RG evolution of neutrino parameters

(In TeV scale seesaw models)

Sasmita Mishra

Physical Research Laboratory, Ahmedabad, India

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Outline of the talk

- Introduction
- Neutrino masses and mixing
- Seesaw mechanism
- Renormalization group effet (RG)
- RG and Tribimaximal mixing matrix
- Analytical and numerical results
- Conclusion

Evidence for Neutrino Mass

- Experimental observations using solar, atmospheric, reactor and accelerator neutrinos have confirmed that neutrinos oscillate between flavours (1996 -2006)
- This is possible if neutrinos have mass and mixing
- Neutrino mass cannot be accommodated naturally in Standard Model
- Need to go for a theory Beyond Standard Model
- Only those theories which can successfully explain all the neutrino oscillation parameters are allowed

The Neutrino Mass Matrix

- The neutrino mass matrix at low energy:

$$m_\nu = U^*_{PMNS} Diag(m_1, m_2, m_3) U^\dagger_{PMNS}$$

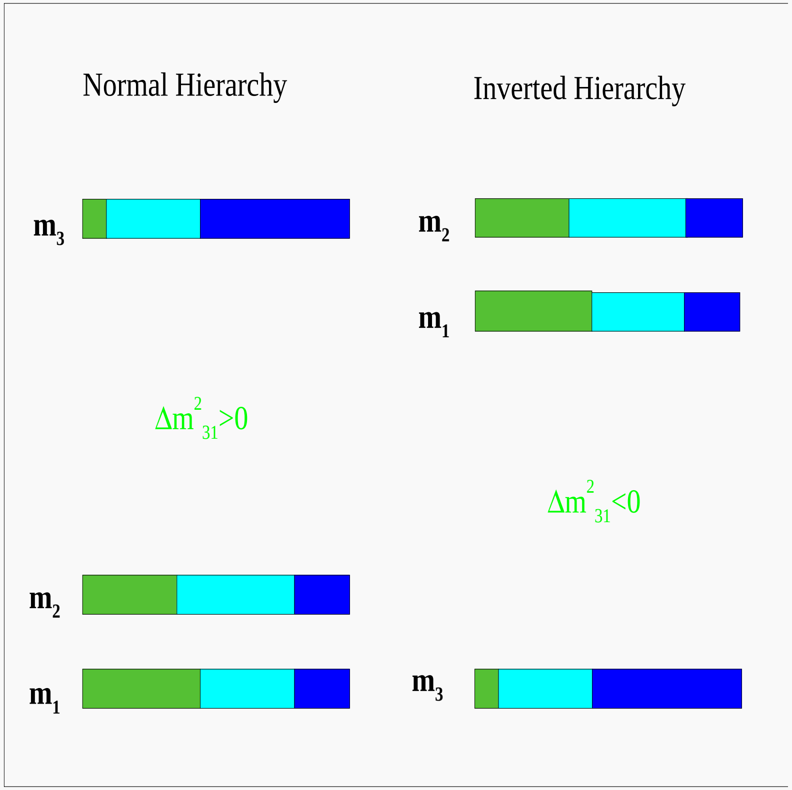
- $U_{PMNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) P(\sigma, \rho)$
  $\rightarrow$ Leptonic mixing matrix relating the flavour and mass eigenstates

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P$$

Parameters of the Neutrino Mass Matrix

- 9 unknown parameters for three neutrino flavours
  - 3 masses, m_1, m_2 and m_3
  - 3 mixing angles
  - 3 phases, 1 Dirac, 2 Majorana
- Oscillation experiments sensitive to
  - 2 mass squared differences (Δm^2_{21}, $\Delta m^2_{31} \approx \Delta m^2_{32}$)
  - 3 mixing angles (θ_{12}, θ_{13}, θ_{23})
  - 1 Dirac phase

Neutrino Masses: Ordering



- Solar data tells $\Delta m^2_{21} > 0$
- But sign of $|\Delta m^2_{31}|$ not known
- Normal Ordering :

$$m_3 >> m_2 >> m_1$$

- Inverted Ordering :

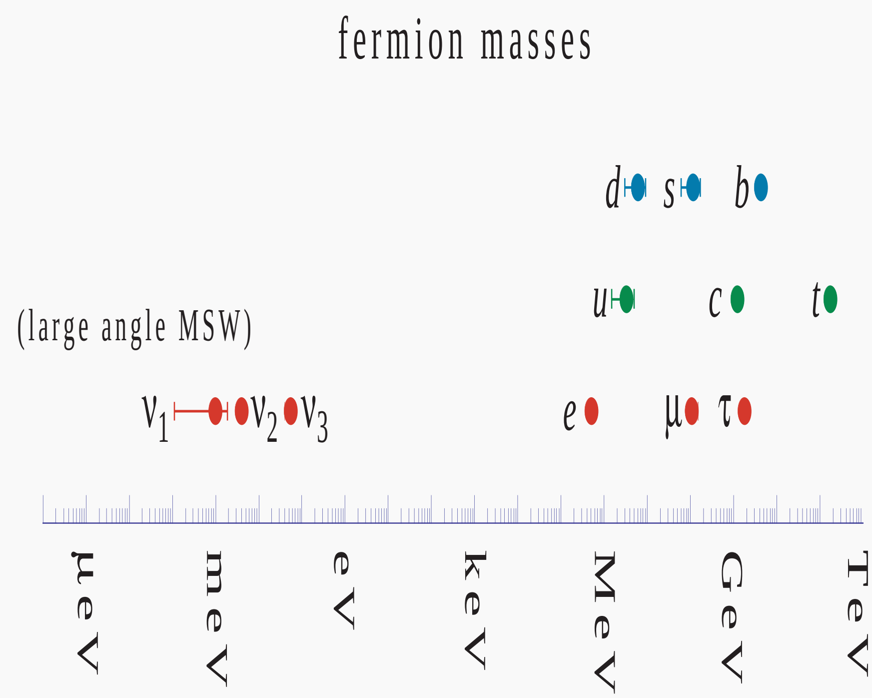
$$m_3^2 << m_2^2 \approx m_1^2 \approx \Delta m^2_{atm}$$

- Quasi-Degenerate

$$m_3 \approx m_2 \approx m_1 >> \sqrt{\Delta m^2_{atm}}$$

Neutrino Masses

- For normal hierarchy: $m_1 \approx 0$, $m_2 \approx 0.009$ eV and $m_3 \approx 0.05$ eV



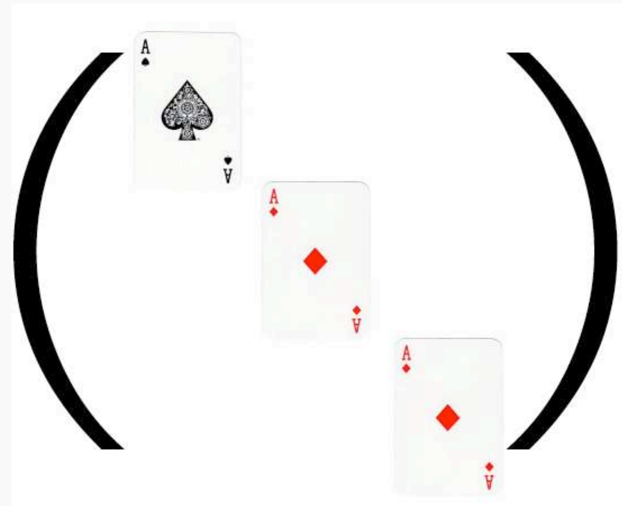
- Neutrino masses much smaller than quark and charged lepton masses
- Hierarchy of neutrino masses not strong : $m_3/m_2 \leq 6$
- Completely different from quark sector
- Inverted hierarchy and quasi-degeneracy has no analogue in quark sector

Models for Neutrino Mass: Key Issues

- Why two large and one small mixing angle unlike in quark sector where all mixing angles are small

$U_{PMNS} \sim$

$V_{CKM} \sim$



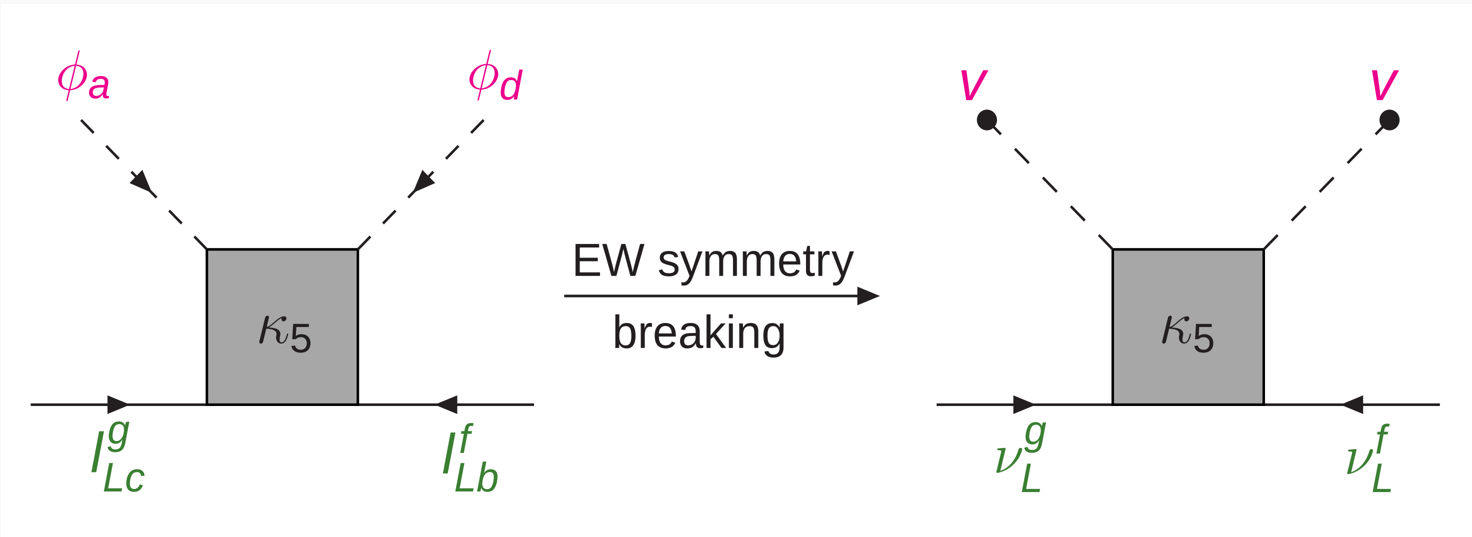
- Current data $\Rightarrow \sin^2\theta_{23} = 0.5$, $\sin^2\theta_{12} = 0.33$, $\sin^2\theta_{13} = 0.02(\simeq 0) \rightarrow$ Tri-Bimaximal Form

Seesaw Mechanism

- Relates the smallness of neutrino mass to some new physics at high scale
- $\mathcal{M} = m_D^T M_R^{-1} m_D; \quad m_D \sim v Y_\nu$
- $m_\nu \sim 0.05$ eV for $M_R = 10^{16}$ GeV, $m_D \sim 100$ GeV, $Y_\nu \sim 1$

Origin of Seesaw

- Heavy field present at high scale Λ
- Tree level exchange of this heavy particle $\Longrightarrow$ effective dimension 5 operator at low scale
  $\mathcal{L} = \kappa_5 l_L l_L \phi\phi, \;\; \kappa_5 = \kappa/\Lambda$



Weinberg, 79

- Majorana Mass : $\kappa_5 v^2$
- In the seesaw approximation $\kappa = {Y_\nu}^T \Lambda^{-1} Y_\nu$
- κ depends on Heavy particles and their couplings

Three types of Seesaw

$$\mathcal{L}_{eff} = \kappa_5 l\, l\, \phi\, \phi$$

- Three ways to form a gauge singlet with the SM doublets l and ϕ
  - Type-I : l and ϕ form a singlet $[\ 2 \times 2 = 3 \oplus 1]$
    $\longrightarrow$ Mediated by singlet Fermions
  - Type-II: l and l (and ϕ and ϕ) form a triplet $[\ 3 \times 3 = 5 \oplus 3 \oplus 1]$
    $\longrightarrow$ Mediated by $SU(2)$ triplet Higgs
  - Type-III: l and ϕ form a triplet $[\ 3 \times 3 = 5 \oplus 3 \oplus 1]$
    $\longrightarrow$ Mediated by $SU(2)$ triplet fermions
- Mediated by $\Rightarrow$ tree level exchange at a high scale $\sim$ heavy particle mass
- Predictive theory at high scale (Type I, II SO(10); Type III SU(5))
- How heavy ? TeV scale seesaw — Low energy signatures ?
- Probing seesaw at LHC ?

Type-I seesaw

- $\mathcal{L} = \mathcal{L}_{\mathcal{SM}} + (Y_\nu)\overline{N_R}\,\tilde{\phi}^\dagger l_L \;+\; \frac{1}{2}\overline{N_R{}^c}(M_R)_{ij}N_R \;+\; \text{h.c}$
- The Majorana mass matrix at seesaw scale

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^{\mathrm{T}} & M_R \end{pmatrix}; \quad m_D = Y_\nu v$$

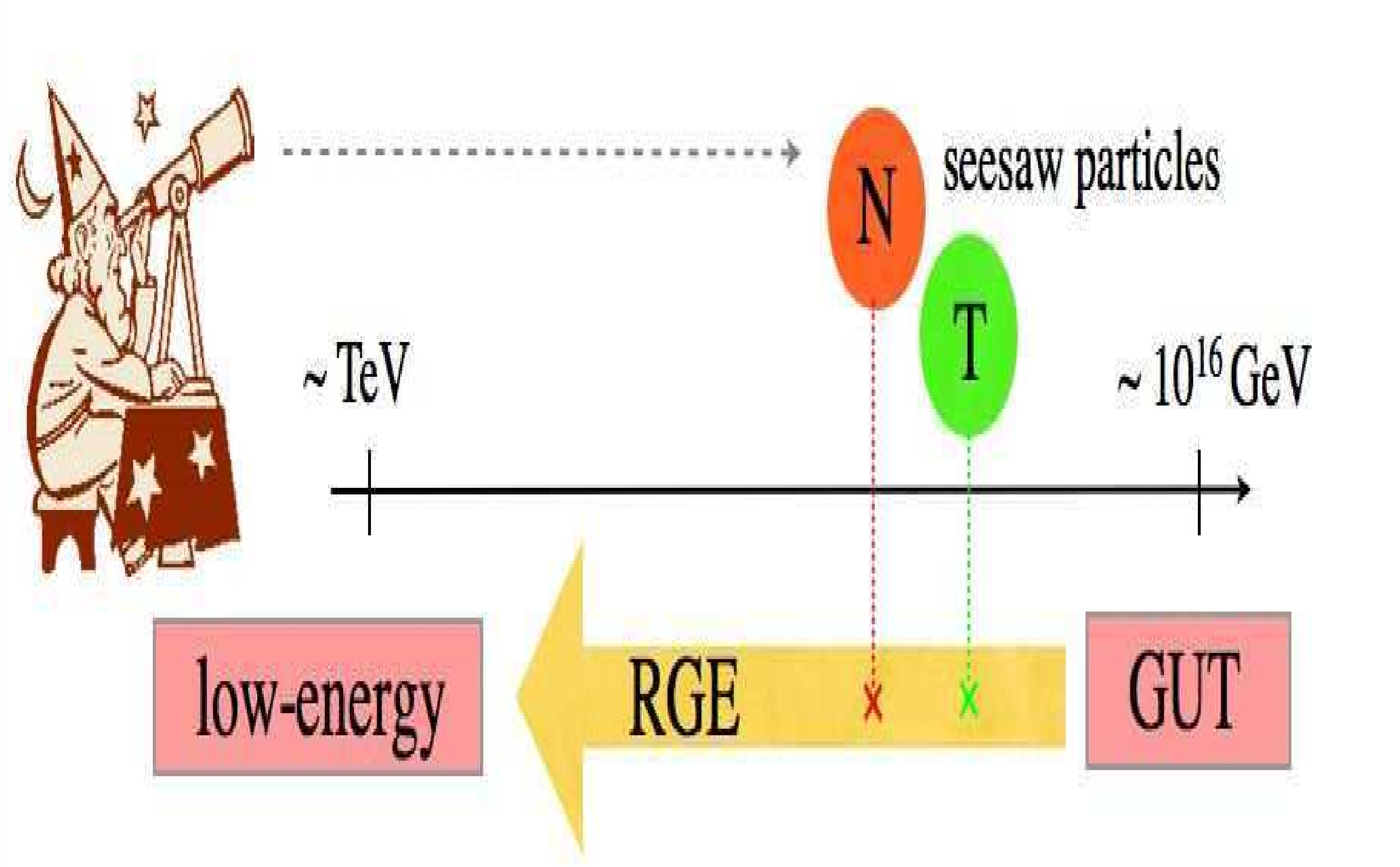
- The light neutrino mass matrix after the seesaw diagonalization assuming $M_R \gg m_D$ is given by
$\mathcal{M}_\nu = -m_D^T M_R^{-1} m_D$
- The Majorana neutrino mass matrix $\mathcal{M}_\nu$ is symmetric and can be diagonalized as $V_\nu^{\mathrm{T}}\mathcal{M}V_\nu = \mathrm{diag}(m_1, m_2, m_3)$ $V_\nu \rightarrow$ the neutrino mixing matrix
- $\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\overline{l_L{}'}\gamma^\mu \nu_L{}' W_\mu^- + h.c = \overline{l_L} V_l^\dagger V_\nu \nu_L W_\mu^-$ $\quad U_{PMNS} = V_l^\dagger V_\nu$
- In a basis where the charged lepton mass matrix is diagonal
$U_{PMNS} = V_\nu$
- Connects flavour states to mass states

Renormalization group effects

- The neutrino mass matrix at the high scale Λ originates from a dimension-5 operator

$$M^{\Lambda}_{\alpha\beta} = \kappa_{\alpha\beta} \frac{(\ell_\alpha . H)(\ell_\beta . H)}{\Lambda} \; .$$

- $\kappa_{\alpha\beta} \longrightarrow$ runs with energy scale



RG equations

$$16\pi^2 \frac{d\kappa}{dt} = C\kappa(Y_\ell^\dagger Y_\ell - Y_\nu^\dagger Y_\nu) + C(Y_\ell^\dagger Y_\ell - Y_\nu^\dagger Y_\nu)^T \kappa + K\kappa$$

$$t = ln\mu$$

MSSM

C = 1

$$K = -(6g_2^2 + 2g_Y^2 + 2S)$$

$$S = Tr(3Y_u^\dagger Y_u)$$

- The first two terms contain the charged lepton Yukawa coupling and thus distinguish between generations
- Responsible for the running of mixing parameters
- K is flavour blind
- One has to consider the running of all the quantities simultaneously
- Can be solved numerically

SM

C = -3/2

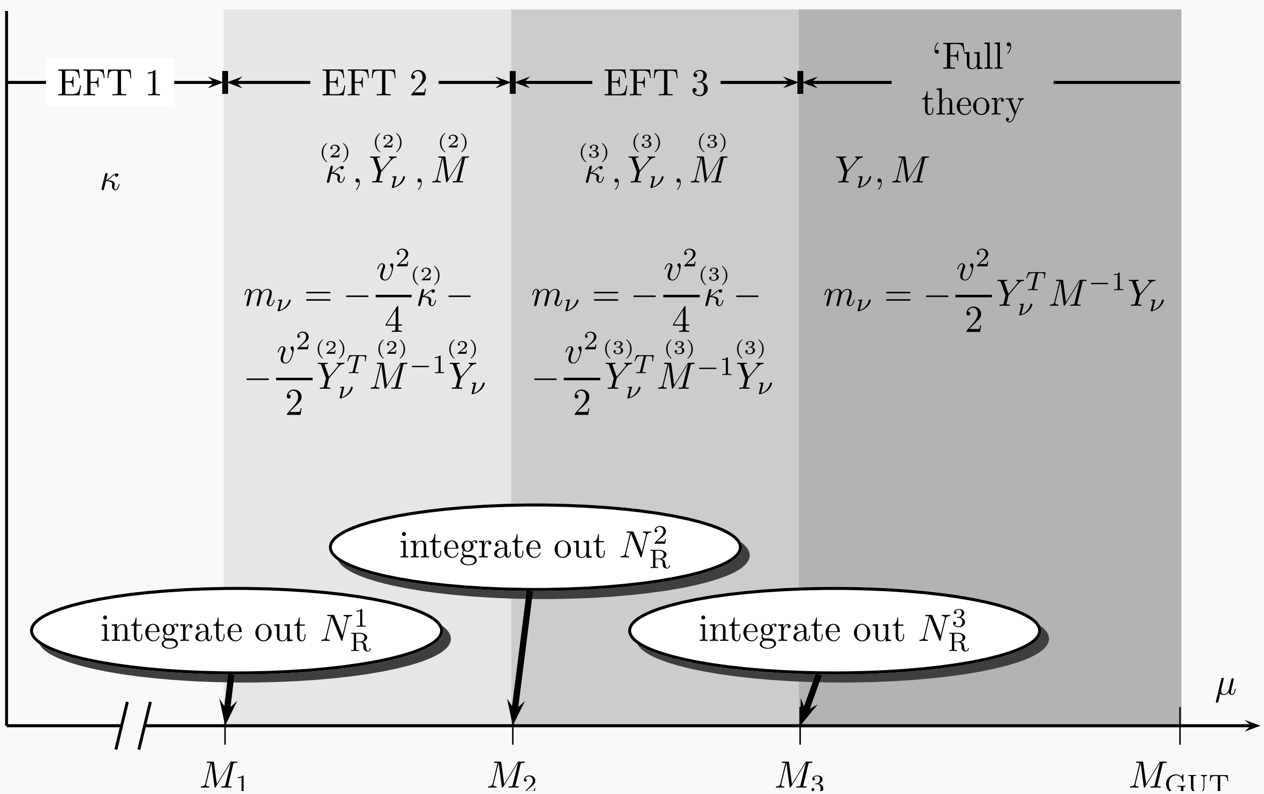
$$K = -(3g_2^2 + 2\lambda + 2S)$$

$$S = Tr(Y_\ell^\dagger Y_\ell + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d)$$

Babu,Leung,Pantaleone, 1993; Drees, Kersten, Lindner,Ratz, 2001,2002

Threshold effect

- M_1, M_2, M_3 : heavy particles coupled to the theory (SM)
- At scale $\mu >> M_2$: particles of masses, $>> M_2$ coupled.
- At scale $\mu << M_2$: particles of masses, $>> M_2$ decoupled.
- At $\mu = M_2$, parameters are continuous.
- Matching conditions are needed.



Analytic Method

- The RG equation: $16\pi^2 \frac{d\kappa}{dt} = C\kappa(Y_\ell^\dagger Y_\ell - Y_\nu^\dagger Y_\nu) + C(Y_\ell^\dagger Y_\ell - Y_\nu^\dagger Y_\nu)^T \kappa + K\kappa$
- $\kappa(\lambda) = I_K \mathcal{I}_\kappa^T \kappa(\Lambda) \mathcal{I}_\kappa \Longrightarrow \mathcal{M}_\nu^\lambda = I_K \mathcal{I}_\kappa^T \mathcal{M}_\nu^\Lambda \mathcal{I}_\kappa$,
- Where we define the integrals :

$$\mathcal{I}_K \equiv \exp\left[-\int_{t(\Lambda)}^{t(\lambda)} K(t)dt\right]$$
$$\mathcal{I}_\kappa \equiv \exp\left[-\int_{t(\Lambda)}^{t(\lambda)} (Y_l^\dagger Y_l - Y_\nu^\dagger Y_\nu)(t)dt\right] ,$$
with, $Y_\ell^\dagger Y_\ell = \mathrm{Diag}\ (y_e^2, y_\mu^2, y_\tau^2)$.
$$t(Q) \equiv (16\pi^2)^{-1} \ln(Q/Q_0)$$

- Assuming $y_{e,\mu}^2 << y_\tau^2$,

$\mathcal{I}_\kappa \approx$ Diag$(1, 1, e^{-\Delta_\tau}) = \mathrm{Diag}(1, 1, 1 - \Delta_\tau) + \mathcal{O}(\Delta_\tau^2)$,

$$\Delta_\tau = \int_{t(\Lambda)}^{t(\lambda)} |y_\tau(t) - Y_\nu^\dagger Y_\nu|^2 = \begin{cases} 5.2 \times 10^{-3} & \text{with} Y_\nu \\ 3.9 \times 10^{-5} & \text{without } Y_\nu \end{cases}$$

Ellis and Lola, PLB, 1999; Chankowski & Pokorski MPA, 2001

Numerical Procedure

- Taking all the experimental available results; angles and masses, construct U_{PMNS}. Phases varied randomly.
- Construct $\kappa = \frac{4}{v^2}(m_\nu)$; $m_\nu = U^*_{\mathrm{PMNS}} Diag(m_1, m_2, m_3) U^\dagger_{\mathrm{PMNS}}$
- Run κ from the SM scale to 1 TeV. Matrix equations $\Longrightarrow$ diagonalize at each step.
- Extract the parameters; angles, masses and phases
- At scale 1TeV, new degrees of freedom appear; Fermion singlets
- The RG equation changes and matching condition is imposed
- Presence of fermion singlets $\Longrightarrow$ new Yukawa couplings
- Again run all the parameters, diagonalize at each step and extract the parameters.
- Our case; upto 10^{12}GeV

Tri-bimaximal Mixing

- Present data gives two large and one small mixing
- Close to Tri-bimaximal form
- $$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i\alpha_3/2}) \; ,$$

  Harrison, Perkins and Scott

- $\sin^2\theta_{23} = 0.5, \sin^2\theta_{12} = 0.33, \sin^2\theta_{13} = 0.03$
- Key Question : Is there some underlying Symmetry ?
- Hint of non-zero $\theta_{13} \Longrightarrow$ symmetry is approximate, Renormalization Group effects

TBM: Radiative corrections to mixing angles

- At low scale :

  $\theta_{ij} = \theta^{\Lambda}_{ij} + k_{e ij}\Delta_e + k_{\mu_{ij}}\Delta_\mu + k_{\tau ij}\Delta_\tau$

  $\delta = \delta^{\Lambda} + d_e\Delta_e + d_\mu\Delta_\mu + d_\tau\Delta_\tau$

  $\alpha^{\lambda}_i = \alpha^{\Lambda}_i + a_{e_i}\Delta_e + a_{\mu_i}\Delta_\mu + a_{\tau_i}\Delta_\tau;\ i = 1, 2$

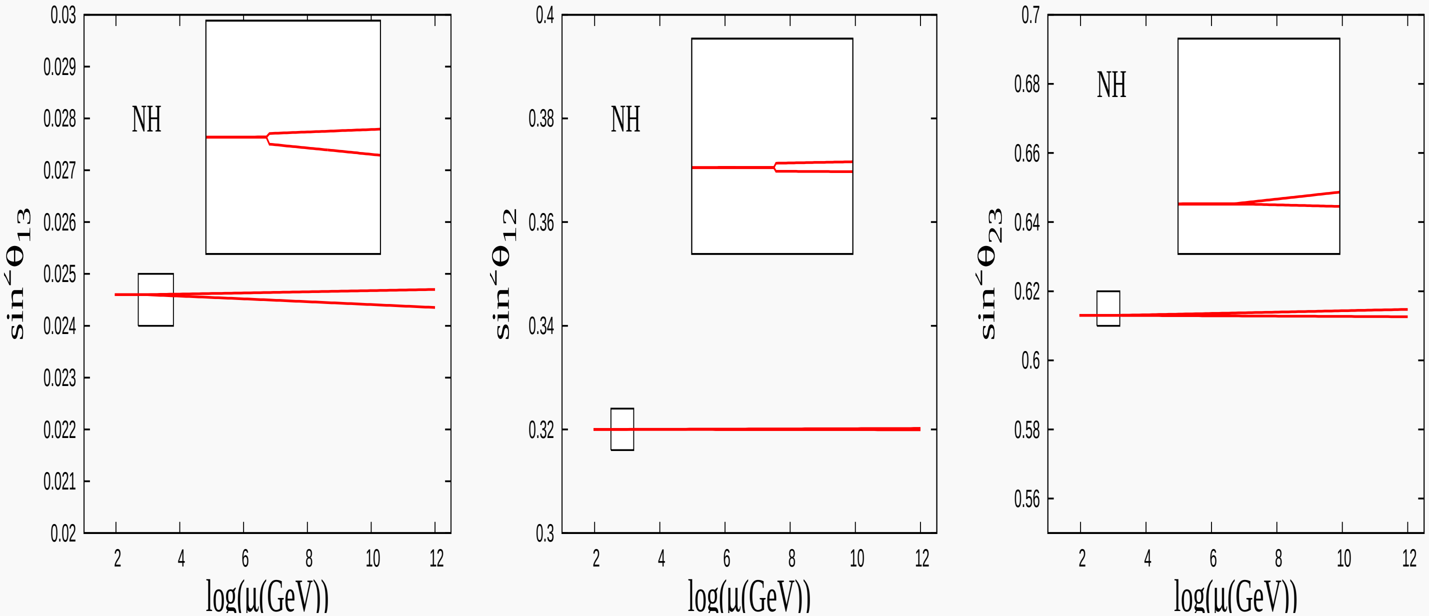
- $\sin^2\theta_{12} - \sin^2\theta^{\Lambda}_{12} \simeq k_{e12}\left(\Delta_e - \cos^2\theta^{\Lambda}_{23}\Delta_\mu - \sin^2\theta^{\Lambda}_{23}\Delta_\tau\right)$

  $\sin^2\theta_{23} - \sin^2\theta^{\Lambda}_{23} \simeq k_{\mu_{23}}\left(\Delta_\mu - \Delta_\tau\right)$

  $\sin^2\theta_{13} - \sin^2\theta^{\Lambda}_{13} \simeq k_{\mu_{23}}\left(\Delta_\mu - \Delta_\tau\right)$

- Two cases:
  - Hierarchical Neutrinos; NH and IH
    - Adding two singlets with opposite lepton number to the SM.
  - Quasidegenerate
    - Adding three fermion singlets to the SM
- Both models give TeV scale seesaw

Normal ordering:



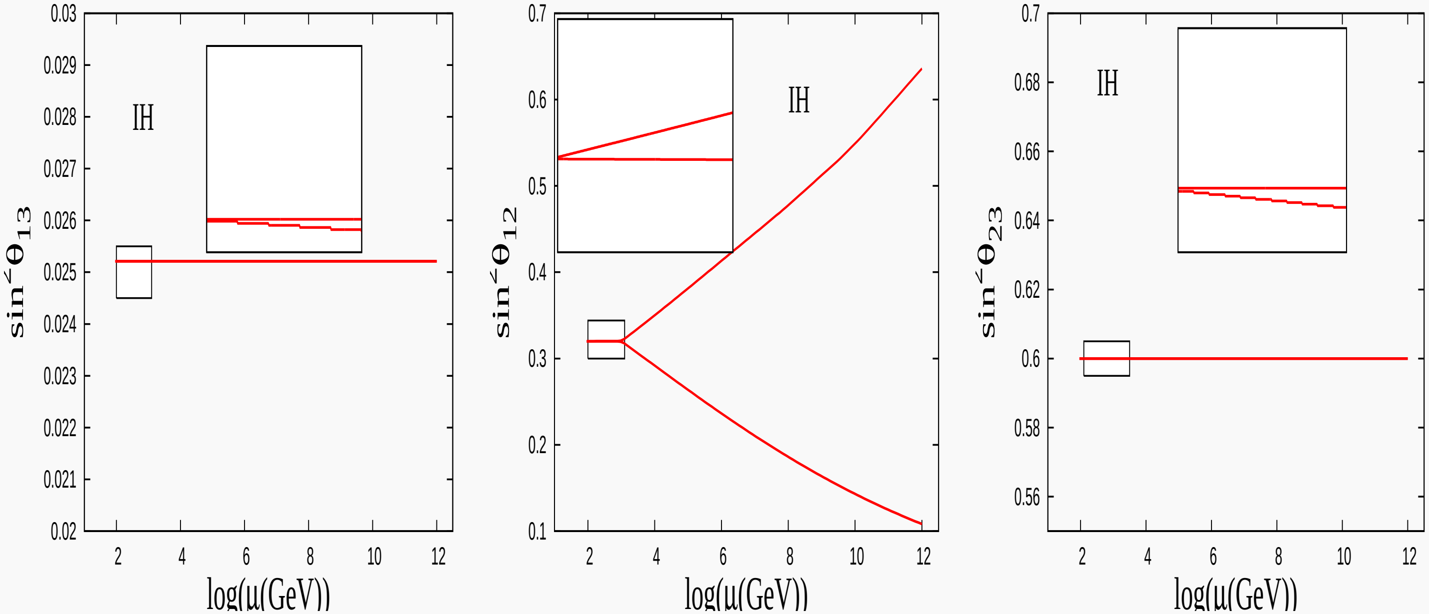
$$\sin^2\theta_{12} - \frac{1}{3} \approx -\frac{1}{2}\sin 2\theta_{12}^{\Lambda}\left(\Delta_e - \cos^2\theta_{23}^{\Lambda}\Delta_\mu - \sin^2\theta_{23}^{\Lambda}\Delta_\tau\right)$$

$$\sin^2\theta_{23} - \frac{1}{2} \approx -\frac{1}{2}\sin 2\theta_{12}^{\Lambda}\sin 2\theta_{23}^{\Lambda}[r + \sqrt{r}\cos(\alpha_2^{\Lambda})]\,(\Delta_\mu - \Delta_\tau)$$

$$\sin^2\theta_{13} \approx -\frac{1}{2}\sin 2\theta_{23}^{\Lambda}[1 + 2\cos^2\theta_{12}^{\Lambda}(r + \sqrt{r}\cos(\alpha_2^{\Lambda}))]\,(\Delta_\mu - \Delta_\tau)$$

$$r = \frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}} \simeq 0.18$$

Inverted ordering:

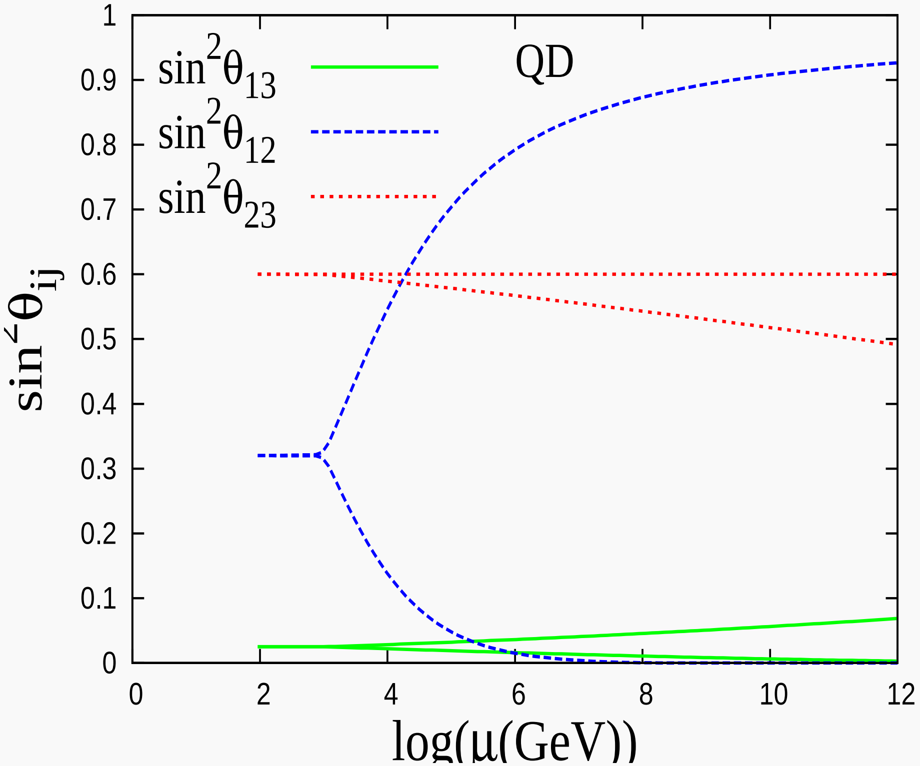


$$\sin^2\theta_{12} - \frac{1}{3} \approx -\sin 2\theta^\Lambda_{12}\left(\frac{1+\cos(\alpha^\Lambda_1-\alpha^\Lambda_2)}{r}\right)\left(\Delta_e - \cos^2\theta^\Lambda_{23}\Delta_\mu - \sin^2\theta^\Lambda_{23}\Delta_\tau\right)$$

$$\sin^2\theta_{23} - \frac{1}{2} \approx \frac{1}{4}\sin 2\theta^\Lambda_{12}\sin 2\theta^\Lambda_{23}\frac{m_3}{\sqrt{\Delta m^2_{13}}}[\cos\alpha^\Lambda_2 - \cos\alpha^\Lambda_1]\left(\Delta_\mu - \Delta_\tau\right)$$

$$\sin^2\theta_{13} \approx \frac{1}{2}\sin 2\theta^\Lambda_{23}[1 + \frac{2m_3}{\sqrt{\Delta m^2_{13}}}(\cos\alpha^\Lambda_2\cos^2\theta^\Lambda_{12} + \cos\alpha^\Lambda_1\sin^2\theta^\Lambda_{12})]\left(\Delta_\mu - \Delta_\tau\right)$$

Quasidegenerate neutrinos:



$$\sin^2\theta_{12} - \frac{1}{3} \approx$$
$$-\sin 2\theta_{12}^{\Lambda} \frac{{m_0}^2}{\Delta m_{21}^2} [1+\cos(\alpha_1^{\Lambda} - \alpha_2^{\Lambda})] \left(\Delta_e - \cos^2\theta_{23}^{\Lambda}\Delta_\mu - \sin^2\theta_{23}^{\Lambda}\Delta_\tau\right)$$

$$\sin^2\theta_{23} - \frac{1}{2} \approx -\frac{1}{2}\sin 2\theta_{12}^{\Lambda} \sin 2\theta_{23}^{\Lambda} \frac{m_0^2}{\Delta m_{32}^2} [\cos\alpha_2^{\Lambda} - \cos\alpha_1^{\Lambda}] \left(\Delta_\mu - \Delta_\tau\right)$$

$$\sin^2\theta_{13} \approx -\frac{m_0^2}{\Delta m_{32}^2} \sin 2\theta_{23}^{\Lambda} [1+\cos\alpha_2^{\Lambda}\cos^2\theta_{12}^{\Lambda} + \cos\alpha_1^{\Lambda}\sin^2\theta_{12}^{\Lambda}] \left(\Delta_\mu - \Delta_\tau\right)$$

General remark

- In general the runnig effect on angles is more in quasidegenerate case than in the hierarchical case.
- So starting with a zero value for θ_{13} it is possible to generate nonzero θ_{13} at low scale $\longrightarrow$ compatible with experiments.
- In hierarchical case: in IH θ_{12} runs considerably. Other angles do not change even taking threshold effects into account.

Ongoing work

- $$\Delta_\tau \approx \begin{cases} 6 \times 10^{-3} & \text{MSSM}, \tan\beta = 30 \\ -2.3 \times 10^{-5} & \text{SM} \end{cases} \Longrightarrow \mathcal{O}(\Delta_\tau^2) \text{ terms negligible}$$
- Running is opposite for SM and MSSM
- Running is very small in SM, in MSSM enhancement by $\tan^2\beta$

Uplifted Spersymmetry

- Couplings constants in MSSM to be perturbative up to Planck scale requires $2 \leq tan\beta \leq 50$
- MSSM is a viable theory when $tan\beta \gg 50$
- The holomorphy of the superpotential demands that the up-type fermions couple to H_u and down-type fermions couple only to H_d
- Once supersymmetry is broken the non-holomorphic terms of the form $Qu^c H_d^*$ and $QD^c H_u^*$ are allowed
- Yukawa couplings get modified: $y_l' = \frac{y_l\,\alpha}{8\pi} G$
- The resulting lepton mass is given by : $m_l = y_l\, v_d + y_l' v_u$
- $\Delta_\tau = \frac{m_\tau^2\,(1+tan^2\beta)}{8\pi^2 v^2 (1+\frac{\alpha G}{8\pi}\, tan\beta)^2} \ln \Lambda/\lambda$
- But $\frac{\alpha G}{8\pi} = 1.778 \times 10^{-3}$
- Can be constrained from neutrino data.

Conclusions

- Global oscillation data give
  - Two large (θ_{12} and θ_{23}) and one small (θ_{13}) angle
  - Weak hierarchy of masses (for NH) can also be inverted hierarchy or quasi degenerate
- Seesaw mechanism can explain smallness of neutrino mass
- Renormalization group effects including threshold effect are important in view of the onset of precision era in neutrino physics
- In MSSM enhancement by $\tan^2\beta$
- The non-zero hint of θ_{13} cannot be accommodated in TBM + RG effects if the effective theory is SM
- Runnig effect is more in quasidegenerate case than in the hierarchical case.

Future plan

- Implementing the threshold effect in left right symmetric models.
- Also in versions of two higgs doublet models and supersymmetric models.

Thank you