Inflationary model building, reconstructing parameters and observational limits

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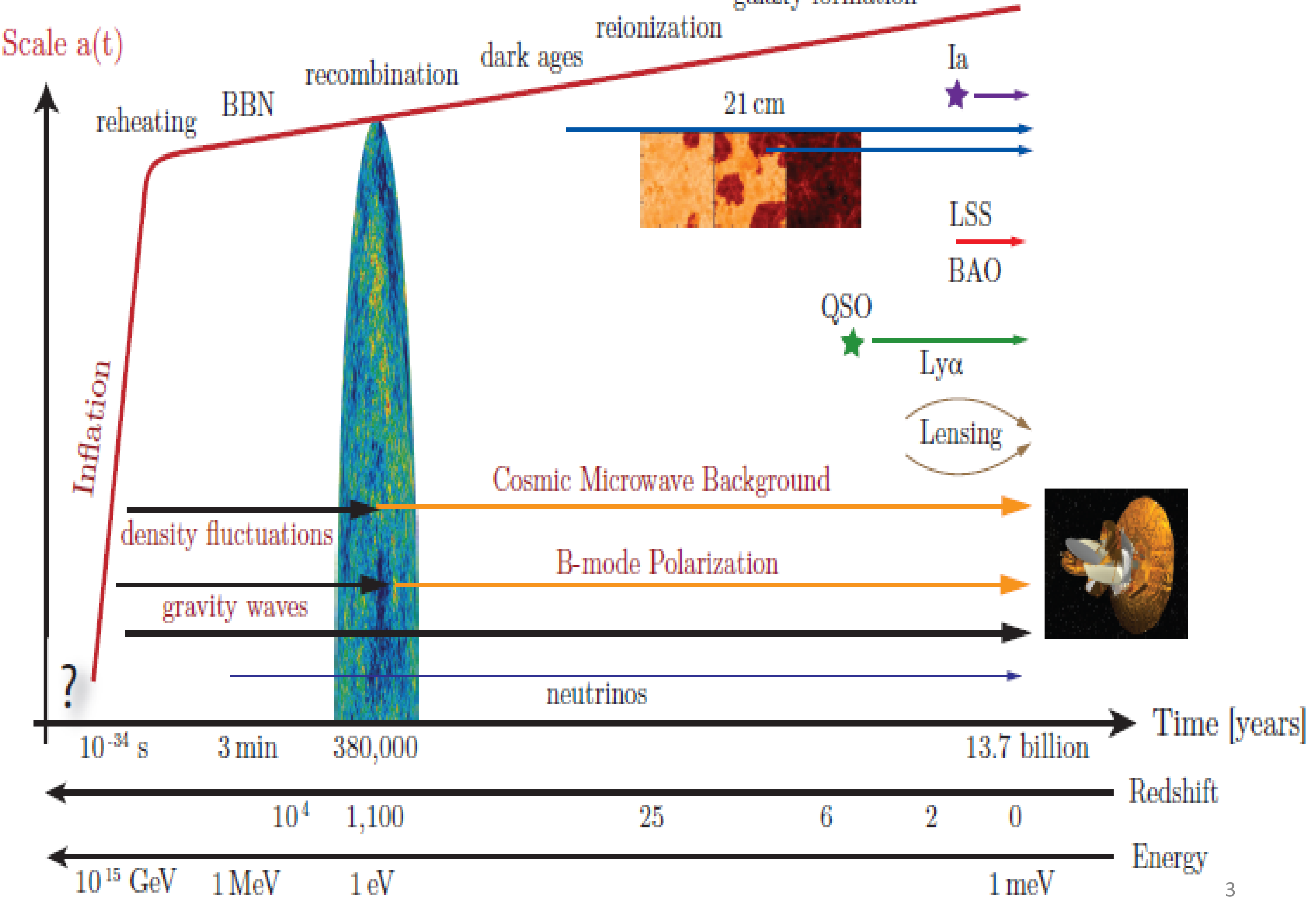
Outline of talk

- Inflationary paradigm and allied issues.
- Observational limits.
- Modeling inflation and parameter estimation.
- Reconstruction of inflationary potential.
- Bottom lines.
- Open issues and future prospects.

DB,arXiv:0907.5424



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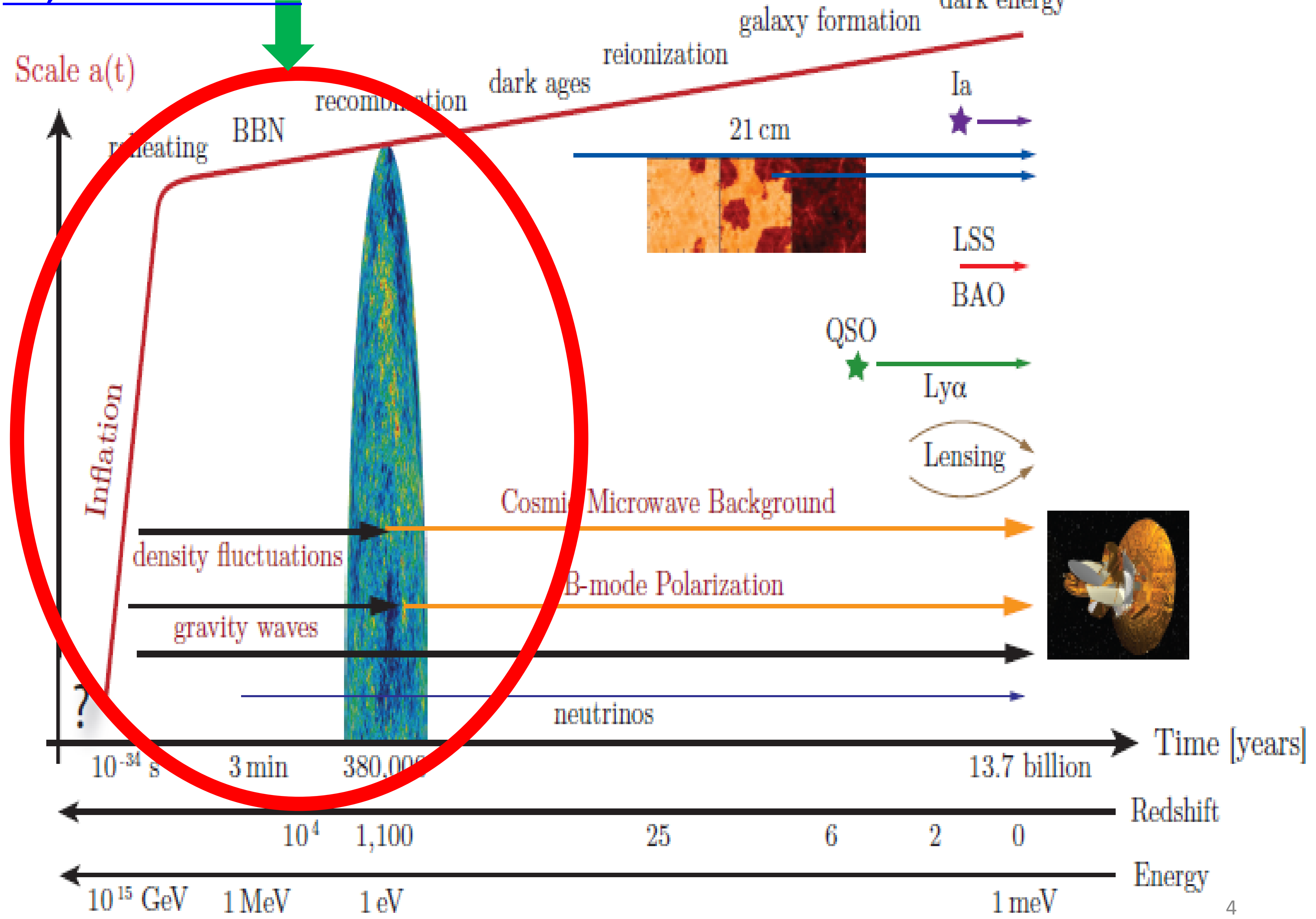

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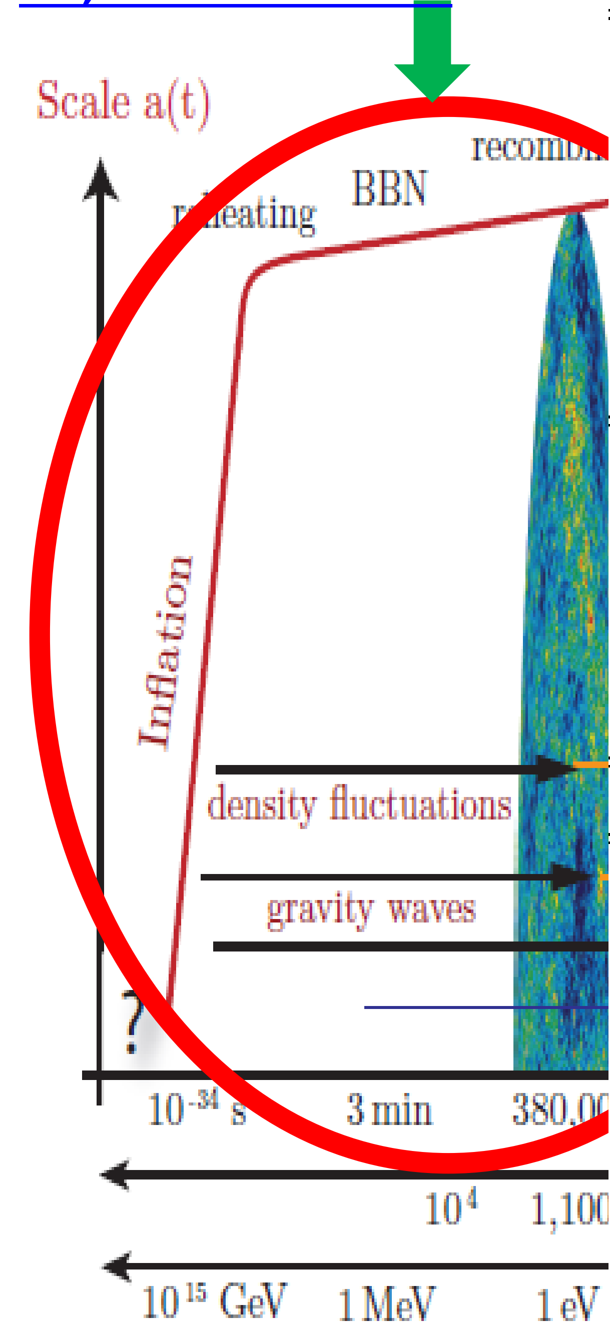

	Time	Energy
Planck Epoch?	$< 10^{-43}$ s	10^{18} GeV
String Scale?	$\gtrsim 10^{-43}$ s	$\lesssim 10^{18}$ GeV
Grand Unification?	$\sim 10^{-36}$ s	10^{15} GeV
Inflation?	$\gtrsim 10^{-34}$ s	$\lesssim 10^{15}$ GeV
SUSY Breaking?	$< 10^{-10}$ s	> 1 TeV
Baryogenesis?	$< 10^{-10}$ s	> 1 TeV
Electroweak Unification	10^{-10} s	1 TeV
Quark-Hadron Transition	10^{-4} s	10^2 MeV
Nucleon Freeze-Out	0.01 s	10 MeV
Neutrino Decoupling	1 s	1 MeV
BBN	3 min	0.1 MeV
Matter-Radiation Equality	10^4 yrs	1 eV
Recombination	10^5 yrs	0.1 eV
Dark Ages	$10^5 - 10^8$ yrs	
Reionization	10^8 yrs	
Galaxy Formation	$\sim 6 \times 10^8$ yrs	
Dark Energy	$\sim 10^9$ yrs	
Solar System	8×10^9 yrs	
Albert Einstein born	14×10^9 yrs	1 meV

Basics of SBBC

- *Homogeneous and isotropic universe: FRW metric (for spatially flat k=0):*

$$ds^2 = -dt^2 + a^2(t)\overrightarrow{dx_3}^2$$

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$$\dot{H} + H^2 = \left(\frac{\ddot{a}(t)}{a(t)}\right) = -\frac{(\rho + 3p)}{6M^2{}_p}$$

- Equation of continuity in GR: $\dot{\rho} + 3H(\rho + p) = 0$
- Equation of state: $w = p/\rho$

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- *FRW Solutions:*

Type	w	ρ(a)	a(t)
RD	1/3	a^{-4}	$t^{1/2}$
MD	0	a^{-3}	$t^{2/3}$
Λ	-1	a^{0}	e^{Ht}

- ***Big Bang Puzzles:***
  1. ***Horizon problem***
  2. ***Flatness problem***
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Inflation !

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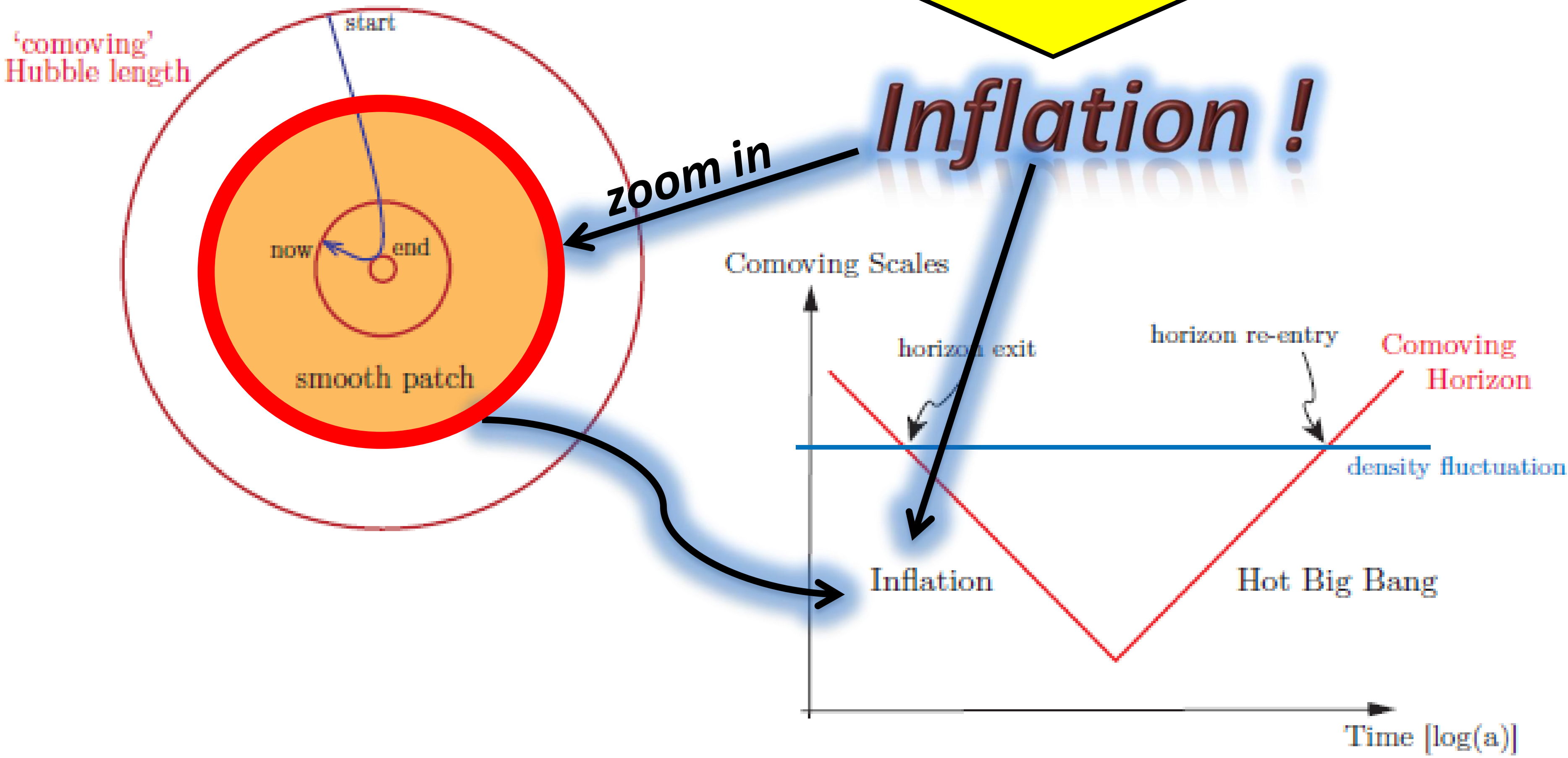

Inflationary paradigm and allied issues

- ***Condition for inflation:***

$$\ddot{a} > 0 \Longleftrightarrow \frac{d}{dt}(aH)^{-1} < 0 \Longleftrightarrow -\frac{\dot{H}}{H^2}\left(= \frac{d\ln H}{dN}\right) > 0 \Longleftrightarrow (\rho + 3P) < 0.$$

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$$V(\phi) = V_{ren}(\phi) + \sum_{\alpha=5}^{\infty} c_\alpha \frac{\phi^\alpha}{\Lambda_{UV}^{\alpha-4}}$$

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$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,,$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,.$$

Scalar Field Eqn.: $$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

Inflation via Slow-roll:

1. $|\ddot{\phi}| << |3H\dot{\phi}|, V^{'}(\phi) \Rightarrow 3H\dot{\phi} \approx -V^{'}(\phi)$.
2. $\dot{\phi}^2 << V(\phi) \Rightarrow H^2 \approx \frac{V(\phi)}{3M_p^2}$.

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$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V = M_P^2\left(\frac{V''}{V}\right)$$

$$\xi_V^2 = M_p^4\left(\frac{V'V'''}{V^2}\right), \quad \sigma_V^3 = M_p^6\left(\frac{V'^2V''''}{V^3}\right)$$

Flatness/ Slow-Roll parameter

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Flatness/ Slow-Roll parameter

End of inflation $\max_{\phi}\{\epsilon_V, |\eta_V|, |\xi_V^2|, |\sigma_V^3|\} = 1$

$$N_{cmb} = -\frac{1}{M_p}\int_{\phi_{cmb}}^{\phi_f}\frac{d\phi}{\sqrt{2\epsilon_V}} \approx 50 - 70.$$

Necessarily required to solve "horizon problem"

Inflationary Model Building

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ALGORITHM

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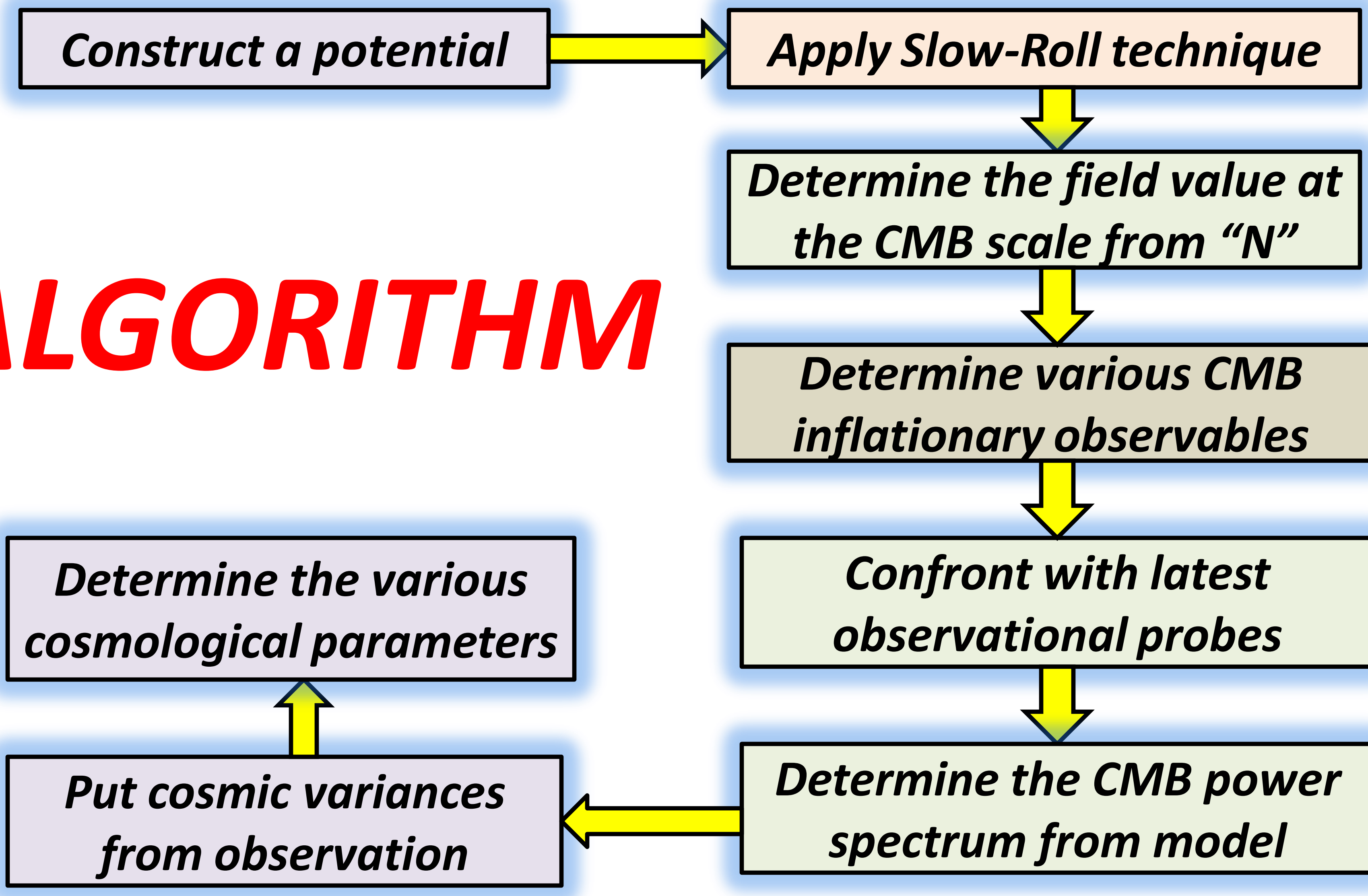
ALGORITHM



Inflationary observables:

CMB measures distortions in space

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Metric perturbation = Curvature (Scalar)+
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ζ scalar mode

$$\langle \zeta(\vec{k})\zeta(\vec{k}')\rangle = (2\pi)^3 P_S(k)\delta^3(\vec{k}+\vec{k}')$$

h_{ij} tensor mode

$$\langle h_{ij}(\vec{k})h_{jk}(\vec{k}')\rangle = (2\pi)^3\delta_{ik}P_T(k)\delta^3(\vec{k}+\vec{k}')$$

Inflationary observables: arXiv:1404.2601

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$$P_S(k) = P_S(k_\star)\left(\frac{k}{k_\star}\right)^{n_S - 1 + \frac{\alpha_S}{2!}\ln\left(\frac{k}{k_\star}\right) + \frac{\kappa_S}{3!}\ln^2\left(\frac{k}{k_\star}\right) + \cdots}$$

DB,LM,arXiv:1404.2601

$$P_T(k) = P_T(k_\star)\left(\frac{k}{k_\star}\right)^{n_T + \frac{\alpha_T}{2!}\ln\left(\frac{k}{k_\star}\right) + \frac{\kappa_T}{3!}\ln^2\left(\frac{k}{k_\star}\right) + \cdots}$$

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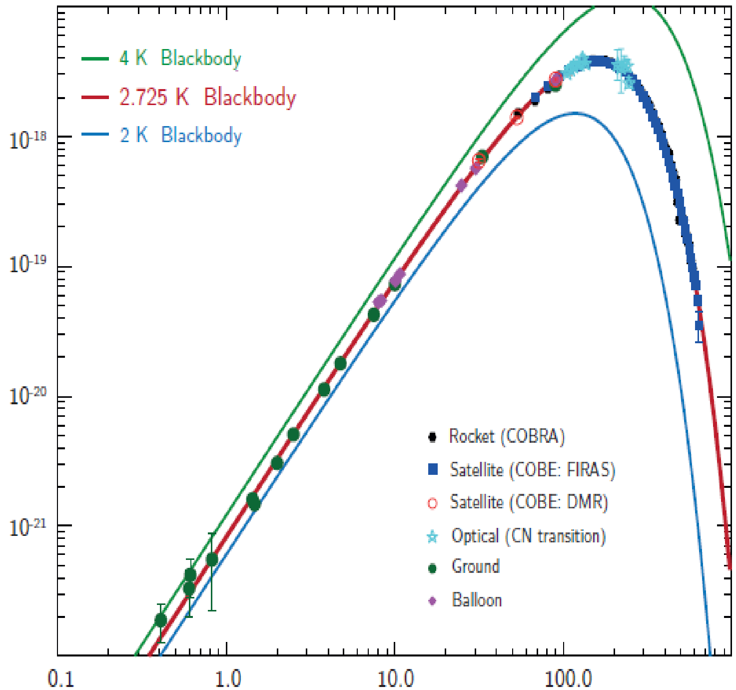
Inflationary observables via flow eqn within slow-roll

1. Scalar power spectrum: $P_S(k_\star) = \frac{V}{24\pi^2 \epsilon_V M_p^4}$
2. Tensor power spectrum: $P_T(k_\star) = \frac{2V}{3\pi^2 M_p^4}$
3. Tensor-to-scalar ratio: $r(k_\star) = \frac{P_T(k_\star)}{P_S(k_\star)} = 16\epsilon_V$
4. Scalar spectral tilt : $n_S(k_\star) - 1 = \frac{d\ln P_S(k)}{d\ln k}|_\star = 2\eta_V - 6\epsilon_V$
5. Tensor spectral tilt: $n_T(k_\star) = \frac{d\ln P_T(k)}{d\ln k}|_\star = -2\epsilon_V$
6. Running of scalar spectral tilt:
$$\alpha_S(k_\star) = \frac{dn_S}{d\ln k}|_\star = 16\eta_V\epsilon_V - 24\epsilon_V^2 - 2\xi_V^2$$
7. Running of tensor spectral tilt:
$$\alpha_T(k_\star) = \frac{dn_T}{d\ln k}|_\star = 4\eta_V\epsilon_V - 8\epsilon_V^2$$
8. Running of the running of scalar spectral tilt:
$$\kappa_S(k_\star) = \frac{d^2 n_S}{d\ln k^2}|_\star = 192\epsilon_V^2\eta_V - 192\epsilon_V^3 + 2\sigma_V^3 - 24\epsilon_V\xi_V^2 + 2\eta_V\xi_V^2 - 32\eta_V^2\epsilon_V$$
9. Running of the running of tensor spectral tilt:
$$\kappa_T(k_\star) = \frac{d^2 n_T}{d\ln k^2}|_\star = 56\eta_V\epsilon_V^2 - 64\epsilon_V^3 - 8\eta_V^2\epsilon_V - 4\epsilon_V\xi_V^2$$

Cosmic Microwave Background

$$I_\nu = \frac{2h\nu^3}{c^2} f(\nu)$$

intensity

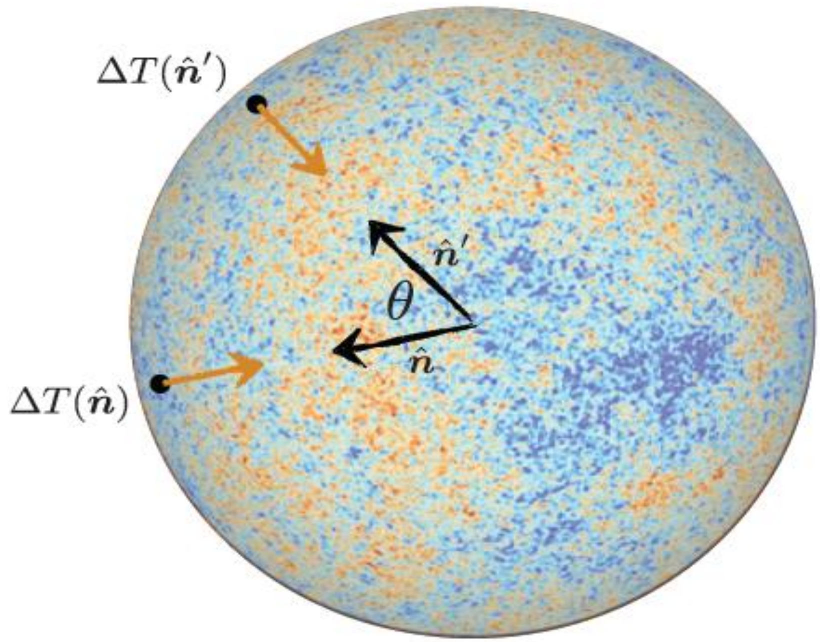


ν [GHz]

frequency

CMB Anisotropies

$$f(\nu, \hat{\boldsymbol{n}}) = [\exp(2\pi\nu/T(\hat{\boldsymbol{n}})) - 1]^{-1}$$



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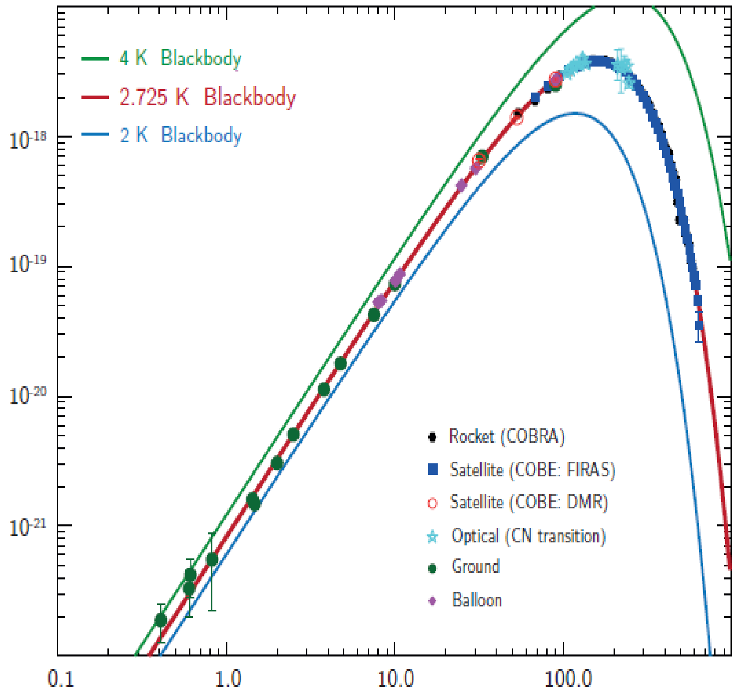

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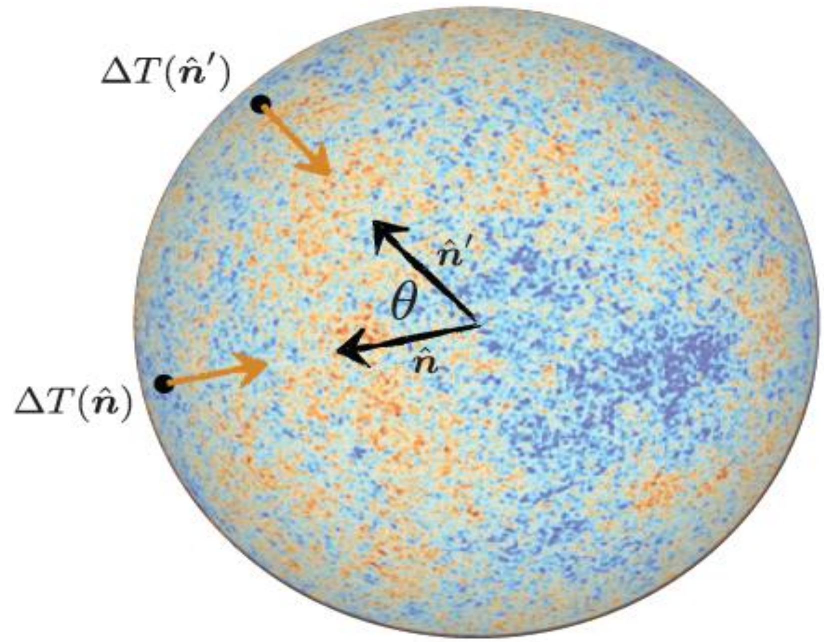
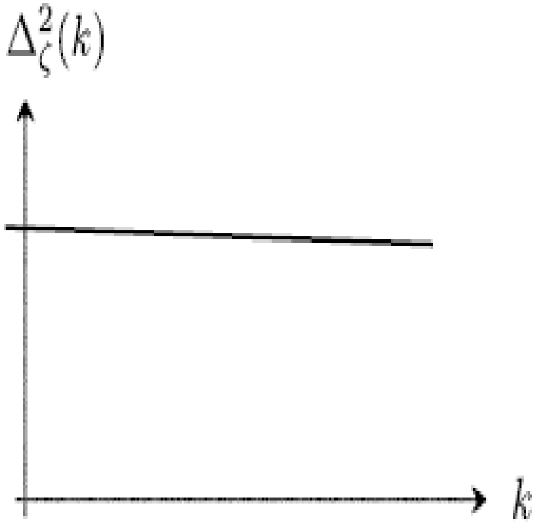
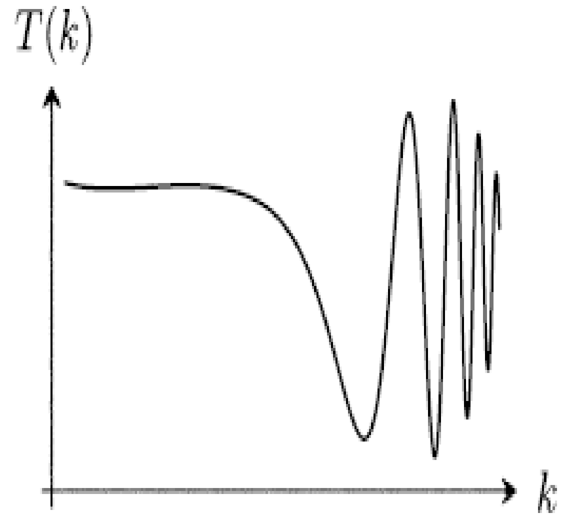


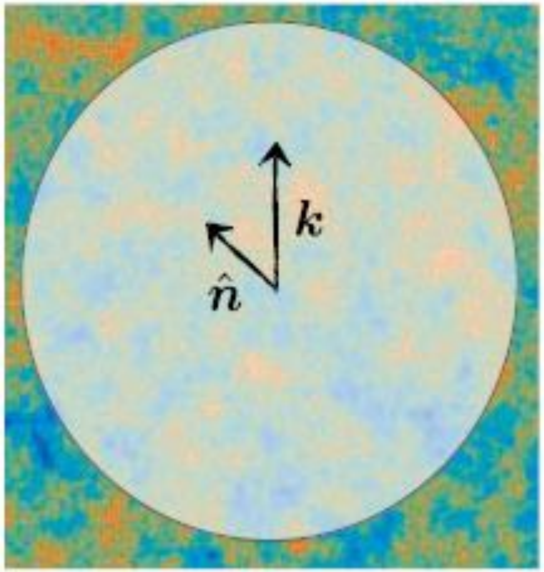
For Gaussian fluctuations, the statistics is determined by the 2-pt function:

$$< (a^X_{lm})^* a^Y_{l'm'} > = C_l^{XY} \delta_{ll'} \delta_{mm'}$$ **where** $m = -l, ..., +l$, $X, Y = T, E, B$

Initial Conditions + Evolution + Projection







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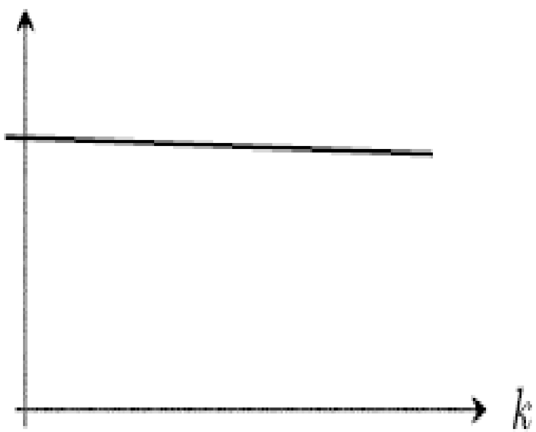
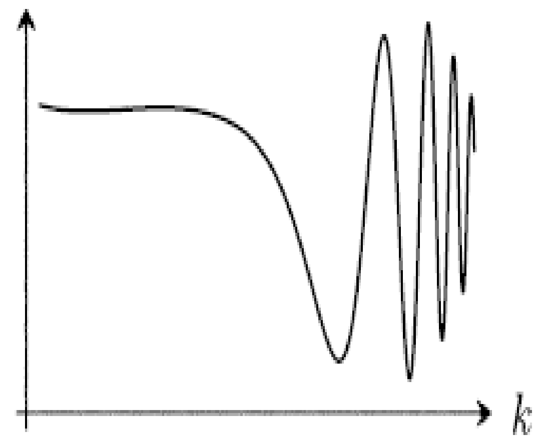
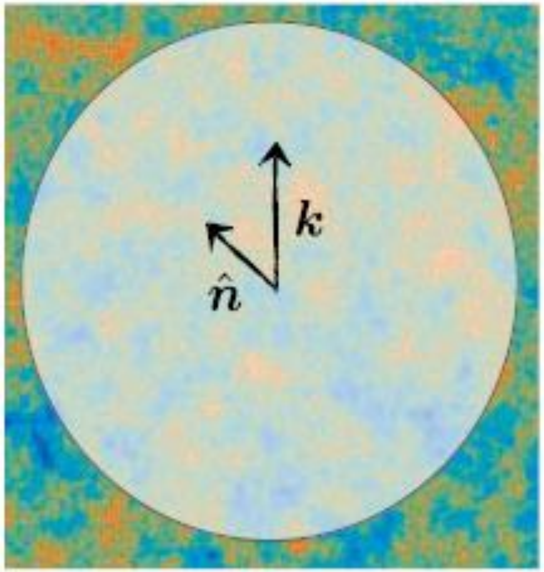


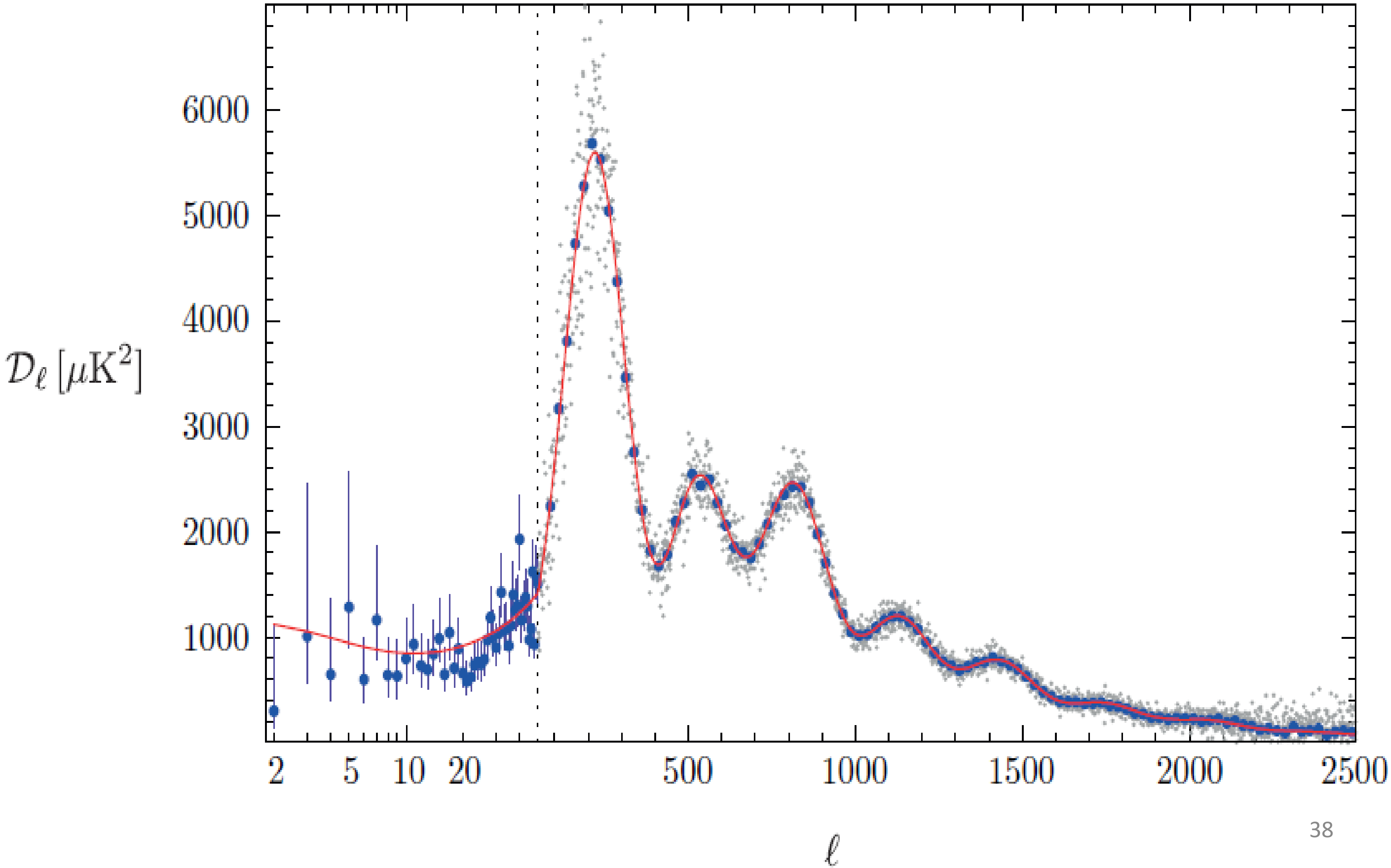




$$C_l^{XY} = \frac{2}{\pi}\int_{k_L(=k_e)}^{k_H(>k_\star)} dk\, k^2 \underbrace{P(k)}_{INFLATION}\ \underbrace{\Delta_{Xl}(k)\Delta_{Yl}(k)}_{ANISOTROPIES}$$

$$\Delta_{Xl}(k) = \int_0^{\eta_0} d\eta\ \underbrace{S_X(k,\eta)}_{SOURCES}\ \underbrace{P_{Xl}(k[\eta_0-\eta])}_{PROJECTION}$$

CMB TT Power spectrum



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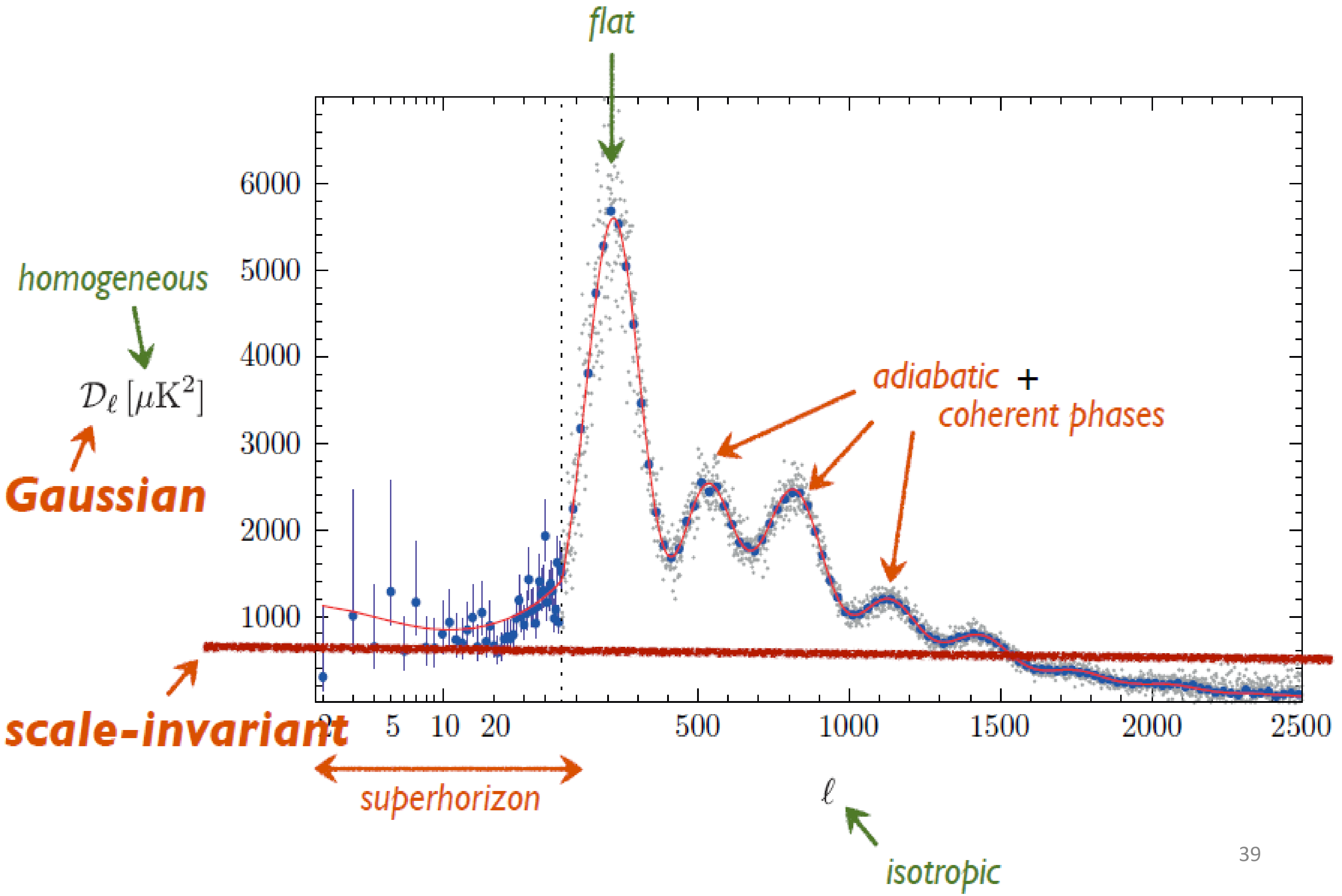

Various probes

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(Upto z=3300)
Early Universe

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6-Parameter Fit

Baseline ΛCDM Model

4 parameters for the ***background***:

$$\Omega_b = 0.045 \pm 0.001 \quad \text{baryons}$$

$$\Omega_m = 0.315 \pm 0.016 \quad \text{dark matter}$$

$$\Omega_\Lambda = 0.685 \pm 0.018 \quad \text{dark energy}$$

$$\tau = 0.089 \pm 0.014 \quad \text{optical depth}$$

2 parameters for the ***perturbations***:

(assuming $r = 0$ as of now)

$$10^9 A_s = 2.20 \pm 0.11 \quad \text{amplitude}$$

$$n_s = 0.960 \pm 0.014 \quad \text{spectral index}$$

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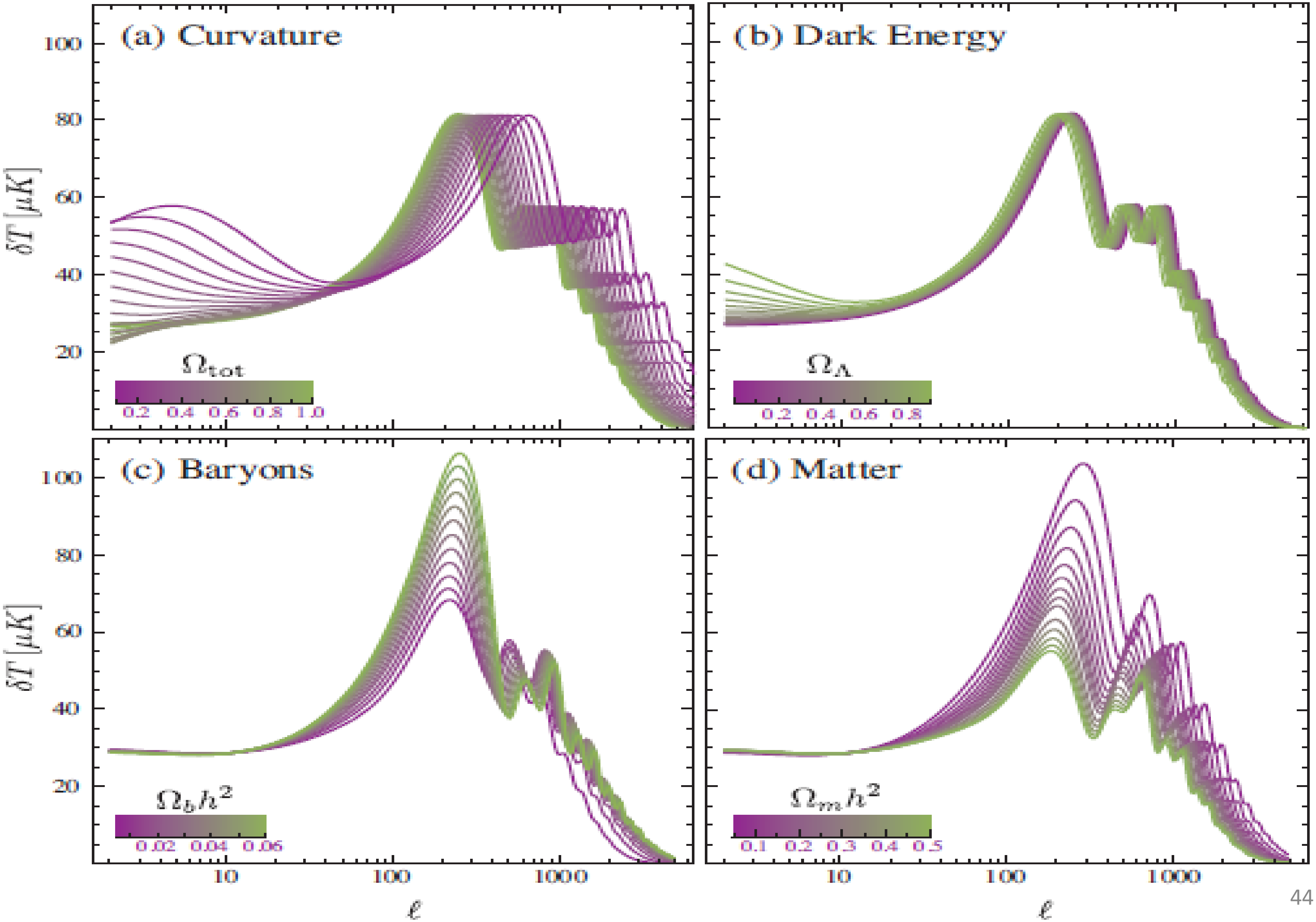
Cosmological Parameter Dependences



Observational limits

Sr. No.	Inflationary observables	PLANCK+WP+BICEP2	PLANCK+WP	WP
1	$\ln(10^{10} P_S)$	$3.089^{+0.024}_{-0.027}$	$3.089^{+0.024}_{-0.027}$	$3.204^{+0.328}_{-0.328}$
2	n_S	0.9600 ± 0.0071	0.9603 ± 0.0073	0.9608 ± 0.008
3	α_S	-0.022 ± 0.010	-0.013 ± 0.009	-0.023 ± 0.011
4	κ_S	$0.020^{+0.016}_{-0.015}$	$0.020^{+0.016}_{-0.015}$	?
5	r	$0.2^{+0.07}_{-0.05}$ ($r = 0$ ruled out at 7σ)	< 0.12	< 0.36
6	n_T	1.36 ± 0.83 (Blue) > -0.76 (Red) ($n_T = 0$ ruled out at 3σ)	? ?	? > -0.048 (Red)

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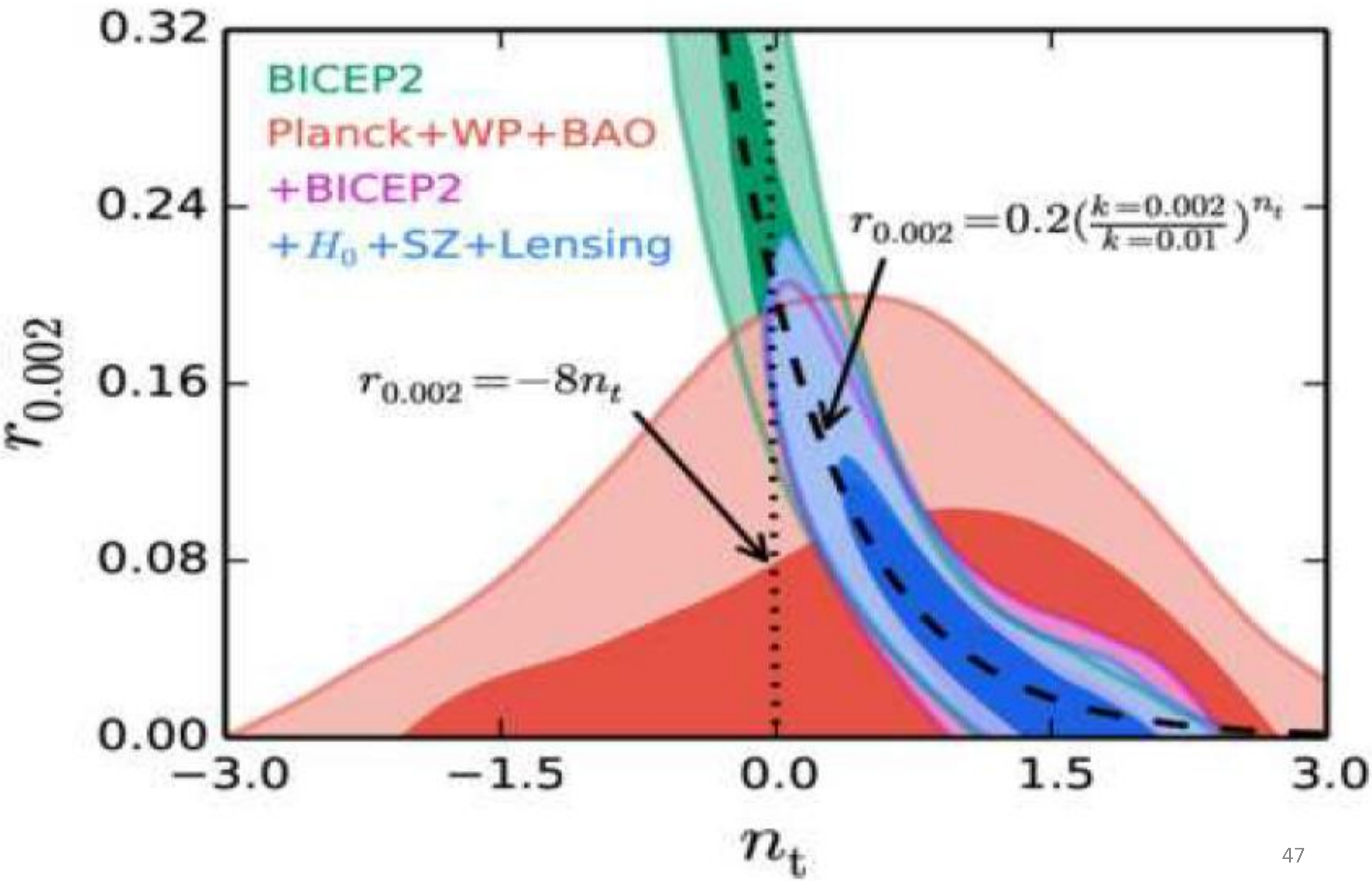
Primordial Gravity Waves: If blue????



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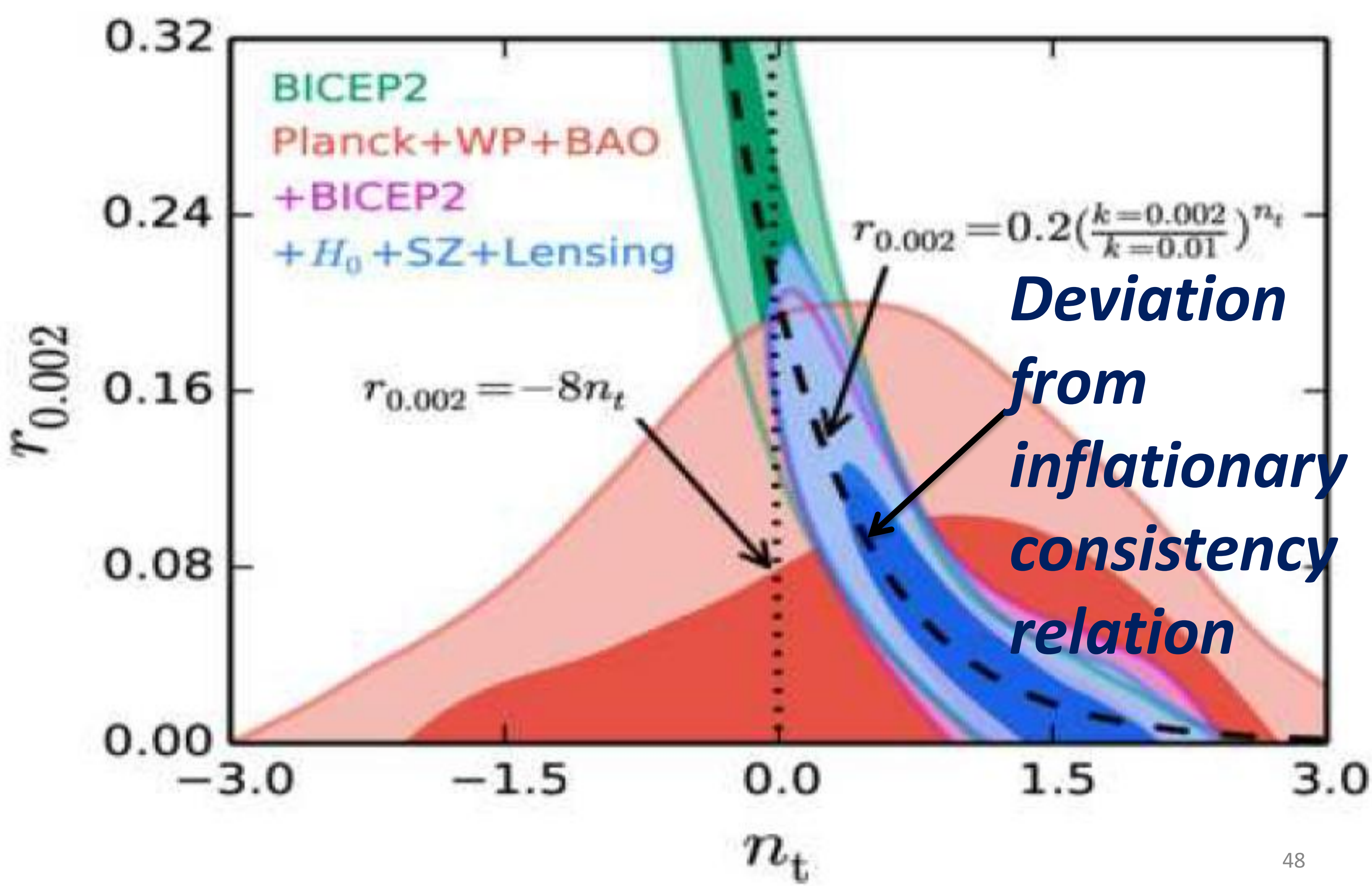
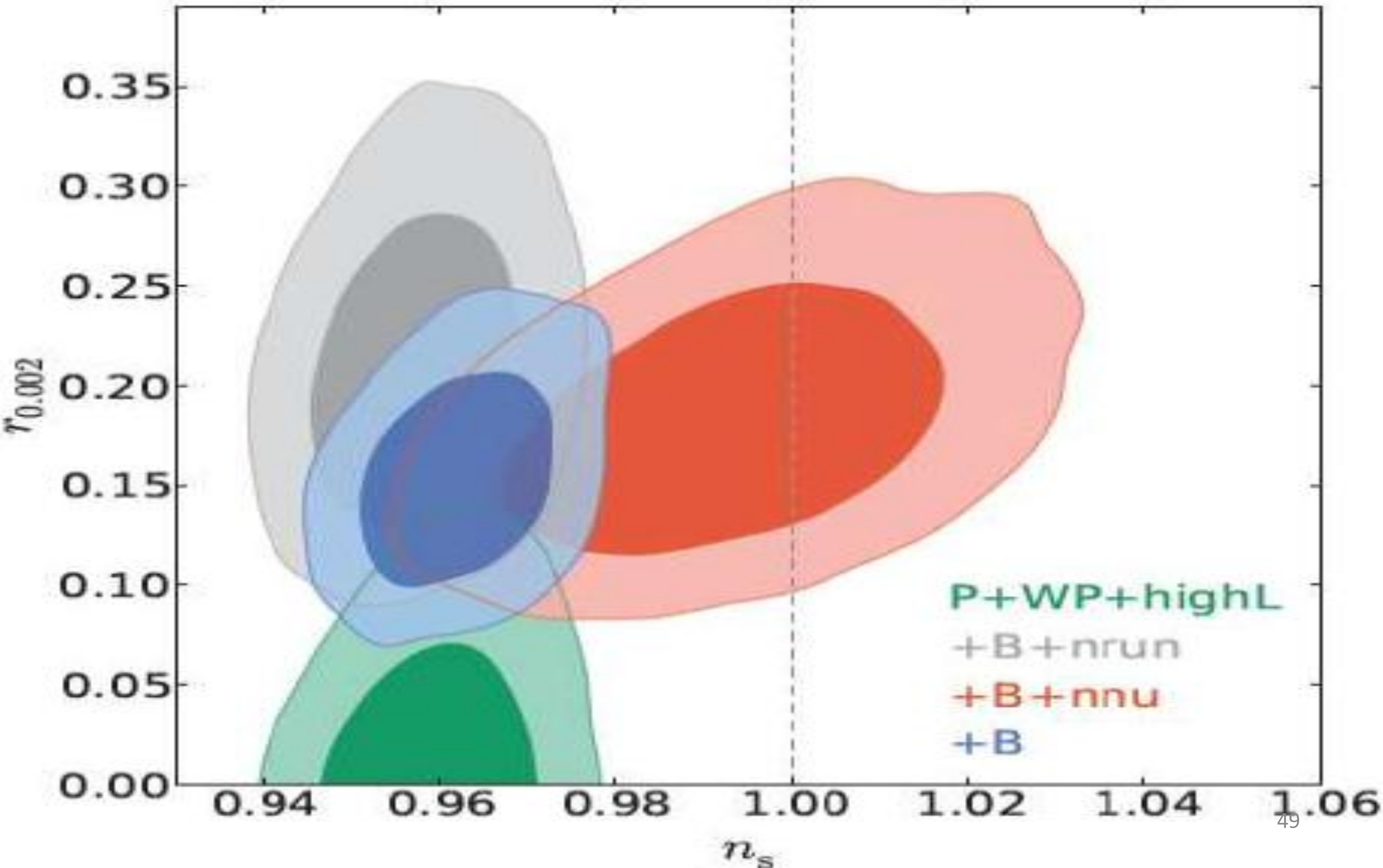
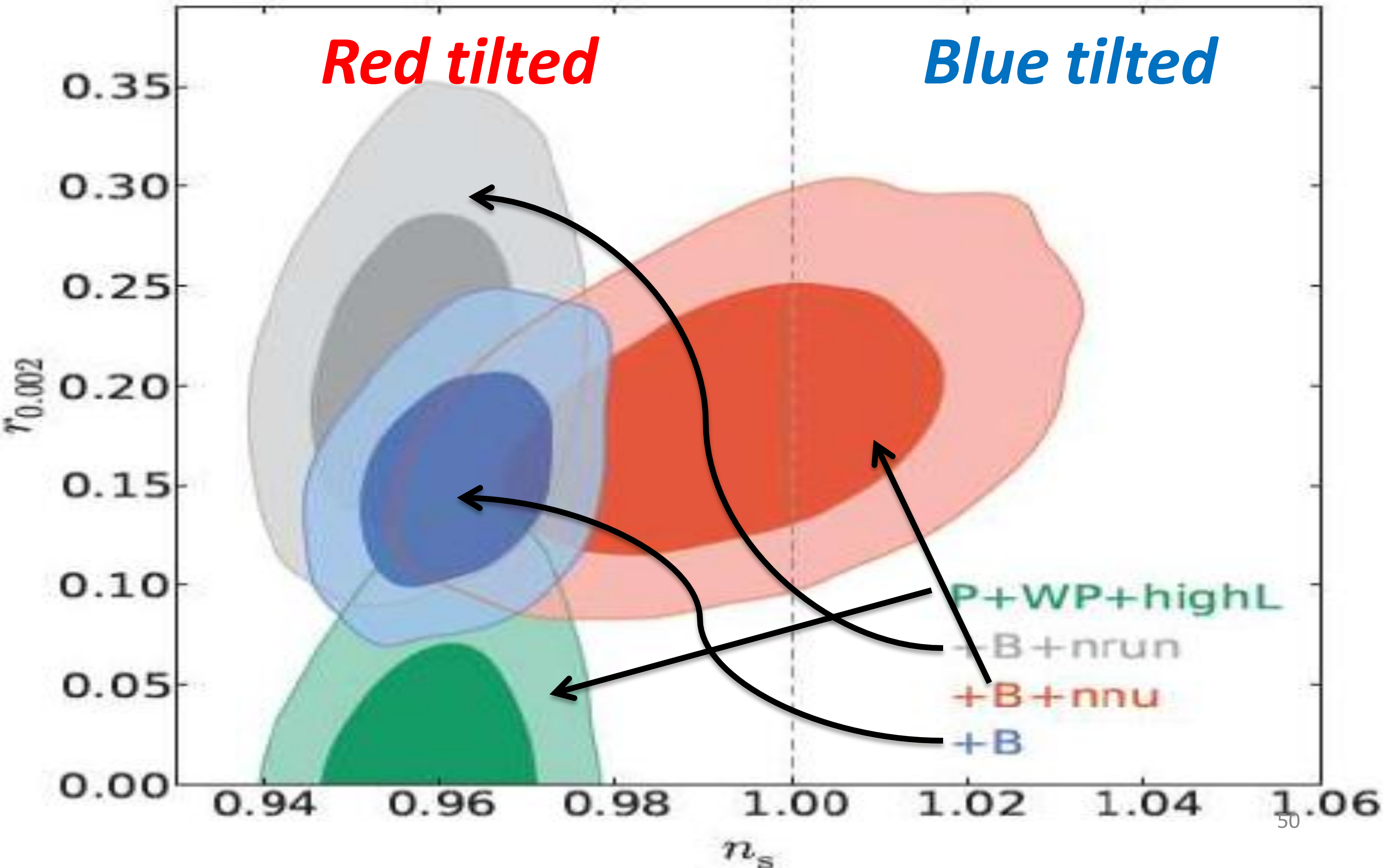


Present status of joint constraints



Present status of joint constraints



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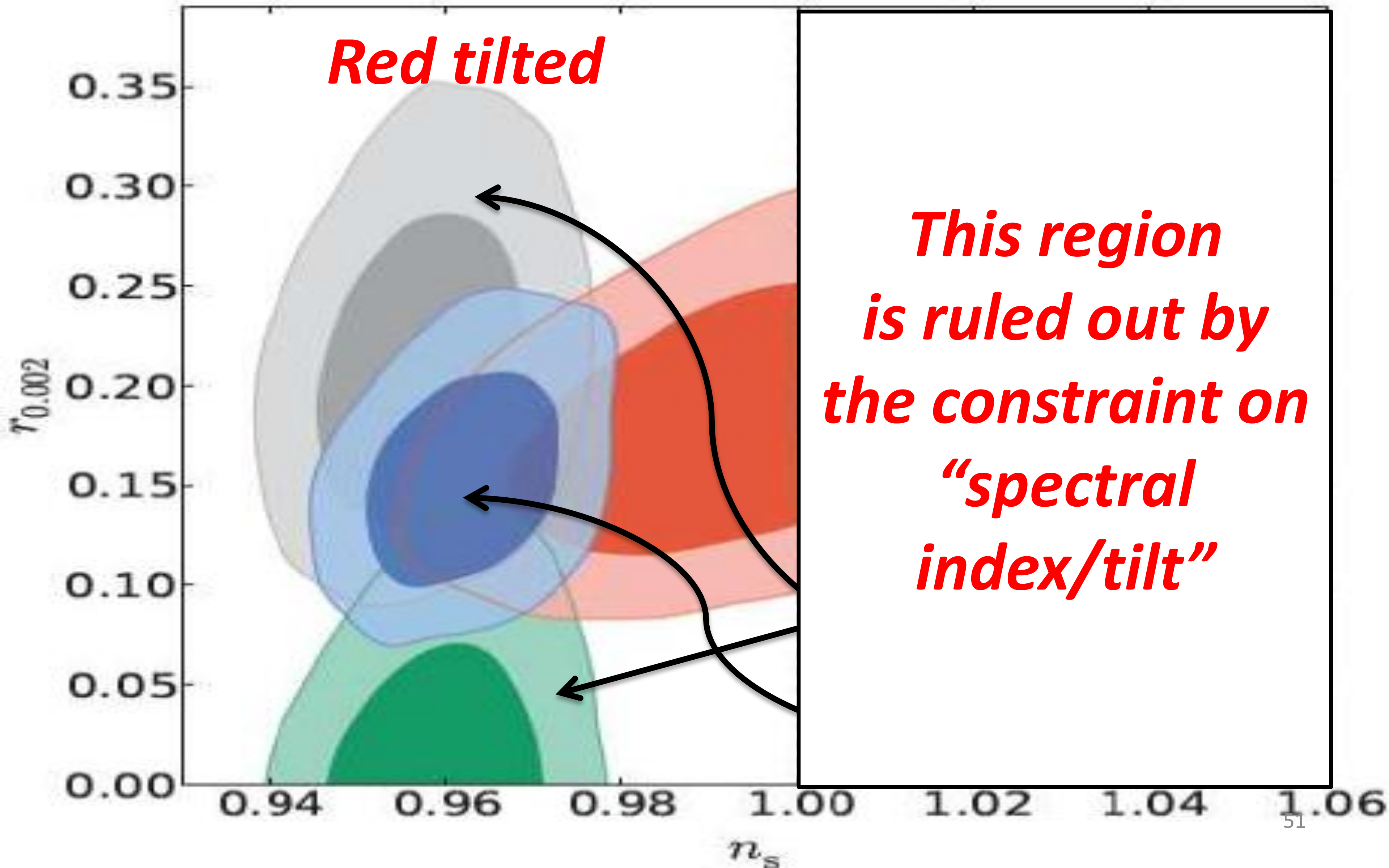

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↓

Reheating temperature

$$T_{rh} \leq \left(\frac{30}{\pi^2 g_*}\right)^{\frac{1}{4}} (1.96 \times 10^{16}\mathrm{GeV}) \sqrt[4]{\frac{r}{0.12}}$$

SC,AM,EP, JHEP 04 (2014) 077,
SC,AM,SP, JCAP 07 (2013) 041,

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Rehe

temp

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But if "r" is fixed via detection of tensor modes at present then scale of inflation lie around the GUT (O($10^{16}\mathrm{GeV}$)) scale.

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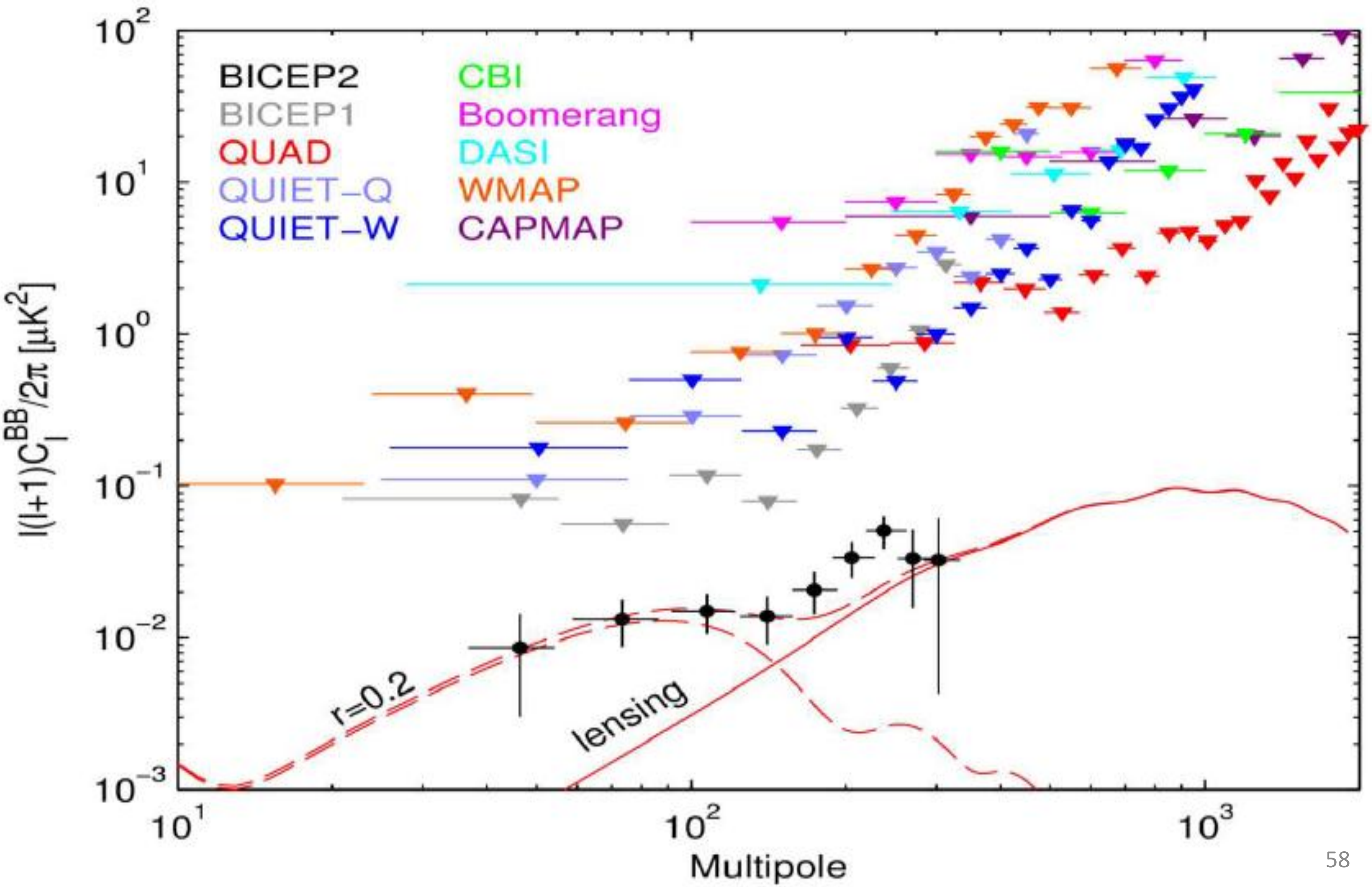
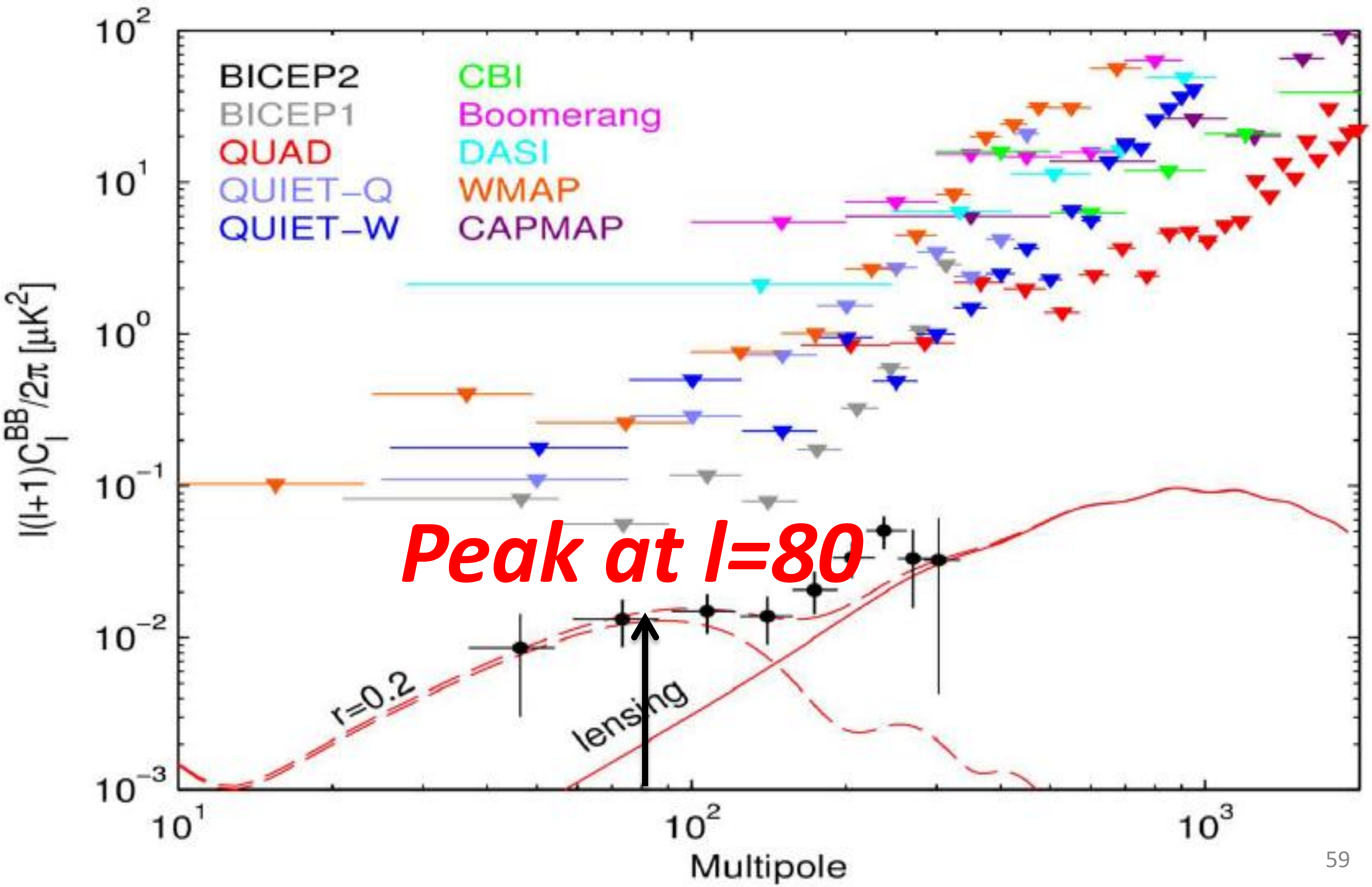
CMB B-modes ......????


BICEP2
BICEP1
QUAD
QUIET–Q
QUIET–W
CBI
Boomerang
DASI
WMAP
CAPMAP
r=0.2
lensing
$l(l+1)C_l^{BB}/2\pi\ [\mu K^2]$
Multipole
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^1
10^2
10^3

CMB B-modes ……????



BICEP2
BICEP1
QUAD
QUIET–Q
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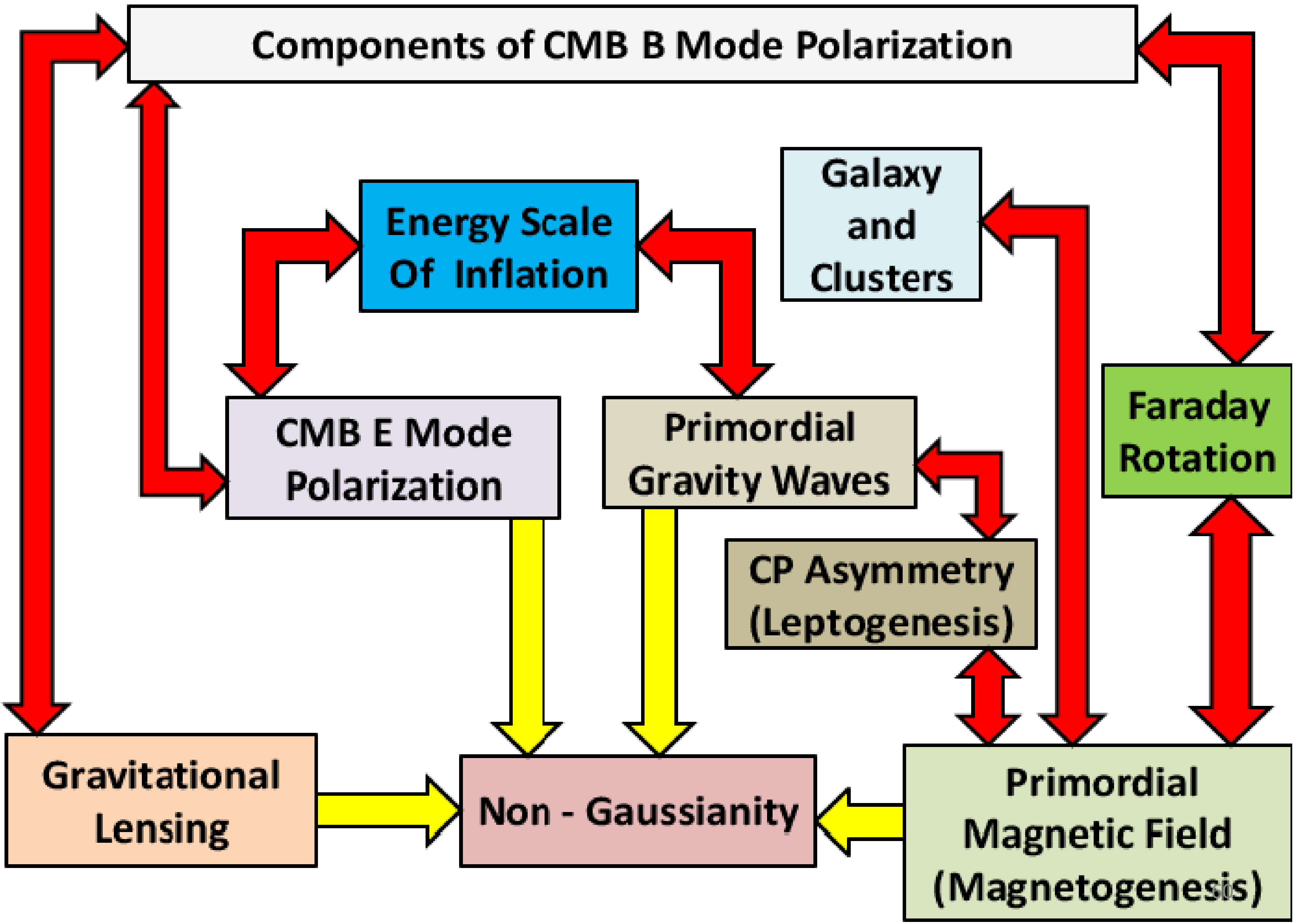
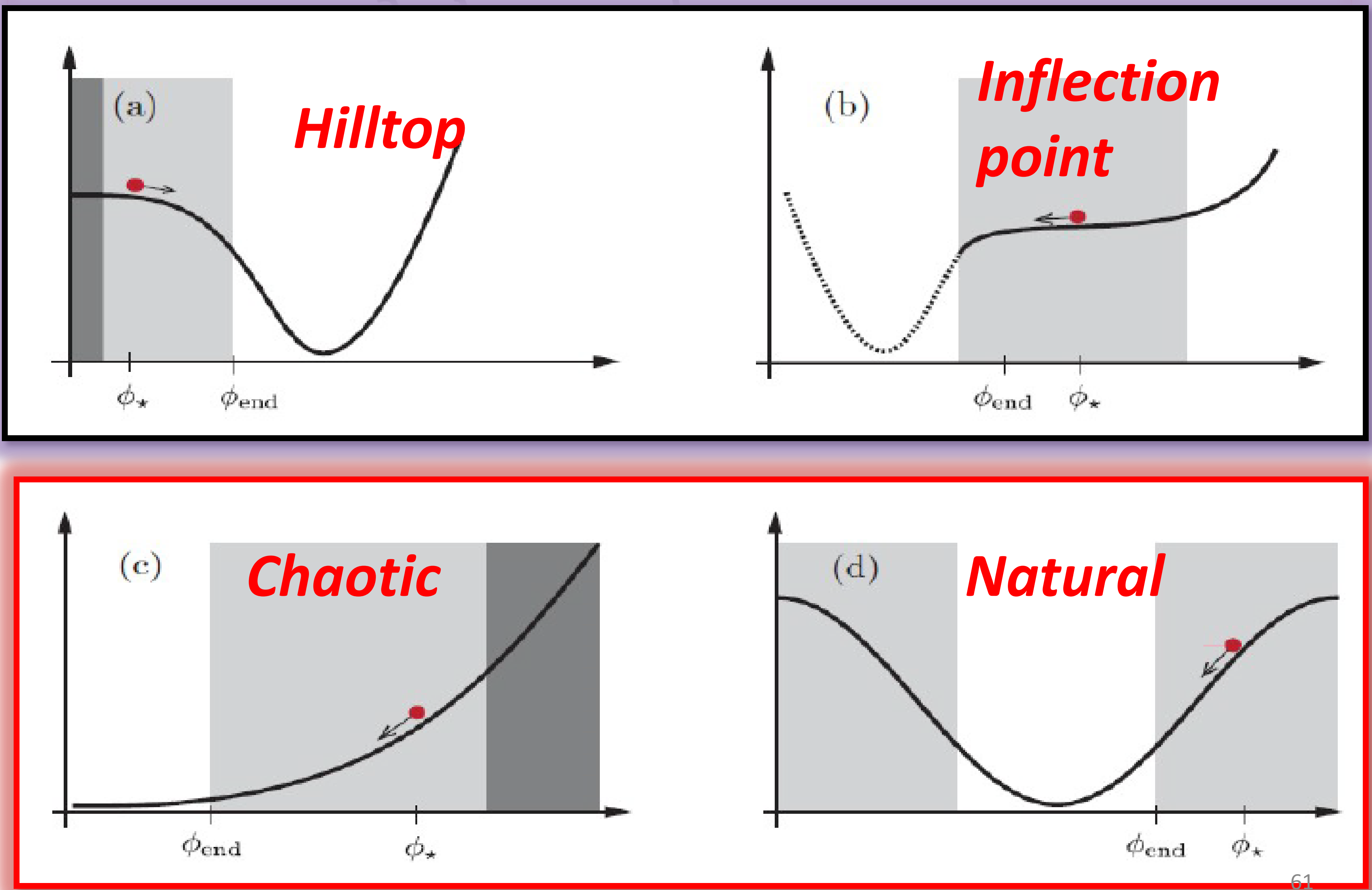
Peak at l=80
r=0.2
lensing
$l(l+1)C_l^{BB}/2\pi$ [μK^2]
Multipole
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{1}
10^{2}
10^{3}


SC,PLB 735 (2014) 138
Components of CMB B Mode Polarization
Galaxy and Clusters
Energy Scale Of Inflation
Faraday Rotation
CMB E Mode Polarization
Primordial Gravity Waves
CP Asymmetry (Leptogenesis)
Gravitational Lensing
Non - Gaussianity
Primordial Magnetic Field (Magnetogenesis)

Modeling inflation & parameter estimation



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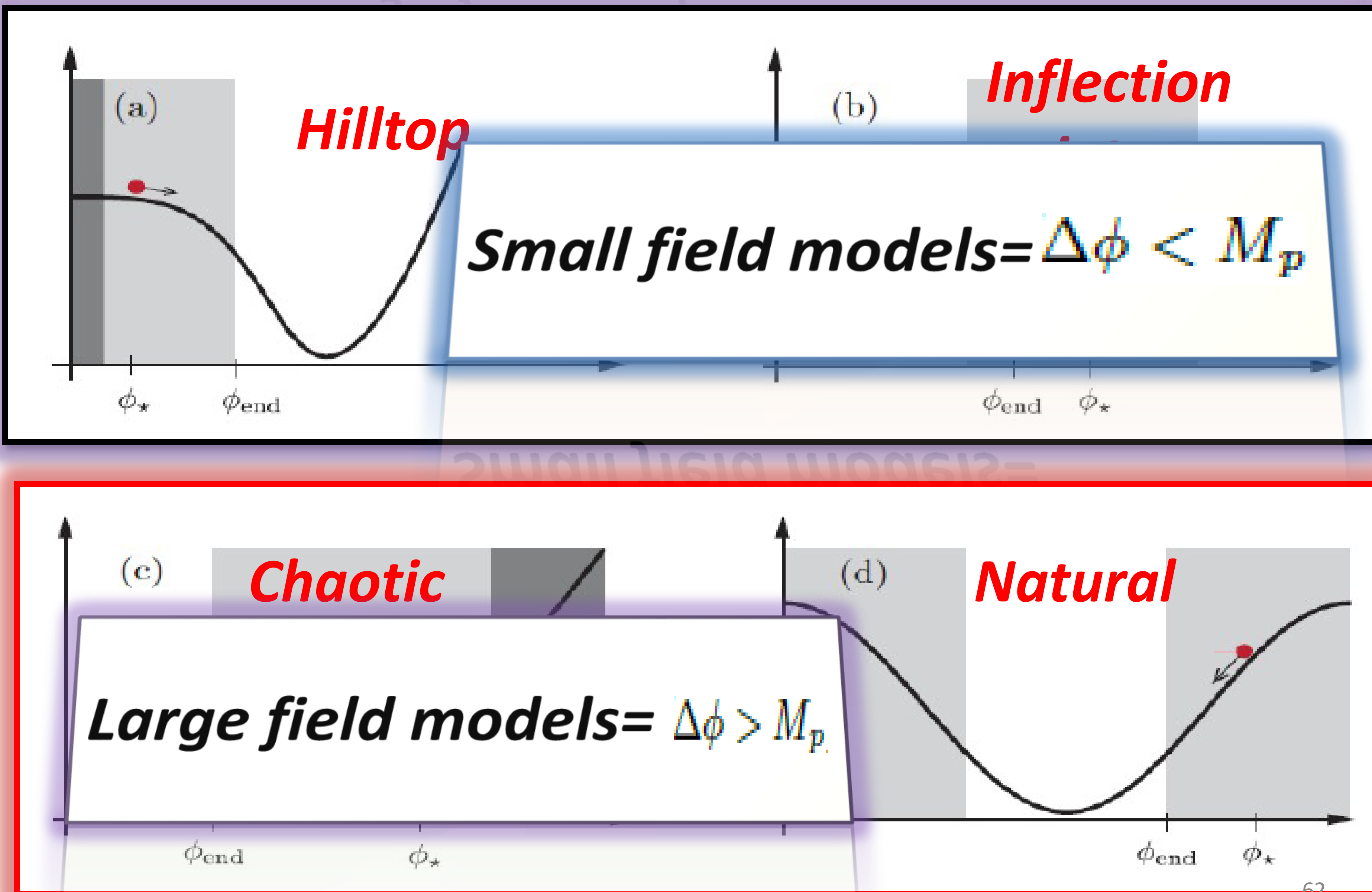

Small Field Model: MSSM inflation

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- MSSM is valid below a certain scale

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- The A-term
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Example of Low scale visible sector model of inflation

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SC,JHEP 04 (2014) 105,
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$$V(\phi,\theta) = V_0 + \frac{(m_\phi^2 + c_H H^2)}{2}|\phi|^2 + (a_H H + a_\lambda m_\phi)\frac{\lambda\phi^n}{nM_p^{n-3}}\cos(n\theta + \theta_{a_H} + \theta_{a_\lambda}) + \frac{\lambda^2|\phi|^{2(n-1)}}{M_p^{2(n-3)}}$$

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SC,JHEP 04 (2014) 105,
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Origin??? → **Hidden Sector : Heavy fields from String sector**

Small Field Model: MSSM inflation

Example of High scale model of inflation

Cosmological Constant

Hubble induced terms

SC,JHEP 04 (2014) 105,
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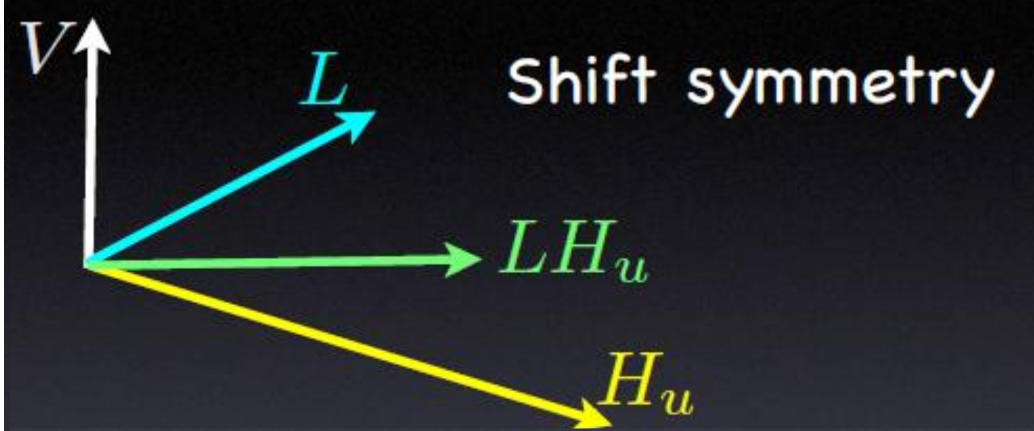
MSSM Flat directions

300 such combinations

	B−L	Always lifted by $W_{\rm renorm}$?
LH_u	−1	
H_uH_d	0	
udd	−1	
LLe	−1	
QdL	−1	
QuH_u	0	√
QdH_d	0	√
LH_de	0	√
$QQQL$	0	
$QuQd$	0	
$QuLe$	0	
$uude$	0	
$QQQH_d$	1	√
QuH_de	1	√
$dddLL$	−3	
$uuuee$	1	
$QuQue$	1	
$QQQQu$	1	
$dddLH_d$	−2	√
$uudQdH_u$	−1	√
$(QQQ)_4LLH_u$	−1	√
$(QQQ)_4LH_uH_d$	0	√
$(QQQ)_4H_uH_dH_d$	1	√
$(QQQ)_4LLLe$	−1	
$uudQdQd$	−1	
$(QQQ)_4LLH_de$	0	√
$(QQQ)_4LH_dH_de$	1	√
$(QQQ)_4H_dH_dH_de$	2	√



$$H_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi \\ 0 \end{pmatrix}$$

$$\Phi = LH_u \equiv c\phi^2$$

$$\textit{In general} \quad \Phi = c\phi^m$$

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QuQue	1	
QQQQu	1	
$dddLH_d$	−2	√
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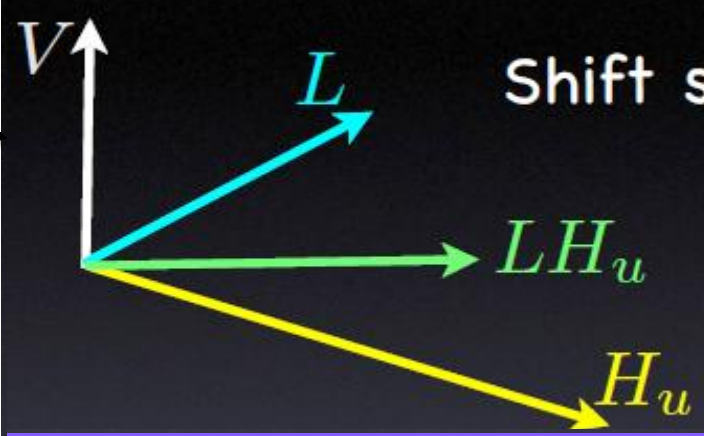

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MSSM INFLATON CANDIDATE

n=4 flat directions:

QQQL, uude, QuQd, QuLe

$$Q_a^{I_1} = \frac{1}{\sqrt{2}}(\Phi, 0)^T, \quad Q_b^{I_2} = \frac{1}{\sqrt{2}}(\Phi, 0)^T, \quad Q_c^{I_3} = \frac{1}{\sqrt{2}}(\Phi, 0)^T,$$

$$L_3^{I_4} = \frac{1}{\sqrt{2}} P_d (0, \Phi)^T, \quad d_a^{B_1} = \frac{\Phi}{\sqrt{2}}, \quad u_b^{B_2} = \frac{\Phi}{\sqrt{2}},$$

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Gauge invariant superpotential:

$$W_4 \approx \frac{\lambda_4}{4M}\Phi^4$$

MSSM INFLATON CANDIDATE

n=6 flat directions:

udd, LLe

$$u_i^\alpha = \frac{1}{\sqrt{3}}\phi\,, \quad d_j^\beta = \frac{1}{\sqrt{3}}\phi\,, \quad d_k^\gamma = \frac{1}{\sqrt{3}}\phi\,.$$

Baryonic

$$L_i^a = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 \\ \phi \end{pmatrix}\,, \quad L_j^b = \frac{1}{\sqrt{3}}\begin{pmatrix} \phi \\ 0 \end{pmatrix}\,, \quad e_k = \frac{1}{\sqrt{3}}\phi\,,$$

Leptonic

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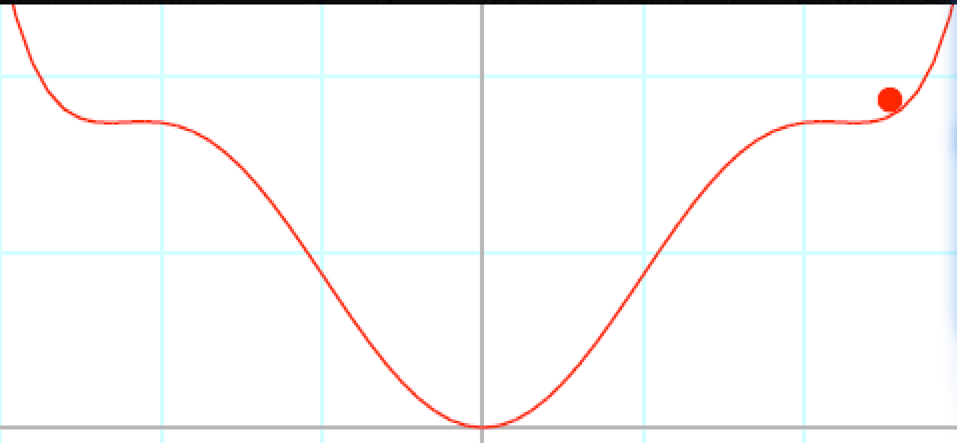
Leptonic

Gauge invariant Superpotential:

$$W_6 \approx \frac{\lambda}{6M_{PL}}\Phi^6$$

Cosmologically Flat Potential



$$\phi = \phi_0 \sim \left(m_\phi M_P^{n-3}\right)^{1/n-2} \ll M_P$$

MSSM superpotential

$$W_{MSSM} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d$$

$$A^2 = 8(n-1)m_\phi^2$$

Saddle point condition: $V'(\phi_0) = 0, \quad V''(\phi_0) = 0$

$$V'''(\phi_0) \neq 0$$

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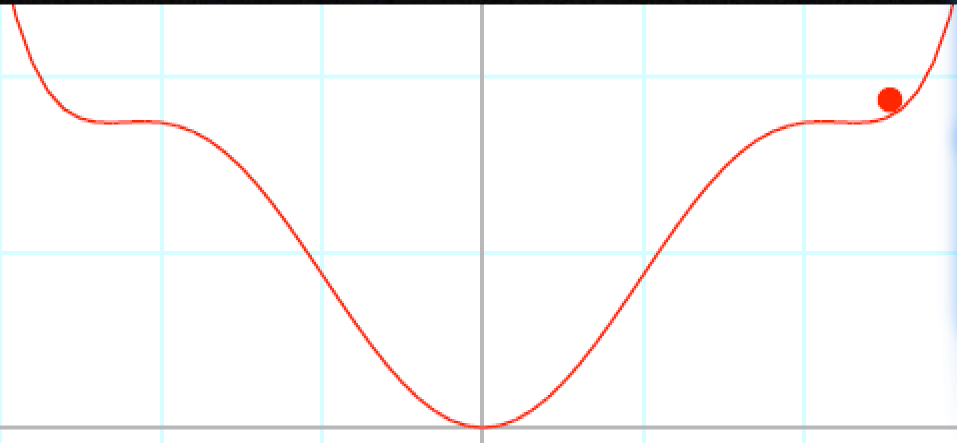

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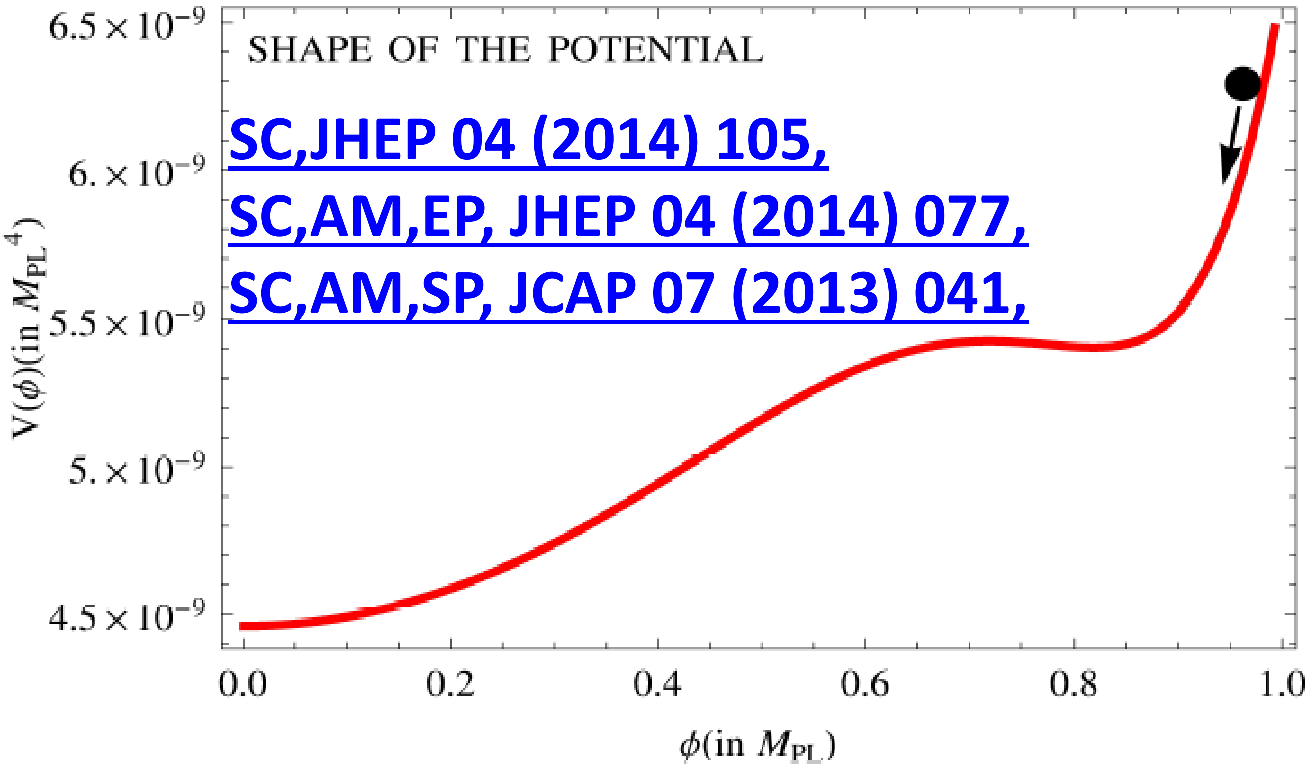
$$m_\phi \sim 1\ \mathrm{TeV}$$
$$H_{inf} \sim 1\ \mathrm{GeV}$$
$$\phi_0 = 3 \times 10^{14}\ GeV$$

Sub-Planckian

$$\Delta\phi \sim \frac{H_{inf}^2}{V'''(\phi_0)} \sim \left(\frac{\phi_0^3}{M_P^2}\right) \gg H_{inf}$$

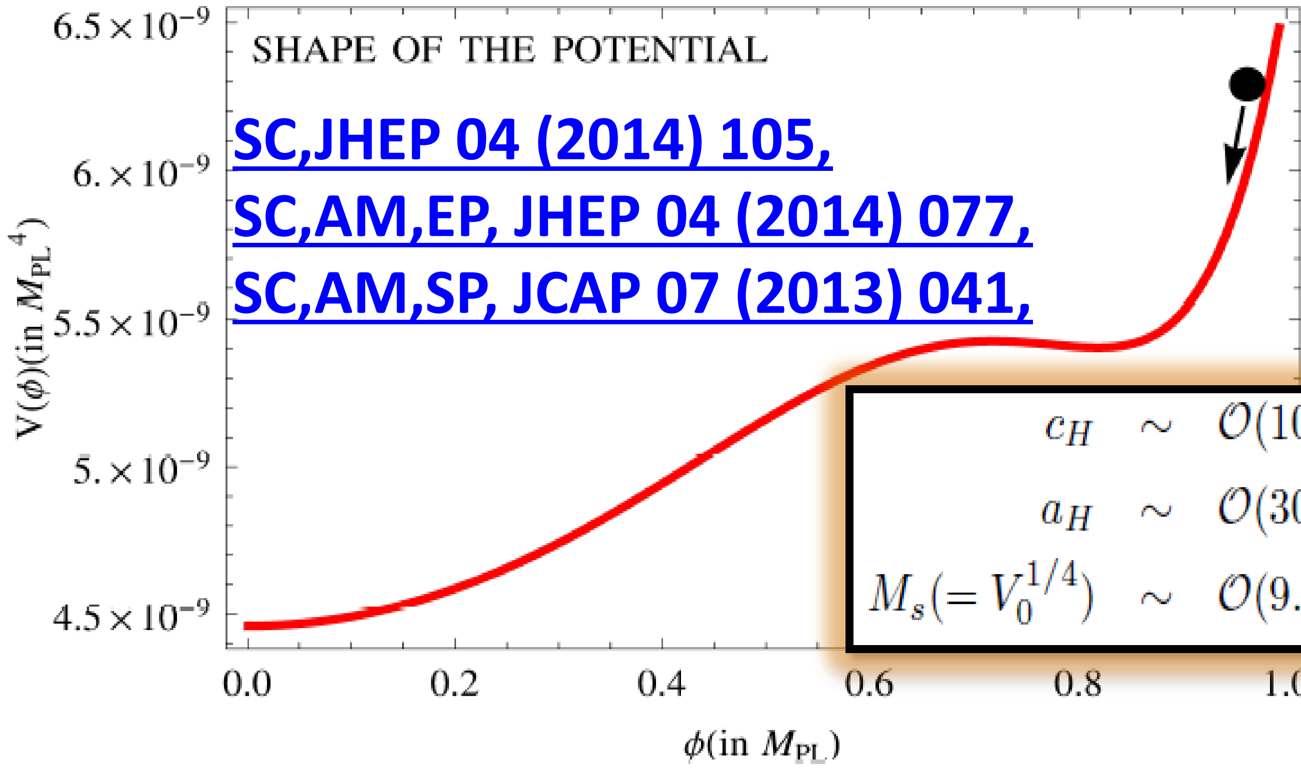


▶ **Inflection point constraints**

(1) **Tuning** $\Rightarrow$ $$\frac{a_H^2}{40c_H^2} = 1 - 4\delta^2$$

(2) **Flatness** $\Rightarrow$ $$V''(\phi_0) = 0$$

(3) **VEV/IP** $\Rightarrow$ $$\phi_0 = \left(\sqrt{\frac{c_H}{10}} H M_{PL}^3\right)^{1/4}$$



SC,JHEP 04 (2014) 105,
SC,AM,EP, JHEP 04 (2014) 077,
SC,AM,SP, JCAP 07 (2013) 041,

$$
\begin{aligned}
c_H &\sim \mathcal{O}(10-10^{-6})\,,\\
a_H &\sim \mathcal{O}(30-10^{-3})\,,\\
M_s(=V_0^{1/4}) &\sim \mathcal{O}(9.50\times10^{10}-1.77\times10^{16})\ \mathrm{GeV}
\end{aligned}
$$

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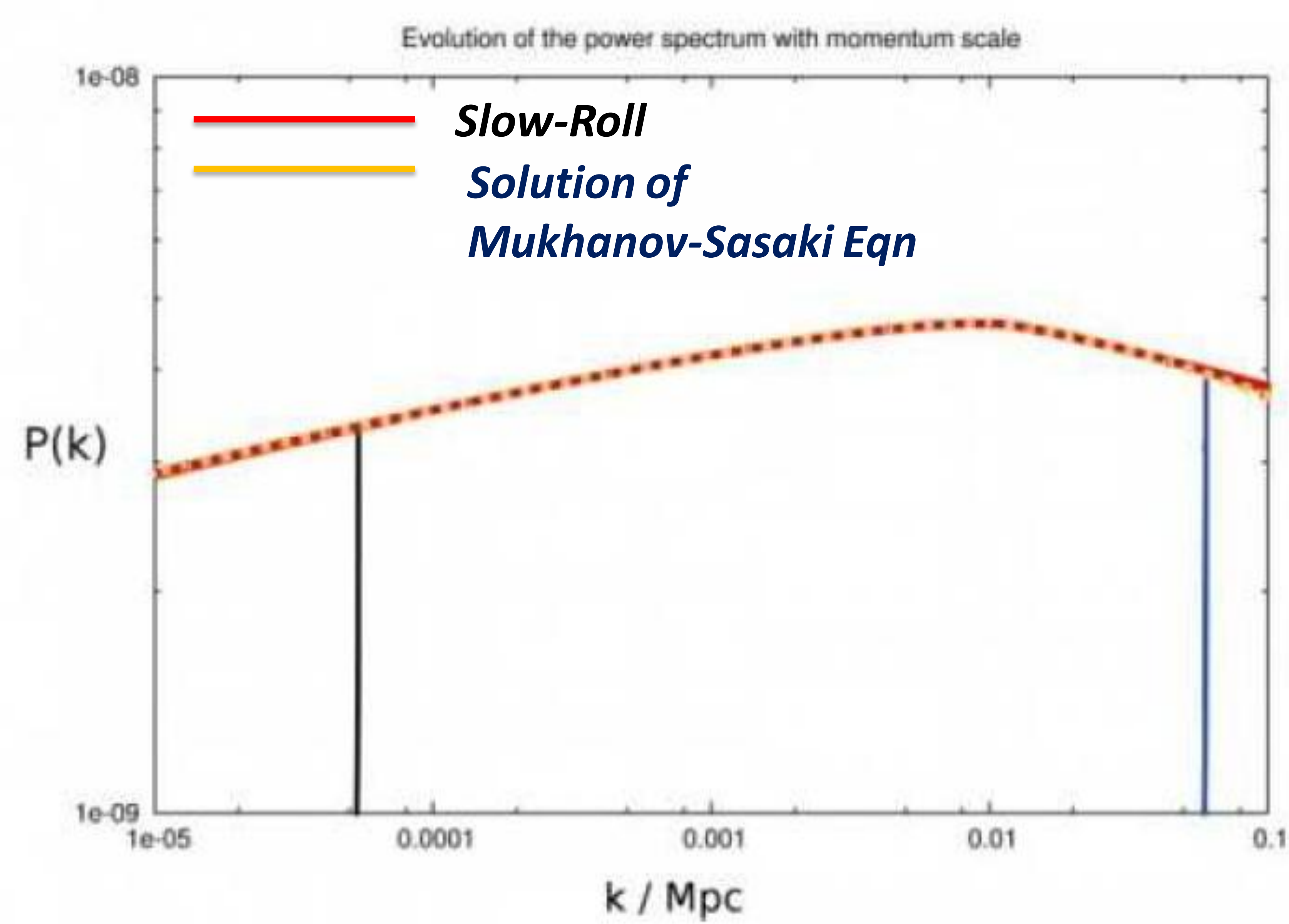
SC,AM,SP, JCAP 07 (2013) 041

Validity of slow-roll approximation



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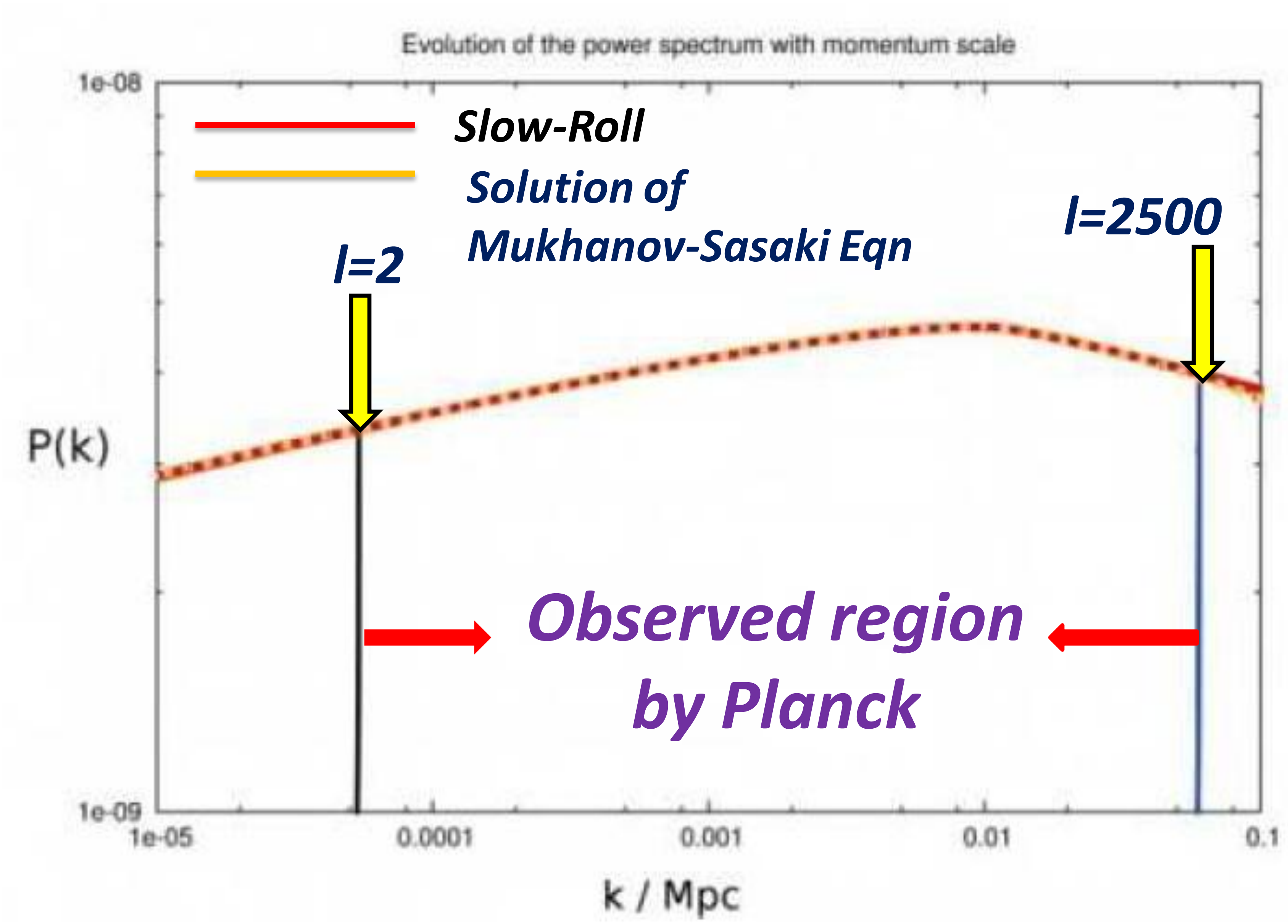
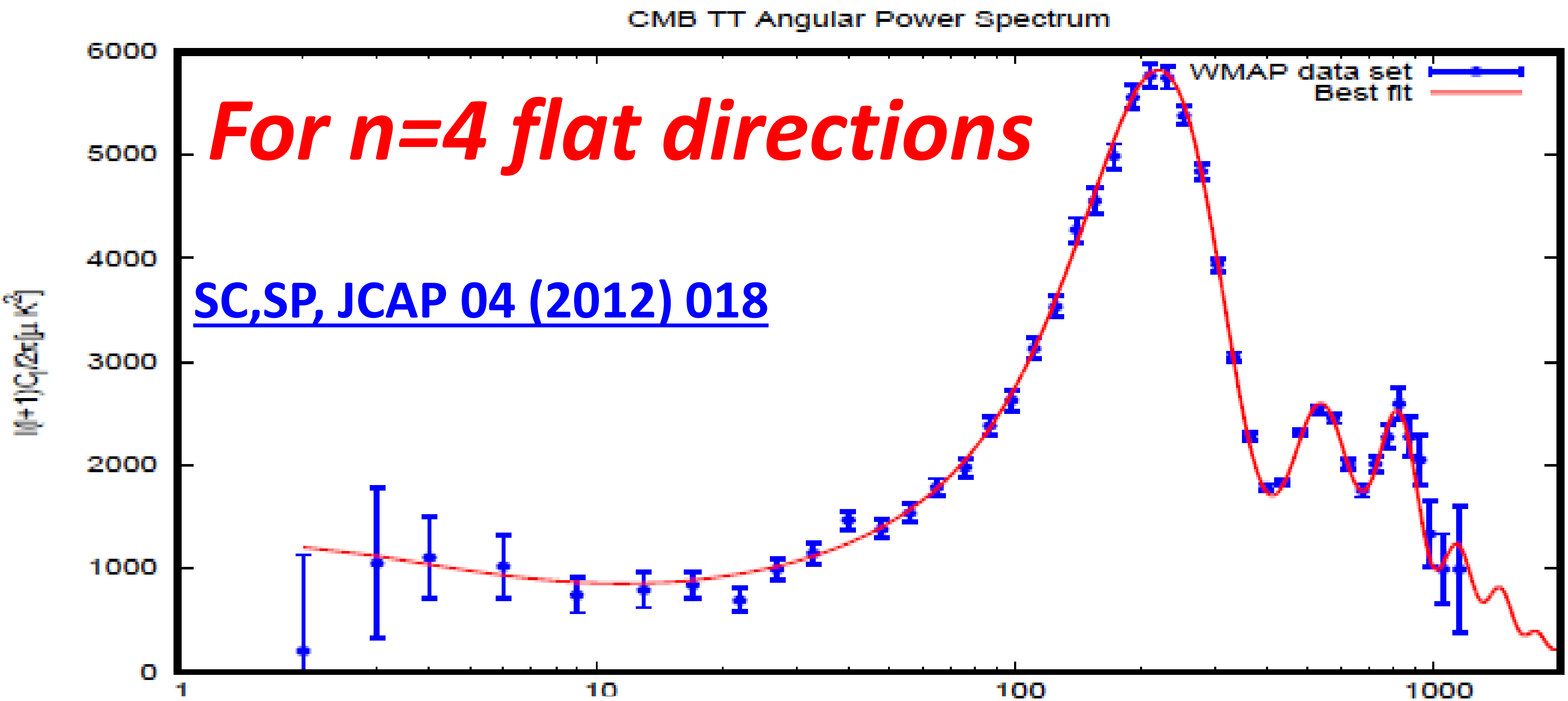


For n=4 flat directions

SC,SP, JCAP 04 (2012) 018

$$P_s(=\Delta_s^2) = 2.498 \times 10^{-9},\, n_s = 0.957,\, n_t = -1.550 \times 10^{-30}$$
$$r = 1.240 \times 10^{-29},\, \alpha_s = -0.612 \times 10^{-3},\, \kappa_s = 1.749 \times 10^{-5}$$

} ***Inflationary observables***



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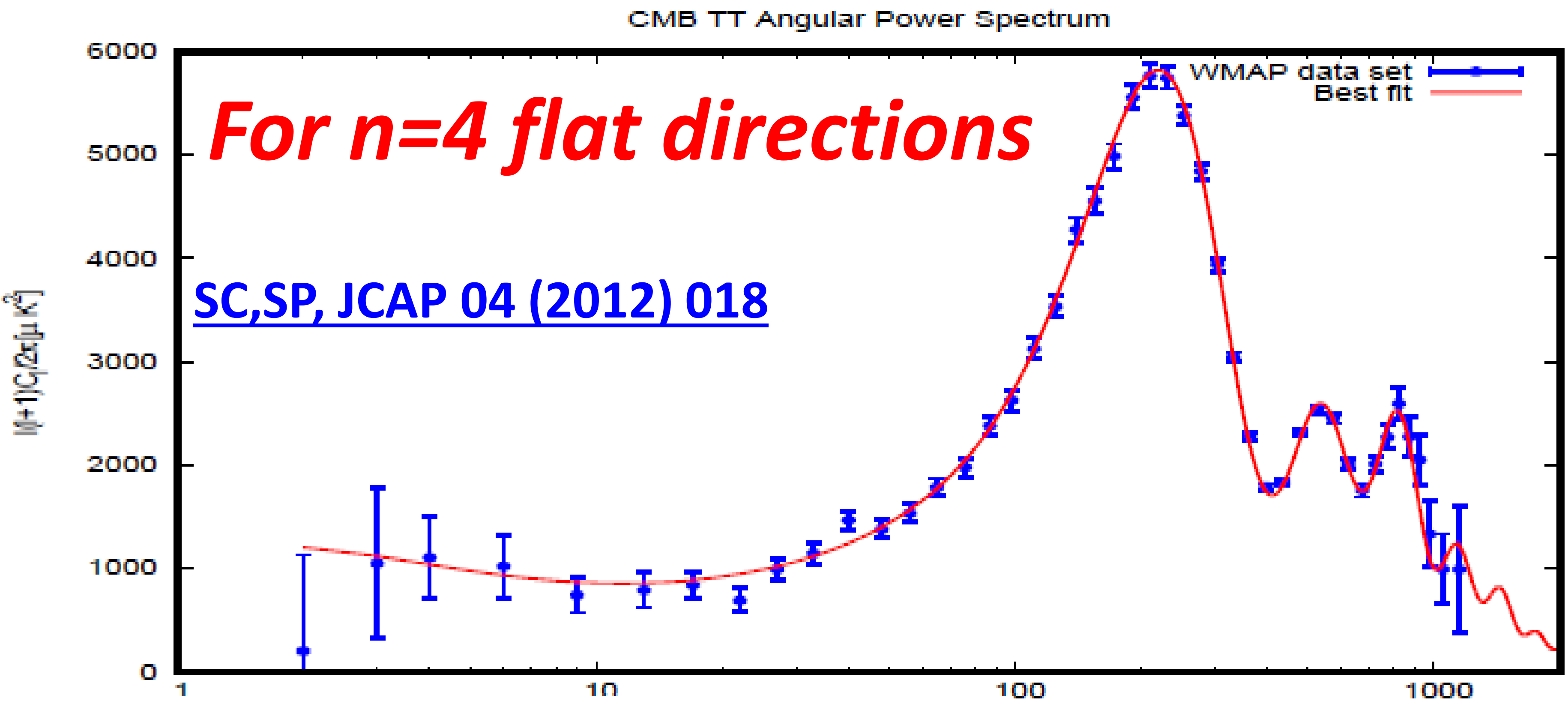

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Inflationary observables

t_0 Gyr	z_{Reion}	Ω_m	Ω_b	Ω_Λ	Ω_k	η_{Rec} Mpc	η_0 Mpc
13.707	10.704	0.2670	0.04	0.7329	0.0	285.10	14345.1

For n=6 flat directions

[SC,AM,SP, JCAP 07 (2013) 041](#)

+ HOSL

$$2.092 < 10^9 P_S < 2.297\,, \quad 0.958 < n_S < 0.963\,, \quad r < 0.12\,,$$
$$-0.0098 < \alpha_S < 0.0003\,, \quad -0.0007 < \kappa_S < 0.006$$

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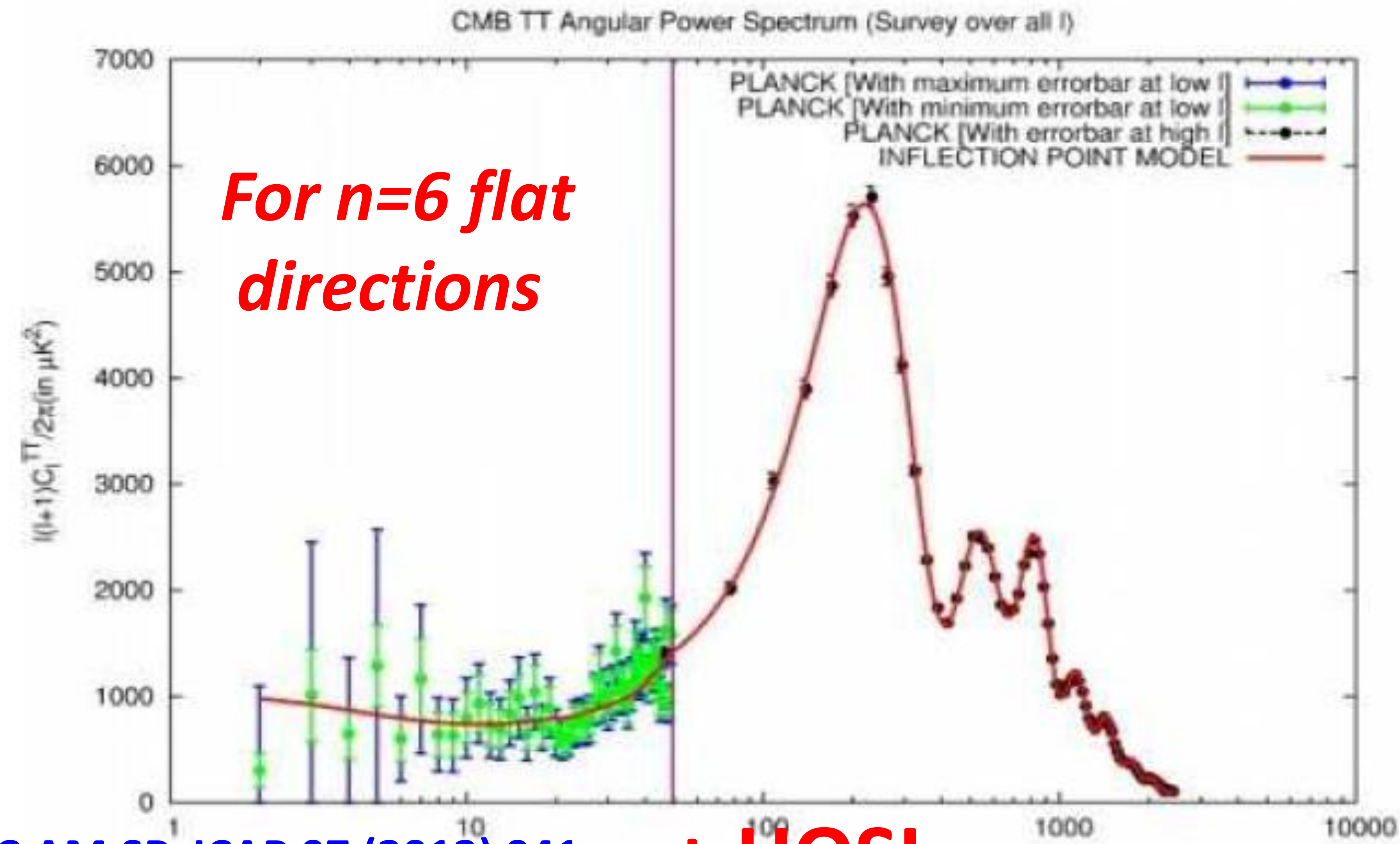

SC,AM,SP, JCAP 07 (2013) 041

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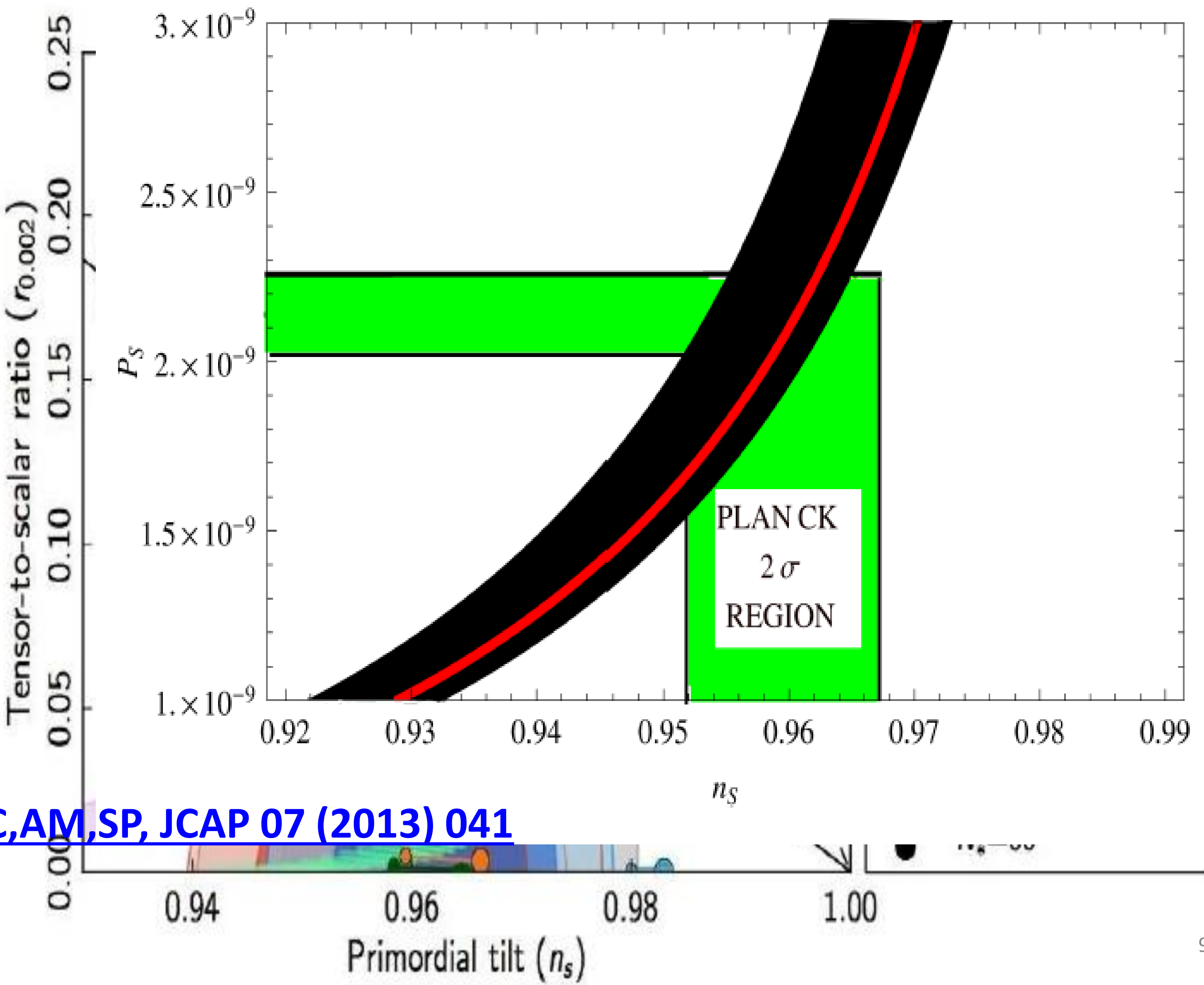

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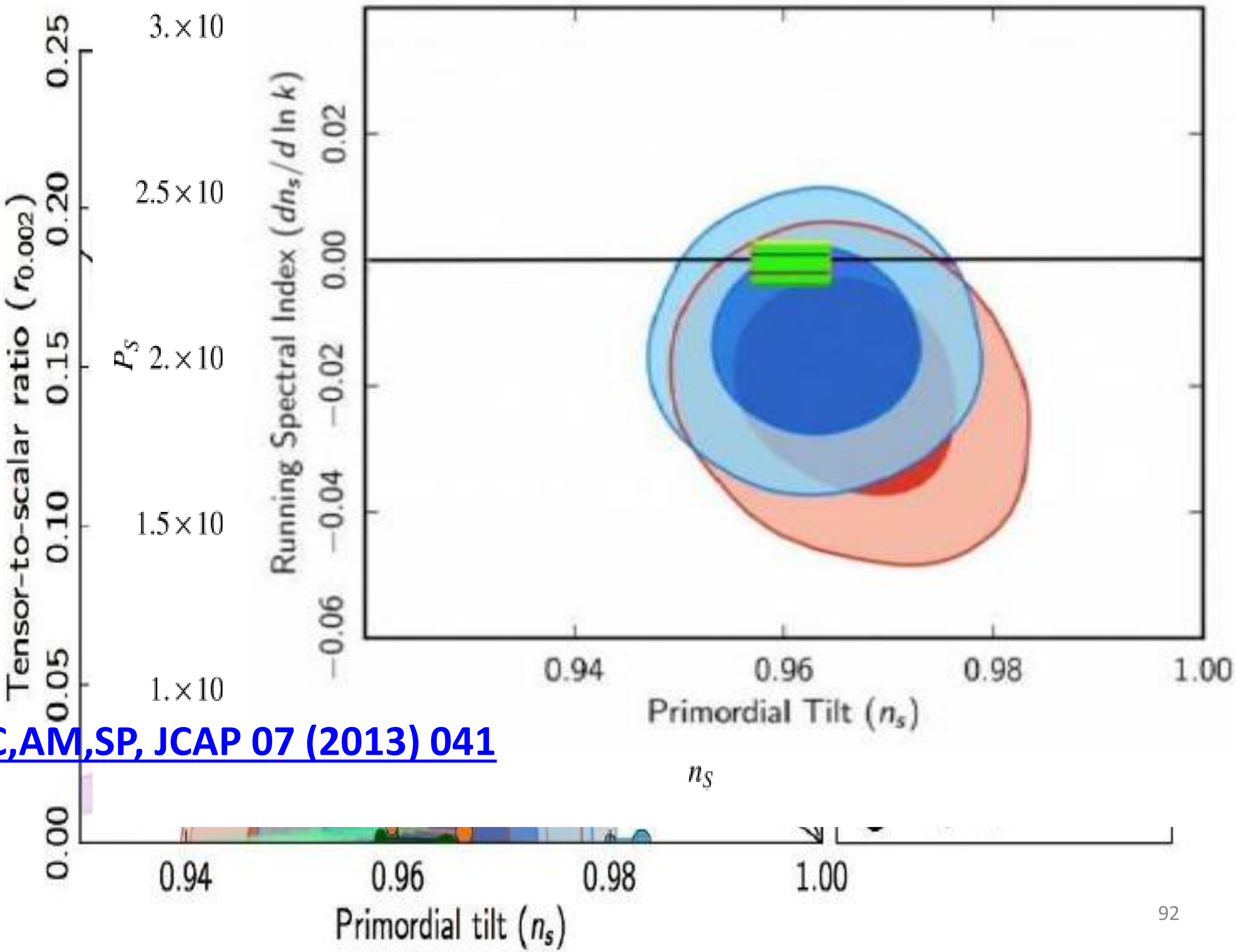

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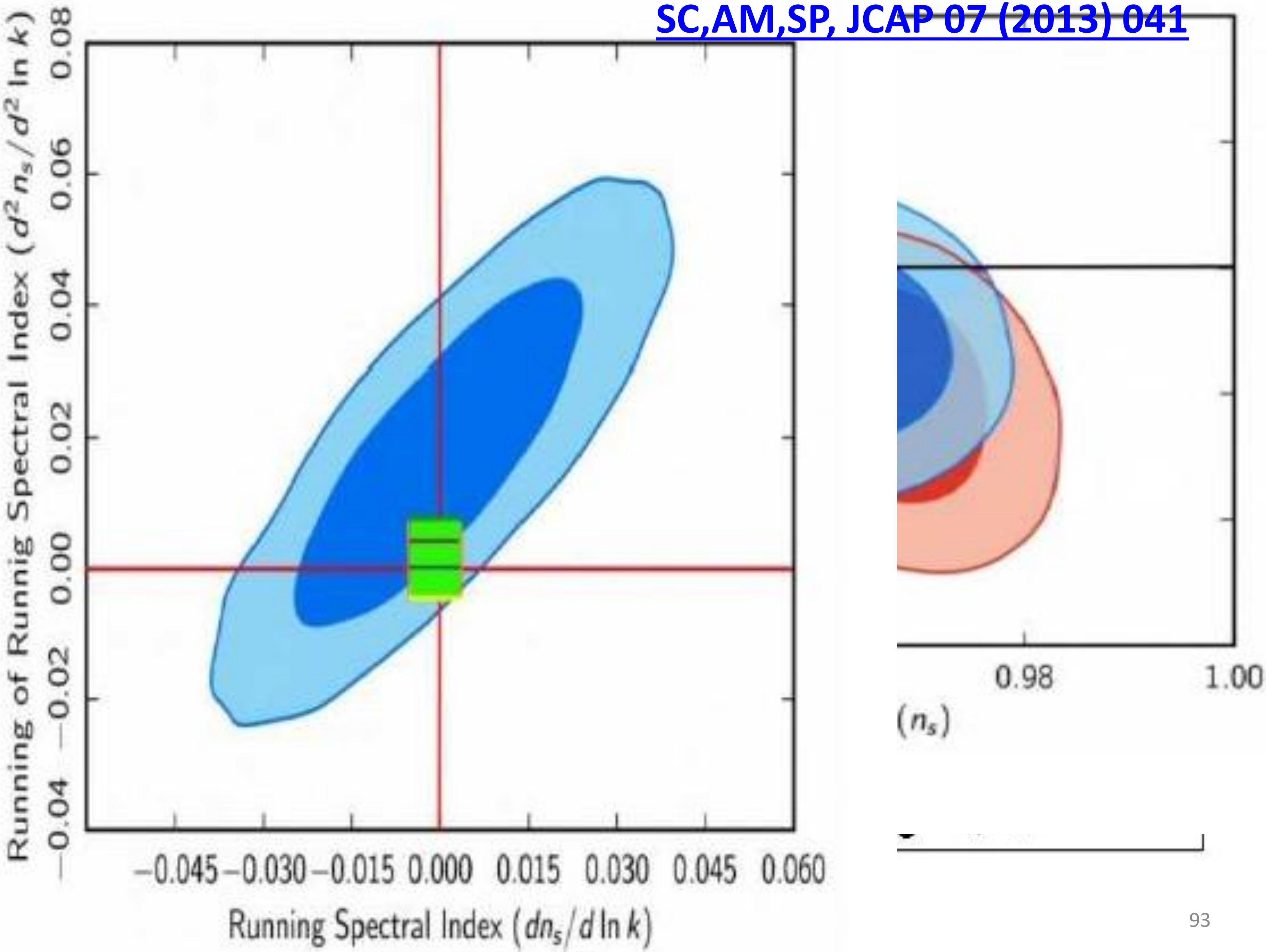
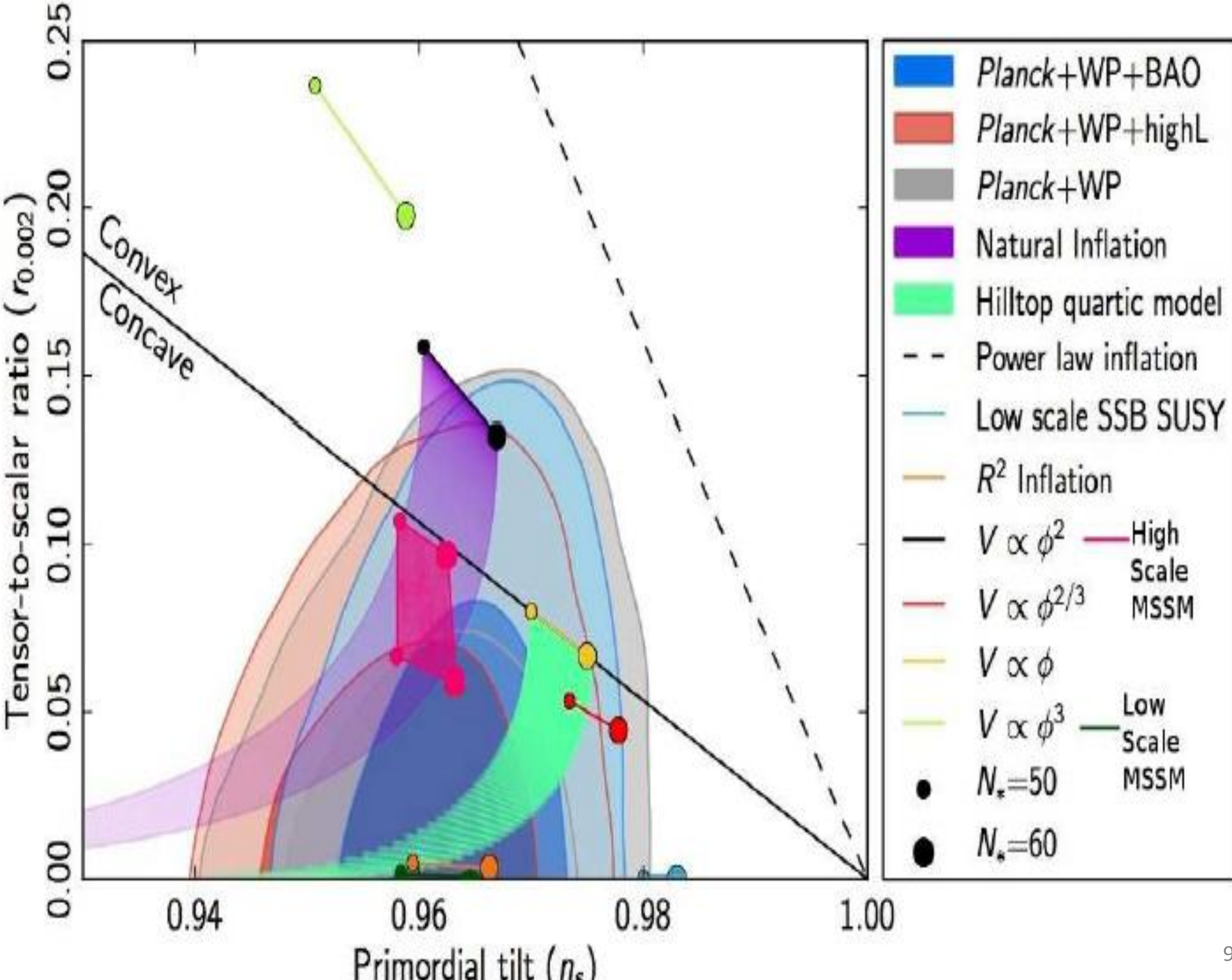



0.25
0.20
0.15
0.10
0.05
0.00
Tensor-to-scalar ratio ($r_{0.002}$)
0.94
0.96
0.98
1.00
Primordial tilt (n_s)
Convex
Concave
Planck+WP+BAO
Planck+WP+highL
Planck+WP
Natural Inflation
Hilltop quartic model
Power law inflation
Low scale SSB SUSY
R^2 Inflation
$V \propto \phi^2$
$V \propto \phi^{2/3}$
$V \propto \phi$
$V \propto \phi^3$
High Scale MSSM
Low Scale MSSM
$N_* = 50$
$N_* = 60$

Reconstruction of inflationary potential

ALGORITHM

Reconstruction of inflationary potential

Inflationary observables at CMB scale

ALGORITHM

Reconstruction of inflationary potential

Inflationary observables at CMB scale → *Assume Slow-Roll*

ALGORITHM

Reconstruction of inflationary potential

Inflationary observables at CMB scale → *Assume Slow-Roll* → *Construct potential and its derivatives at CMB scale*

ALGORITHM

Reconstruction of inflationary potential

Inflationary observables at CMB scale

Assume Slow-Roll

Construct potential and its derivatives at CMB scale

Using matrix inversion technique further construct the co-efficient of Taylor expansion

ALGORITHM

Reconstruction of inflationary potential

ALGORITHM

1. *Inflationary observables at CMB scale*
2. *Assume Slow-Roll*
3. *Construct potential and its derivatives at CMB scale*
4. *Using matrix inversion technique further construct the co-efficient of Taylor expansion*
5. *Reconstructed potential*

Reconstruction of inflationary potential

Inflationary observables at CMB scale → *Assume Slow-Roll*

ALGORITHM

Construct potential and

$$V(\phi) = V(\phi_0) + V'(\phi_0)(\phi - \phi_0) + \frac{V''(\phi_0)}{2}(\phi - \phi_0)^2 + \frac{V'''(\phi_0)}{6}(\phi - \phi_0)^3 + \frac{V''''(\phi_0)}{24}(\phi - \phi_0)^4 + \cdots$$

SC,AM,arXiv:1403.5549,1404.3398,
SC,arXiv:1406.7618

potential

technique further construct the co-efficient of Taylor expansion

Reconstruction of inflationary potential

SC,AM,arXiv:1403.5549,1404.3398,
SC,arXiv:1406.7618

$$V(\phi_\star) = \tfrac{3}{2}P_S(k_\star)r(k_\star)\pi^2 M_p^4,$$

$$V^{'}(\phi_\star) = \tfrac{3}{2}P_S(k_\star)r(k_\star)\pi^2\sqrt{\tfrac{r(k_\star)}{8}}M_p^3,$$

$$V^{''}(\phi_\star) = \tfrac{3}{4}P_S(k_\star)r(k_\star)\pi^2\left(n_S(k_\star)-1+\tfrac{3r(k_\star)}{8}\right)M_p^2,$$

$$V^{'''}(\phi_\star) = \tfrac{3}{2}P_S(k_\star)r(k_\star)\pi^2\left[\sqrt{2r(k_\star)}\left(n_S(k_\star)-1+\tfrac{3r(k_\star)}{8}\right) - \tfrac{1}{2}\left(\tfrac{r(k_\star)}{8}\right)^{\frac{3}{2}} - \alpha_S(k_\star)\sqrt{\tfrac{2}{r(k_\star)}}\right]M_p,$$

$$V^{''''}(\phi_\star) = 12P_S(k_\star)\pi^2\left\{\frac{\kappa_S(k_\star)}{2} - \frac{1}{2}\left(\frac{r(k_\star)}{8}\right)^2\left(n_S(k_\star)-1+\frac{3r(k_\star)}{8}\right) + 12\left(\frac{r(k_\star)}{8}\right)^3 + r(k_\star)\left(n_S(k_\star)-1+\frac{3r(k_\star)}{8}\right)^2 + \left[\sqrt{2r(k_\star)}\left(n_S(k_\star)-1+\frac{3r(k_\star)}{8}\right) - \frac{1}{2}\left(\frac{r(k_\star)}{8}\right)^{\frac{3}{2}} - \alpha_S(k_\star)\frac{2}{r(k_\star)}\right] \times\left[\sqrt{\frac{r(k_\star)}{8}}\left(n_S(k_\star)-1+\frac{3r(k_\star)}{8}\right) - 6\left(\frac{r(k_\star)}{8}\right)^{\frac{3}{2}}\right]\right\}$$

$$\begin{pmatrix} 1 & \Theta_* & \frac{\Theta_*^2}{2} & \frac{\Theta_*^3}{6} & \frac{\Theta_*^4}{24} & \cdots & \cdots \\ 0 & 1 & \Theta_* & \frac{\Theta_*^2}{2} & \frac{\Theta_*^3}{6} & \cdots & \cdots \\ 0 & 0 & 1 & \Theta_* & \frac{\Theta_*^2}{2} & \cdots & \cdots \\ 0 & 0 & 0 & 1 & \Theta_* & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} V(\phi_0) \\ V'(\phi_0) \\ V''(\phi_0) \\ V'''(\phi_0) \\ V''''(\phi_0) \\ \cdots \\ \cdots \end{pmatrix} = \begin{pmatrix} V(\phi_*) \\ V'(\phi_*) \\ V''(\phi_*) \\ V'''(\phi_*) \\ V''''(\phi_*) \\ \cdots \\ \cdots \end{pmatrix}$$

$$
\begin{pmatrix}
1 & \Theta_* & \frac{\Theta_*^2}{2} & \frac{\Theta_*^3}{6} & \frac{\Theta_*^4}{24} & \cdots & \cdots \\
0 & 1 & \Theta_* & \frac{\Theta_*^2}{2} & \frac{\Theta_*^3}{6} & \cdots & \cdots \\
0 & 0 & 1 & \Theta_* & \frac{\Theta_*^2}{2} & \cdots & \cdots \\
0 & 0 & 0 & 1 & \Theta_* & \cdots & \cdots \\
0 & 0 & 0 & 0 & 1 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}
\begin{pmatrix}
V(\phi_0) \\ V'(\phi_0) \\ V''(\phi_0) \\ V'''(\phi_0) \\ V''''(\phi_0) \\ \cdots \\ \cdots
\end{pmatrix}
=
\begin{pmatrix}
V(\phi_*) \\ V'(\phi_*) \\ V''(\phi_*) \\ V'''(\phi_*) \\ V''''(\phi_*) \\ \cdots \\ \cdots
\end{pmatrix}
$$

INVERSION PROCEDURE

$$
\begin{pmatrix}
V(\phi_0) \\ V'(\phi_0) \\ V''(\phi_0) \\ V'''(\phi_0) \\ V''''(\phi_0) \\ \cdots \\ \cdots
\end{pmatrix}
=
\begin{pmatrix}
1 & -\Theta_* & \frac{\Theta_*^2}{2} & -\frac{\Theta_*^3}{6} & \frac{\Theta_*^4}{24} & \cdots & \cdots \\
0 & 1 & -\Theta_* & \frac{\Theta_*^2}{2} & -\frac{\Theta_*^3}{6} & \cdots & \cdots \\
0 & 0 & 1 & -\Theta_* & \frac{\Theta_*^2}{2} & \cdots & \cdots \\
0 & 0 & 0 & 1 & -\Theta_* & \cdots & \cdots \\
0 & 0 & 0 & 0 & 1 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}
\begin{pmatrix}
V(\phi_*) \\ V'(\phi_*) \\ V''(\phi_*) \\ V'''(\phi_*) \\ V''''(\phi_*) \\ \cdots \\ \cdots
\end{pmatrix}
$$

Let us take Planck+WP+High L +BICEP2:

Let us take Planck+WP+High L +BICEP2:

$$5.26\times10^{-9}M_p^4 \le V(\phi_0) \le 9.50\times10^{-9}M_p^4,$$
$$2.44\times10^{-10}M_p^3 \le V'(\phi_0) \le 1.74\times10^{-9}M_p^3,$$
$$4.19\times10^{-11}M_p^2 \le V''(\phi_0) \le 6.44\times10^{-10}M_p^2,$$
$$6.29\times10^{-10}M_p \le V'''(\phi_0) \le 7.08\times10^{-10}M_p,$$
$$5.56\times10^{-10} \le V''''(\phi_0) \le 4.82\times10^{-9}.$$

$$\epsilon_V \sim \mathcal{O}(0.10-1.69)\times10^{-2},$$
$$|\eta_V| \sim \mathcal{O}(9.14\times10^{-3}-0.06),$$
$$|\xi_V^2| \sim \mathcal{O}(5.60\times10^{-3}-0.014),$$
$$|\sigma_V^3| \sim \mathcal{O}(2.28\times10^{-4}-0.017).$$

Let us take Planck+WP+High L +BICEP2:

New consistency relations

SC,AM,arXiv:1403.5549

$$n_T = -\frac{r}{8}\left(2 - \frac{r}{8} - n_S\right) + \cdots\,,$$

$$\alpha_T = \frac{dn_T}{d\ln k} = \frac{r}{8}\left(\frac{r}{8} + n_S - 1\right) + \cdots\,,$$

$$n_r = \frac{dr}{d\ln k} = \frac{16}{9}\left(n_S - 1 + \frac{3r}{4}\right)\left(2n_S - 2 + \frac{3r}{8}\right) + \cdots\,,$$

$$\kappa_T = \frac{d^2 n_T}{d\ln k^2} = \frac{2}{9}\left(n_S - 1 + \frac{3r}{4}\right)\left(2n_S - 2 + \frac{3r}{8}\right)\left(\frac{r}{8} + n_S - 1\right) + \frac{r}{8}\left[\alpha_S + \frac{2}{9}\left(n_S - 1 + \frac{3r}{4}\right)\left(2n_S - 2 + \frac{3r}{8}\right)\right] + \cdots\,,$$

$$\kappa_r = \frac{d^2 r}{d\ln k^2} = \frac{16}{9}\left(2n_S - 2 + \frac{3r}{8}\right)\left\{\alpha_S + \frac{4}{3}\left(n_S - 1 + \frac{3r}{4}\right)\left(2n_S - 2 + \frac{3r}{8}\right)\right\} + \frac{16}{9}\left(n_S - 1 + \frac{3r}{4}\right)\left\{2\alpha_S + \frac{2}{3}\left(n_S - 1 + \frac{3r}{4}\right)\left(2n_S - 2 + \frac{3r}{8}\right)\right\}$$

Let us take Planck+WP+High L +BICEP2:

Estimated inflationary parameters

$$n_T = -\frac{r}{8}\left(2 - \frac{r}{8} - n_S\right) + \cdots,$$

$$\alpha_T = \frac{dn_T}{d\ln k} = \frac{r}{8}\left(\frac{r}{8} + n_S - 1\right) + \cdots,$$

$$n_r = \frac{dr}{d\ln k} = \ldots$$

$$\kappa_T = \frac{d^2 n_T}{d\ln k^2} = \ldots$$

$$\kappa_r = \frac{d^2 r}{d\ln k^2} = \frac{16}{9}\left(2n_S - \ldots\right. + \frac{16}{9}\ldots$$

SC,AM,arXiv:1403.5549

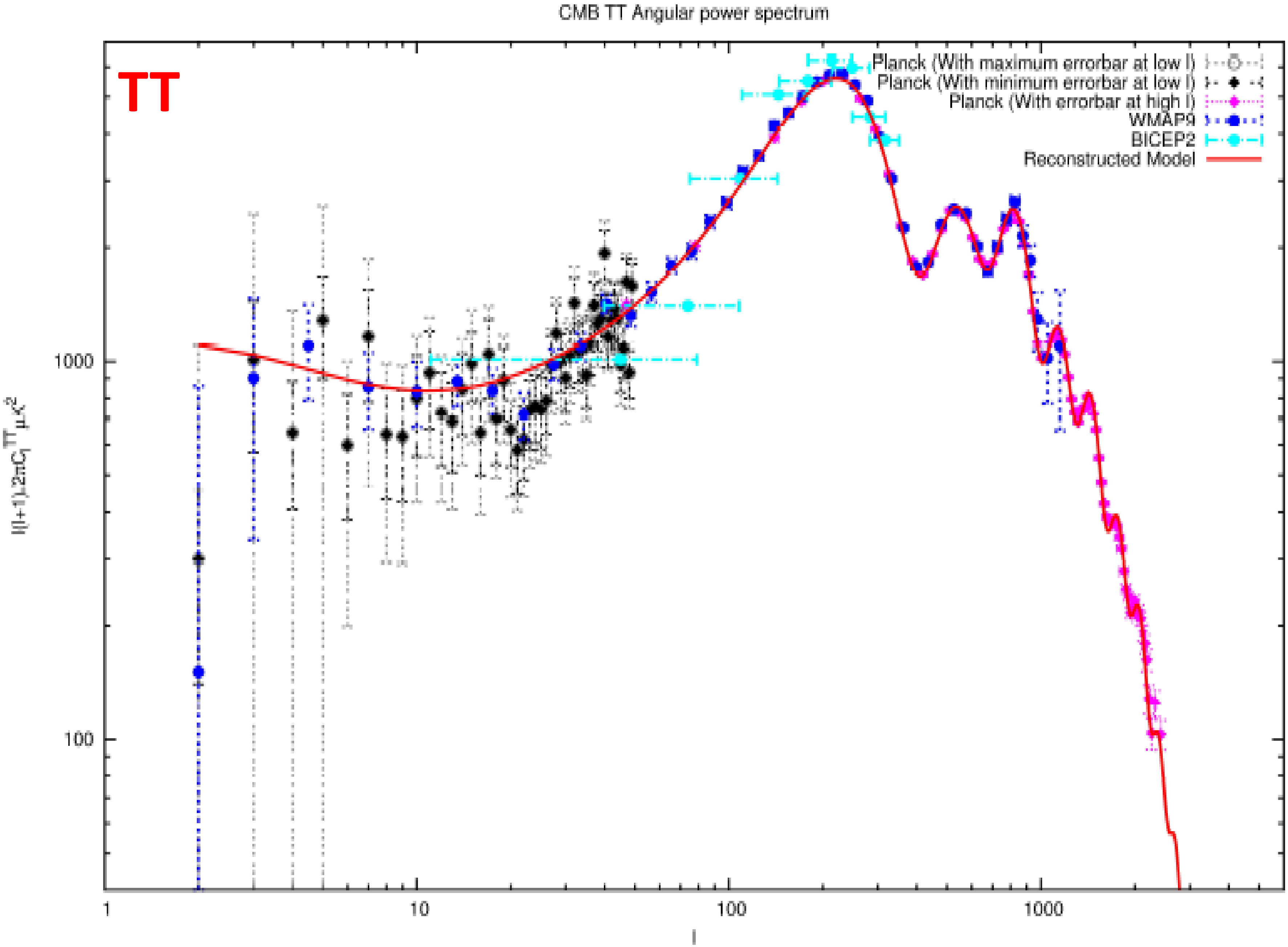
$$-0.019 < n_T < -0.033,$$

$$-2.97 \times 10^{-4} < \alpha_T < 2.86 \times 10^{-5},$$

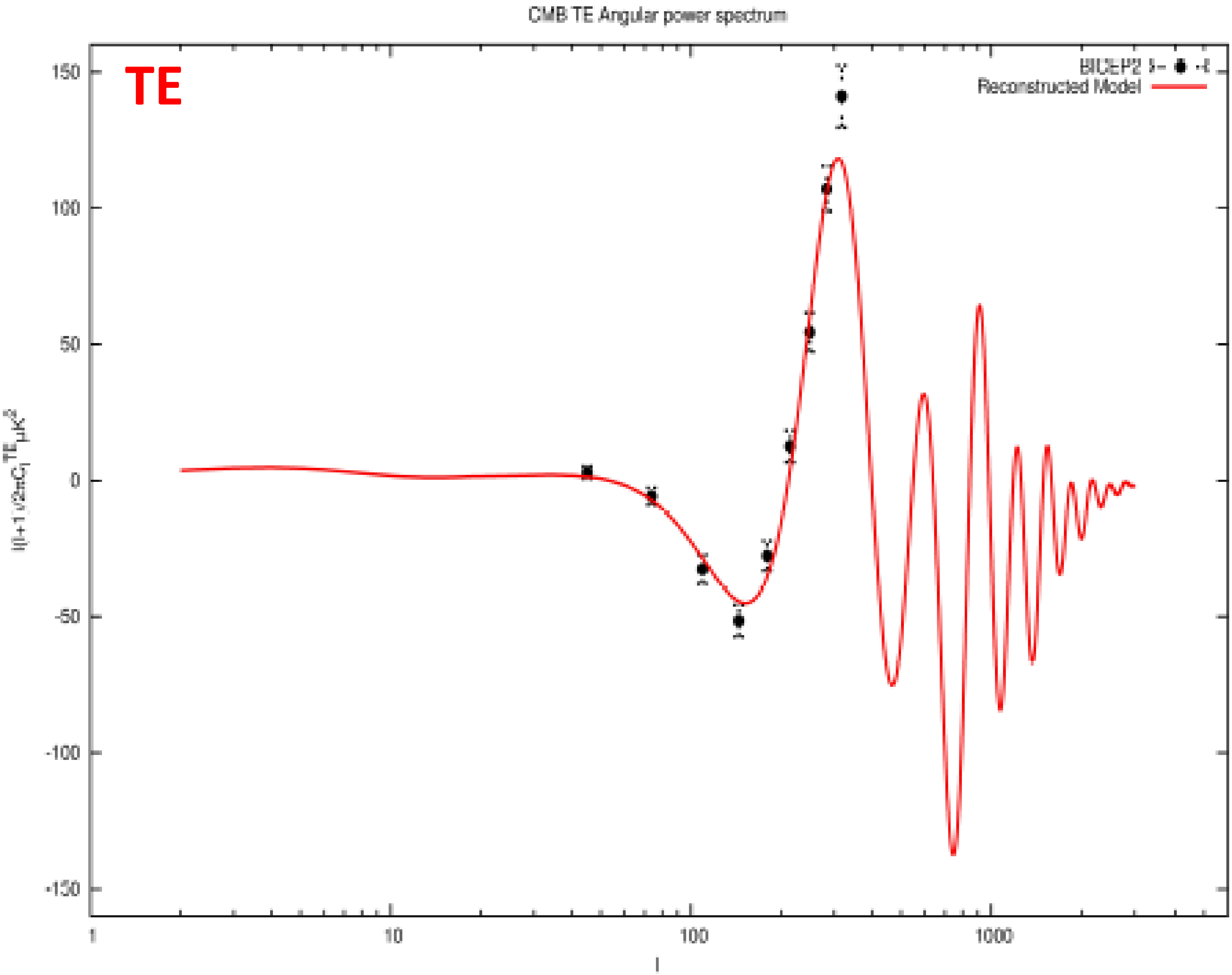
$$2.28 \times 10^{-4} < |n_r| < 0.010,$$

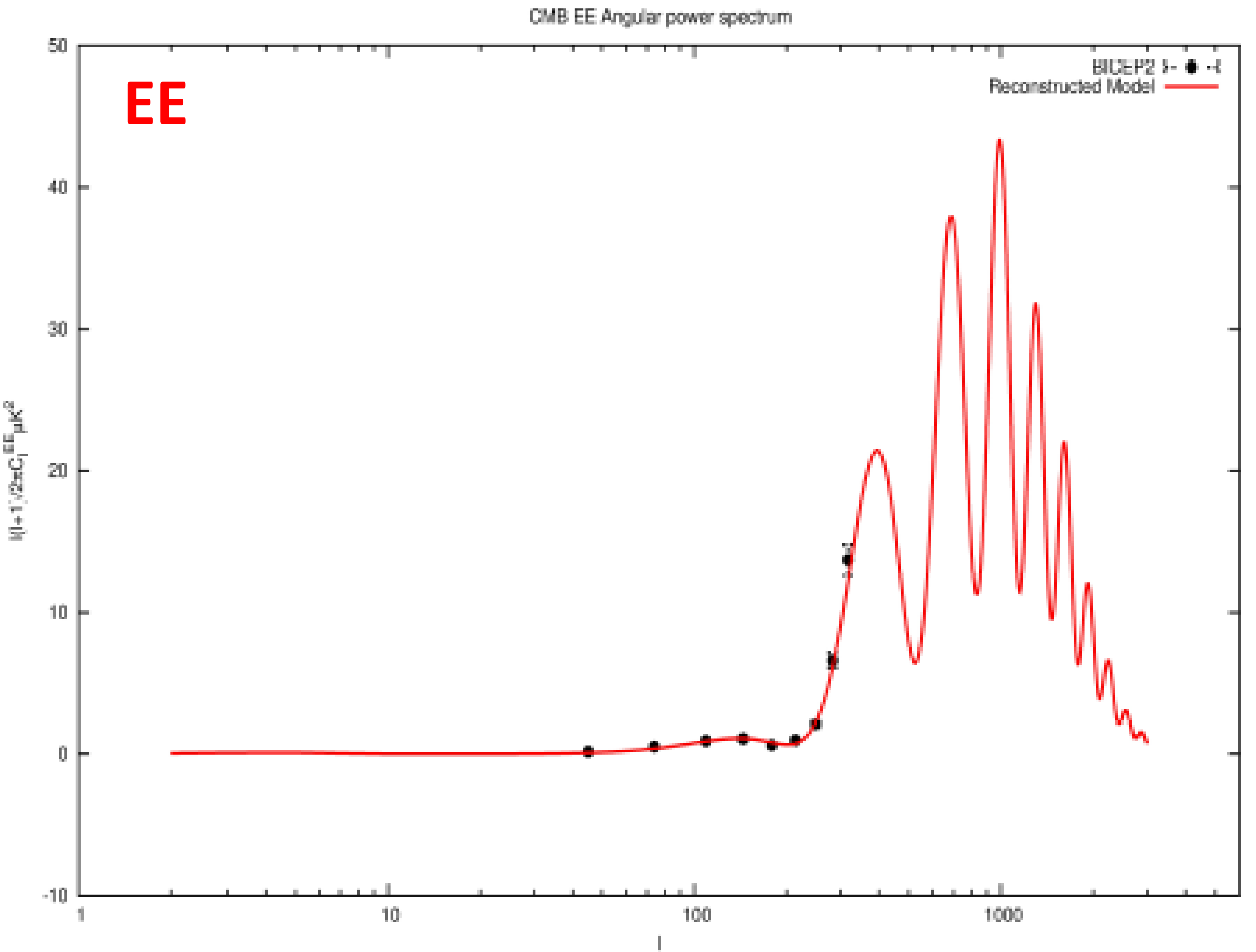
$$-0.11 \times 10^{-4} < \kappa_T < -3.58 \times 10^{-4},$$

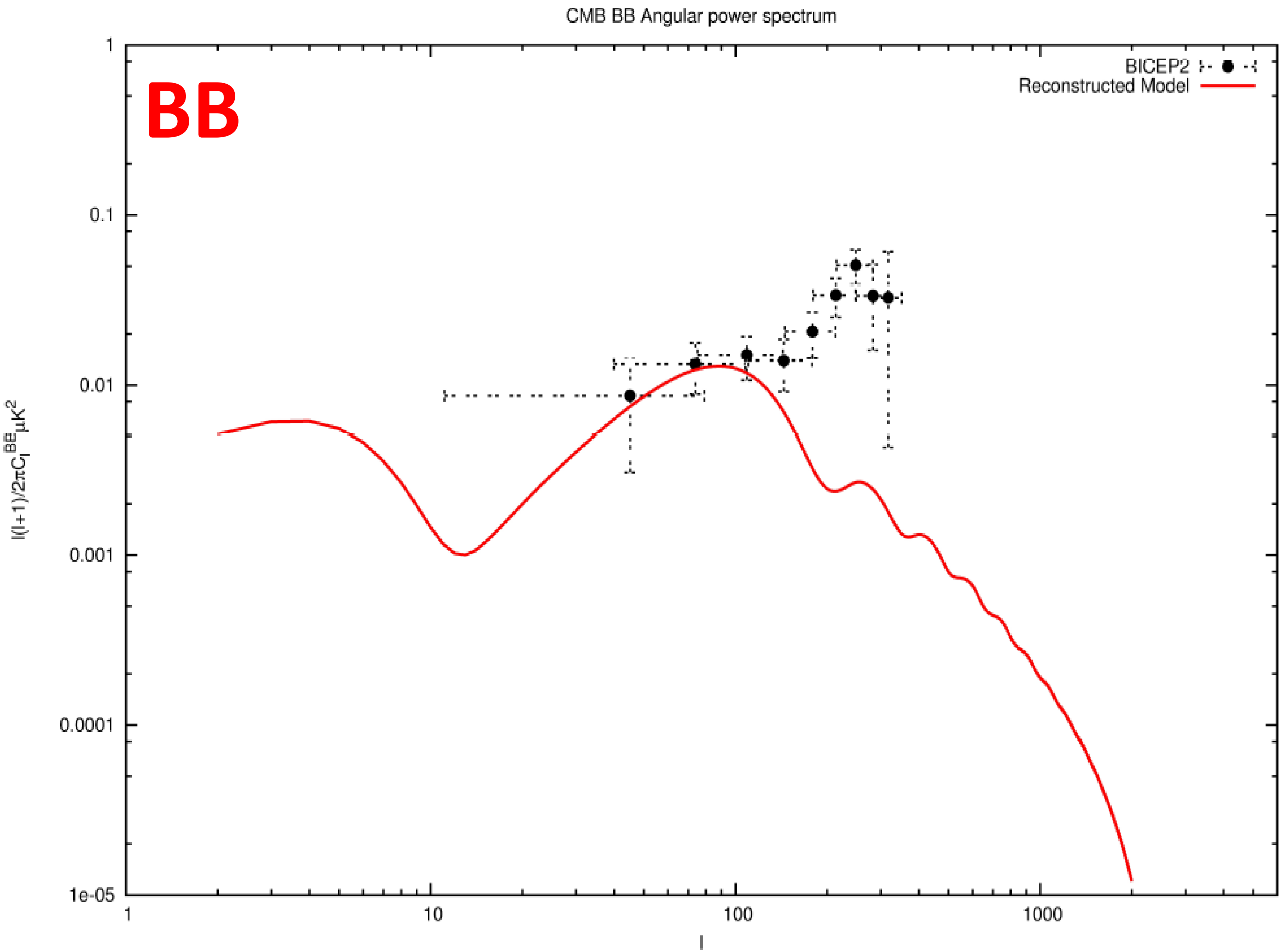
$$-5.25 \times 10^{-3} < \kappa_r < -6.27 \times 10^{-3}$$



CMB TT Angular power spectrum
TT
Planck (With maximum errorbar at low l)
Planck (With minimum errorbar at low l)
Planck (With errorbar at high l)
WMAP9
BICEP2
Reconstructed Model
1000
100
1
10
100
1000
l


CMB TE Angular power spectrum
TE
BICEP2
Reconstructed Model
150
100
50
0
-50
-100
-150
1
10
100
1000
l

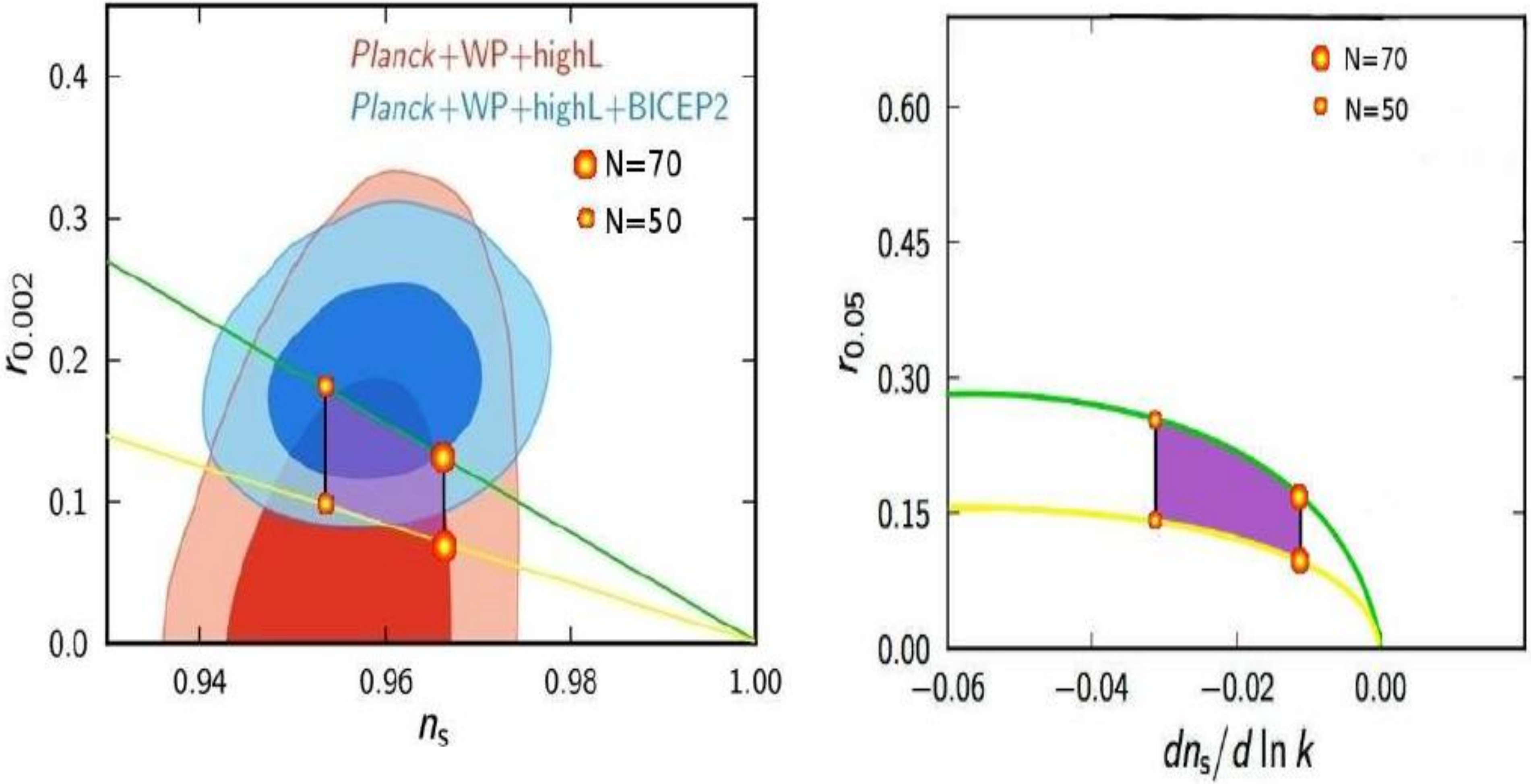

CMB EE Angular power spectrum
EE
BICEP2
Reconstructed Model
l(l+1)/2πC_l^EE μK^2
l
50
40
30
20
10
0
-10
1
10
100
1000

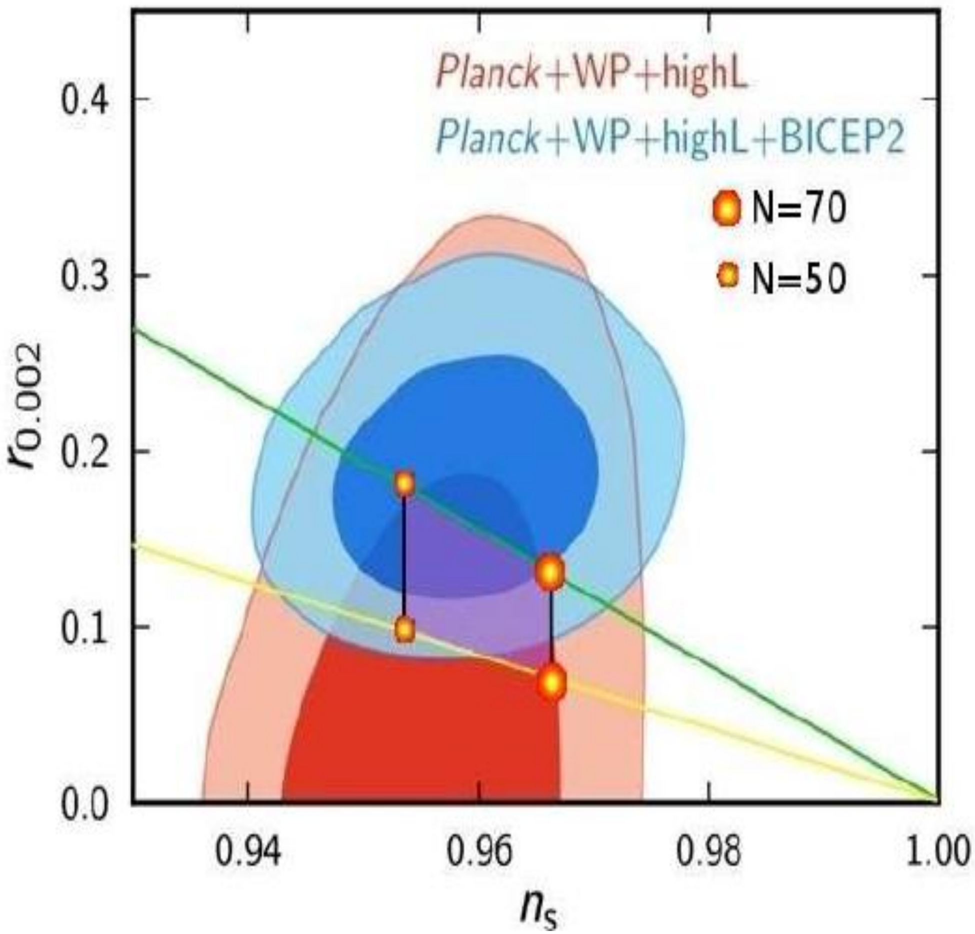


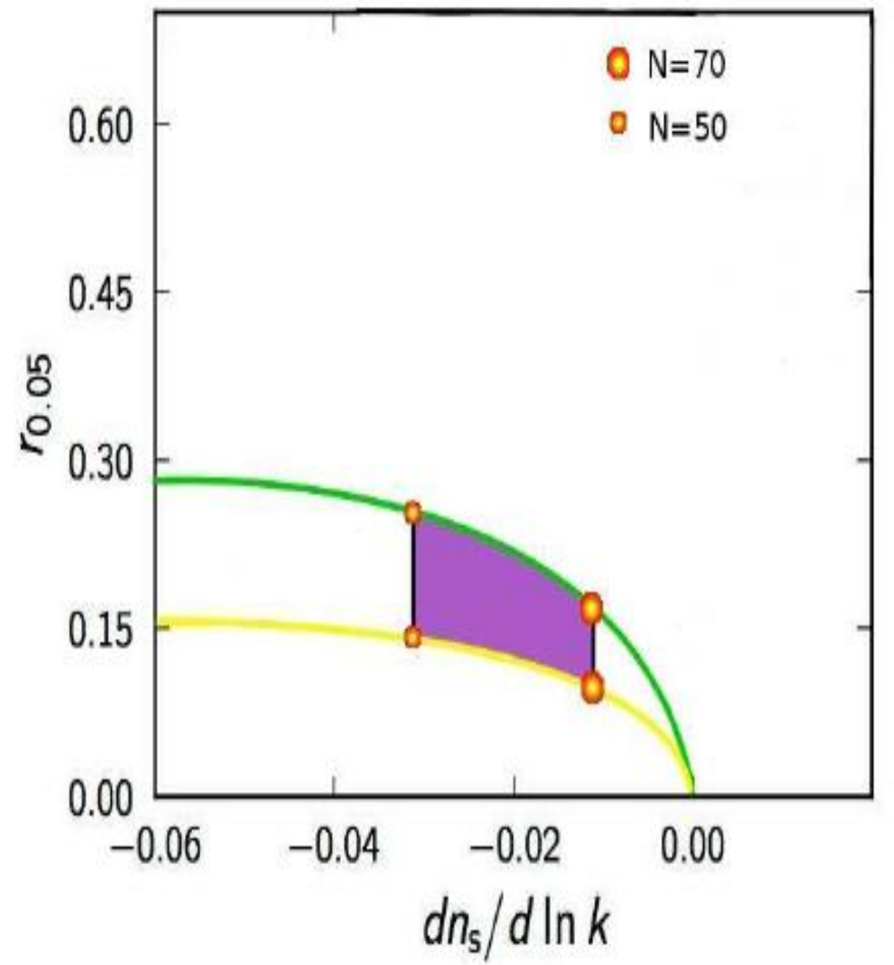
CMB BB Angular power spectrum
BB
BICEP2
Reconstructed Model
1
0.1
0.01
0.001
0.0001
1e-05
l(l+1)/2πC_l^BB μK^2
1
10
100
1000
l

Ifs and Buts in the formalism…….

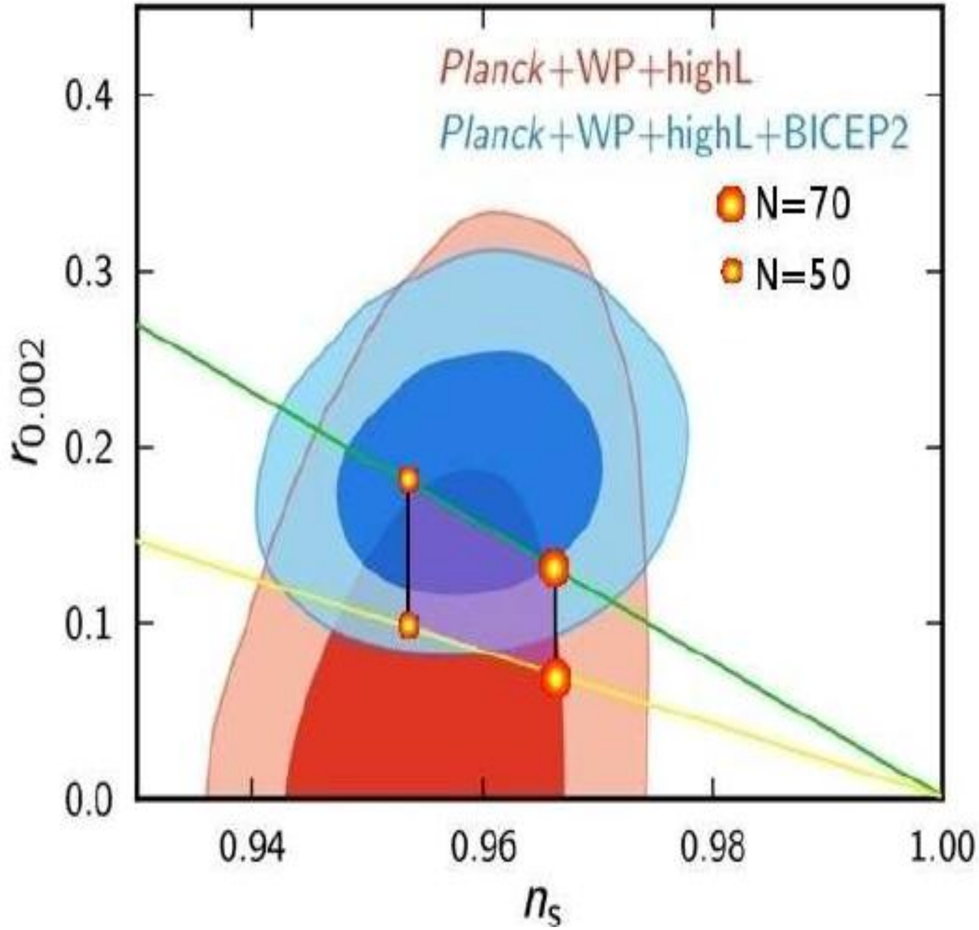
1. *Need to further determine the values of the cosmological parameters….*
2. *Need to check the proper thermal history can be explained….*
3. *Need to check which class of models are favoured…..*
4. *To rule out models need to increase the statistical accuracy level by upgrading the tool…..*
5. *Need to break the degeneracy between the cosmological parameters….*

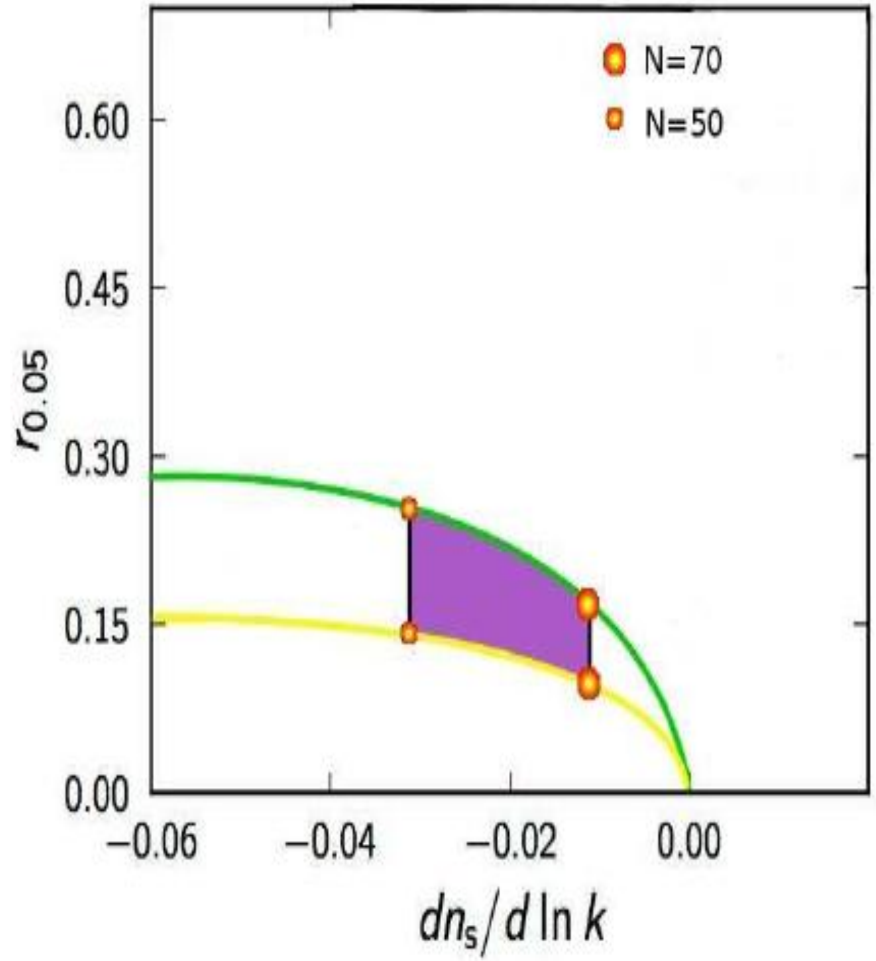

Planck+WP+highL
Planck+WP+highL+BICEP2
N=70
N=50
0.4
0.3
0.2
0.1
0.0
$r_{0.002}$
0.94
0.96
0.98
1.00
n_s
N=70
N=50
0.60
0.45
0.30
0.15
0.00
$r_{0.05}$
−0.06
−0.04
−0.02
0.00
$dn_s/d\ln k$





Field excursion is super-Planckian or Sub-Planckian???





Field excursion is super-Planckian or Sub-Planckian???

Effective field theory prescription Valid???

EFT with Large “r” ?????

Tensor-to scalar ratio:

$$r_b(k) = \begin{cases} r_b(k_*) \\ r_b(k_*)\left(\dfrac{k}{k_*}\right)^{n_T(k_*)-n_S(k_*)+1} \\ r_b(k_*)\left(\dfrac{k}{k_*}\right)^{n_T(k_*)-n_S(k_*)+1+\frac{\alpha_T(k_*)-\alpha_S(k_*)}{2!}\ln\left(\frac{k}{k_*}\right)} \\ r_b(k_*)\left(\dfrac{k}{k_*}\right)^{n_T(k_*)-n_S(k_*)+1+\frac{\alpha_T(k_*)-\alpha_S(k_*)}{2!}\ln\left(\frac{k}{k_*}\right)+\frac{\kappa_T(k_*)-\kappa_S(k_*)}{3!}\ln^2\left(\frac{k}{k_*}\right)} \end{cases}$$

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SC,AM,arXiv:1403.5549,1404.3398,

SC,arXiv:1406.7618

$$\mathrm{Field-excursion\ (in\ GR):} \quad \left|\frac{\Delta\phi}{M_p}\right| = \begin{cases} \mathcal{O}(2.7-5.1) \\ \mathcal{O}(2.7-4.6) \\ \mathcal{O}(0.6-1.8) \\ \mathcal{O}(0.2-0.3) \end{cases}$$

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Note:

Large (detectable) r+ $|\Delta\phi| < M_p$ (EFT)= running/Beyond GR (RS)/muiltifield/….

In RS Model:

$$\mathbf{H^2} \approx \frac{\mathbf{V}(\phi)}{3\mathbf{M_p^2}}\left(1+\frac{\mathbf{V}(\phi)}{2\sigma}\right)$$

$$\mathbf{M_5^3} = \sqrt{\frac{4\pi\sigma}{3}}\mathbf{M_p}$$

$$\sigma = \sqrt{-\frac{3}{4\pi}\mathbf{M_5^3}\mathbf{\Lambda_5}} > 0$$

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SC,arXiv:1406.7618

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$$\text{Brane tension}: \quad \sigma \leq \mathcal{O}(10^{-9})\, M_p^4,$$

$$\text{5D Scale}: \quad M_5 \leq \mathcal{O}(0.04)\, M_p,$$

$$\text{5D Cosmological Constant}: \Lambda_5 \geq -\mathcal{O}(10^{-15})\, M_p^5$$

Bottom lines

- **High scale models of inflations are favoured after BICEP2.**

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- Validity of EFT prescription requires: running +HOSL,Mutifield, RS or more sophisticated parametrization for powspec to get large "r".
- Proposed semi-analytical reconstruction technique is valid for any preferred choice of input observational data.
- High scale MSSM model fits well with CMB TT spectra within $2<l<2500$ (Planck).
- New sets of infla consistency relations and values of the tensor mode parameters are proposed.

Open issues and future prospects

- **To comment on the correct value of "r" from any observation requires new tools for separating various components of CMB B-modes (=Infla+PMF+NG+Lensing etc.).**

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- If BICEP results are correct then need to clarify the issue of getting blue tilted gravity waves.

Open issues and future prospects

- The primordial non-Gaussianity is not yet been detected with high statistical accuracy ($f_{NL}^{local} = 2.7 \pm 5.8$ & $\tau_{Nl}^{local} \leq 2800$). But if it is detected in near future experiments then using this tool it is possible to rule out various inflationary models.

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- **To increase the numerical convergence of the proposed reconstruction technique need to incorporate the numerically integrated powspec by solving MS eqn using various numerical methods.**

Open issues and future prospects

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- Need to work on thermal features. Ex: Reheating, Dark matter etc.
- Need to propose an unified approach through which it is possible to unify inflation, dark matter & dark energy. Need also to check how Reconstruction business works here.

Thanks for your time......