NRQCD – How effective a theory of

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QCD

- The theory of strong interactions based on a non-abelian colour $SU(3)$ symmetry.
- QCD is asymptotically free.
- QCD factorisation.

Charmonium Family

		M (GeV)	
η_c	$^1S_0$	2.98	$\to \gamma\gamma$
J/ψ	$^3S_1$	3.096	$\to ee, \mu\mu$
$\chi_{0,1,2}$	$^3P_{0,1,2}$	3.41, 3.51, 3.55	$\to J/\psi\gamma$
h_c	$^1P_1$	3.52	$\to J/\psi\pi$
ψ'	2^3S_1	3.686	$\to ee, \mu\mu$

Pre-NRQCD models

- Colour-evaporation
- Colour-singlet

Colour Singlet Model

- Colour singlet model worked reasonably for low-energy (ISR) production
- At higher energies, problems with b quark initiated states.
- At Tevatron, prompt J/ψ production disagreed seriously with colour singlet model predictions.

NRQCD

- Non-Relativistic QCD (NRQCD) is an effective theory obtained from QCD.
- Used to model bound state dynamics and study production and decay of quarkonia.
- Obtained by treating QCD with an ultraviolet cutoff $\sim M$.
- Neglecting states above M and adding new operators to account for this exclusion.

Velocity expansion

- Other scale is $Mv \ll M$ with $v \ll 1$.
- Suggests an expansion of the quarkonium wavefunction in v.

$$|J/\psi\rangle = |c\bar{c}(^3S_1^{[1]})\rangle + v^2|c\bar{c}(^3P_J^{[8]})g\rangle + \ldots \tag{1}$$

- So there is an octet state in the J/ψ with P-state quantum numbers – which connects to the physical state through the emission of a non-perturbative gluon.

Electric and Magnetic transitions

- So, in NRQCD quarkonium production and decay involves intermediate states where the $Q\bar{Q}$ pair has quantum numbers different from those of the physical quarkonium.
- Forms the physical state via chromo-electric or chromo-magnetic transitions. More explicitly,

$$|c\bar{c}(^3S_1^{[1]})\rangle + v^2|c\bar{c}(^3P_J^{[8]})g\rangle + v^2|c\bar{c}(^3S_1^{[8]})gg)\rangle + v^2|c\bar{c}(^1S_0^{[8]})g\rangle + \ldots$$

P-state decays

- Consider the χ states:

(2) $$|\chi\rangle = v|c\bar{c}(^3P_J^{[1]})\rangle + v|c\bar{c}(^3S_1^{[8]})g\rangle$$

- In the colour-singlet model the amplitude for χ decays into hadrons has a divergence. This is due to neglecting the colour-octet component.
- Colour-singlet model is flawed.

J/ψ at CDF – I

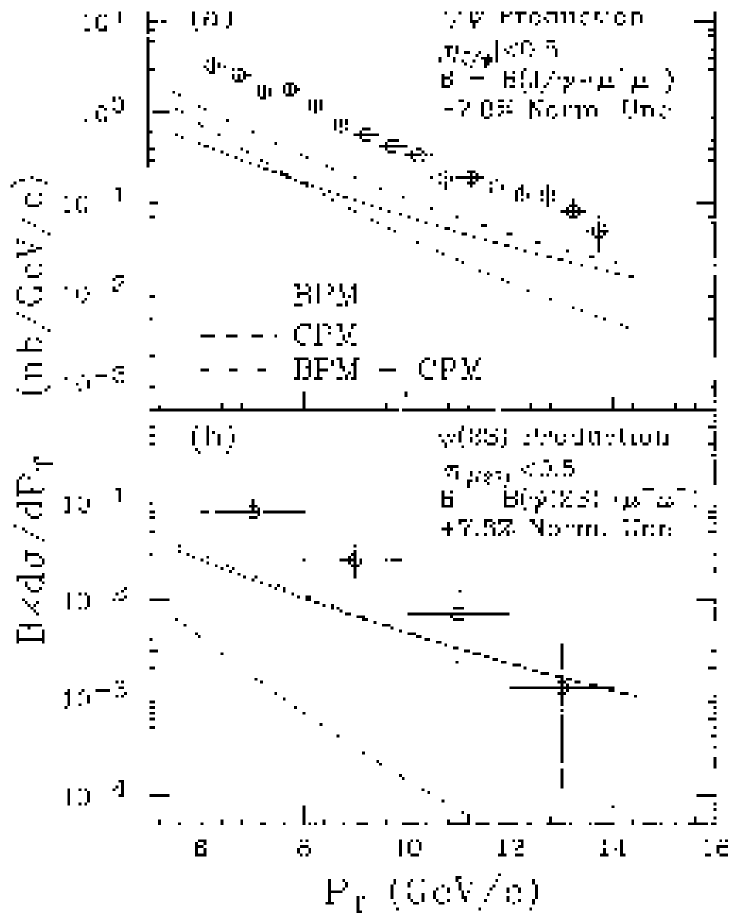


FIG. 2. The product $B(d\sigma/dP_T)$ vs P_T for (a) $J/\psi \to \mu^+\mu^-$
nd (b) $\psi(2S) \to \mu^+\mu^-$. The circles correspond to the data.
he solid curve corresponds to J/ψ's [$\psi(2S)$'s] produced from B
eson decays by using Refs. [6–9]. The dashed curve corre-
ponds to J/ψ's [$\psi(2S)$'s] from direct charmonium production
rom the model of Ref. [1]. The dot-dashed curve is their sum.

NRQCD factorization

- The cross section for production of a quarkonium state H is:

$$\sigma(H) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{{M_Q}^{d_n-4}} \langle \mathcal{O}_n^H({}^{2S+1}L_J) \rangle, \tag{3}$$

- F_n's are the perturbatively computable short-distance coefficients
- $\mathcal{O}_n$ are operators of naive dimension d_n, describing the long-distance effects.
- Factorization $\longrightarrow$ momentum-independence of the non-perturbative elements.

Tevatron data

- NRQCD gives a good description of the cross-sections for J/ψ and other charmonium states measured at the Tevatron.
- One of the crucial features of the data is the large p_T tail which is due to gluon fragmentation.
- Fragmentation becomes important when $p_T > M$ and is naturally incorporated in NRQCD through colour-octet components.

J/ψ at CDF – II

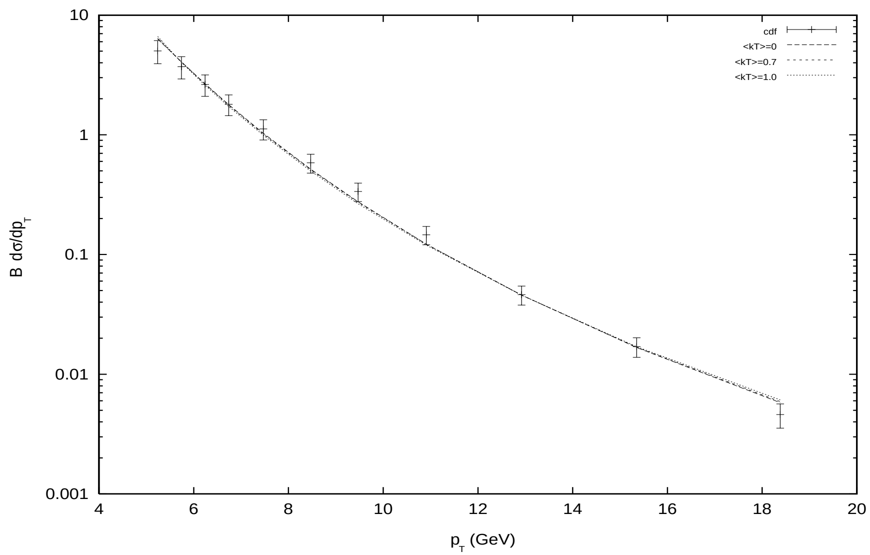


Figure 2: J/ψ *at CDF*

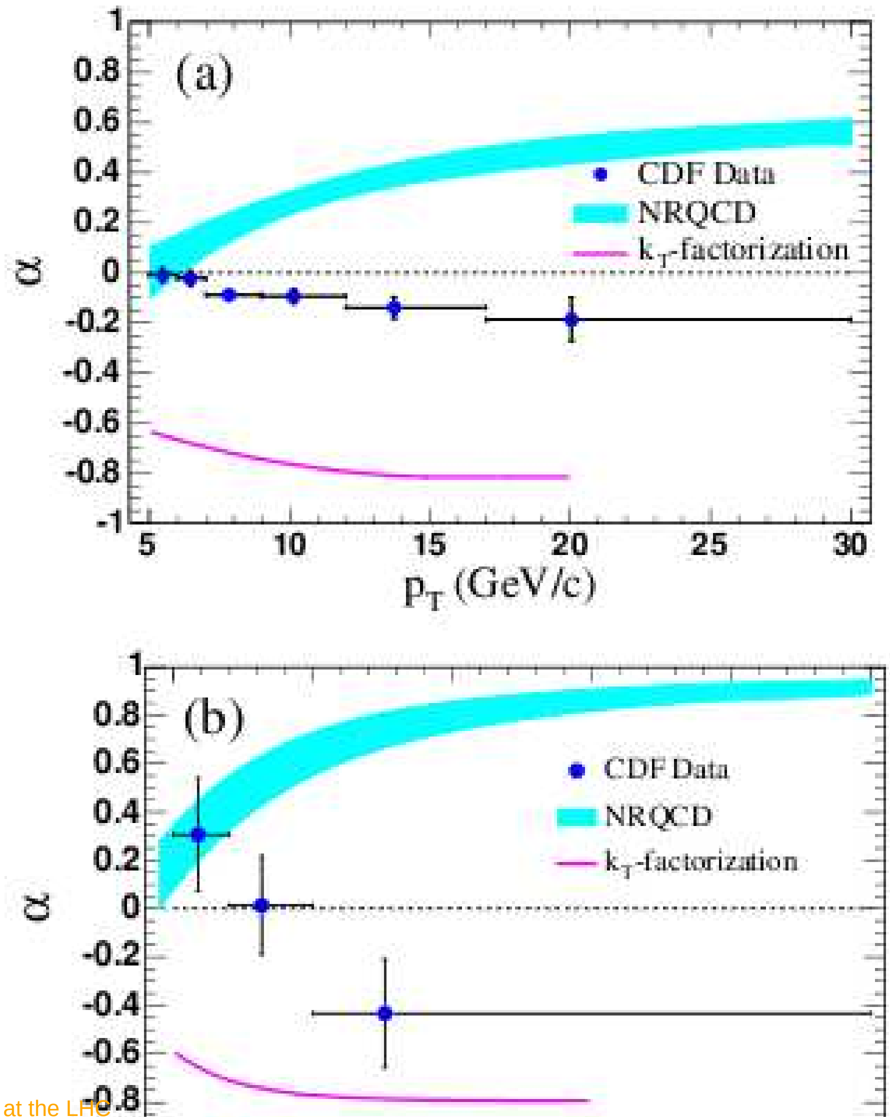
J/ψ polarisation

- In fragmentation, the gluon transfers all its transverse polarisation to the $c\bar{c}$ pair.
- NRQCD has a heavy-quark symmetry – the spin and flavour degrees of freedom are irrelevant in the non-perturbative soft interactions – due to which the J/ψ inherits the transverse polarisation of the $c\bar{c}$ pair.
- The J/ψ at large-p_T should be transversely polarised.

Measuring polarisation

- Experimentally the $\cos\theta^*$ distribution is measured where θ^* is the angle of the decay lepton in the J/ψ rest frame with respect to the J/ψ boost direction in the lab.
  Then
  $$\frac{d\sigma}{d\cos\theta^*} \sim (1 + \alpha\cos\theta^*) \tag{4}$$
  where α is the polarisation parameter.
- $\alpha = 1 \longrightarrow$ Transverse polarisation
- $\alpha = -1 \longrightarrow$ Longitudinal polarisation

CDF polarisation data



Alternate test of NRQCD

- The heavy-quark symmetry of NRQCD implies that the non-perturbative matrix elements are related to each other.
- For example, for η_c production there are three contributions: from a colour-singlet ${}^1S_0$ state and from colour-octet ${}^1P_1$ and ${}^3S_1$ channels.
- We need to know three non-perturbative parameters to predict the η_c cross-section.

Heavy-quark symmetry relations

$$\langle 0 | \mathcal{O}_1^{\eta_c}[{}^1S_0] | 0 \rangle = \frac{1}{3} \langle 0 | \mathcal{O}_1^{J/\psi}[{}^3S_1] | 0 \rangle \; (1 + O(v^2)),$$

$$\langle 0 | \mathcal{O}_8^{\eta_c}[{}^1P_1] | 0 \rangle = \langle 0 | \mathcal{O}_8^{J/\psi}[{}^3P_0] | 0 \rangle \; (1 + O(v^2)),$$

$$\langle 0 | \mathcal{O}_8^{\eta_c}[{}^3S_1] | 0 \rangle = \langle 0 | \mathcal{O}_8^{J/\psi}[{}^1S_0] | 0 \rangle \; (1 + O(v^2)).$$

This allows us to make predictions for η_c production at the LHC.

η_c Production

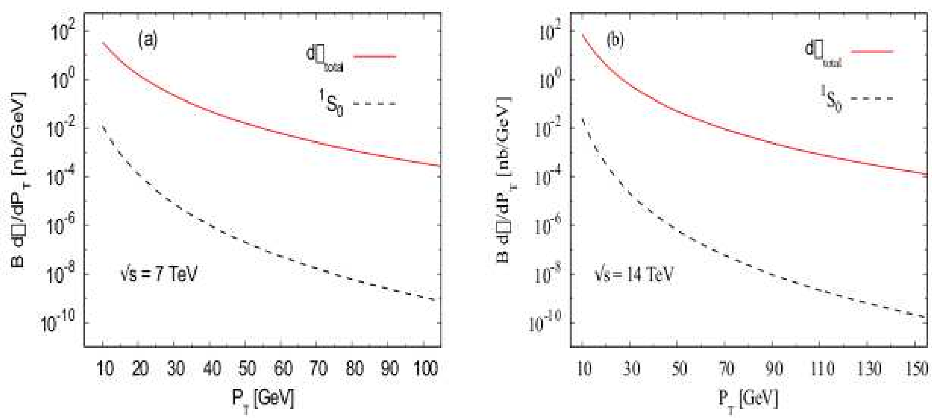


Figure 1: $d\sigma/dp_T$ (in nb/GeV) for η_c production (after folding in with $\text{Br}(\eta_c \to \gamma\gamma) = 3.0 \times 10^{-4}$) in pp collisions at $\sqrt{s} = 7$ TeV and 14 TeV with $-2 \le y \le 2$.

h_c production

- A similar strategy may be exploited for h_c production.
- More difficult resonance to study – has never been seen in hadron collisions.
- But large enough cross-sections for this state to be detected at the LHC. Will help study its properties more accurately.