General Relativity

Institute of Physics Bhubaneshwar

Homework 7

Textbook: Sean Carroll's *Spacetime and Geometry*

Remember each homework carries weight.

Late submissions will not be accepted.

1. In this problem I help you to verify a deep property of gravitational plane waves in transverse traceless gauge. The property is that as the gravitational wave passes, <u>coordinates</u> of a slowly moving, freely falling, particle do not change. So it does not make sense to make a gravitational wave detector with only one particle. With two particles, although the <u>coordinates</u> of each particle do not change, the <u>proper distance</u> between them can change. This change is what LIGO measures (LIGO has two mirrors).

   Now the problem: Consider a particle moving on a geodesic in a gravitational plane wave in the transverse traceless gauge. Let τ be the proper time of the particle. The geodesic equation is

   $$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau^2}\frac{dx^\sigma}{d\tau} = 0. \tag{0.1}$$

   Rewrite these equations in such a way that the time coordinate becomes the parameter, and you get an expression for the coordinate acceleration $\ddot{x}^i = \frac{d^2x^i}{dt^2}$. The answer is

   $$\ddot{x}^i = -\left[\Gamma^i_{tt} + 2\Gamma^i_{tj}v^j + \Gamma^i_{jk}v^jv^k\right] + \left[\Gamma^t_{tt} + 2\Gamma^t_{ti}v^i + \Gamma^t_{jk}v^jv^k\right]v^i, \tag{0.2}$$

   where $v^i = \dot{x}^i = \frac{dx^i}{dt}$. Now make the assumption $v^i \ll 1$ and compute the relevant components of Γ's to show that the resulting equation is

   $$\ddot{x}^i = 0. \tag{0.3}$$

   Hence the particle doesn't suffer any acceleration. If it is stationary it remains stationary. The gravitational wave doesn't effect the location of the particle.