General Relativity

Institute of Physics Bhubaneshwar

Homework 5

Textbook: Sean Carroll's *Spacetime and Geometry*

Remember each homework carries weight.

Late submissions will not be accepted.

1. Consider the metric in $d+1$ spacetime dimensions,

$$ds^2 = -dt^2 + R^2(t)h_{ij}dx^i dx^j, \tag{0.1}$$

   where h_{ij} is a round metric on the $d-$dimensional sphere of unit radius, i.e., its Ricci scalar is $d(d-1)$, and $R(t)$ is a general function of time t. Compute the $(d+1)$ dimensional Ricci scalar for this metric.

   [HINTS: (i) Do not be lazy. (ii) The answer is: $2d\ddot{R}R^{-1} + d(d-1)(1+\dot{R}^2)R^{-2}$.]

2. Follow all steps and show that the variation of the Hilbert action (4.55) gives the Einstein's equations (4.72).

3. Compute the stress-tensor (4.79) of the scalar field starting with the Lagrangian (4.52) and considering variations with respect to the inverse metric $g^{\mu\nu}$.