Correlations Between Random Observables

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Quantum entanglement from random measurements
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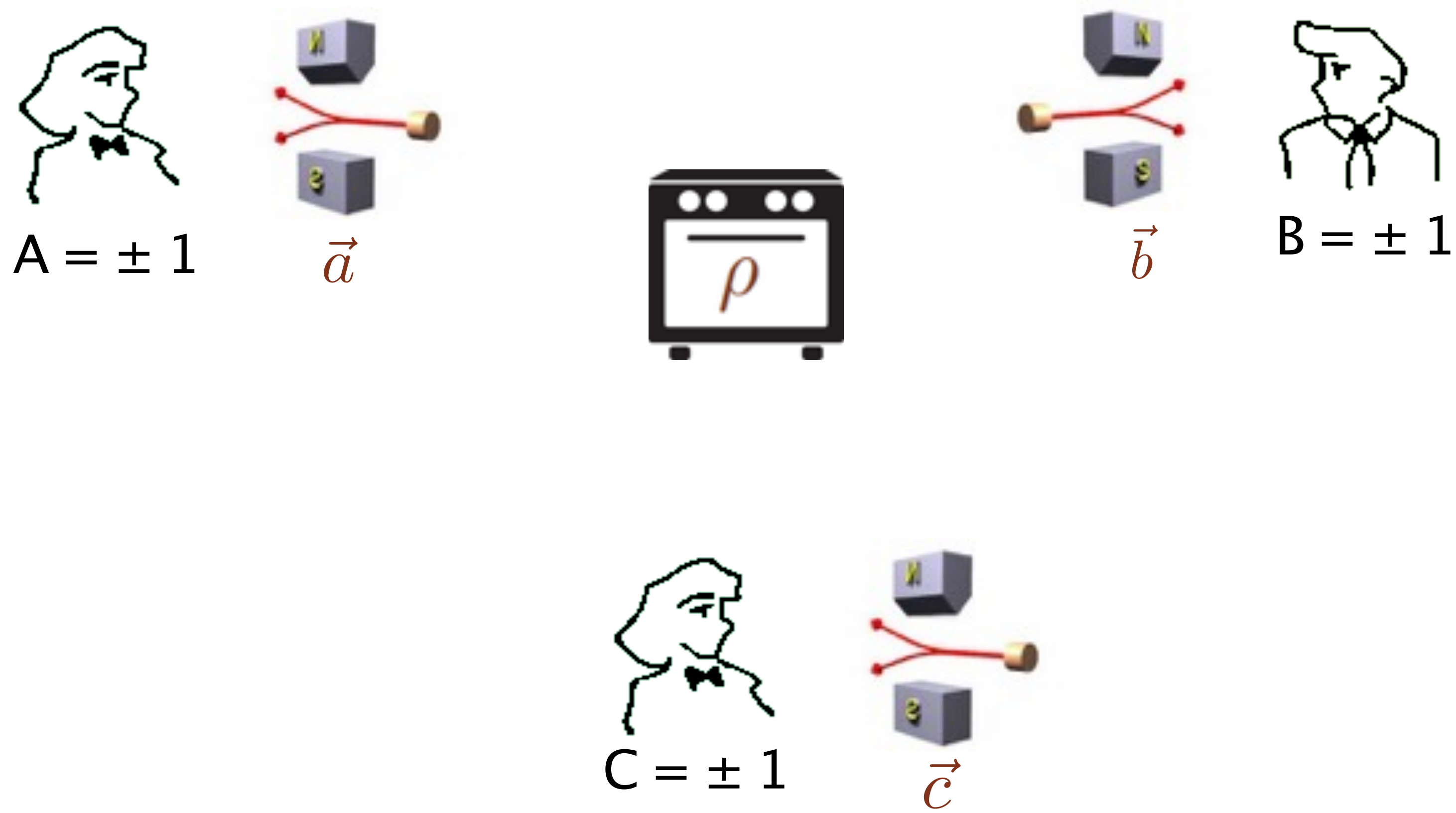
CORRELATION FUNCTIONS

INTUITION: TWO SYSTEMS ARE CORRELATED IF MEASURING ONE TELLS SOMETHING ABOUT THE OTHER

EXPECTATION VALUE OF PRODUCT OF LOCAL RESULTS

QUANTUM CORRELATION FUNCTIONS



$$E(\vec{a},\vec{b},\vec{c}) = \langle ABC \rangle = \mathrm{Tr}(\rho\, \vec{a}\cdot\vec{\sigma} \otimes \vec{b}\cdot\vec{\sigma} \otimes \vec{c}\cdot\vec{\sigma})$$

DIRECTLY EXPERIMENTALLY ACCESSIBLE

ALTERNATIVE REPRESENTATION OF A QUANTUM STATE

RANDOM CORRELATIONS

$$\mathcal{R} \equiv \frac{1}{(4\pi)^N} \int d\vec{u}_1 \ldots \int d\vec{u}_N \; E^2(\vec{u}_1, \ldots, \vec{u}_N)$$

Squared correlation functions averaged over uniform choices of settings for each individual observer.

LENGTH OF CORRELATIONS

$$\mathcal{R} = \frac{1}{3^N} \sum_{j_1,\dots,j_N=1}^{3} T^2_{j_1\dots j_N}$$

$$\rho = \frac{1}{2^N} \sum_{\mu_1,\dots,\mu_N=0}^{3} T_{\mu_1\dots\mu_N}\, \sigma_{\mu_1} \otimes \dots \otimes \sigma_{\mu_N}$$

1. *Length of correlations is invariant under local unitary operations.*
2. *Local unitary operations are isomorphic to local rotations.*
3. *R is the average of any squared correlation over random rotations.*
4. *There are 3^N correlations summed, hence the pre-factor.*

$$\mathcal{R} = \frac{1}{3^N} \sum_{j_1,\dots,j_N=1}^{3} T^2_{j_1\dots j_N}$$

Linearisation

$$T^2_{j_1\dots j_N} = \langle\psi|\sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle\langle\psi|\sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle$$

$$T^2_{j_1\dots j_N} = \langle\psi|\langle\psi|\sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N} \otimes \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N}|\psi\rangle|\psi\rangle$$

$$\mathcal{R} = \frac{1}{3^N}\langle\psi|\langle\psi|H_{11'} \otimes \cdots \otimes H_{NN'}|\psi\rangle|\psi\rangle$$

$$H_{nn'} = \sum_{j_n=1}^{3} \sigma^{(n)}_{j_n} \otimes \sigma^{(n')}_{j_n}$$

Universal lower bound:
For all pure states of N qubits

$$\mathcal{R} \geq 1/3^N$$

1. *Heisenberg hamiltonian has eigenvalues -3 (singlet) and +1 (triplet).*
2. *We have to restrict the solution to symmetric subspace.*
3. *Hence only even number of singlets is allowed, 2k.*
4. *Therefore the eigenvalues of* $H_{11'} \dots H_{NN'}$ *are given by* $(-3)^{2k}$.
5. *The expectation of* $H_{11'} \dots H_{NN'}$ *is not below the smallest eigenvalue.*

A pure state is entangled if and only if $\mathcal{R} > 1/3^N$

$$|\psi_{1\ldots N}\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_N\rangle$$

- *Operational significance of entanglement*
- *Entanglement is solely characterised by correlations between all parties*
- *Entangled states are more correlated in random measurements*

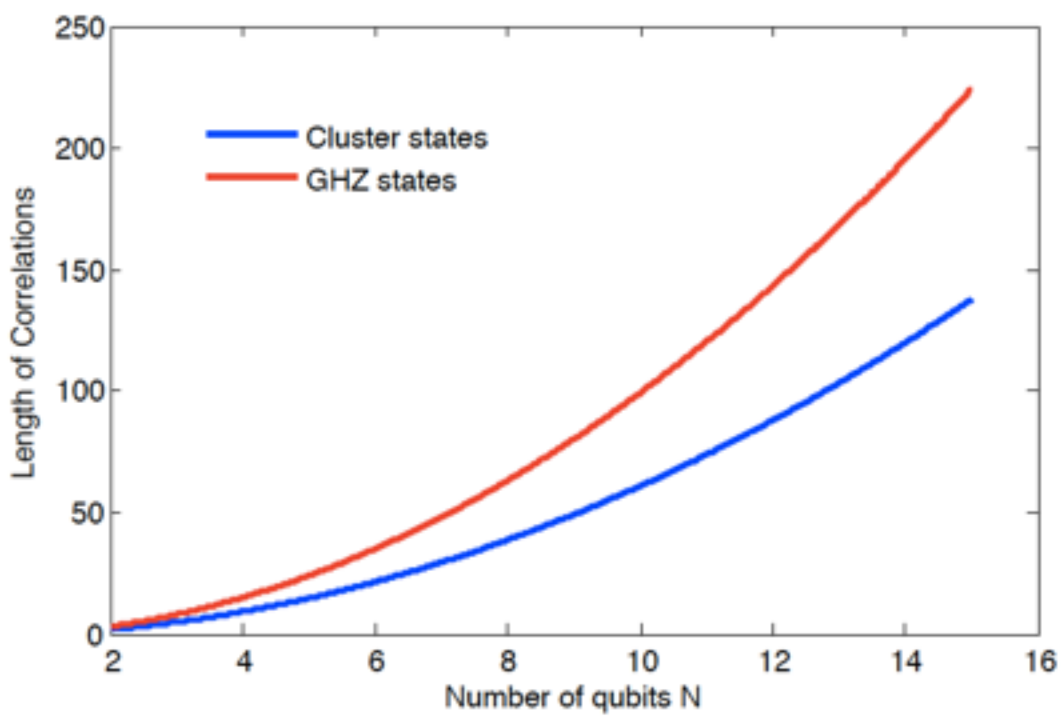
A. S. M. Hassan, P. S. Joag QIC **8**, 773 (2007); PRA **77**, 062334 (2008); PRA **80**, 042302 (2009)

DO RANDOM CORRELATIONS MEASURE ENTANGLEMENT?

MAXIMALLY CORRELATED STATES

$$|\psi_{\max}\rangle = \frac{1}{\sqrt{2}}(|0\ldots0\rangle + |1\ldots1\rangle)$$

2D CLUSTER STATES MODERATE CORRELATIONS



ENTANGLEMENT DOES NOT INCREASE UNDER LOCC

IN PARTICULAR THIS APPLIES TO LOCAL MEASUREMENTS

$$\mathcal{E}(\psi) \geq \sum_j p_j \mathcal{E}(\psi_j)$$

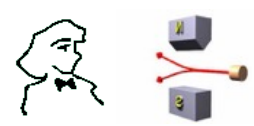
state before measurement — $\mathcal{E}(\psi)$

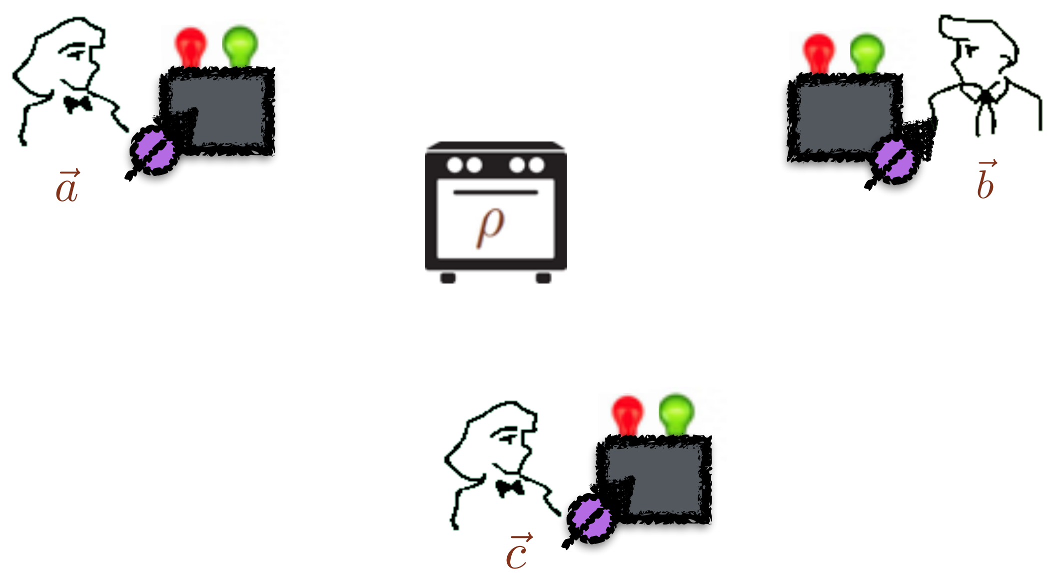
probability of jth result — p_j

state after result j is obtained — $\mathcal{E}(\psi_j)$

5-QUBIT COUNTEREXAMPLE

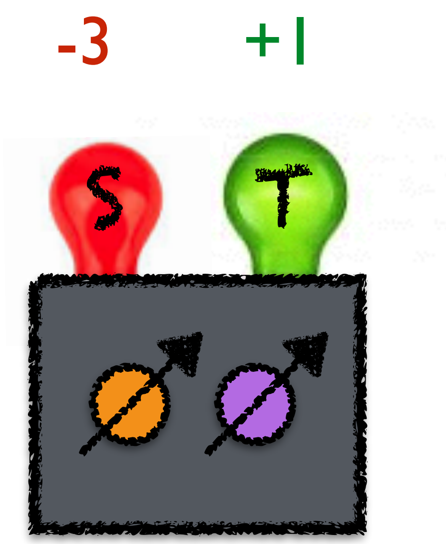
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\psi_0\rangle + |1\rangle|\psi_1\rangle) \qquad \mathcal{E}(\psi) = 8 \qquad \mathcal{E}(\psi_0) = \mathcal{E}(\psi_1) = 9$$





$$\rho = \frac{1}{2^3} \sum_{\mu,\nu,\eta=0}^{3} T_{\mu\nu\eta}\, \sigma_\mu \otimes \sigma_\nu \otimes \sigma_\eta$$

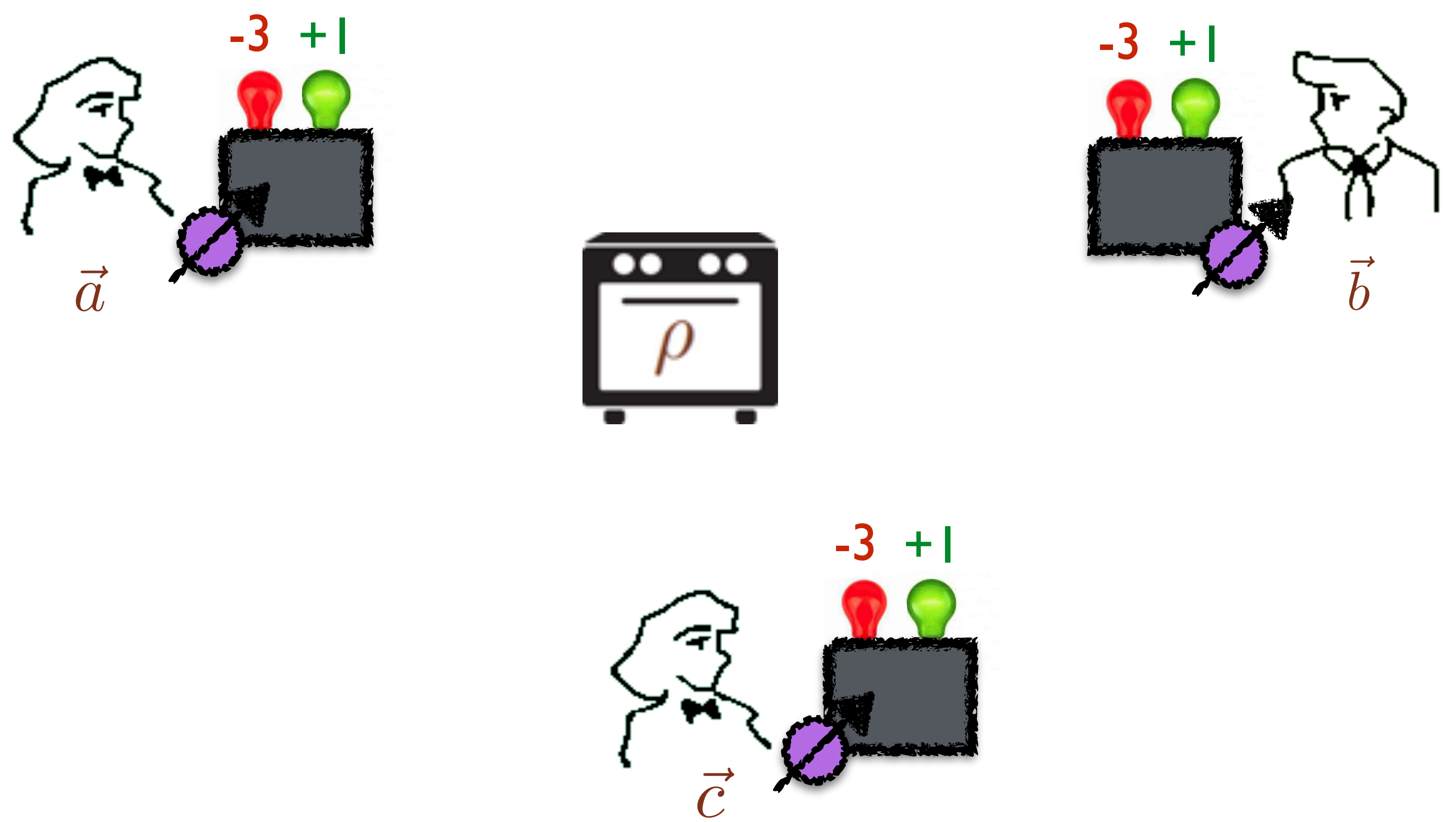
$$E(\vec{a},\vec{b},\vec{c}) = \sum_{j,k,l=1}^{3} T_{jkl}\, a_j b_k c_l$$



Observable of the nth party

$$\mathcal{M} = +1(\mathbb{1} - |\psi^-\rangle\langle\psi^-|) - 3|\psi^-\rangle\langle\psi^-| = \sum_{j=1}^{3} \sigma_j \otimes \sigma_j$$

CORRELATIONS WITH MICROSCOPIC REFERENCES

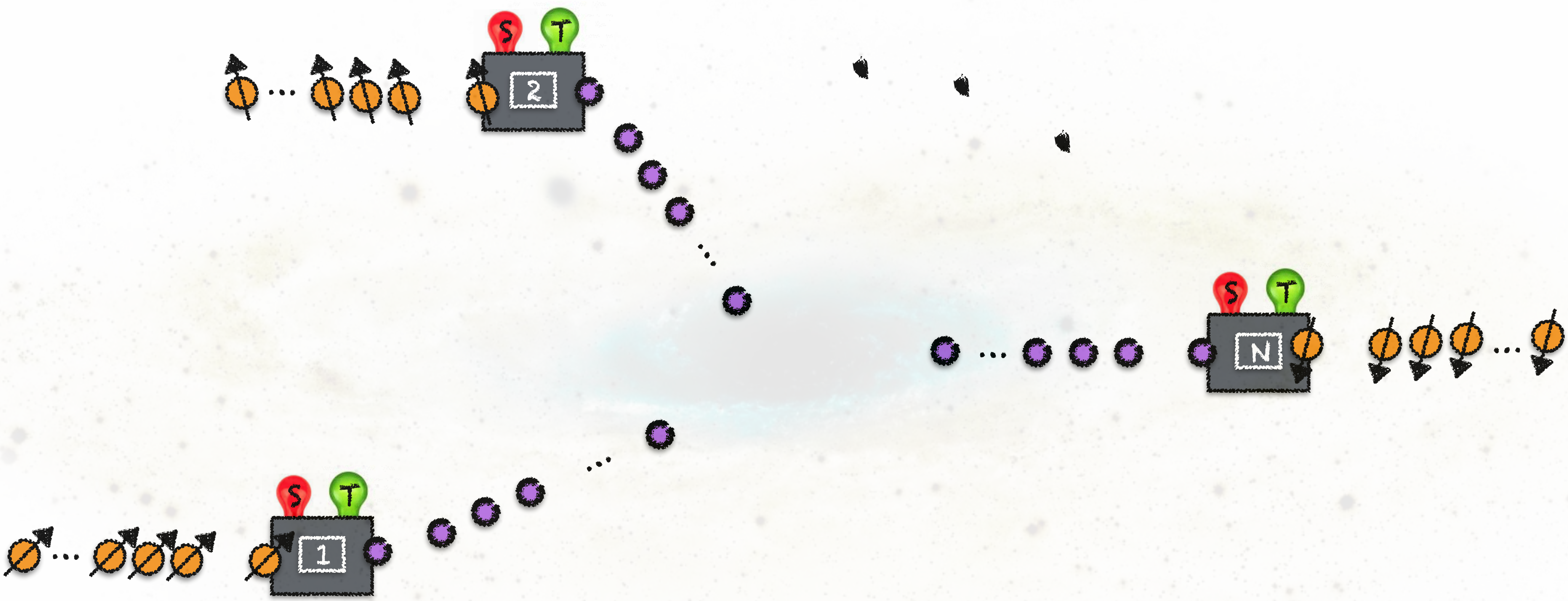


$$E(\vec{a},\vec{b},\vec{c}) \longrightarrow \rho_{123} \otimes a \otimes b \otimes c \longrightarrow \sum_{j,k,l=1}^{3} T_{jkl}\, a_j b_k c_l$$

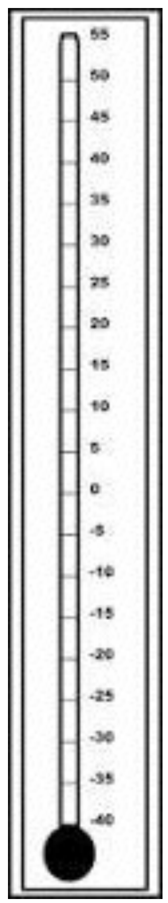
$\mathcal{M}_A$ $\mathcal{M}_C$

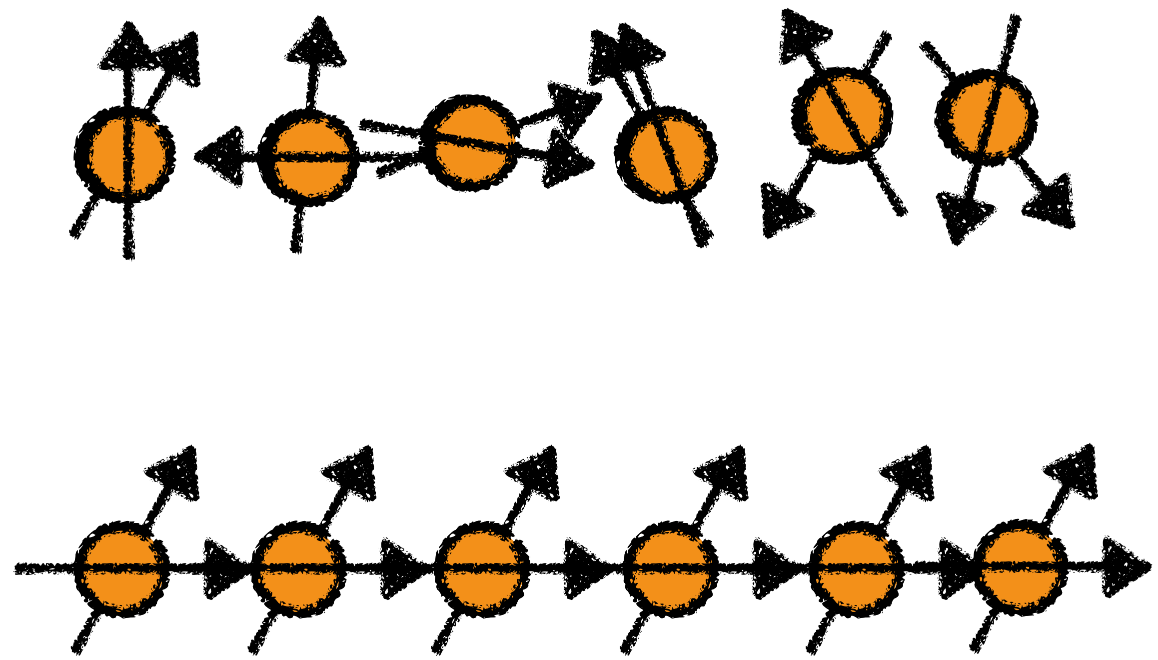
$\mathcal{M}_B$

ENTANGLEMENT DETECTION WITH MINIMAL INDEPENDENT FRAMES

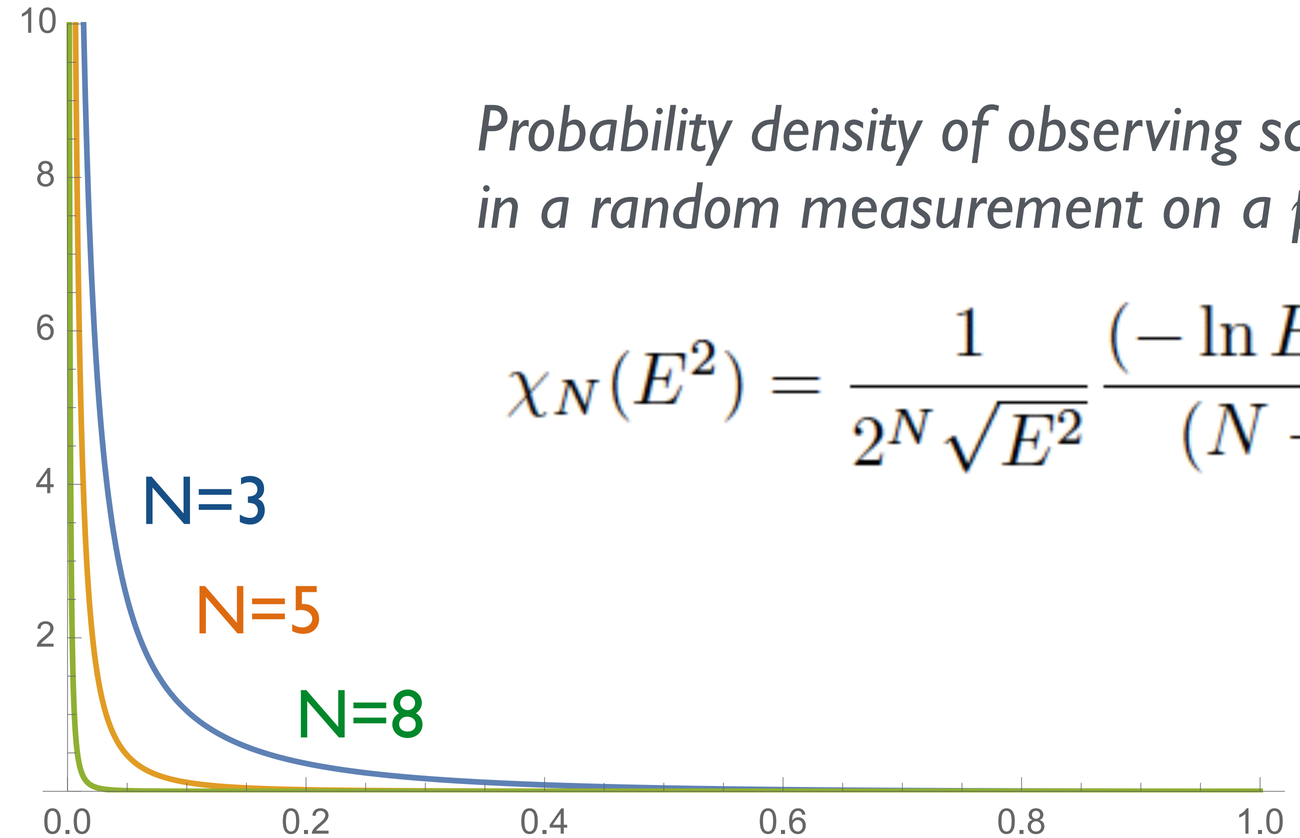


SPONTANEOUS MAGNETISATION





ENTANGLEMENT DETECTION WITH A SINGLE RANDOM SETTING



Probability density of observing squared correlations in a random measurement on a product state

$$\chi_N(E^2) = \frac{1}{2^N\sqrt{E^2}}\frac{(-\ln E^2)^{N-1}}{(N-1)!}$$

Probability to detect GHZ entanglement with confidence 95.4%

N	3	4	5	6	7	8	9	10
	26%	44%	48%	63%	67%	77%	80%	86%

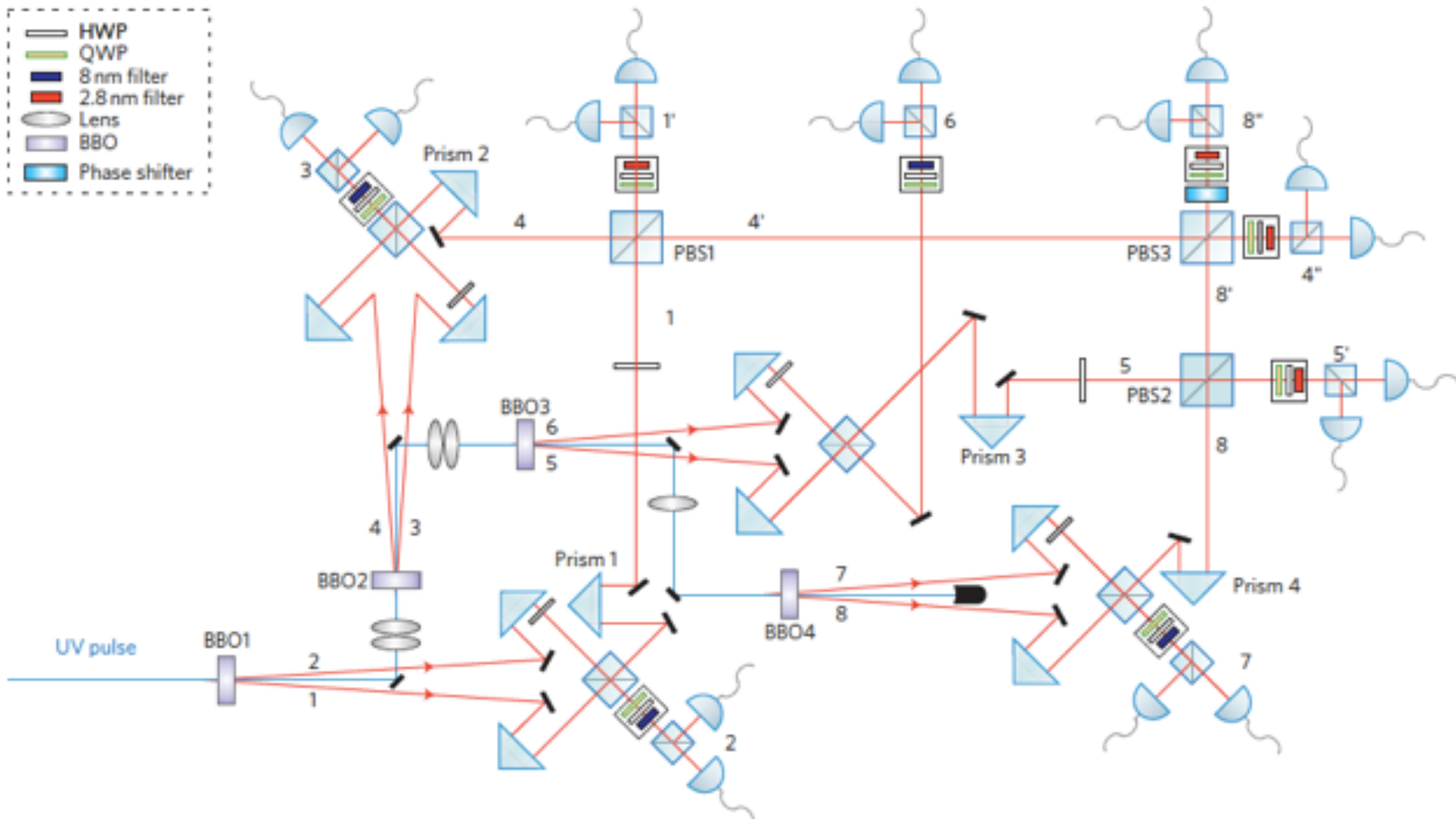
M: *number of random measurement settings*
K: *number of experimental runs to estimate correlations*

$$\mathcal{R}_{M,K} > 1/3^N + \gamma\,\Delta_{M,K}$$

then likely state is entangled

Probability to detect GHZ entanglement in 1000 trials with confidence 95.4%

N	3	4	5	6	7	8	9	10
	26%	44%	47%	57%	52%	48%	41%	34%
	26%	44%	48%	63%	67%	77%	80%	86%



Coincidence click only about every 6 minutes!

Density matrix reconstruction would take 75 years!

With the random setting you likely detect entanglement in 4.5 days

X.-C. Yao, T.-X. Wang, …, J.-W. Pan, Nature Phot. **6**, 225 (2012)

SUMMARY

Pure state entanglement is solely characterised by N-party correlations

Entangled states are more correlated in random measurements

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