Clustering in light, heavy & superheavy nuclei using Relativistic Mean Field Formalism

S.K. Patra

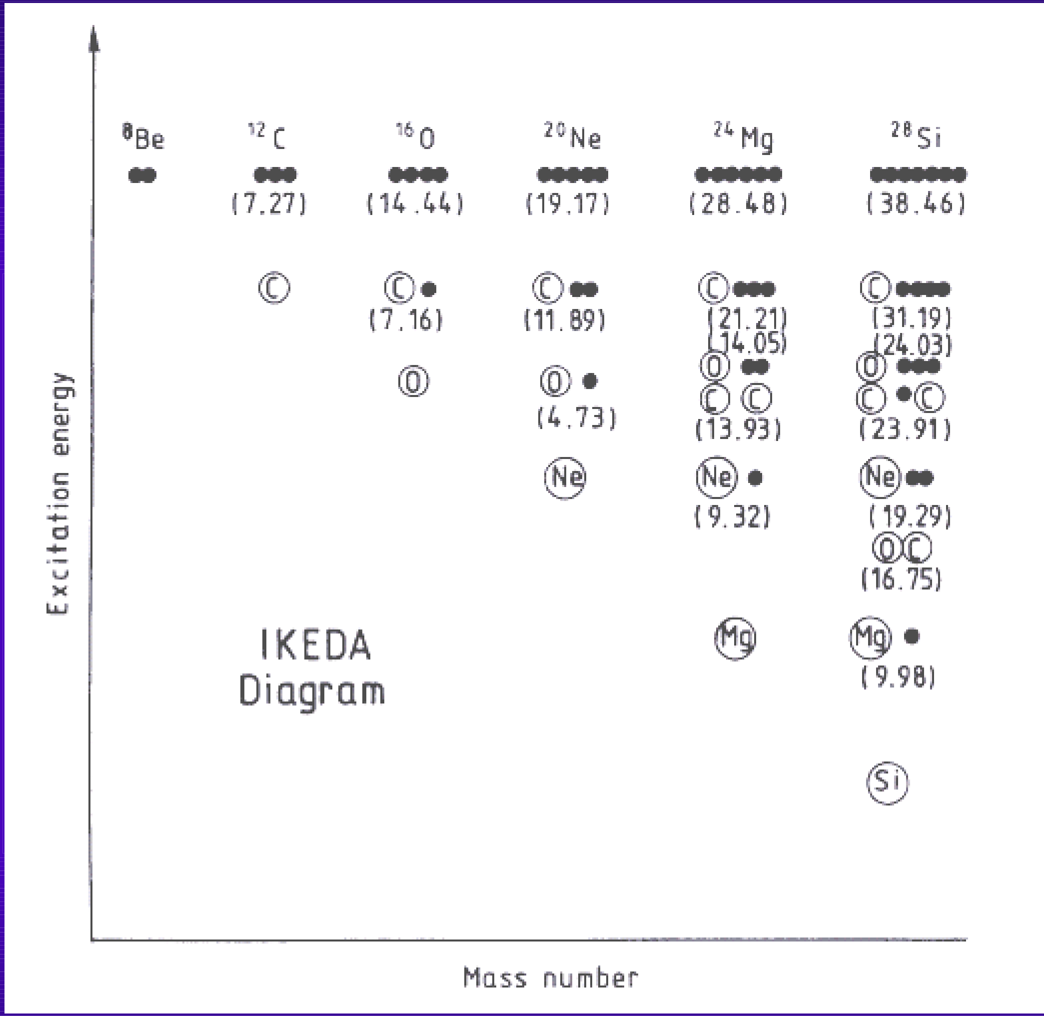
Institute of Physics, Bhubaneswar, INDIA

Alumni day 2006

Importance of clustering phenomena (light nuclei)

- The discovery of highly deformed and superdeformed shapes in N=Z light nuclei , PRL 85 (2000) 2693; for a review, IJMPE 13 (2004) 9.
- Several theoretical predictions as well as of experimental discovery of clustering structures from last 40 years.
- Quasimolecular resonances have also been observed for α–like nuclei (ex. $^{28}Si+^{28}Si$ reaction).
- Interpretation of resonant structures with an excitation energy of 30-50 MeV is a subject of contemporary debate.

Schematic Structure of α-nuclei



K. Ikeda, N. Takigawa and H. Horiuchi, Suppl. Prog. Phys., 464 (1968)

Models

- Mean-field models such as the shell model becomes insufficient.
- Most of the cluster models the existence of clusters is assumed *apriori* (e.g. cranked cluster model)
- *ab-initio* calculations also could explain clustering.

  Fermionic molecular dynamics (FMD) and antisymmetrized molecular dynamics (AMD) which describes well, several nuclei and their excited states in the lighter mass region.
- Here we analyze how well RMF calculations could be useful in studying the clustering.

RMF formalism

$$\mathcal{L} = \overline{\psi_i}\{i\gamma^\mu \partial_\mu - M\}\psi_i + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_s\overline{\psi_i}\psi_i\sigma$$
$$-\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_w^2 V^\mu V_\mu + \frac{1}{4}c_3(V_\mu V^\mu)^2 - g_w\overline{\psi_i}\gamma^\mu\psi_i V_\mu - \frac{1}{4}\vec{B}^{\mu\nu}.\vec{B}_{\mu\nu}$$
$$+\frac{1}{2}m_\rho^2\vec{R}^\mu.\vec{R}_\mu - g_\rho\overline{\psi_i}\gamma^\mu\vec{\tau}\psi_i.\vec{R}^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\psi_i}\gamma^\mu\frac{(1-\tau_{3i})}{2}\psi_i A_\mu.$$

$$\rho(r) = \sum_\alpha \varphi_\alpha^\dagger(r)\varphi_\alpha(r)$$

$$\rho_{\mathrm{p}}(r) = \sum_\alpha \varphi_\alpha^\dagger(r)\left(\frac{1+\tau_3}{2}\right)\varphi_\alpha(r)$$

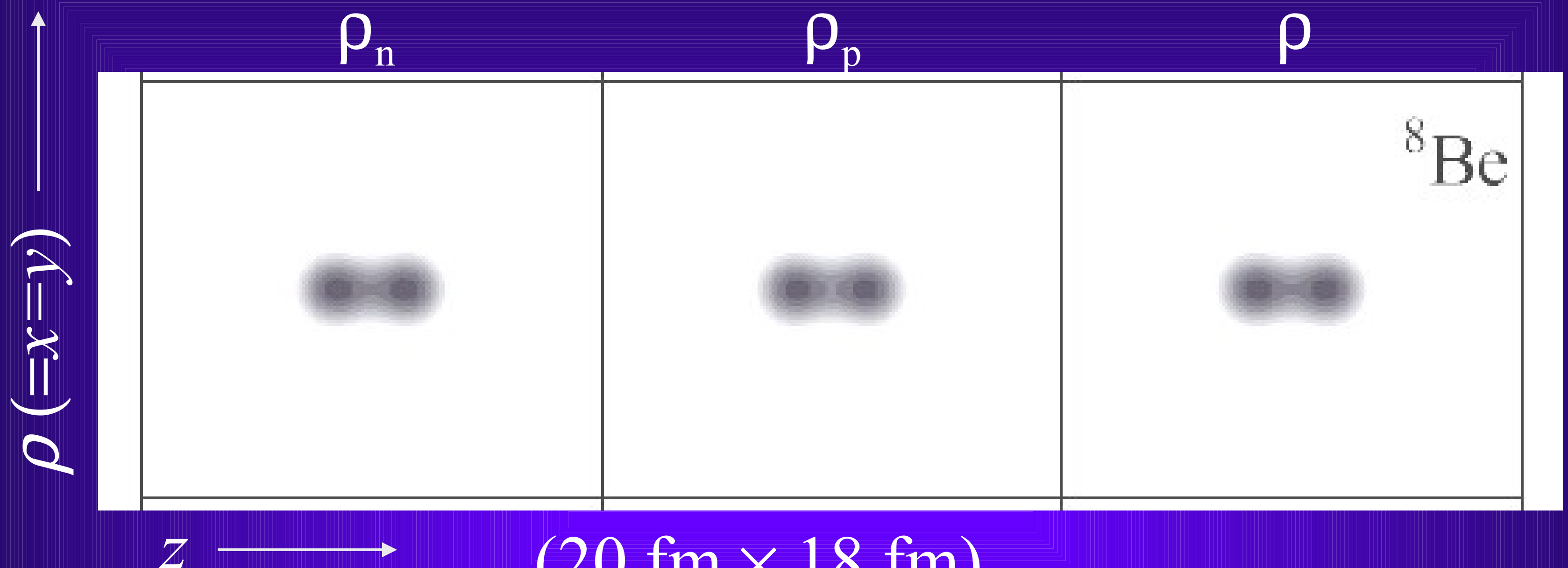
- NL3 & NL-SH parameter sets
- No pairing for light nuclei, BCS pairing for heavy & superheavy nuclei
- Axially deformed oscillator basis
- **Densities are obtained in $z\rho$ - plane**

Details formalism & results:

Are available in:

- P. Arumugam et al., Phys. Rev. C71 (2005) 064308.
- B.K. Sharma et al., J. Phys. G32 (2006) L1.
- S.K. Patra et al., Phys. Rev. C (communicated).
- R.K. Gupta et al., J. Phys. G (communicated)
- Some other works in preparation.

Clusters in RMF



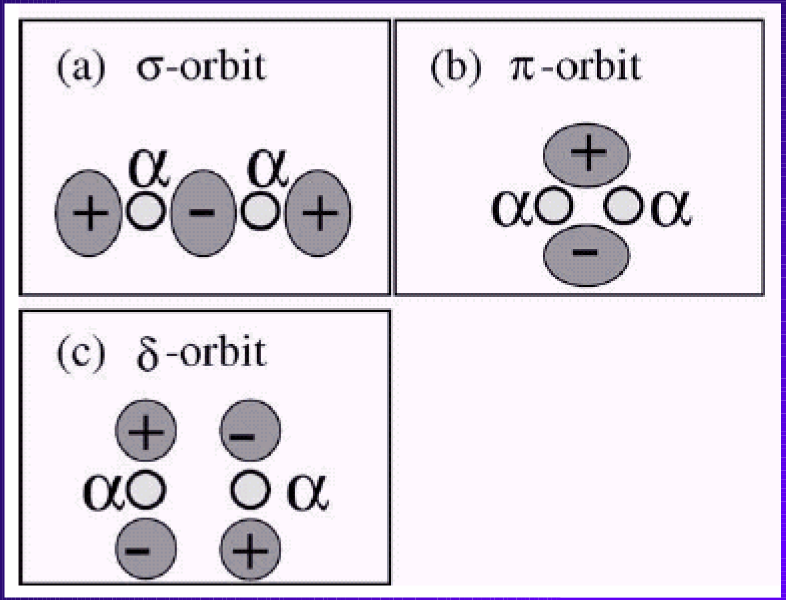
(20 fm × 18 fm)

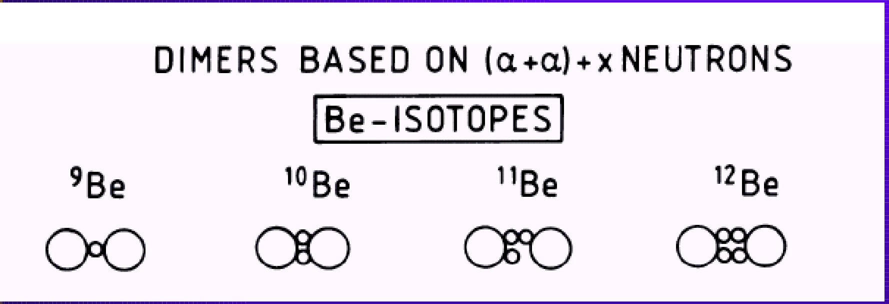
Clusters do appear in RMF calculations

Present limitations

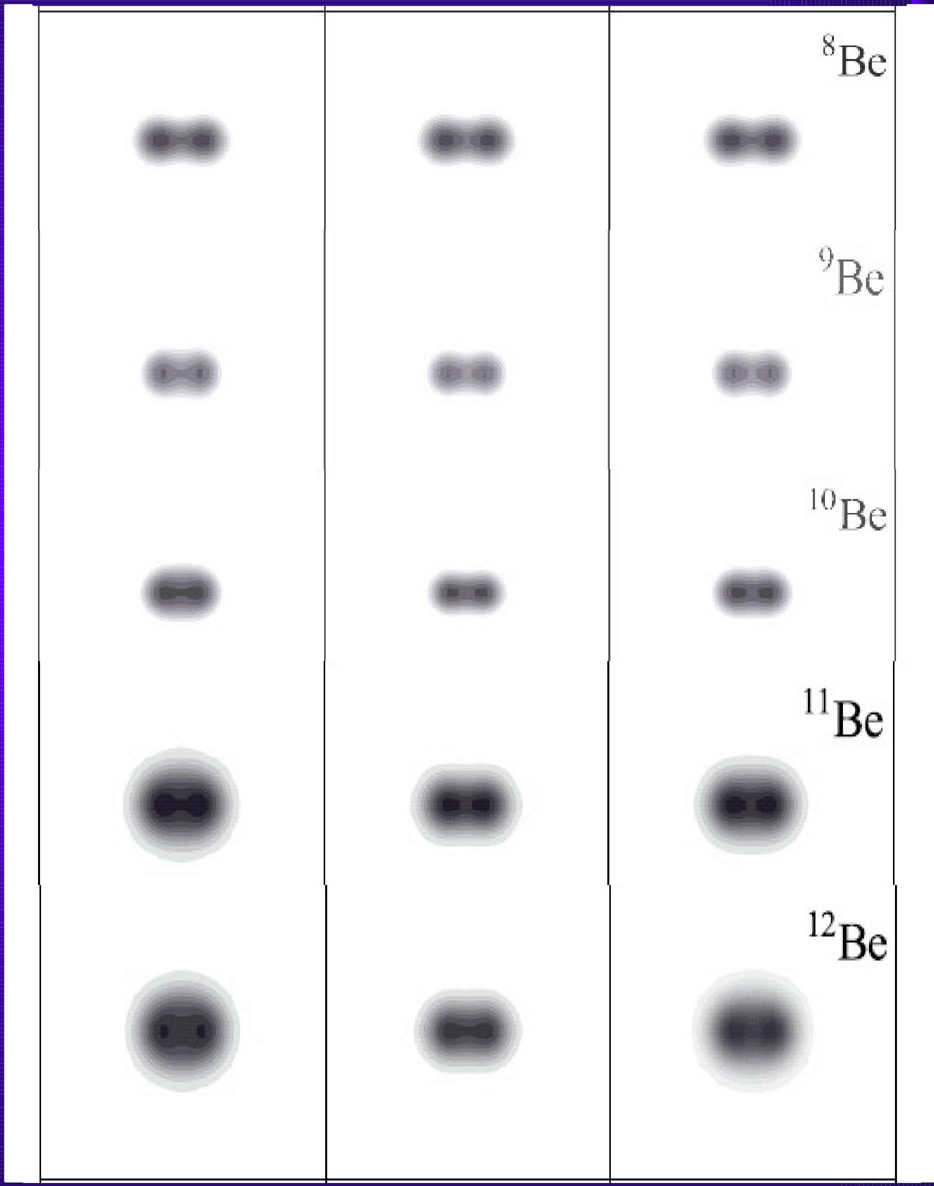
- Only axially deformed shapes
- No parity asymmetric structures

GS bonding (Dimers)

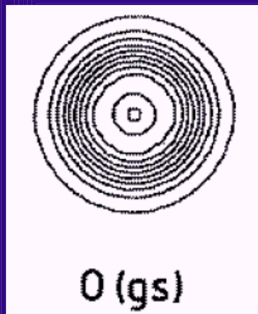




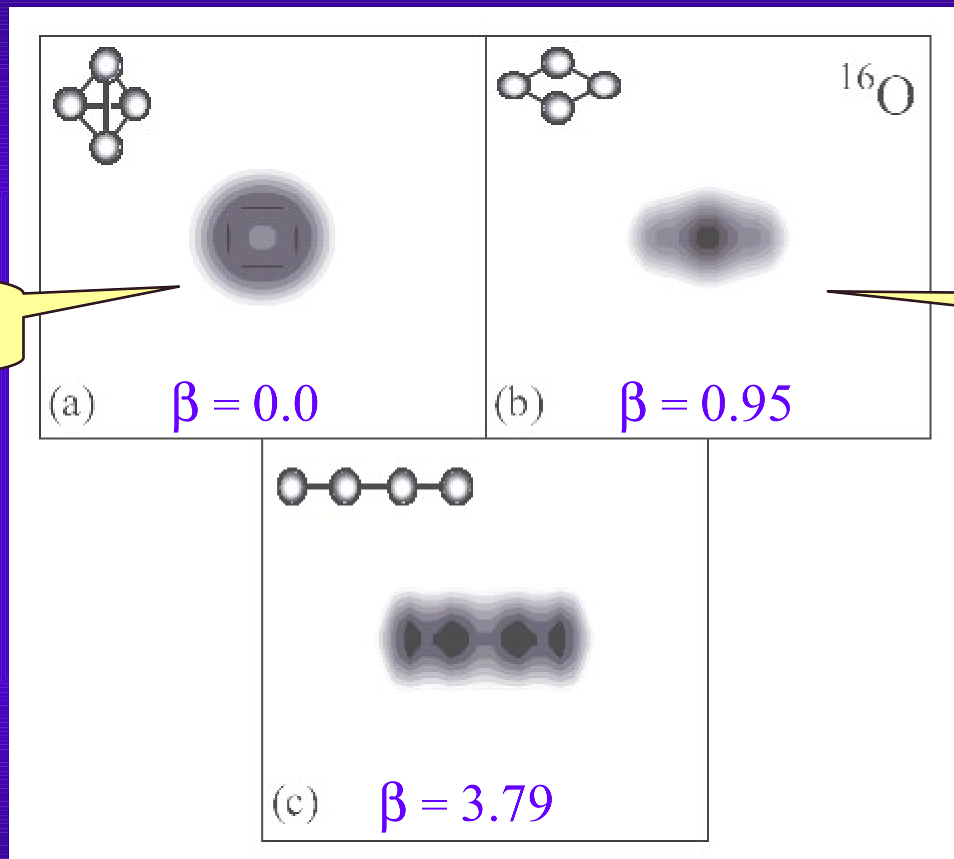
W.v. Oertzen, Nuovo Cimento **110A** 895 (1997).

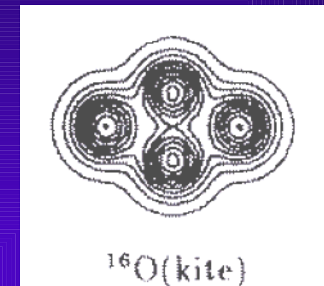


Clustering in α-nuclei



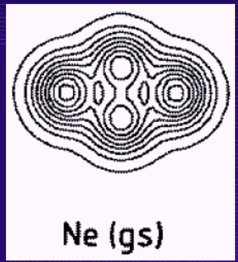
Tetrahedron



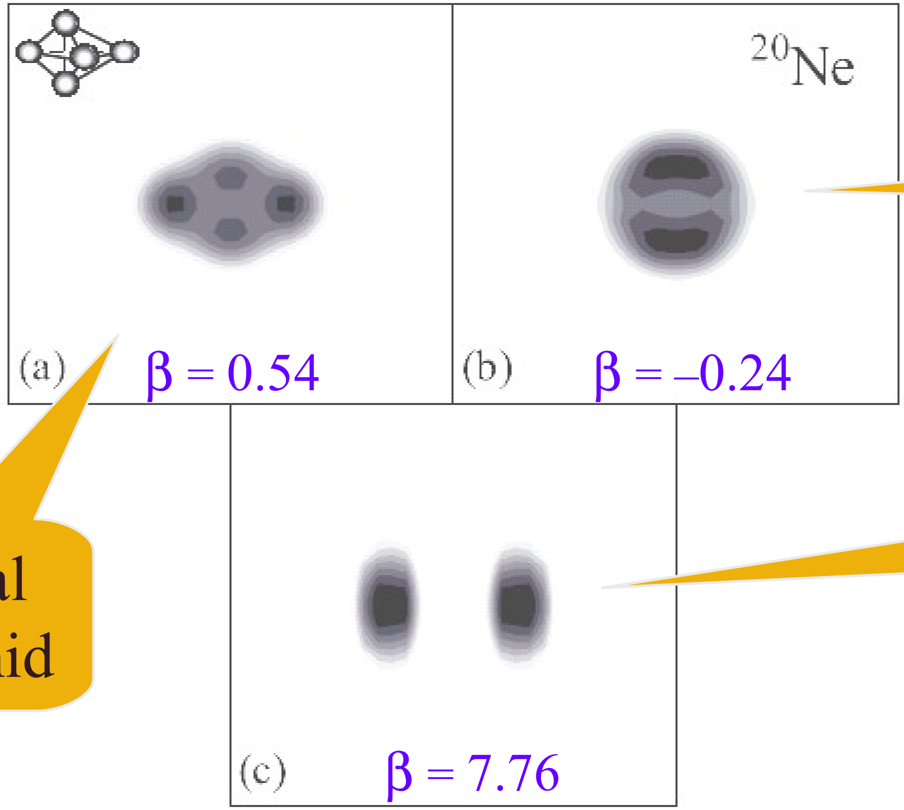


Kite

CCM: NPA575, 61 (1994), NPA564, 252 (1993)



CCM



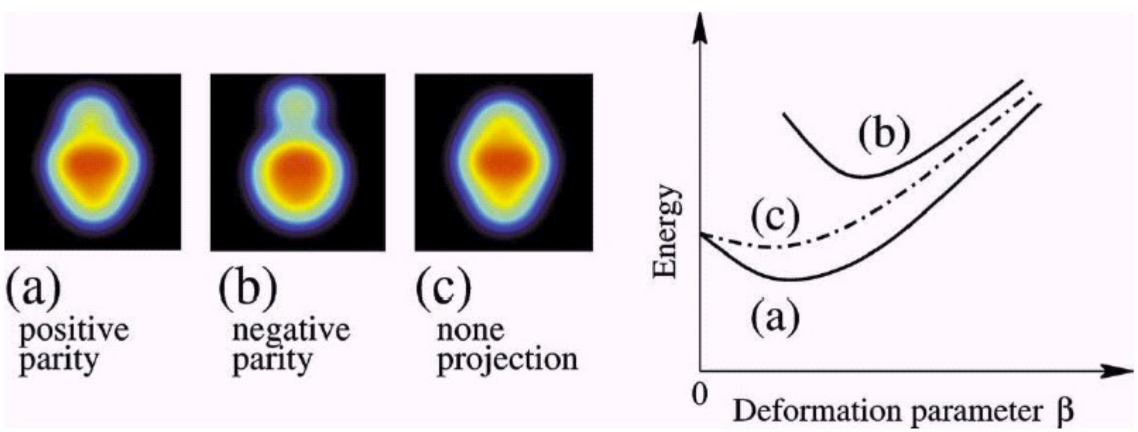
$^{10}B + ^{10}B$

Trigonal Bipyramid

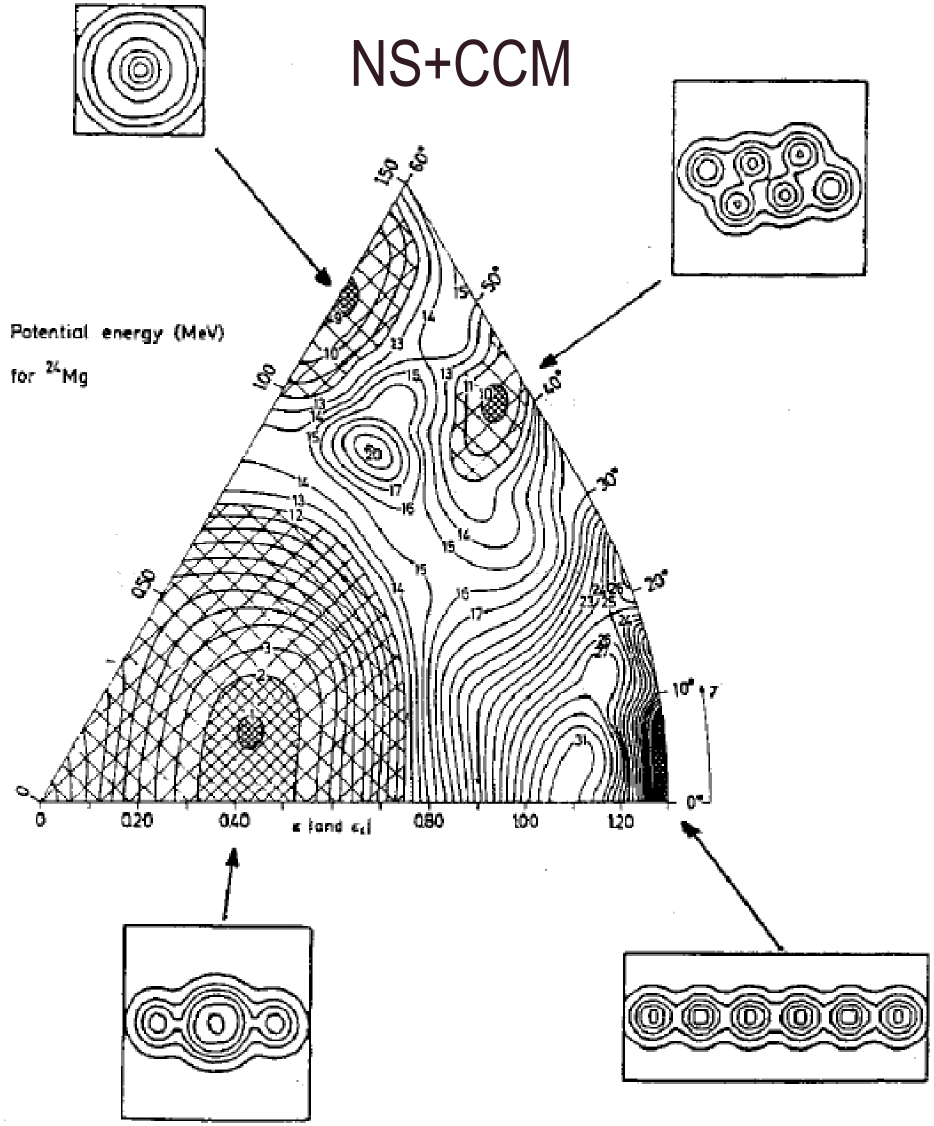
Fission Fragments

No 5α chain

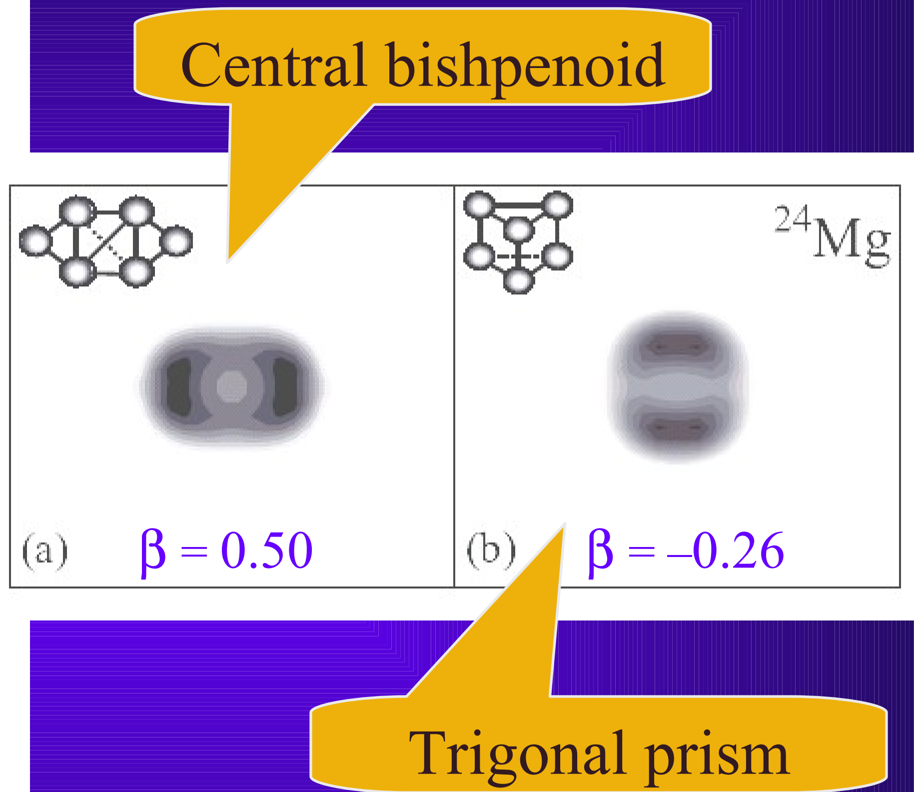
AMD



NS+CCM

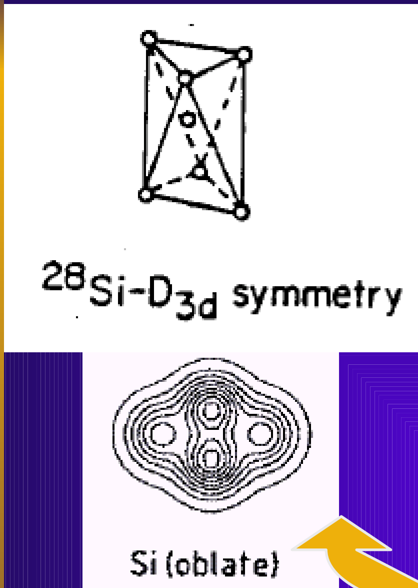


Potential energy (MeV)
for 24Mg
0
0.20
0.40
0.80
1.00
1.20

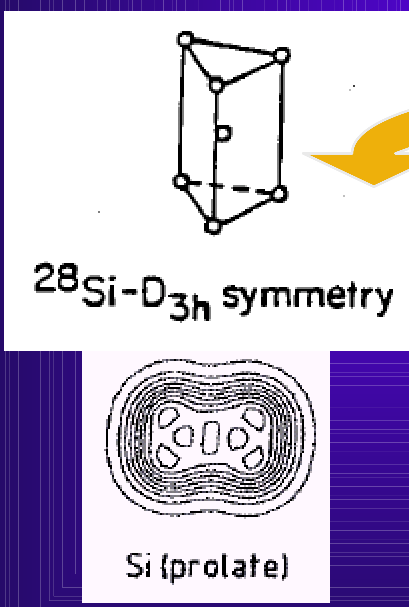


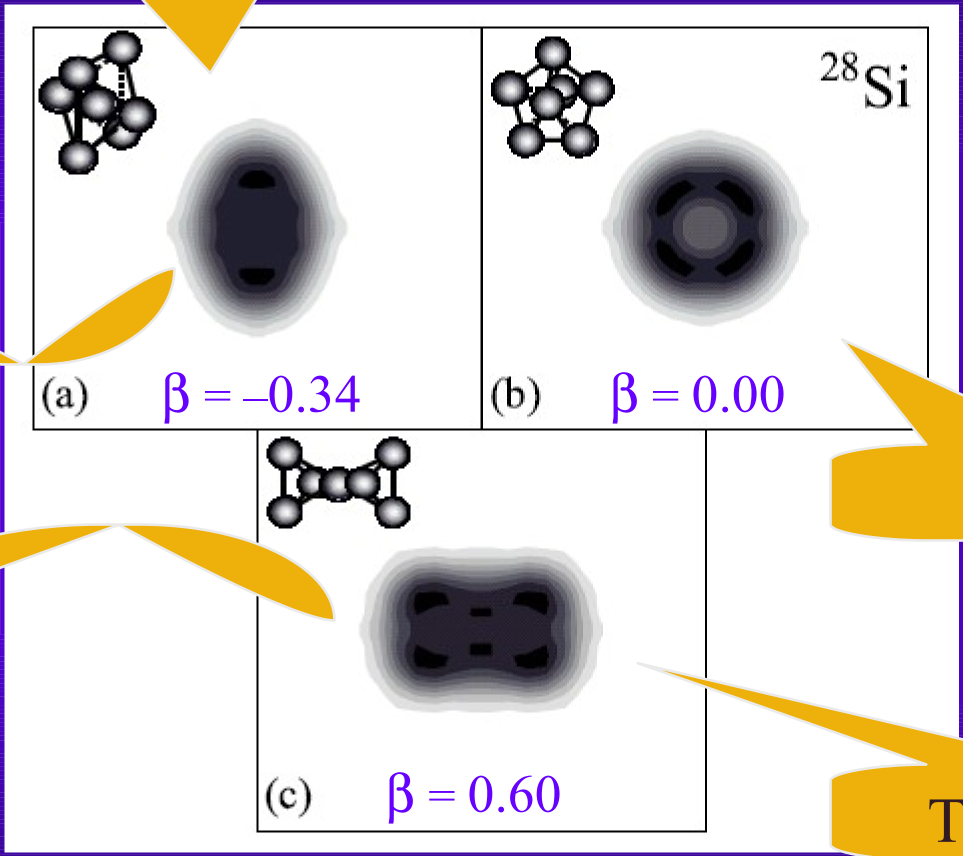
Central bishpenoid
24Mg
(a) β = 0.50
(b) β = –0.26
Trigonal prism

Tetragonal bipyramid



CCM





Pentagonal biprism

Trigonal prism

Nucleus	B.E. (MeV) Theo.	B.E. (MeV) Expt.	β_2 Theo.	β_2 Expt.	Probable structure
^{12}C	89.74	92.16	−0.29	0.58	3α - Equilateral triangle
	89.63		0.00		Spherical
	72.55		2.33		3α - Linear chain
^{16}O	128.84	127.62	0.00		4α - Tetrahedron
	112.95		0.95		4α - Kite
	92.28		3.79		4α - Linear chain
^{20}Ne	156.70	160.64	0.54	0.73	5α - Trigonal bipyramid
	151.96		−0.24		^{10}B+^{10}B
	108.24		7.76		^{10}B+^{10}B (fragments)
^{24}Mg	194.37	198.26	0.50	0.61	^{12}C+^{12}C (- Central bishpenoid)
	186.82		−0.26		^{12}C+^{12}C - Trigonal biprism
^{28}Si	232.08	236.54	−0.34	0.41	D_{3d} symmetry
	231.18		0.00		Hollow sphere (Pentagonal bipyramid)
	224.11		0.60		^{12}C+α+^{12}C Trigonal biprism
^{32}S	265.96	271.78	0.25	0.31	^{16}O+^{16}O (Kite)
	256.38		1.03		^{16}O+^{16}O (Tetrahedron)

Results for Be & B isotopes

TABLE I: Calculated binding energies and deformation parameters for the Be and B isotopes along with experimental data.

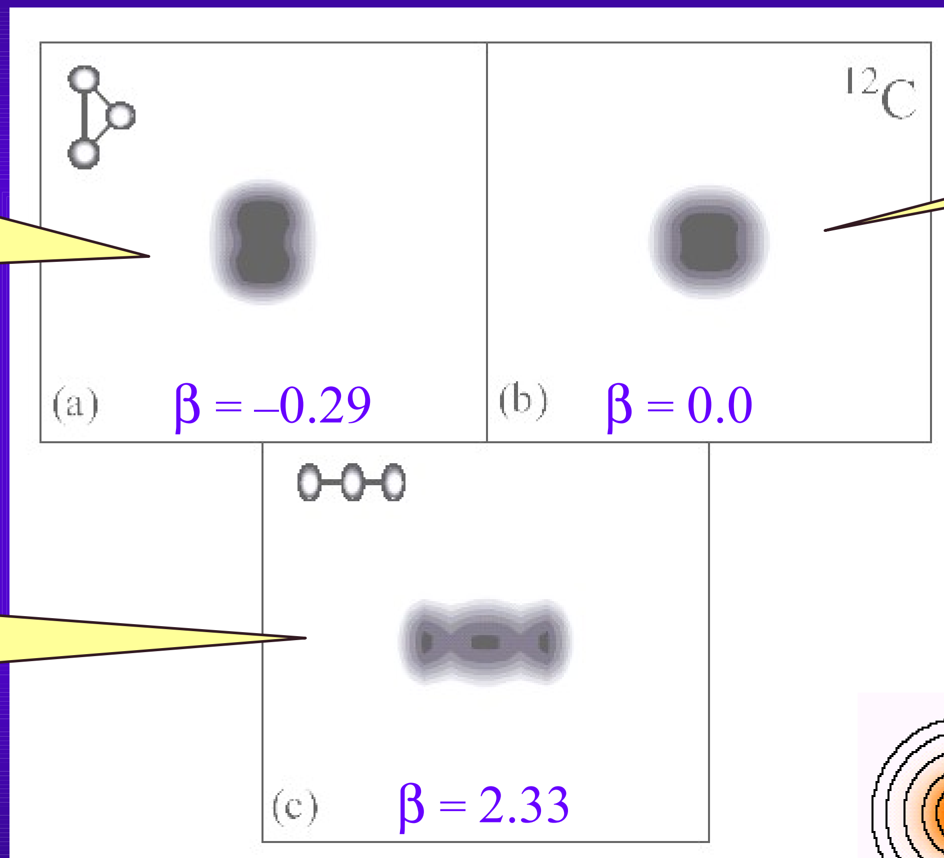
	B.E. (MeV)		β_2 (Theo.)		
Nucleus	Theo.	Expt.	Neutron	Proton	Total
^{6}Be	31.28	26.92	0.23	1.15	0.84
^{7}Be	40.69	37.60	0.90	1.20	1.07
^{8}Be	52.76	56.50	1.18	1.20	1.19
^{9}Be	58.02	58.16	0.70	0.90	0.79
^{10}Be	64.87	64.98	0.40	0.67	0.51
^{11}Be	67.74	65.48	0.25	0.58	0.37
^{12}Be	71.80	68.65	0.13	0.48	0.25
^{13}Be	72.28	68.55	0.52	0.62	0.55
^{14}Be	74.37	69.91	0.83	0.74	0.80
^{11}B	76.90	76.20	0.20	0.28	0.23
^{13}B	88.85	84.45	0.05	0.17	0.10
^{15}B	92.48	88.19	0.67	0.44	0.59
^{17}B	94.64	89.53	0.69	0.48	0.62
^{19}B	94.73	90.08	0.43	0.39	0.42

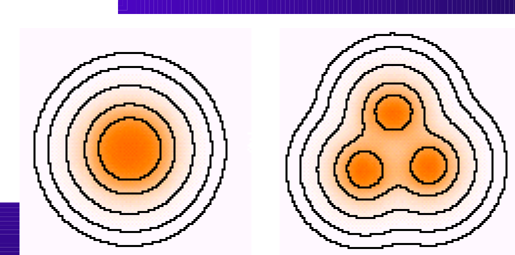
GS & Excited states of α-nuclei

Equilateral triangle

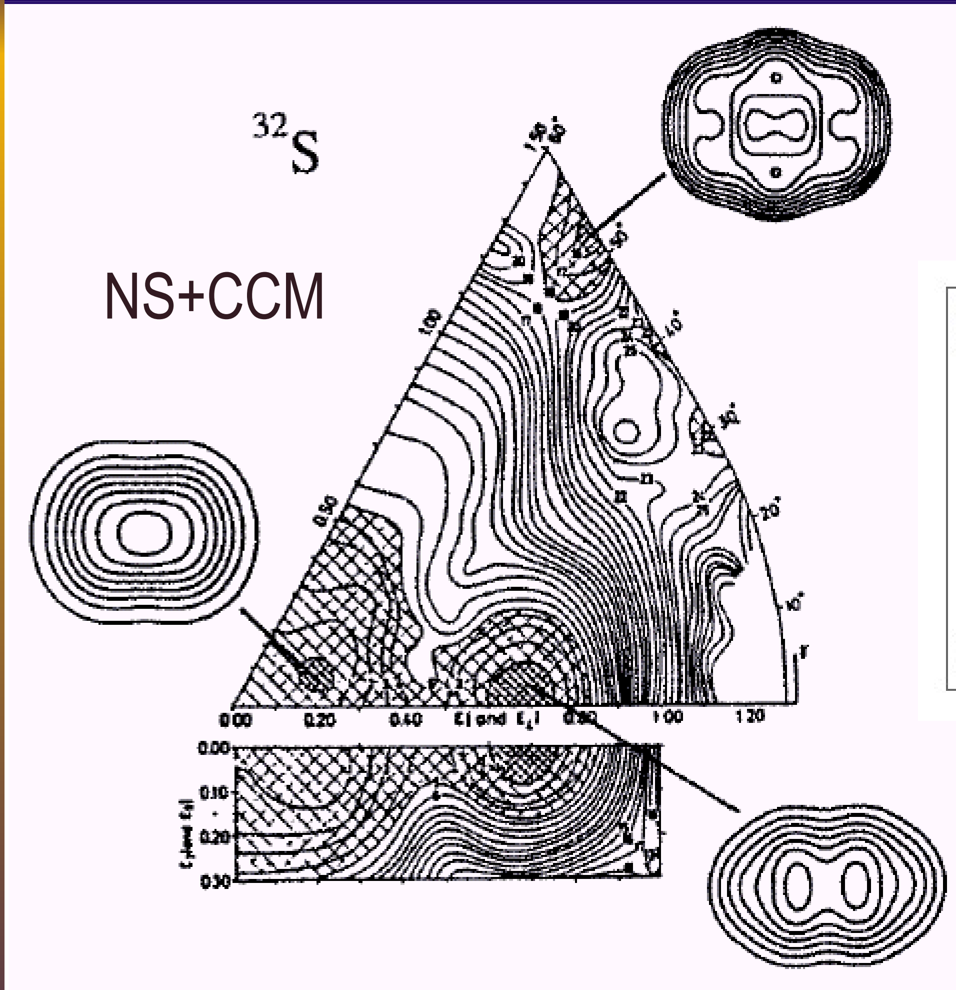
Spherical

3α – linear chain

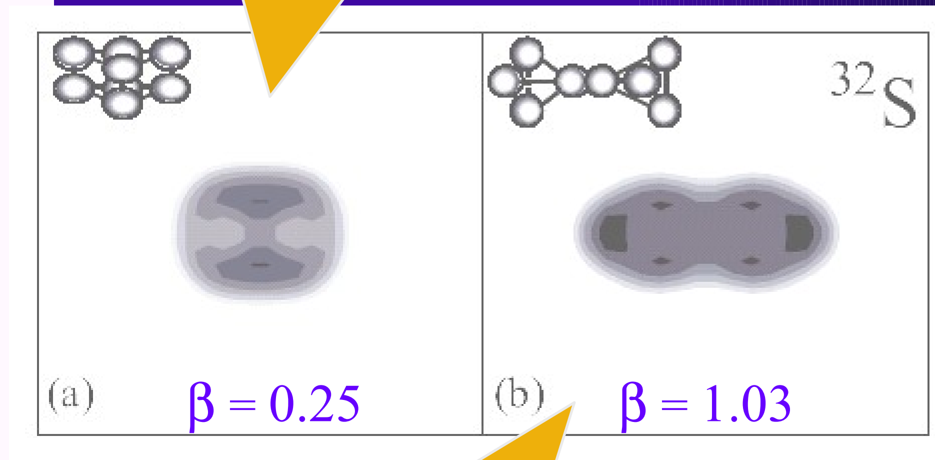




FMD: T. Neff and H. Feldmeier, Arxiv:nucl-th/0312130



$^{16}O+^{16}O$ (Kite)



$^{16}O+^{16}O$ (Tetrahedron)

Exotic Clustering

- Z>87 exotic clustering radioactivity
- Quantum mechanical fragmentation theory predicts, Greiner et al. PRL32 (1974) 548.

  ^{14}C, ^{20}O, ^{23}F, ^{22,24}Ne, ^{28,30}Mg, ^{32,34}Si from various Ra, Ac, Th, U, Pa, Pu and Cm.
- Experimentally observed by Rose & Jones, Nature 307 (1984) 245.
- Prediction of $^{48\text{-}51}$Ca, ^{34}Si, ^{14}C and ^{10}B in the Superheavy region are also there.

Actinides properties (RMF & SHF)

TABLE I: The calculated binding energies BE, the deformation parameters β and the root-mean-square charge radii r_c for some radioactive nuclei, using the relativistic NL3 (upper panel) and non-relativistic SkI4 (lower panel) parameter sets, with pairing interactions included. The experimental data are from Refs. [34–36]; the numbers marked as (*) are for neighbouring nuclei. The energies are in MeV and the radii in fm.

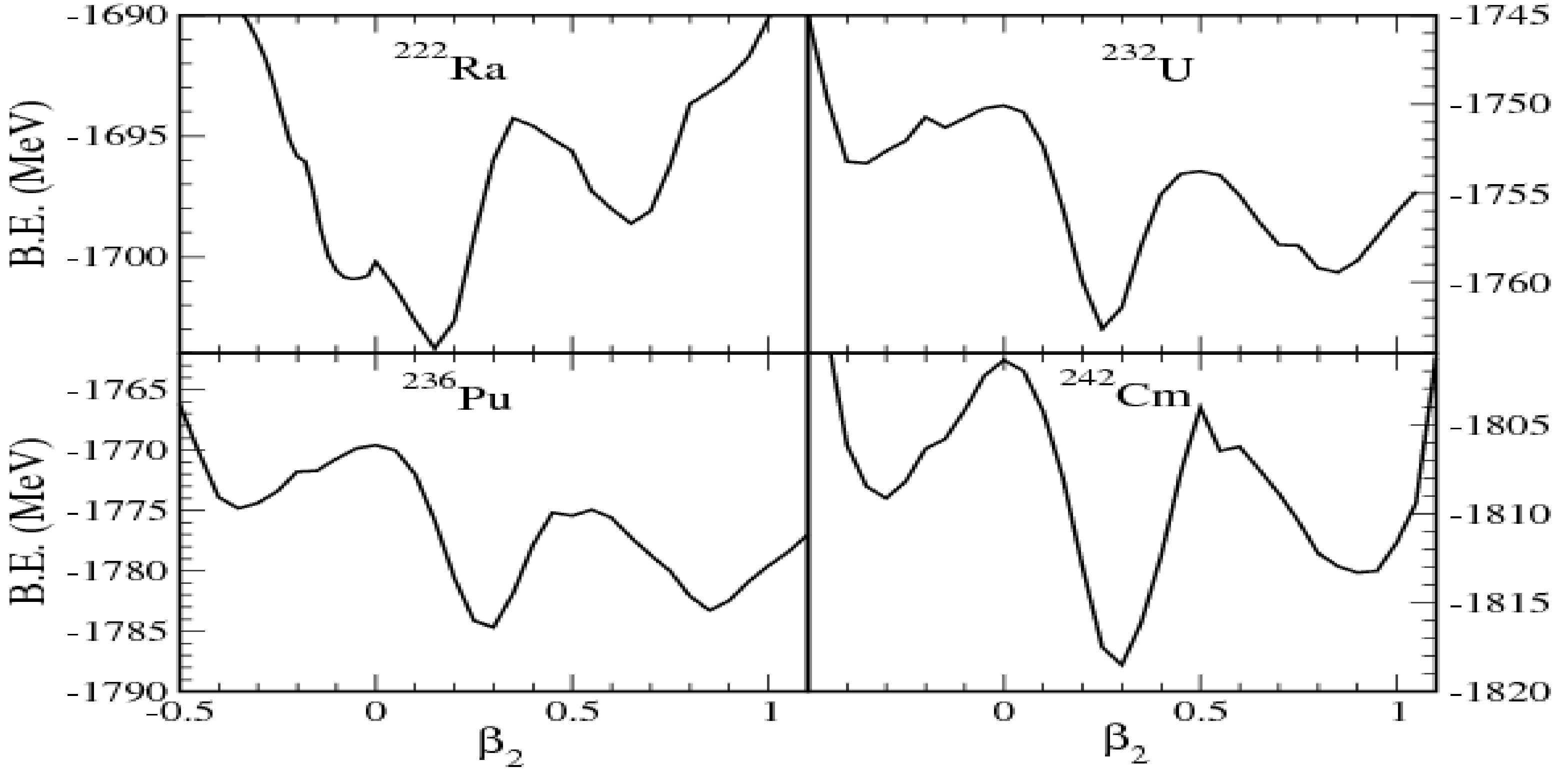
nucleus	RMF(NL3) BE	β	r_c	Expt. BE	β	r_c
^{222}Ra	1711.9	0.082	5.701	1708.734	0.192	5.650
	1708.1	0.658	6.056			
	1704.6	−0.365	5.862			
^{232}U	1768.2	0.251	5.839	1765.984	0.264	5.814*
	1765.4	0.688	6.188			
	1761.3	−0.243	5.866			
^{236}Pu	1791.8	0.272	5.886	1788.408	0.286*	5.825*
	1789.7	0.877	6.402			
	1783.6	−0.339	5.962			
^{242}Cm	1827.1	0.291	5.943	1823.228	0.297*	5.785
	1825.4	0.879	6.466			
	1818.4	−0.276	5.973			
	SHF(SkI4)					
^{222}Ra	1704.6	0.154	5.667			
	1698.9	0.769	6.136			
^{232}U	1762.7	0.259	5.800			
	1759.5	0.837	6.300			
	1753.5	−0.367	5.916			
^{236}Pu	1784.9	0.281	5.848			
	1783.2	0.852	6.355			
	1774.9	−0.357	5.940			
^{242}Cm	1819.5	0.294	5.902			
	1817.3	0.931	6.489			
	1808.9	−0.305	5.944			

SHE properties (RMF & SHF)

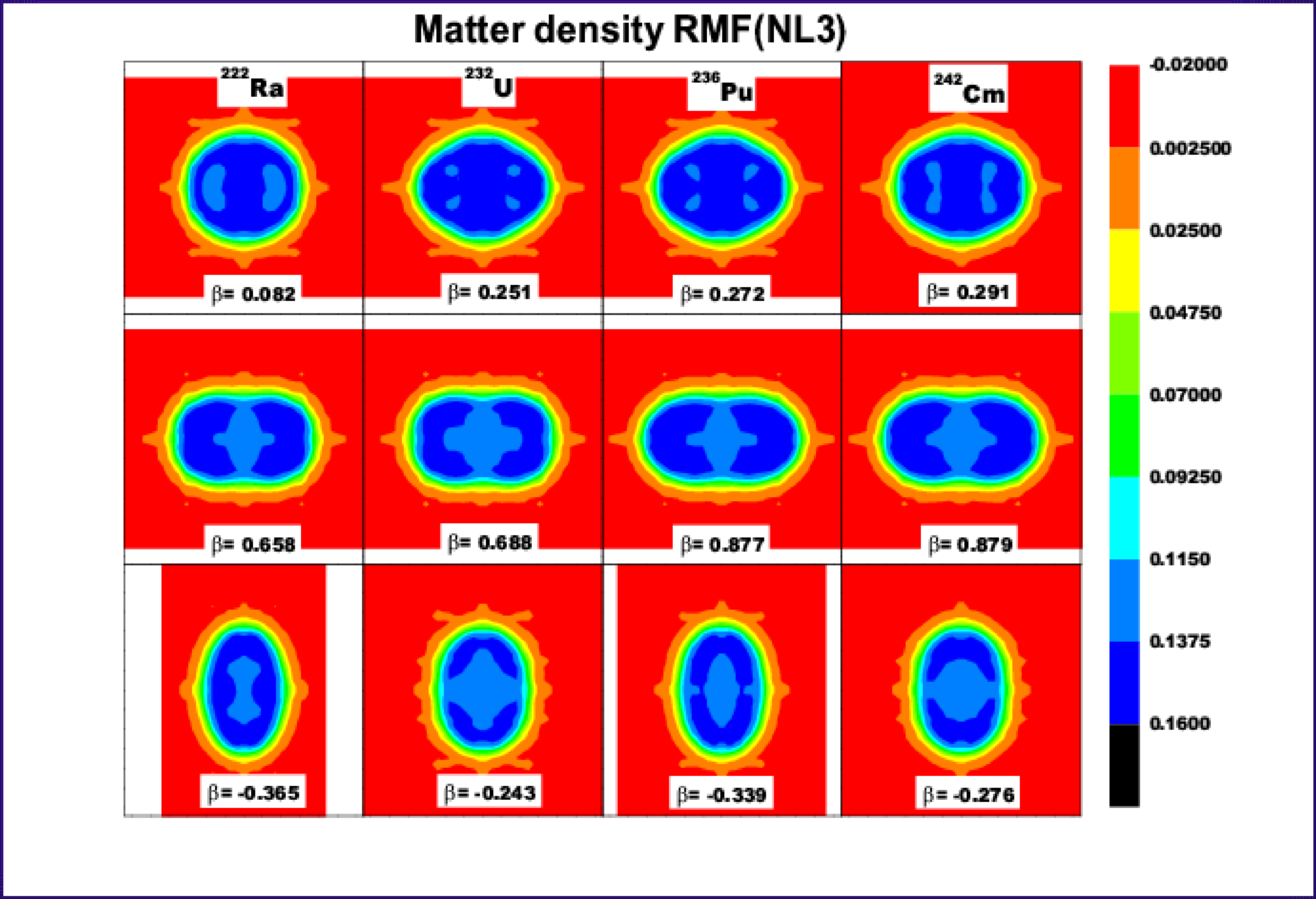
TABLE II: The same as for Table I, but for some superheavy nuclei. No experimental data is available.

nucleus	RMF(NL3) BE	β	r_c
$^{286}114$	2053.5	0.502	6.466
	2052.0	0.000	6.234
	2049.6	−0.346	6.390
$^{290}114$	2078.9	0.505	6.491
	2077.1	−0.212	6.305
	2076.6	0.001	6.246
$^{294}114$	2101.3	0.522	6.526
	2100.1	0.000	6.257
	2099.5	−0.234	6.343
$^{298}114$	2122.4	0.570	6.593
	2121.9	0.000	6.271
	2119.5	−0.312	6.435
	SHF(SkI4)		
$^{286}114$	2043.3	0.152	6.192
	2041.1	0.557	6.462
$^{290}114$	2069.6	−0.009	6.190
	2066.5	0.517	6.452
$^{294}114$	2096.2	−0.01	6.202
	2090.9	0.567	6.519
$^{298}114$	2121.0	0.000	6.215
	2113.3	0.562	6.539

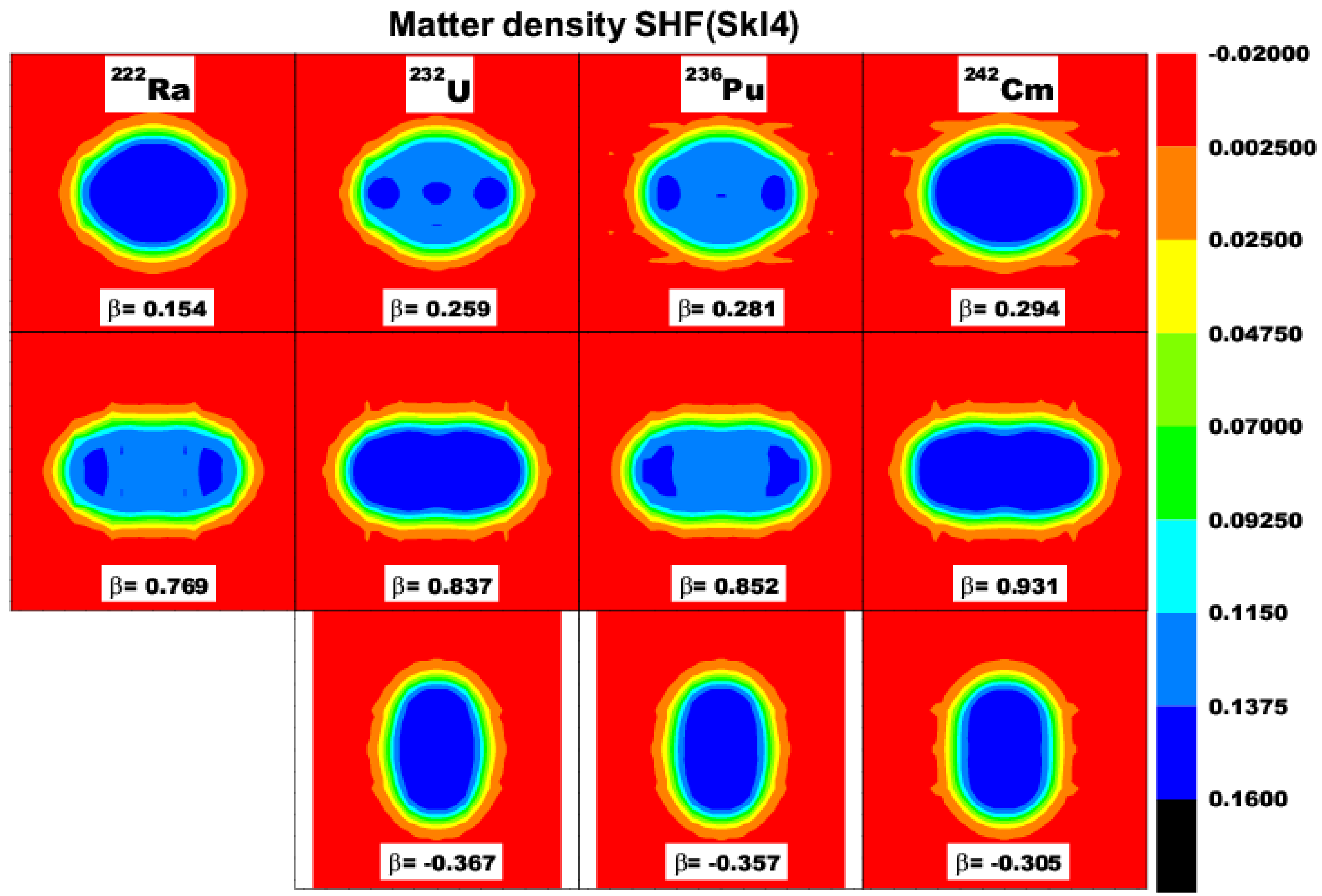
PES for cluster radioactive nuclei



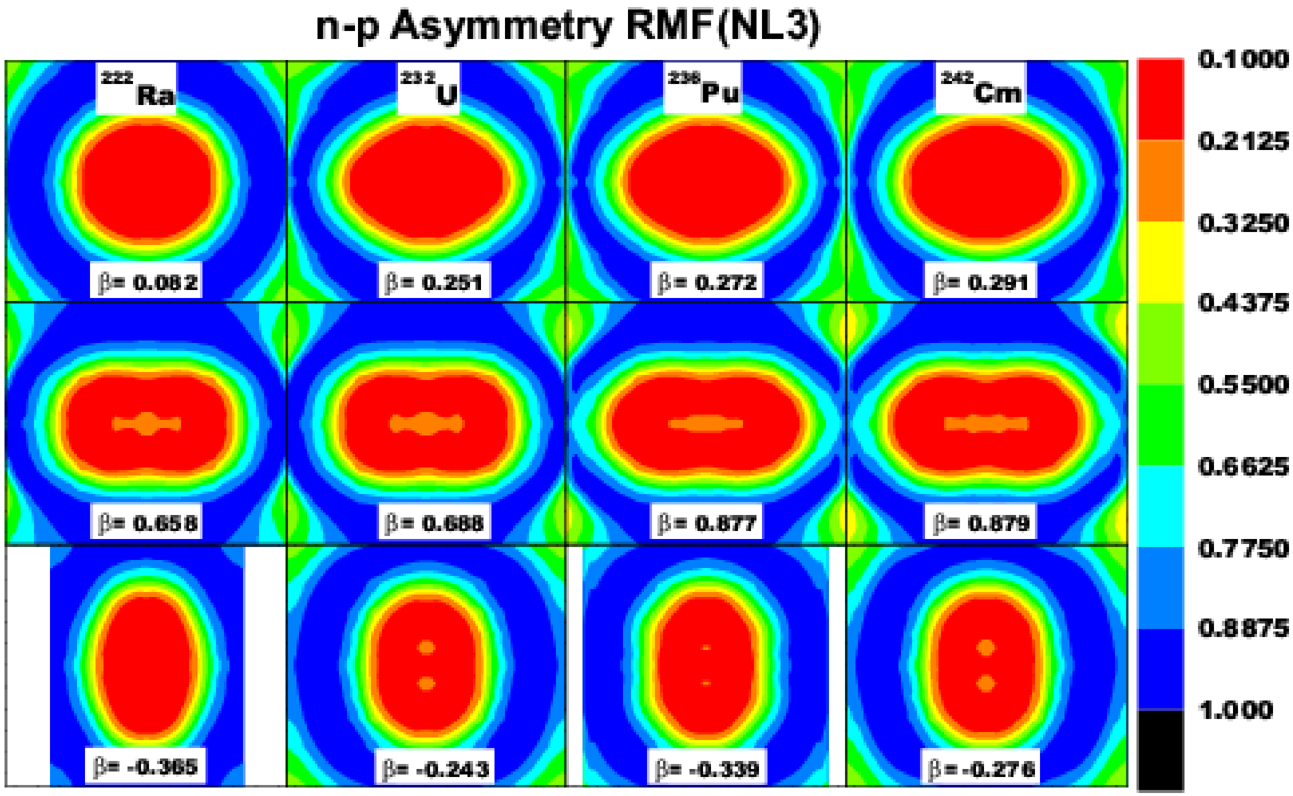
Clustering in actinides



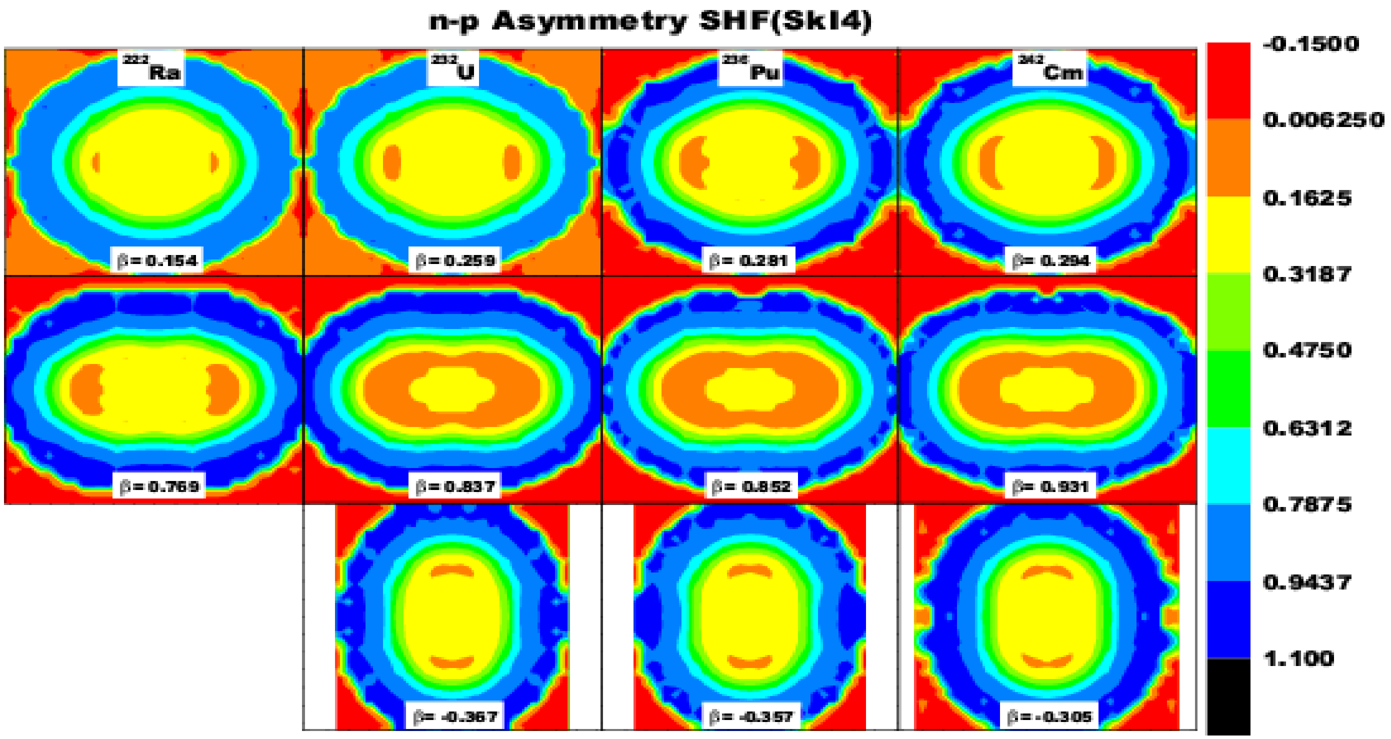
Clustering in actinides



$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}.$$



$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$



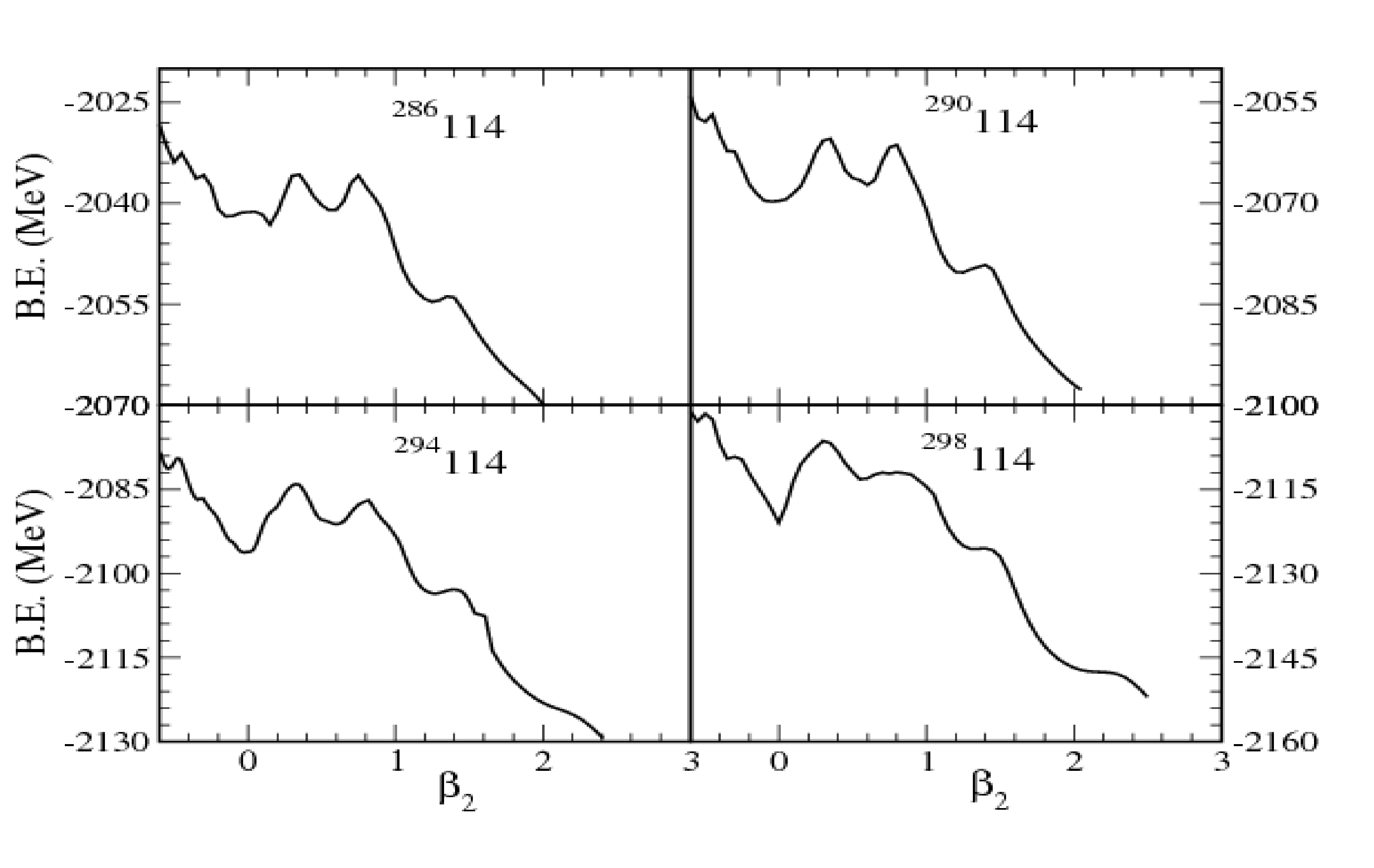
α-decay & cluster decay

- The signature of fragmented n-(or p)-rich matter starts appearing for the oblate excited state, in the form of two small fragments for ^{232}U, ^{236}Pu and ^{242}Cm.
- The g.s. of all nuclei contain pure α-particle like matter and the g.s. decays prefer α-like fragments due to the WIGNER term in liquid drop energy.
- For exotic cluster decays, the appearance of a single asymmetry (N not equal to Z) fragment at the centre of the N=Z matter of S.D. states of all 4 nuclei.
- The S.D. are the excited states, which might represent the surface excitations of these nuclei due to α-particle emission. (The observed half-life of α-decay is much smaller than the exotic cluster decay.)

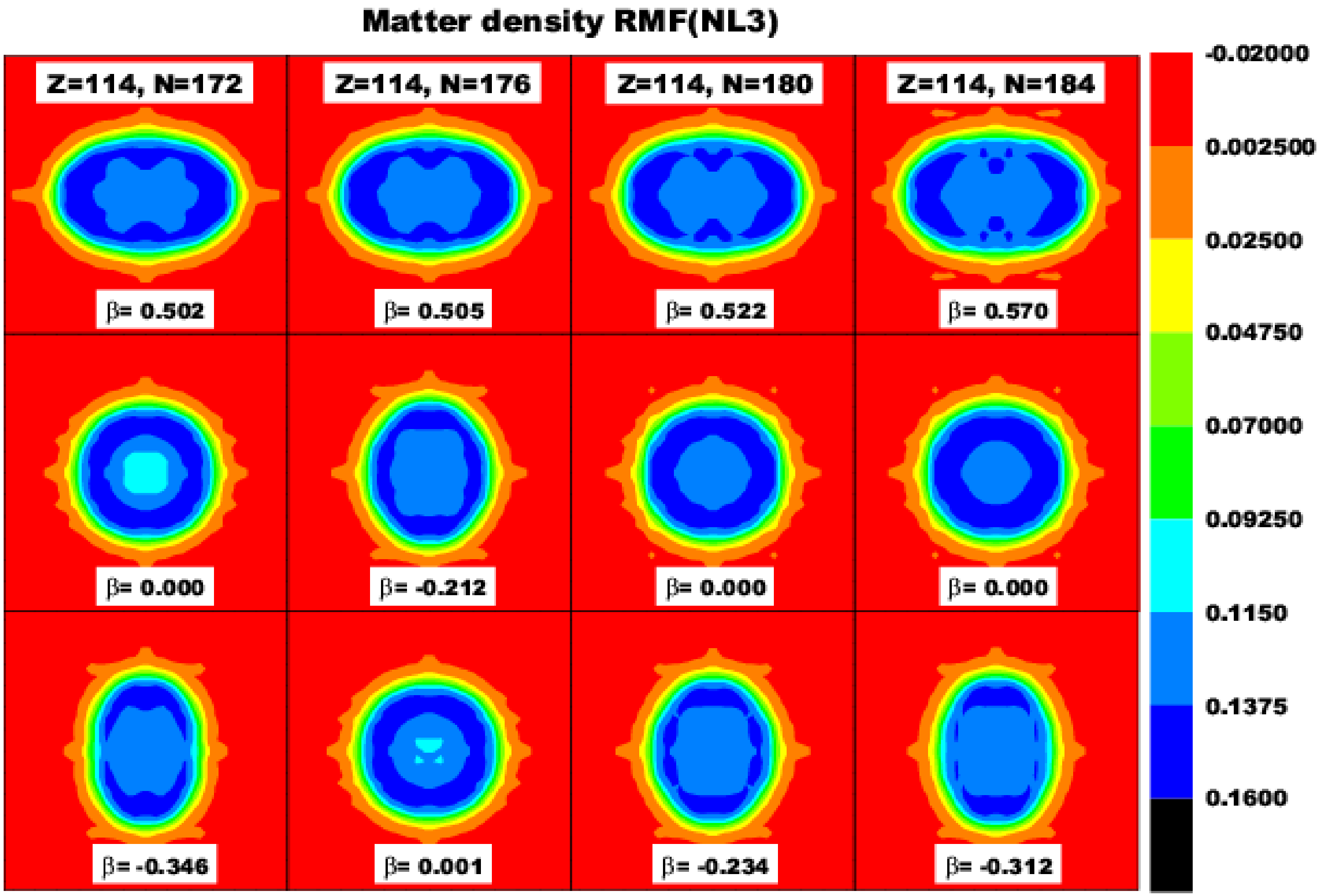
Extension of the formalism to SHE (Z=114 & 120)

- Clustering in SHE
- Potential Energy Surface

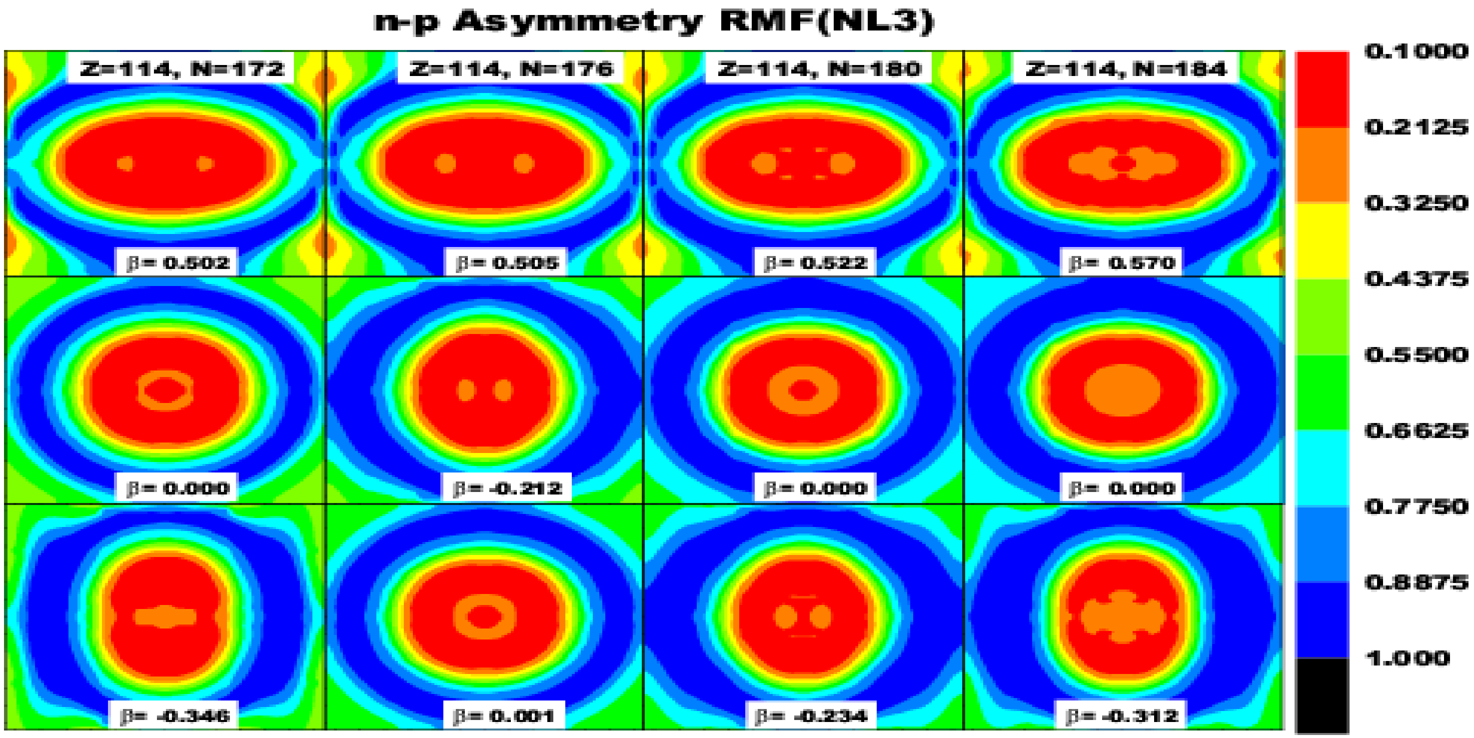
PES for SHE nuclei (SkI4)



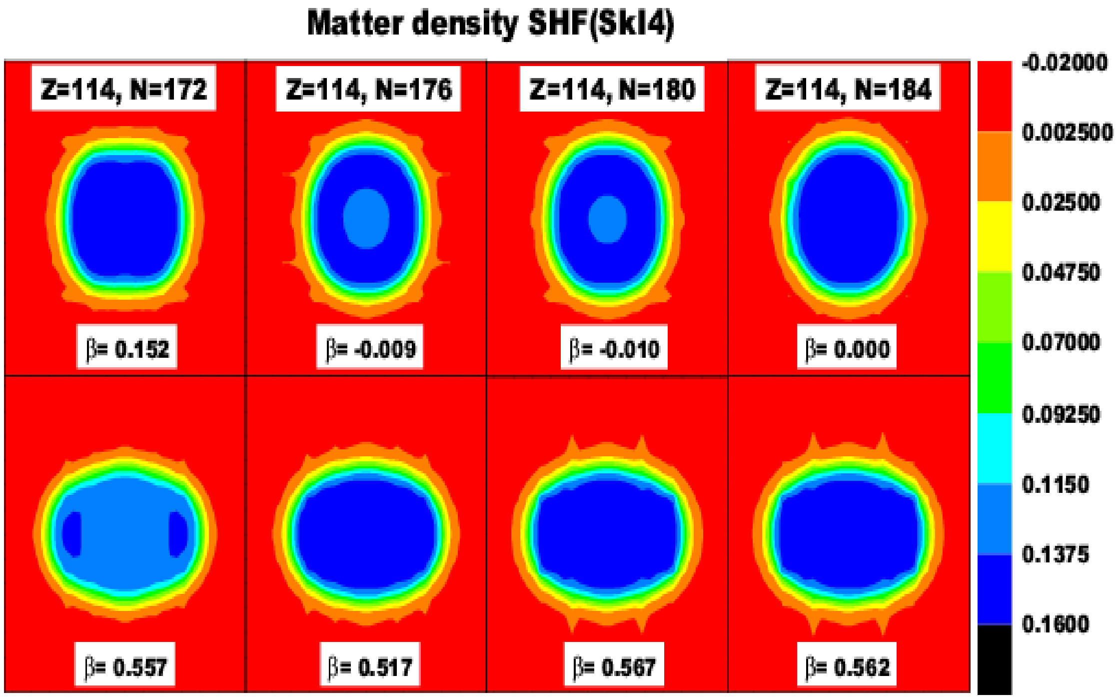
Clustering in SHE (Z=114)



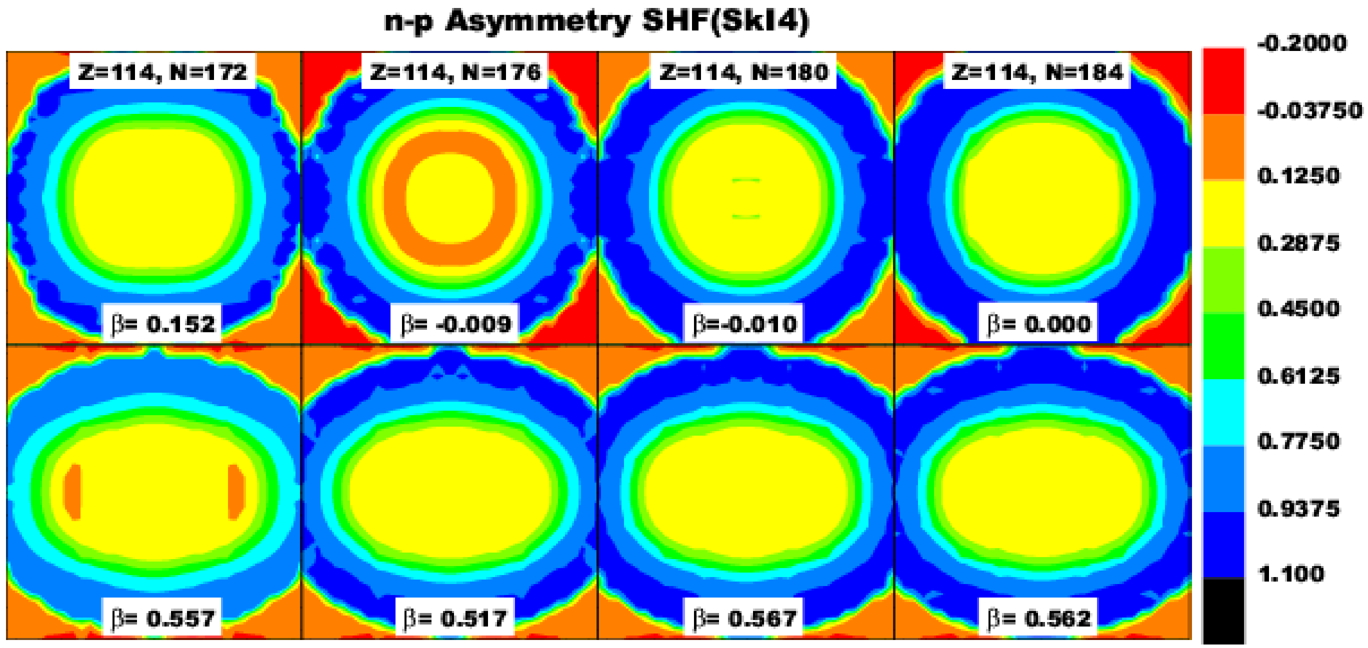
$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$



Clustering in SHE (Z=114) SkI4



$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}.$$



Clustering in SHE (Z=120)

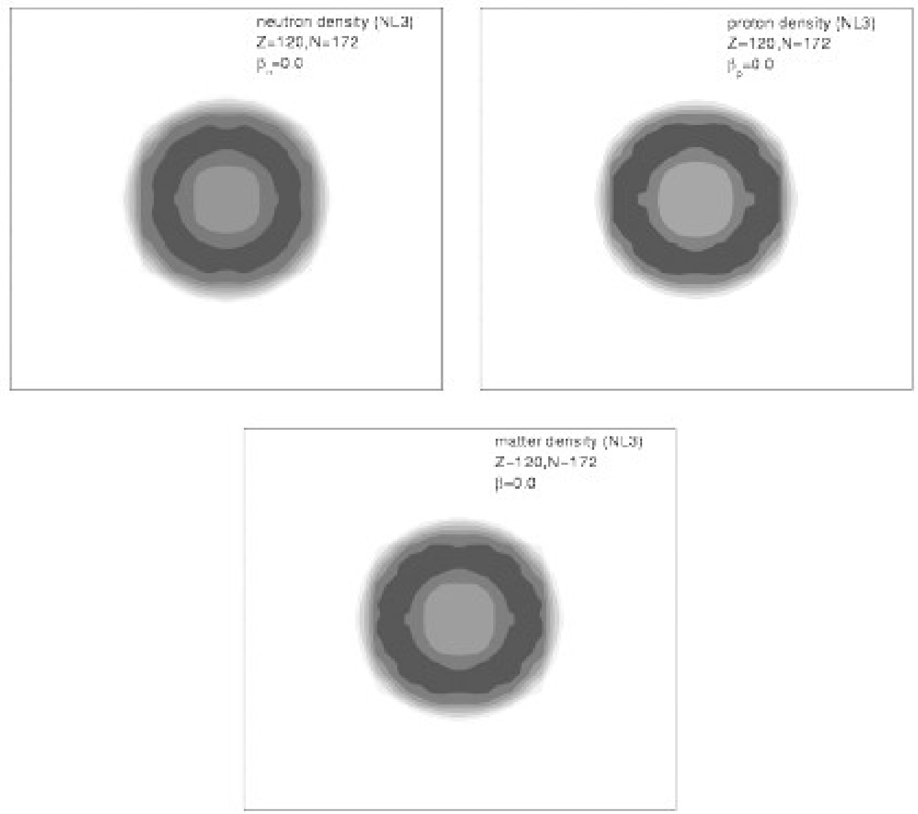


Figure 1. The total matter and nucleon density distributions for the ground-state spherical solution of the $Z = 120$, $N = 172$, $^{292}120$ compound system. The full black portion is the maximum density region and the full white one is the zero-density region.

Oblate SHE (Z=120)

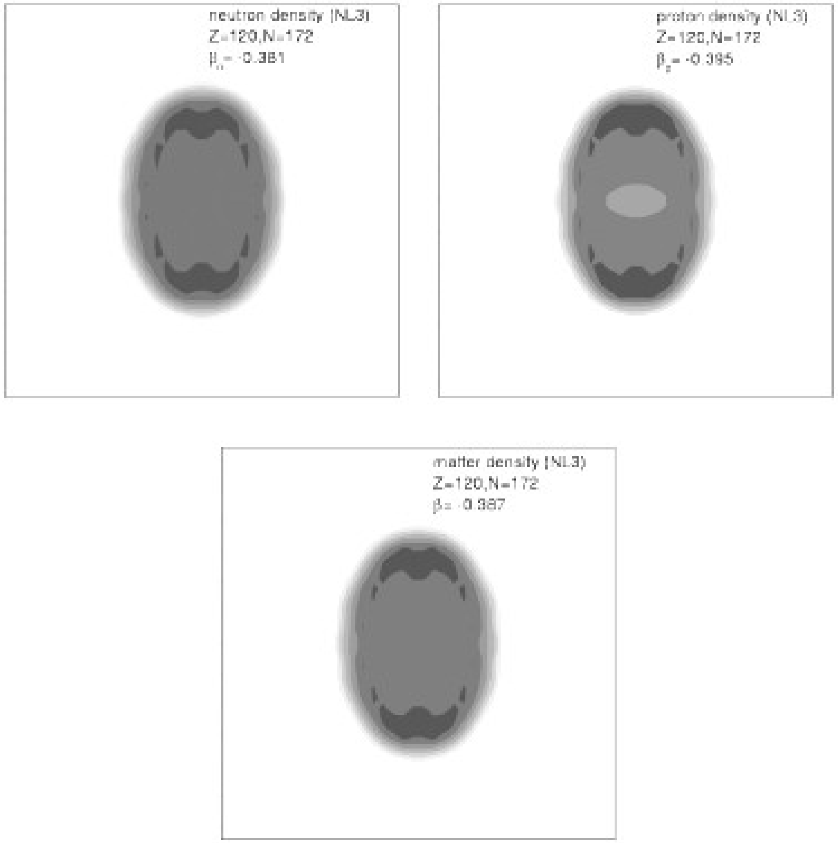


Figure 2. Same as for figure 1, but for the normal-oblate-deformed intrinsic excited state solution of the $Z = 120$, $N = 172$, $^{292}120$ compound system.

Shape co-existence SHE (Z=120)

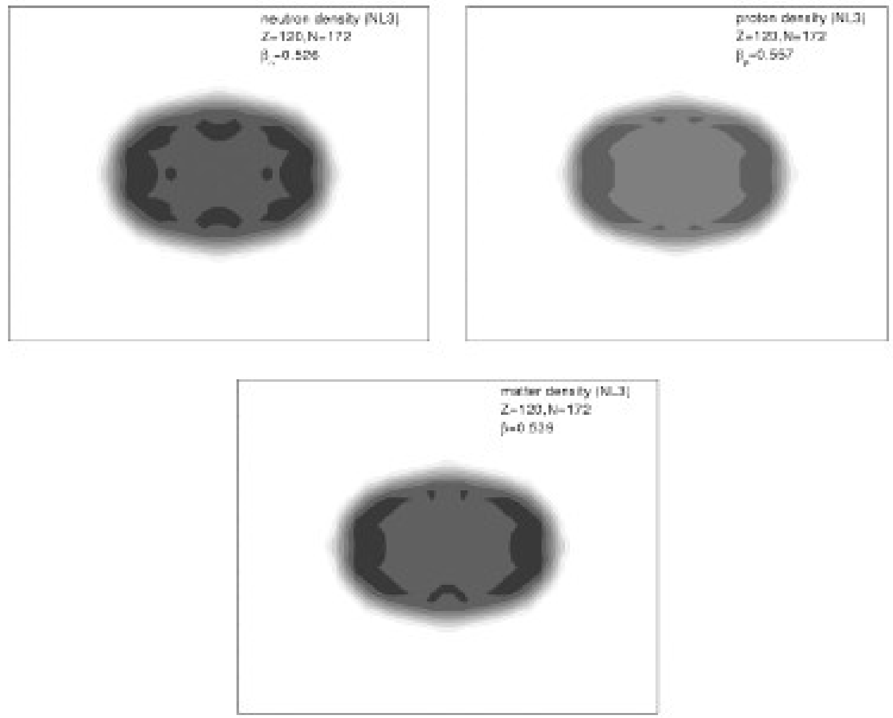


Figure 3. Same as for figure 1, but for the shape-coexisting, superdeformed solution of the $Z = 120$, $N = 172$, $^{292}120$ compound system.

Superdeformed SHE (Z=120)

Figure 4. Same as for figure 1, but for only the total matter density distributions of superdeformed ground-state solutions of $Z = 120$, $N = 176, 180, 184$, $^{296-304}120$ compound systems.

$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}.$$

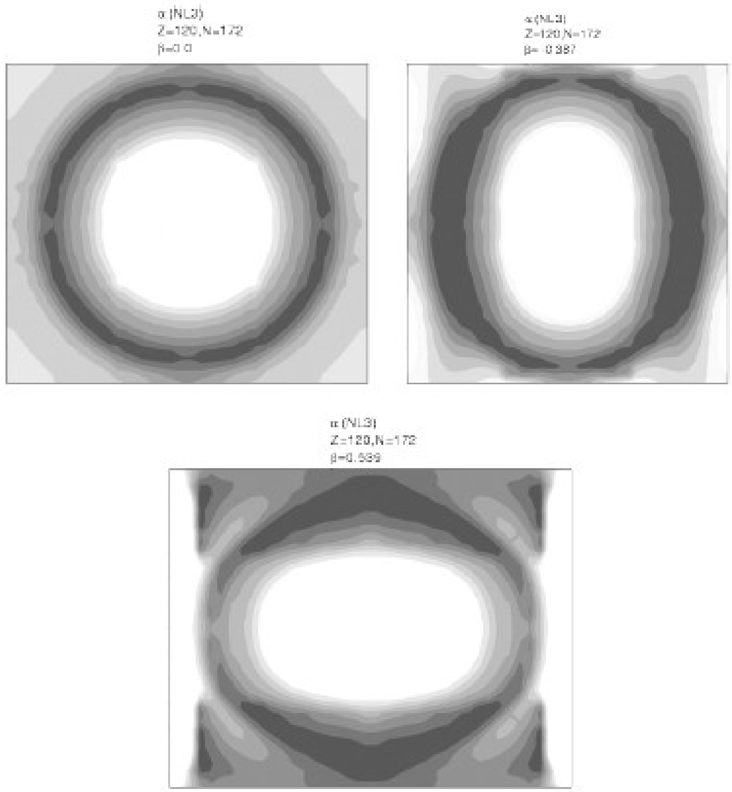


Figure 5. The asymmetry α for the three (spherical, somewhat superdeformed, superdeformed) solutions of $^{292}120$. The full black portion is the pure neutron (or proton) matter and the full white one is the $N = Z$ matter.

$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}.$$

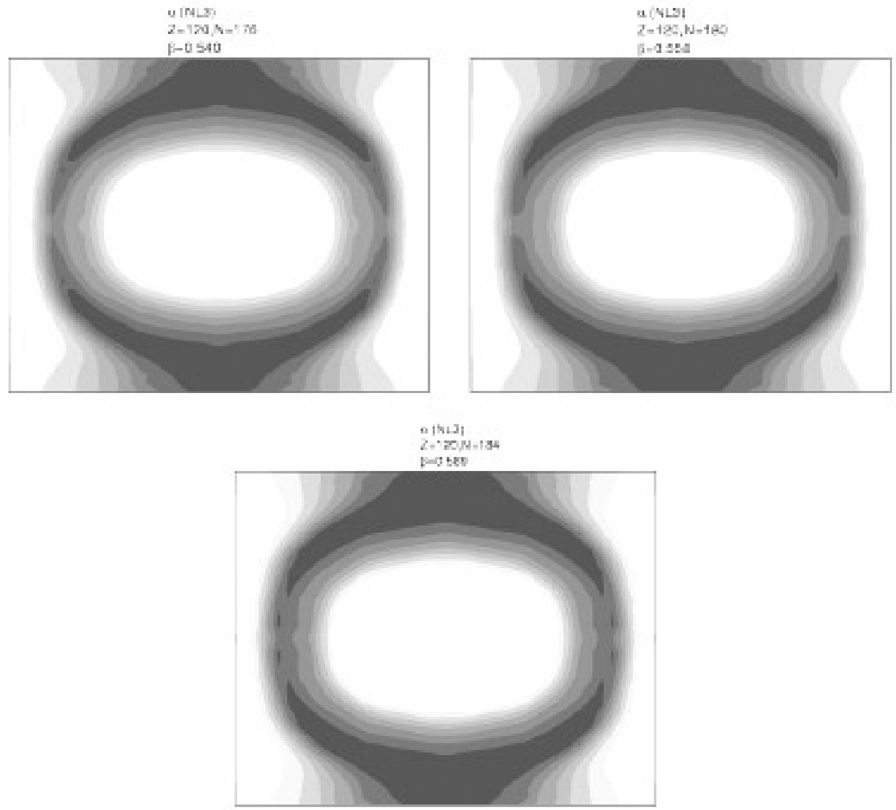
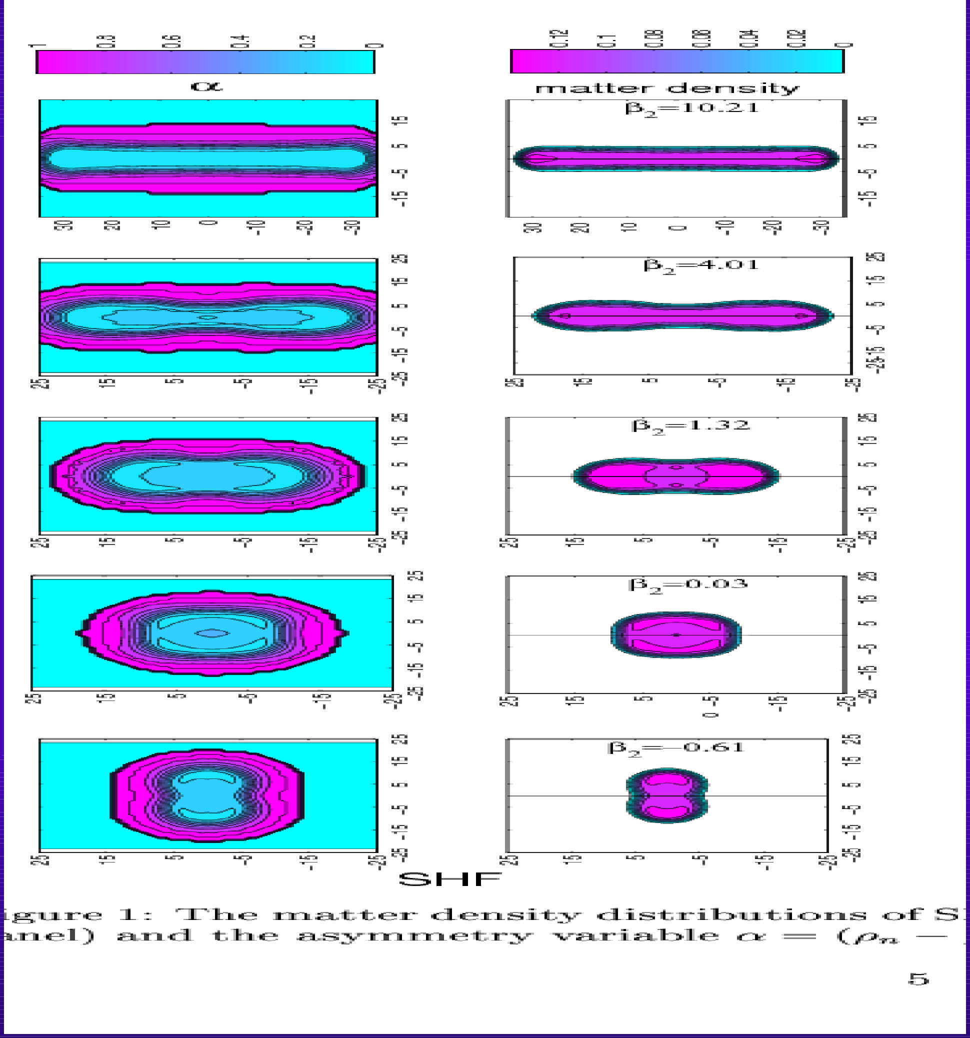


Figure 6. Same as for figure 5, but for superdeformed ground states of $^{296-304}120$.

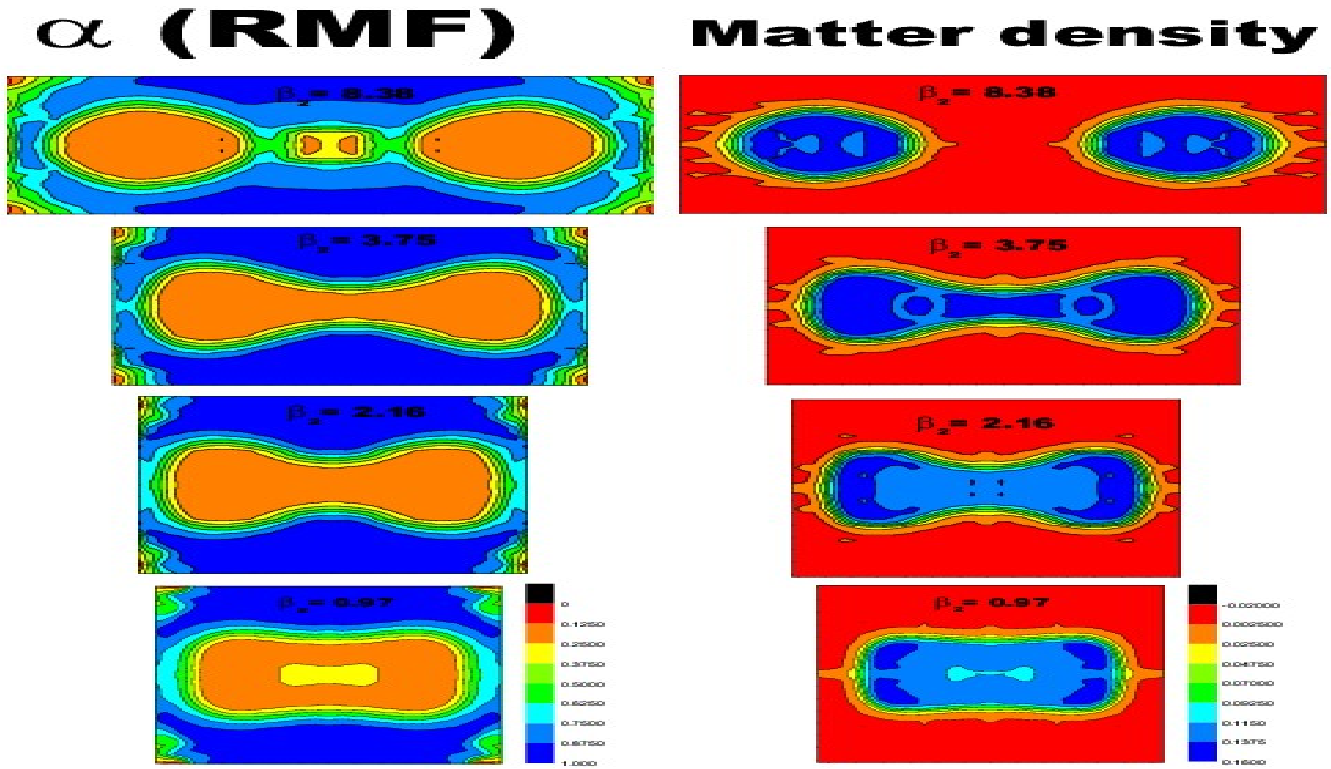
For SHE elements (z=114 & 120):

- We have applied our method to Z=114, Z=120 with N=172-184 and bubble-like structure for Z=114, N=184 & Z=120, N=172 as signature of doubly close shell.
- Two or multi-cluster for other nuclei in the ground and excited states are some of the interesting results.
- Possibility of exotic clusters at the centre of the SHE are predicted.
- Clustering is more clearly seen in RMF than SHF.

Super-superheavy compound system



Super-superheavy compound system



Super-superheavy nuclei

- We studied the clustering phenomenon in various resonance states of the compound nucleus $^{476}184^*$ formed in $^{238}U+^{238}U$ reaction.
- Both the formalisms indicate clear signatures of clustering in various resonance states of the super-superheavy compound system.
- The clustering are of N=Z, α-like and neutron-rich clusters, whose decay could finally end up in a superheavy nucleus from the un-known region.

These works are done in Collaboration with:

- P.D. Stevenson, University of Surrey, U.K.
- R.K. Gupta, Panjab University, India
- P. Arumugam, CFIF, Portugal
- B.K. Sharma, IOP, Bhubaneswar, India
- W. Greiner, FIAS, Frankfurt, Germany.

THANK YOU