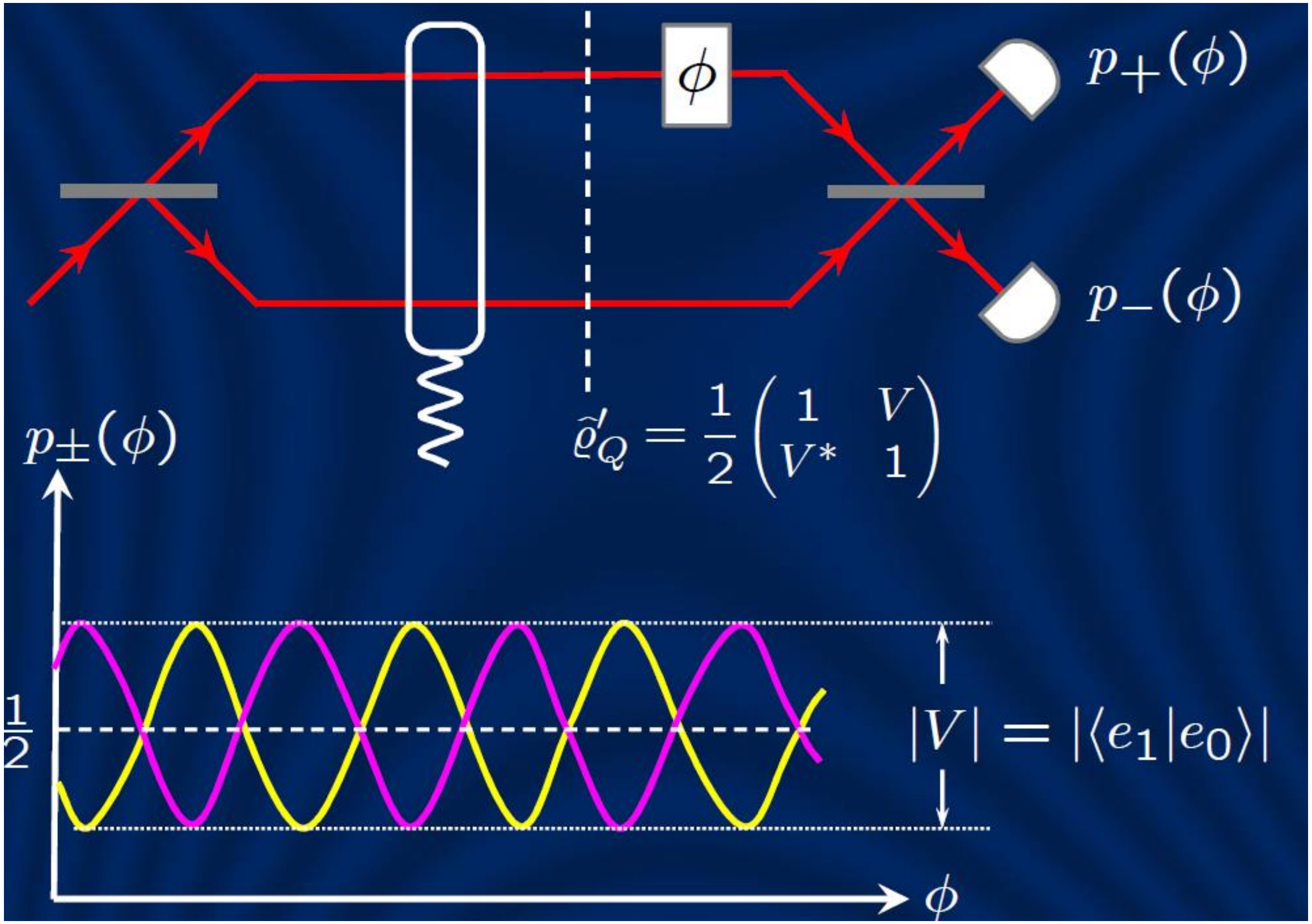
D = distinguishability between two detector states → discriminating two channels

V=interference fringe visibility $= \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Which way information



Visibility |V|



Trade-off

B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996)

Which-way information

$$D = \sqrt{1 - |\langle e_0 | e_1 \rangle|^2}$$

Interference visibility

$$|V| = |\langle e_1 | e_0 \rangle|$$

Bound

$$D^2 + |V|^2 \leq 1$$

$$D = 1 \text{ (particle nature)} \Rightarrow V = 0$$

$$D = 0 \text{ (wave nature)} \Rightarrow V = 1$$

General Scenario

- **A single quantum particle (quanton) Q travelling through a two-path interferometer with equiprobable paths.**
- **The initial state of the quanton and the detector system by $\rho_{QD}^{(\text{in})} = \rho_Q^{(\text{in})} \otimes \rho_D^{(\text{in})}$. When the quanton takes either path 0 or 1 of the interferometer arms, the detector state correspondingly gets transformed into**

$$\rho_D^{(i)} \;=\; U_D^{(i)} \rho_D^{(\text{in})} U_D^{(i)\dagger}, \quad i = 0, 1.$$

$U_D^{(i)} \Rightarrow$ **unitary transformations on the detector states corresponding to the paths of the quanton.**

- **The interaction is constrained such that the quanton paths cannot get transferred into one another due to interaction.**

- **Which-way information is quantified in terms of *distinguishability*** $0 \leq \mathcal{D} \leq 1$,

$$\mathcal{D} \;=\; \frac{1}{2}\,||\rho_D^{(0)} - \rho_D^{(1)}||.$$

- **The paths of the quanton cannot be distinguished when** $\mathcal{D} = 0$ **i.e.,when** $\rho_D^{(0)} \equiv \rho_D^{(1)}$**. The paths are perfectly distinguishable when** $\mathcal{D} = 1$ **i.e., when** $\rho_D^{(0)}$ **and** $\rho_D^{(1)}$ **are orthogonal.**

- **Interference fringe visibility**

$$\mathcal{V} = \left|\mathrm{Tr}[U_D^{(0)}\,\rho_D^{(\mathrm{in})}\,U_D^{(1)\dagger}]\right|.$$

- **Visibility** $0 \leq \mathcal{V} \leq 1$**.**

- **Wave-particle duality or Complementarity is quantified in terms of the trade-off relation:**

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1.$$

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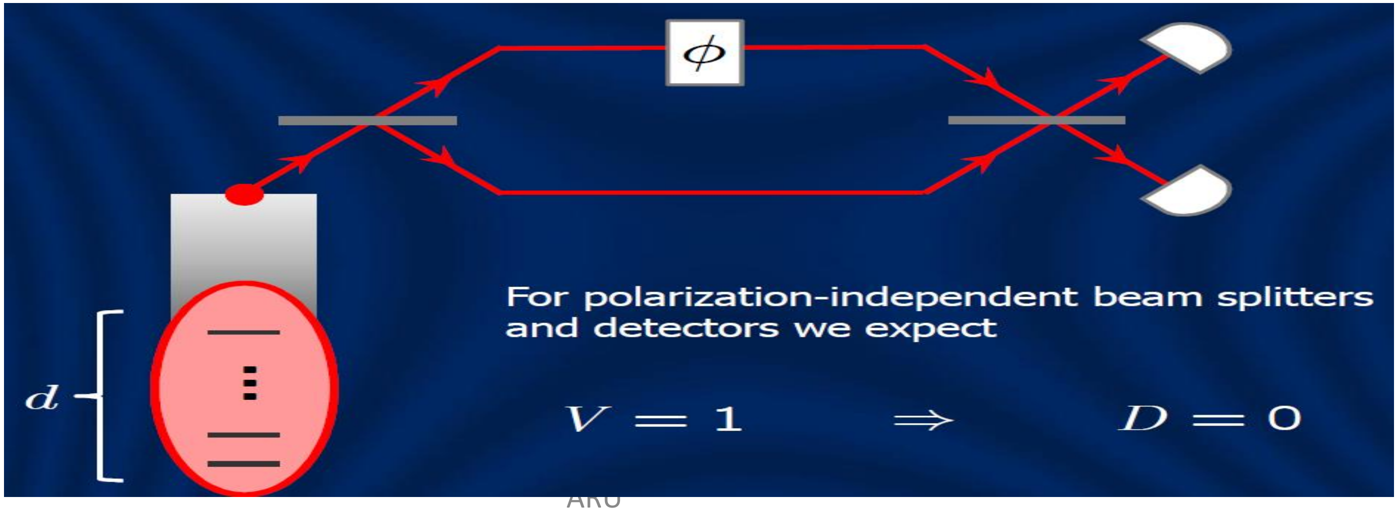
OPEN

Quantum mechanical which-way experiment with an internal degree of freedom

Konrad Banaszek[1], Paweł Horodecki[2,3], Michał Karpiński[1,†] & Czesław Radzewicz[1]

For a particle travelling through an interferometer, the trade-off between the available which-way information and the interference visibility provides a lucid manifestation of the quantum mechanical wave–particle duality. Here we analyse this relation for a particle possessing an internal degree of freedom such as spin. We quantify the trade-off with a general inequality that paints an unexpectedly intricate picture of wave–particle duality when internal states are involved. Strikingly, in some instances which-way information becomes erased by introducing classical uncertainty in the internal degree of freedom. Furthermore, even imperfect interference visibility measured for a suitable set of spin preparations can be sufficient to infer absence of which-way information. General results are illustrated with a proof-of-principle single-photon experiment.



Trade-off relation when the quanton has internal structure

- **Internal degree is characterized by a d_S level quantum system.**

- **Consequence: The which-way information $\mathcal{D}$ on the initial preparation of the internal spin state, in addition to the specific details of its interaction with the detector.**

- **Banaszek et. al. obtained a stringent bound on distinguishability in terms of *generalized fringe visibility*,**

$$\mathcal{D}^2 + \mathcal{V}_G^2 \leq 1$$

- **Distinguishability**

$$\mathcal{D} = \frac{1}{2}\,||\rho_D^{(0)} - \rho_D^{(1)}||$$

$\rightarrow$ leak-out of which-way information to the detector.

- $\rho_D^{(i)} = 2\, {}_Q\langle i|\mathrm{Tr}_S[U_{QSD}\, \rho_{QS}^{(\mathrm{in})} \otimes \rho_D^{(\mathrm{in})}\, U_{QSD}^\dagger]|i\rangle_Q, \quad i = 0, 1$ **are the detector states corresponding to the quanton paths** 0 **and** 1**.**

- **Interaction:**

$$U_{QSD} = \sum_{i=0,1} |i\rangle_Q\langle i| \otimes U_{SD}^{(i)}$$

$\rightarrow$ **the which-way interaction does not shift the quanton between interferometer arms.**

- **Generalized fringe visiblity:**

$$\mathcal{V}_G = d_S\, ||(\mathbb{1} \otimes \Lambda_{01})[(I_S \otimes \sqrt{\rho_{S0}^{(\mathrm{in})}})|\Phi_+\rangle\langle\Phi_+|\,(I_S \otimes \sqrt{\rho_{S1}^{(\mathrm{in})}})]||$$

- **The unitary interaction of the detector with the spin subsystem corresponds to the action of a quantum channel Λ on the internal spin state:**

  Initial state: $\rho_{QSD}^{(\mathrm{in})} = \rho_{QS}^{(\mathrm{in})} \otimes \rho_{D}^{(\mathrm{in})}$

  State after interaction: $\rho_{QSD}^{(\mathrm{fin})} = U_{QSD}\, \rho_{QSD}^{(\mathrm{in})}\, U_{QSD}^{\dagger}$.

- **This unitary interaction on the initial state $\rho_{QSD}^{(\mathrm{in})}$ is viewed as a quantum channel (superoperator) Λ on the input state $\rho_{QS}^{(\mathrm{in})}$:**

$$\Lambda(\rho_{QS}^{(\mathrm{in})}) = \mathrm{Tr}_D[\rho_{QSD}^{(\mathrm{fin})}]$$

- **The quanton should not get switched between the interferometer arms 0 and 1 $\Rightarrow$**

$$\Lambda(|i\rangle_Q\langle j| \otimes \sigma_S) = |i\rangle_Q\langle j| \otimes \Lambda_{ij}(\sigma_S), \qquad i,j = 0,1$$

Here, σ_S corresponds to any operator in the spin space.

- **The states $\rho_{S0}^{(\mathrm{in})},\ \rho_{S1}^{(\mathrm{in})}$ are the initial spin states along the paths 0, 1:**

$$\rho_{S0}^{(\mathrm{in})} = 2\,{}_Q\langle 0|\rho_{QS}^{(\mathrm{in})}|0\rangle_Q$$

$$\rho_{S1}^{(\mathrm{in})} = 2\,{}_Q\langle 1|\rho_{QS}^{(\mathrm{in})}|1\rangle_Q$$

Specific situations

- **No interaction with the detector:**

  The channel reduces to

$$\Lambda_{01} = \mathbb{1}$$

and one obtains

$$\mathcal{V}_G = \sqrt{\mathrm{Tr}[\rho_{S0}^{(\mathrm{in})}]\,\mathrm{Tr}[\rho_{S1}^{(\mathrm{in})}]} = 1,$$

irrespective of the preparation of the initial spin state

Thus, when the detector gains no information about the path, visibility $\mathcal{V}_G$ is 1 and the distinguishability $\mathcal{D}$ is 0.

- **The interaction channel is given by**

$$\Lambda_{01}(\sigma_S) = \mathrm{Tr}[\sigma_S]\,\Sigma_S$$

Σ_S denotes a unit-trace hermitian operator.

The generalized visibility reduces to the fidelity between the spin states $\rho_{S0}^{(\mathrm{in})}$, $\rho_{S1}^{(\mathrm{in})}$ i.e.,

$$\mathcal{V}_G = \mathrm{Tr}[\sqrt{\sqrt{\rho_{S0}^{(\mathrm{in})}}\,\rho_{S1}^{(\mathrm{in})}\,\sqrt{\rho_{S0}^{(\mathrm{in})}}}] = F(\rho_{S0}^{(\mathrm{in})}, \rho_{S1}^{(\mathrm{in})}).$$

$\Rightarrow$ the which-way information can be blocked by preparing identical spin states for both the paths i.e., $\rho_{S0}^{(\mathrm{in})} = \rho_{S1}^{(\mathrm{in})}$, so that the generalized visibility takes its maximum value 1. Consequently, the which-way information is blocked (distinguishability is zero).

- **Interaction $\Lambda_{01}(\sigma_S) = \sigma_S^T/d_S$:**

  In this case, the generalized visibility takes the form

$$\mathcal{V}_G = ||\sqrt{\rho_{S0}^{(\mathrm{in})}}||\,||\sqrt{\rho_{S1}^{(\mathrm{in})}}||/d_S.$$

  The spin states in both the paths, prepared initially in a completely mixed state

$$\rho_{S0}^{(\mathrm{in})} = \rho_{S1}^{(\mathrm{in})} = I_S/d_S$$

  would lead to the generalized fringe visibility $\mathcal{V}_G = 1$ and the which-path information to the detector can thus be blocked.

So far….

The trade-off between interference visibility and which-path distinguishability for a quantum particle possessing an internal structure -- such as spin or polarization is useful to erase 'which-path' information (by appropriate preparations of states of the internal degree of freedom). One can thus recover interference

- **the internal structure could play a manipulative role in controlling the information about which path in the interferometer arms is taken by the particle.**

- **Generalized fringe visibility and detector state distinguishability show complementarity (trade-off)**

- **What happens if detector state has an internal structure??**

Channel discrimination and which-path information in two-slit interference

- **Which-way information is gathered by distinguishing the two detector states $\rho_D^{(0)}$ and $\rho_D^{(1)}$.**

  $\Rightarrow$ discriminating the two quantum channels Φ_0, Φ_1.

- **Measure of distinguishability of two quantum channels is given by their trace distance,**

$$\frac{1}{2}||\Phi_0 - \Phi_1|| =^{\max}_{\rho}\ \frac{1}{2}\,||\Phi_0(\rho) - \Phi_1(\rho)||$$

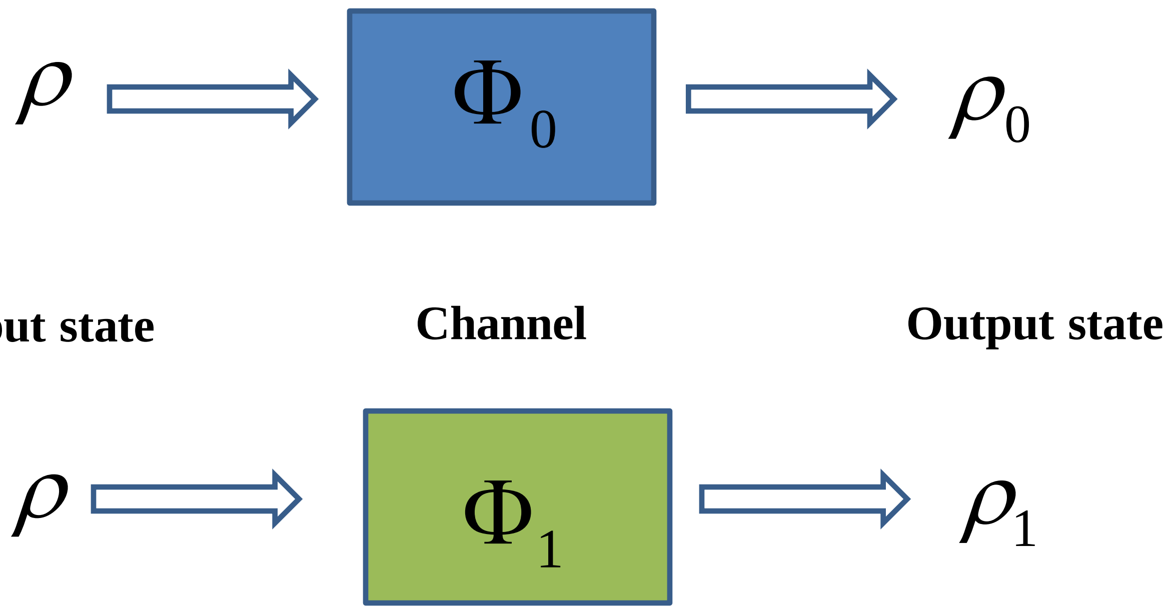
  $\rightarrow$ The maximum is taken over all pure input states ρ .

- **Optimization: Prepare the input state entangled with ancilla and apply one of the channels to the input state (with ancillary subsystem being an idler); then distinguish the resulting output state to identify which of the channels was applied.**

- **There are examples of channels that can be distinguished *perfectly*, when they are applied to one part of a maximally entangled state, while they are *indistinguishable* if the auxilliary system is not employed.**

Channel Discrimination

Distinguishing two channels Φ_0, Φ_1 with input state ρ :



Channel discrimination → which path information

All entangled states are useful for Channel discrimination task

M. F. Sacchi, Phys. Rev. A 71, 062340 (2005); 72, 014305 (2005)
M. Piani and J. Watrous, Phys. Rev. Lett. 102, 250501 (2009)

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Quantum which-way information and fringe visibility when the detector is entangled with an ancilla

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Quantum-mechanical wave-particle duality is quantified in terms of a trade-off relation between the fringe visibility and the which-way distinguishability in an interference experiment. This relation was recently generalized by Banaszek *et al.* [Nat. Commun. **4**, 2594 (2013)] when the particle is equipped with an internal degree of freedom such as spin. Here, we extend the visibility-distinguishability trade-off relation to quantum interference of a particle possessing an internal degree of freedom, when the *which-way* detector state is entangled with an ancillary system. We introduce an *extended which-way distinguishability* $\mathcal{D}_E$ and the associated *extended fringe visibility* $\mathcal{V}_E$, satisfying the inequality $\mathcal{D}_E^2 + \mathcal{V}_E^2 \leqslant 1$ in this scenario. We illustrate, with the help of three

We put forth some instances where distinguishability is 0, yet generalized fringe visibility is not equal to 1. Where is the missing information?

Our work: Tracking missing 'which-path' information via Generalized distinguishability when detector is assisted by an ancilla.

Which-way information using entangled detector-ancilla state

Our work: Investigating the enhancement of which-way information, given that the detector is entangled with an ancilla.

Def: Extended distinguishability

$$\mathcal{D}_E = \frac{1}{2}||(\Phi_0 \otimes \mathbb{1})(\rho_{DD'}^{(\text{in})}) - (\Phi_1 \otimes \mathbb{1})((\rho_{DD'}^{(\text{in})})|| = \frac{1}{2}\,||\rho_{DD'}^{(0)} - \rho_{DD'}^{(1)}||$$

Here,

$$\rho_{DD'}^{(i)} = (\Phi_i \otimes \mathbb{1})(\rho_{DD'}^{(\text{in})})$$

$\rightarrow$ **final detector-ancilla states corresponding to quanton paths $i = 0, 1$.**

Inequality between *Trace distance*, $D(\varrho,\, \tau) = \frac{1}{2}\,||\varrho - \tau||$ and *Fidelity*, $F(\varrho,\, \tau) = \mathrm{Tr}[\sqrt{\sqrt{\varrho}\;\tau\;\sqrt{\varrho}}]$:

(C. A. Fuchs and J. van de Graaf, IEEE Trans. Inf. Theory 45, 1216 (1999))) :

$$D(\varrho, \tau) \leq \sqrt{1 - F^2(\varrho,\, \tau)}$$

$$\begin{aligned} \Rightarrow \quad \mathcal{D}_E \quad &\leq \quad \sqrt{1 - F^2(\rho^{(0)}_{DD'}, \rho^{(1)}_{DD'})} \\ &= \quad \sqrt{1 - \mathcal{V}_E^2} \end{aligned}$$

Extended Visibility: $\mathcal{V}_E = F^2(\rho^{(0)}_{DD'}, \rho^{(1)}_{DD'})$.

New Trade-off relation:

$$\mathcal{D}_E^2 + \mathcal{V}_E^2 \leq 1$$

- **We investigated specific examples of interaction between the quanton-spin and the detector to demonstrate that the extended which-way distinguishability $\mathcal{D}_E$ can assume non-zero values even when the distinguishability $\mathcal{D}$ inferred only by the detector vanishes.**

- **Unitary interaction between the quanton and the detector:**

$$U_{SD}^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad U_{SD}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & e^{-i\phi} & 0 \\ 0 & e^{i\phi} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- **Initial quanton path-spin state:**

$$|\zeta_{QS}\rangle^{(\mathrm{in})} = \frac{1}{2}[|0\rangle_Q + |1\rangle_Q] \otimes |0\rangle_S$$

- **Initial detector-ancilla state:**

$$|\Psi\rangle_{DD'} = \frac{1}{\sqrt{2}}\,(|0\rangle_D\,|0\rangle_{D'} + |1\rangle_D\,|\,1\rangle_{D'}).$$

$\rightarrow$ **Maximally entangled detector-ancilla state.**

- **Detector-ancilla states after interaction**:

$$\rho_{DD'}^{(0)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \rho_{DD'}^{(1)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i\phi} & 0 \\ 0 & e^{i\phi} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- **Extended Distinguishability:**

$$\mathcal{D}_E = |\sin(\phi/2)|$$

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- **Extended fringe visibility:**

$$\mathcal{V}_E = |\cos(\phi/2)|$$

- **Detector states** $\rho_D^{(i)} = \mathrm{Tr}_{D'}[\rho_{DD'}^{(i)}] = I_D/2$ **are indistinguishable (no which-way information)**.
- **Generalized visibility** $\mathcal{V}_G$ **and extended visibility** $\mathcal{V}_E$ **are equal.**

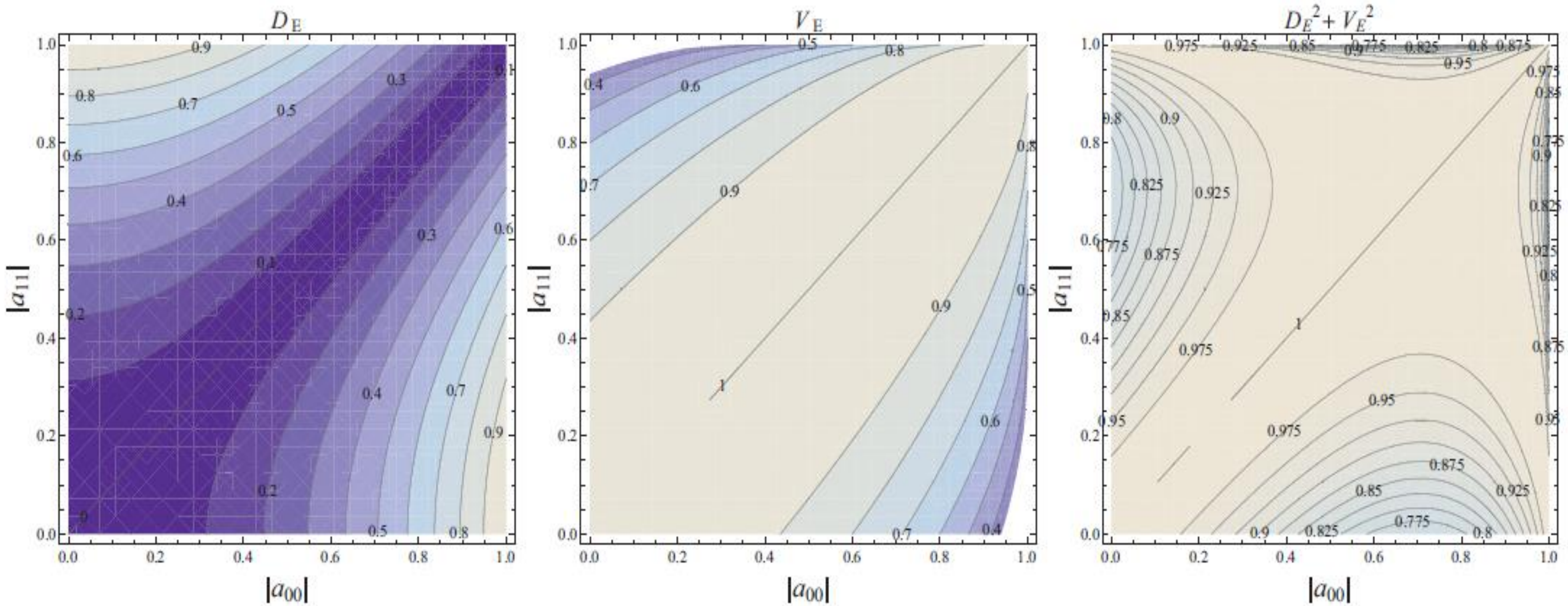

FIG. 1. (Color online) Contour plots of *extended distinguishability* $\mathcal{D}_E$, *extended visibility* $\mathcal{V}_E$ [see (16) and (17)], and $\mathcal{D}_E^2 + \mathcal{V}_E^2$ as functions of $|a_{00}|,|a_{11}|$, the parameters of the initial spin preparation. It is clearly seen that the which-way distinguishability $\mathcal{D}_E$ and fringe visibility $\mathcal{V}_E$ obey the duality relation $\mathcal{D}_E^2 + \mathcal{V}_E^2 \leqslant 1$.

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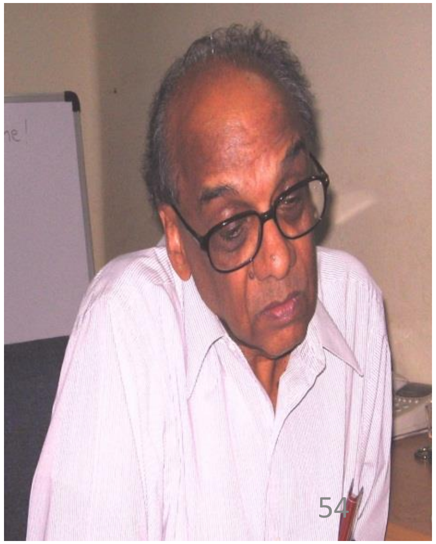

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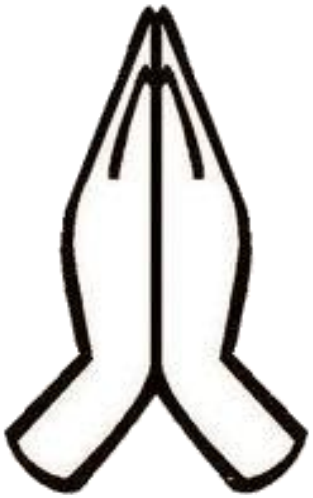

Thanks