# *Origin of brane cosmological constant in warped geometry models*

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The talk is based on my collaborations with

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and

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# Hierarchy Problem

Vast disparity between the weak and Planck scale – Gauge hierarchy problem

$$\delta m_H^2 \sim \Lambda^2$$

where $\Lambda$ is the cutoff scale say Planck scale

To keep $m_H$ within Tev, one needs extreme fine tuning $\sim 10^{-32}$

UNNATURAL

Challenge for standard model – Extra-dimensions ?

ADD model and RS model

# Warped Geometry – Randall-Sundrum Model

The Einstein action in 5 dimensional $ADS_5$ space

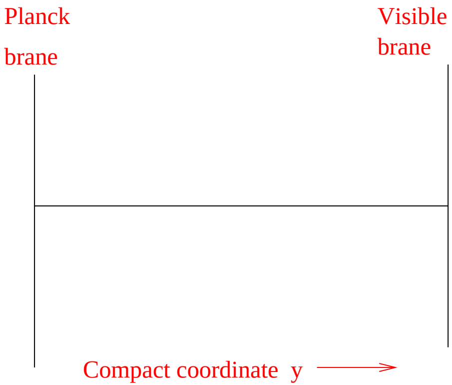
$$S = \frac{1}{16 G_5} \int d^5x \sqrt{-g_5}\, [\mathcal{R} - \Lambda]$$

Compactify the extra coordinate $y = r\phi$ on $S_1/Z_2$ orbifold

Identify $\phi$ to $-\phi$ i.e lower semi-circle to upper semi circle

Place two 3-branes at the two orbifold fixed points $\phi = 0, \pi$

$r$ is the radius of $S_1$




The $Z_2$ orbifolded coordinate $y = r\phi$ with $0 \leq \phi \leq \pi$ and $r$ is the radius of the $S_1$

## Action

$$S = S_{Gravity} + S_{vis} + S_{hid}$$

$$S_{Gravity} = \int d^4x \ r \ d\phi \sqrt{-G} \ [2M^3 R - \underbrace{\Lambda}_{5-dim}]$$

$$S_{vis} = \int d^4x \sqrt{-g_{vis}} \ [L_{vis} - V_{vis}]$$

$$S_{hid} = \int d^4x \sqrt{-g_{hid}} \ [L_{hid} - V_{hid}]$$

Metric ansatz:

$$ds^2 = e^{-A(\phi)}\ \eta_{\mu\nu} dx^\mu dx^\nu + r^2 d\phi^2$$

Computing the warp factor $A(y)$

Warp factor and the brane tensions are found by solving the 5 dimensional Einstein's equation with orbifolded boundary conditions

$$A = 2kr\phi$$

$$V_{hid} = -V_{vis} = 24M^3 k$$

and

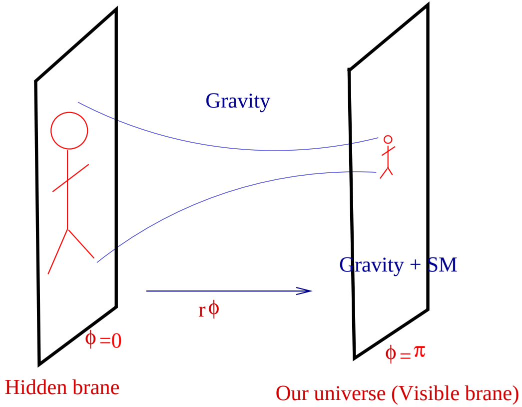
$$k^2 = \frac{-\Lambda}{24M^3}$$

## Warping

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}|_{\phi=\pi} = e^{-2kr\pi} \sim (10^{-16})^2$$

$\Rightarrow kr = \frac{16}{\pi}\ln(10) = 11.6279 \quad \leftarrow$ RS value with $k \sim M_P$ and $r \sim l_P$

So hierarchy problem is resolved without introducing any new scale



# Remarks

1. Warped geometry models have been constructetd in a string background – 'Throat geometry' with fluxes to stabilise the moduli
2. RS model is a simple field theoretic description which captures the essential idea of warped geometry and very useful in estimating various signatures of such models in particle phenomenology/cosmology
3. Modulus can be stabilised by Goldberger-Wise mechanism
4. It is defined on a flat/static visible brane with zero cosmological constant
5. Can we generalize it to include non-flat branes?

In the original RS scenario, it was proposed that the visible 3-brane being flat has zero cosmological constant.

$$ds^2 = e^{-2kry}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + r^2dy^2$$

But such model was generalized to Ricci flat spaces :

$$R_{\mu\nu} = 0$$

and the warp factor turned out to be the same as obtained by RS

See Chamblin, Hawking, Real : Phys.Rev.D, 61,065007 (2000)

In our work we demonstrate that the condition of zero cosmological constant can be relaxed and a more general warp factor can be obtained

S.Das, D.Maity, S.SenGupta

# Generalized Randall Sundrum braneworld with constant modulus

Generalize the RS model to non-flat brane scenario with constant radion field

The metric :

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

The induced metric $q_{\mu\nu}(x, y)$ in the previous section is now taken as : $e^{-2A(y)} g_{\mu\nu}$

The action is :

$$S = \int d^5x \sqrt{-G} (M^3 \mathcal{R} - \Lambda_5) + \int d^4x \sqrt{-g_i}\, \mathcal{V}_i$$

The bulk Einstein's equations away from the 3-branes are as follows :

$$ {}^{(4)}G_{\mu\nu} - g_{\mu\nu}e^{-2A}\left(-6A'^2 + 3A''\right) = -\frac{\Lambda_5}{2M^3}g_{\mu\nu}e^{-2A} $$

and

$$ -\frac{1}{2}e^{2A}\,{}^{(4)}R + 6A'^2 = -\frac{\Lambda_5}{2M^3} $$

with the boundary conditions :

$$ A'(y) = \frac{\epsilon_i}{12M^3}\mathcal{V}_i\epsilon_{pl} = -\epsilon_{vis} = 1 $$

Rearranging terms we get,

$$^{(4)}G_{\mu\nu} = -\Omega g_{\mu\nu}$$

This is the effective four dimensional Einstein's equation with $\Omega$ is the induced cosmological constant

<u>Negative $\Omega$ – ADS case</u> Define the parameter $\omega^2 \equiv -\Omega/3k^2 \geq 0$

The solution for the warp factor,

$$e^{-A} = \omega \cosh\left(\ln \frac{\omega}{c_1} + ky\right)$$

The above solution is an exact solution for the warp factor in presence of $\Omega$.

The RS solution $A = ky$ is recovered in the limit $\omega \to 0$.

## Positive $\Omega$ – de Sitter

$$e^{-A} = \omega_{pl} \sinh\left(\ln \frac{c_2}{\omega_{pl}} - \tilde{k}|y|\right)$$

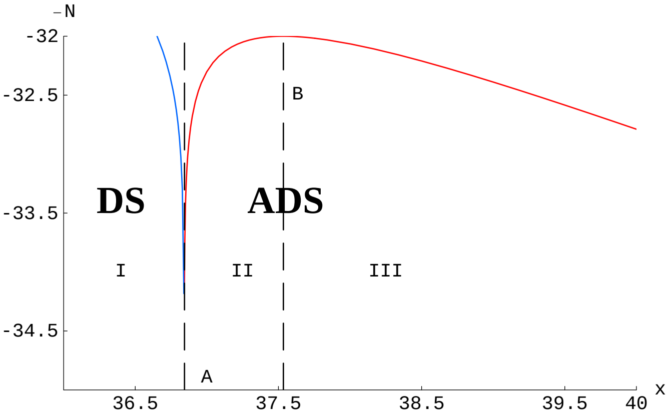
where $\omega_{pl}^2 = \Omega_{pl}/3\tilde{k}^2$ with $c_2 = 1 + \sqrt{1+\omega_{pl}^2}$
Once again for $\omega \to 0$ we retrieve RS solution

The brane tensions on both the branes are:

$$\mathcal{V}_{vis} = -12M^3\tilde{k}\left[\frac{c_2^2+\omega_{vis}^2}{c_2^2-\omega_{vis}^2}\right], \mathcal{V}_{pl} = 12M^3\tilde{k}\left[\frac{c_2^2+\omega_{pl}^2}{c_2^2-\omega_{pl}^2}\right]$$

Here the brane tension in one brane is always positive while the other is negative just as in RS case

In this case, there are no bounds on $\omega^2$, i.e. the (positive) cosmological constant can be of arbitrary magnitude.



−N
-32
-32.5
-33.5
-34.5
B
DS
ADS
I
II
III
A
X
36.5
37.5
38.5
39.5
40

From region I in FIG.1 and it is easy to observe that a small and positive value of the cosmological constant which corresponds to the observed value $\sim 10^{-124}$ in Planckian unit indicates a value for $x$ i.e $kr\pi$ very very close to the RS value 36.84

Here by generalizing the RS model with a non-vanishing cosmological constant on the visible brane we show that

1. Issue of smallness of cosmological constant, smallness of the factor in gauge hierarchy and brane tensions are intimately related in a generalized Randall-Sundrum (RS) type of warped geometry model .

2. Exact solution for the warp factors are determined for both DS and ADS cases.

3. Region of positive cosmological constant on the visible 3-brane ( de-Sitter) strictly implies negative brane tension However visible brane with negative

   cosmological constant ( anti de-Sitter) admits of both positive and negative brane tension.

1. For both the cases the desired warping from Planck to Tev scale can be achieved as a proper resolution of the gauge hierarchy problem.
2. The magnitude of the negative induced cosmological constant on the 3-brane has an upper bound $\sim 10^{-32}$ in Planck unit
3. For a very tiny but negative value of the induced cosmological constant the hierarchy problem can be resolved for two different values of the modulus, one of which corresponds to a positive tension Tev brane alongwith the positive tension Planck brane.
4. In the other region namely $\Omega > 0$ the Tev brane tension turns out to be necessarily negative . The modulus value corresponding to the observed value of the cosmological constant lies very close to the RS value.

Thus in a generalized warp braneworld model the fine tuning problem in connection with the Higgs mass requires that the cosmological constant $\Omega$ (whether positive or negative) on the Tev brane must be tuned to a very very small value.
In other words:

The fine tuning problem in connection with the Higgs mass and the cosmological fine tuning problem are intimately related and one implies the other!

We now try to generalize the model even further by incorporating space-time dependent radion scenario

In such scenario the previous approach fails and one must resort to an alternatice path

EFFECIVE EINSTEIN'S EQUATION ON AN EMBEDDED SURFACE

S.Lahiri and S.SenGupta

# Effective Einstein's equation

1. Consider a system of two 3-branes placed at the orbifold fixed points and embedded in a bulk
2. Bulk is a five dimensional AdS spacetime containing the bulk cosmological constant $\Lambda_5$ only
3. The most general metric is taken through radion field $\phi$ which is a function of both spacetime co-ordinates $x^\mu$ and extra dimensional co-ordinate $y$

Metric Ansatz

$$ds^2 = q_{\mu\nu}(y,x)dx^\mu dx^\nu + e^{2\phi(y,x)}dy^2$$

The proper distance between the two branes within the fixed interval $y = 0$ to $y = r\pi$ is given by:

$$d_0(x) = \int_0^{r\pi} dy e^{\phi(y,x)}$$

# Effective Einstein's equation

The effective Einstein's equations on a 3-brane is given by Gauss-Codacci equations :

$$^{(4)}G^{\mu}_{\nu} = \frac{3}{l^2}\delta^{\mu}_{\nu} + KK^{\mu}_{\nu} - K^{\mu}_{\alpha}K^{\alpha}_{\nu} + \frac{1}{2}\delta^{\mu}_{\nu}\left(K^2 - K^{\alpha}_{\beta}K^{\beta}_{\alpha}\right) - E^{\mu}_{\nu}$$

and

$$D_{\nu}K^{\nu}_{\mu} - D_{\mu}K = 0$$

where
$D_{\mu}$ is the covariant derivative with respect to the induced metric $q_{\mu\nu}$ on a brane and
Bulk curvature radius $l = \sqrt{\frac{-6}{\kappa^2\Lambda_5}}$

$K_{\mu\nu}$ is the extrinsic curvature on $y =$ constant hypersurface and is given by,

$$K_{\mu\nu} = \nabla_\mu n_\nu + n_\mu D_\nu \phi$$

where $n = e^{-\phi}\partial_y$ and $E^\mu_\nu$ is the projected part of five dimensional Weyl tensor

The junction conditions on the 3-branes are as follows :

$$[K^\mu_\nu - \delta^\mu_\nu K]_{y=0} = -\frac{\kappa^2}{2}(-\mathcal{V}_{hid}\,\delta^\mu_\nu + T_1{}^\mu{}_\nu)$$

and

$$[K^\mu_\nu - \delta^\mu_\nu K]_{y=r\pi} = \frac{\kappa^2}{2}(-\mathcal{V}_{vis}\,\delta^\mu_\nu + T_2{}^\mu{}_\nu)$$

$T_1{}^\mu{}_\nu$ and $T_2{}^\mu{}_\nu$ are respective energy momentum tensors on positive and negative tension branes

A perturbative scheme:

Consider the dimensionless perturbation parameter $\epsilon = (\frac{l}{L})^2$ such that $L >> l$, where $L$ is the brane curvature scale

$K^{\mu}_{\nu}$ and $E^{\mu}_{\nu}$ in the bulk are expanded as,

$$K^{\mu}_{\nu} = {}^{(0)}K^{\mu}_{\nu} + {}^{(1)}K^{\mu}_{\nu} + ....$$

and

$$E^{\mu}_{\nu} = {}^{(1)}E^{\mu}_{\nu} + ....$$

# Zero-the order

At the zero-th order ${}^{(0)}E^{\mu}_{\nu} = 0$

Only one evolution equation corresponding to ${}^{(0)}K^{\mu}_{\nu}$ whose solution satisfying the Coddacci equation is given by

$$ {}^{(0)}K^{\mu}_{\nu} \;=\; -\frac{1}{l}\delta^{\mu}_{\nu} $$

The junction conditions at the zeroth order are given by :

$$ \left[ {}^{(0)}K^{\mu}_{\nu} \;-\; \delta^{\mu}_{\nu}\, {}^{(0)}K \right]_{y=0} = \frac{\kappa^2}{2}\,\mathcal{V}_{hid}\,\delta^{\mu}_{\nu} $$

and

$$ \left[ {}^{(0)}K^{\mu}_{\nu} \;-\; \delta^{\mu}_{\nu}\, {}^{(0)}K \right]_{y=r\pi} = -\frac{\kappa^2}{2}\,\mathcal{V}_{vis}\,\delta^{\mu}_{\nu} $$

These imply that the relation between bulk curvature radius $l$ and brane tensions is similar to the fine-tuning condition of the RS model given by :

$$\frac{1}{l} = \frac{1}{6}\kappa^2\mathcal{V}_{hid} = -\frac{1}{6}\kappa^2\mathcal{V}_{vis}$$

Four dimensional cosmological constant

$$\Lambda_4 = 1/2\kappa_5^2(\Lambda_5 + 1/6\kappa_5^2 V_{vis}^2) = 0$$

Thus at the zeroth order the combined effect of bulk cosmological constant and the brane tensions on the 3-branes exactly counterbalance one another to produce a vanishing brane cosmological constant such that ${}^{(4)}G^{\mu}_{\nu} = 0$

This is the static RS model which predicts both static and flat 3-branes

So the curvature of 3-branes can emerge only from higher order correction to $K^{\mu}_{\nu}$ and $E^{\mu}_{\nu}$

# 1-st order

At the first order we have two evolution equations, one for ${}^{(1)}E^{\mu}_{\nu}$ and the other for ${}^{(1)}K^{\mu}_{\nu}$

The corresponding junction conditions at the first order :

$$\left[{}^{(1)}K^{\mu}_{\nu} - \delta^{\mu}_{\nu}\,{}^{(1)}K\right]_{y=0} = -\frac{\kappa^2}{2}\,T_{1}{}^{\mu}{}_{\nu}$$

and

$$\left[{}^{(1)}K^{\mu}_{\nu} - \delta^{\mu}_{\nu}\,{}^{(1)}K\right]_{y=r\pi} = \frac{\kappa^2}{2}\,T_{2}{}^{\mu}{}_{\nu}$$

The Gauss equation at the first order on the visible brane is :

$${}^{(4)}G^{\mu}_{\nu} = -\frac{2}{l}\left({}^{(1)}K^{\mu}_{\nu}(y_0,x) - \delta^{\mu}_{\nu}\,{}^{(1)}K(y_0,x)\right) - {}^{(1)}E^{\mu}_{\nu}(y_0,x) = -\frac{\kappa^2}{l}\,T_{2}{}^{\mu}{}_{\nu} - {}^{(1)}E^{\mu}_{\nu}($$

where $y_0 = r\pi$.

Use the bulk solution of ${}^{(1)}K^{\mu}_{\nu}$ which contains ${}^{(1)}E^{\mu}_{\nu}$

From this ${}^{(1)}E^{\mu}_{\nu}$ can be explicitly determined on the visible brane

Using ${}^{(1)}E^{\mu}_{\nu}$ the Einstein's equations on visible brane:

$$\begin{aligned}
{}^{(4)}G^{\mu}_{\nu} &= \frac{\kappa^2}{l}\frac{1}{\Phi}\,T_{2}{}^{\mu}{}_{\nu} + \frac{\kappa^2}{l}\frac{(1+\Phi)^2}{\Phi}\,T_{1}{}^{\mu}{}_{\nu} \\
&+ \frac{1}{\Phi}\left(D^{\mu}D_{\nu}\Phi - \delta^{\mu}_{\nu}D^2\Phi\right) \\
&+ \frac{\omega(\Phi)}{\Phi^2}\left(D^{\mu}\Phi D_{\nu}\Phi - \frac{1}{2}\delta^{\mu}_{\nu}(D\Phi)^2\right)
\end{aligned}$$

where,

$$\Phi = e^{2d_0/l} - 1, \qquad \omega(\Phi) = -\frac{3}{2}\frac{\Phi}{1+\Phi}$$

$\Phi$ is a function of the brane co-ordinates $x$

Note that an effective brane matter on a 3-brane can originate from :

1. Explicit matter distribution on the brane $T_i^{\mu}{}_{\nu}$, where $i = 1, 2$
2. A matter distribution on the hidden brane i,e. $T_1^{\mu}{}_{\nu}$ can induce a non-zero matter distribution on the visible brane via the bulk curvature
3. A space-time dependent modulus field $e^{2\phi(x,t)}$ can also induce a non-vanishing energy momentum on the visible brane

# Warped cosmological metric in constant radion field

We first consider a constant modulus scenario where the 3-branes are endowed with matter densities $\rho_{vis}$, $\rho_{pl}$ and brane pressures $p_{vis}, p_{pl}$

FRW metric ansatz :

$$ds^2 = e^{-2A(y)}\left[-dt^2 + v^2(t)\delta_{ij}\,dx^i dx^j\right] + dy^2$$

In this case constant radion field implies $\phi = 0$

$q_{\mu\nu}(x,y) = e^{-2A(y)} g_{\mu\nu}$ where $g_{\mu\nu}$ is a FRW metric with a flat spatial curvature

The proper distance along the $y$ direction determined between the interval $y = 0$ to $r\pi$ is given by:

$$d_0 = \int_0^{r\pi} dy = r\pi$$

The five dimensional bulk-brane action is

$$S = \int d^5x\sqrt{-G}(M^3\,\mathcal{R} - \Lambda_5) \quad +$$
$$\int d^5x \left[\sqrt{-g_{pl}}\,(\mathcal{L}_1 - \mathcal{V}_{pl})\,\delta(y) + \sqrt{-g_{vis}}\,(\mathcal{L}_2 - \mathcal{V}_{vis})\,\delta(y-\pi)\right]$$

By varying the action, the five dimensional Einstein's equations

$$\sqrt{-G}\,G_{MN} = -\frac{\Lambda_5}{2M^3}\sqrt{-G}\,g_{MN} + \frac{1}{2M^3}\left[\tilde{T}_1{}^{\gamma}{}_{\mu}\,\delta(y)\sqrt{-g_{pl}} + \tilde{T}_2{}^{\gamma}{}_{\mu}\,\delta(y-\pi)\sqrt{-g_{v}}\right.$$

The energy momentum tensors on the two 3-branes

On the Planck brane ($y = 0$) :

$$\tilde{T}_1{}^{\gamma}{}_{\mu} = diag(-\rho_{pl} + \mathcal{V}_{pl}\,, p_{pl} + \mathcal{V}_{pl}\,, p_{pl} + \mathcal{V}_{pl}\,, p_{pl} + \mathcal{V}_{pl})$$

On the visible brane ($y = \pi$) :

$$\tilde{T}_2{}^{\gamma}{}_{\mu} = diag(-\rho_{vis} + \mathcal{V}_{vis}\,, p_{vis} + \mathcal{V}_{vis}\,, p_{vis} + \mathcal{V}_{vis}\,, p_{vis} + \mathcal{V}_{vis})$$

Einstein's equations:

tt component

$$3\frac{\dot{v}^2}{v^2} + e^{-2A(y)}\left(3A'' - 6A'^2\right) = -\frac{\Lambda_5}{2M^3}(-1)\,e^{-2A(y)}$$

ii component

$$\left(-2\frac{\ddot{v}}{v} - \frac{\dot{v}^2}{v^2}\right) + e^{-2A(y)}\left[-3A'' + 6A'^2\right] = -\frac{\Lambda_5}{2M^3}\,e^{-2A(y)}$$

yy component :

$$6\,A'^2 - 3\,e^{2A}\,\frac{\dot{v}^2 + \ddot{v}}{v^2} = -\frac{\Lambda_5}{2M^3}$$

After rearrangement of terms in *tt* and *ii* components :

$$\frac{{}^{(4)}G_{\mu\nu}}{g_{\mu\nu}} = e^{-2A(y)}\left[-\frac{\Lambda_5}{2M^3} + 3A'' - 6A'^2\right] = -\Omega$$

The effective Einstein's equation on the Planck 3-brane with metric $g^{pl}_{\mu\nu}$ is

$$\frac{{}^{(4)}G^{(pl)}_{\mu\nu}}{g^{(pl)}_{\mu\nu}} = -\Omega_{pl}$$

with

$$-\Omega_{pl} = e^{-2A(y)}\left[-\frac{\Lambda_5}{2M^3} + 3A'' - 6A'^2\right] \tag{1}$$

Similarly for the visible brane

$$\frac{{}^{(4)}G_{\mu\nu}^{(vis)}}{g_{\mu\nu}^{(vis)}} = -\Omega_{vis}$$

From *tt* and *ii* components :

$$\frac{\dot{v}^2}{v^2} + \frac{\ddot{v}}{v} = 0$$

The solution is

$$v(t) = e^{H_0 t}$$

where $H_0$ is an integration constant.

From different component of Einstein's equation

$$\Omega_{pl} = 3H_0^2$$

which indicates a de-Sitter spacetime.

When $H_0 \longrightarrow 0$ the induced cosmological constants on both the branes vanish leading to a static and flat Universe

Once again the warp factor is given by :

$$e^{-A(y)} = \omega_{pl} \sinh\left[-\tilde{k}\,|y| + \ln\frac{c_2}{\omega_{pl}}\right]$$

Normalizing the warp factor to one on the Planck brane at $y = 0$, we have

$$c_2 = 1 + \sqrt{1 + \omega_{pl}^2}$$

# Effective Hubble parameter on the visible brane

The metric:

$$ds^2 = \omega_{pl}^2 \sinh^2\left[-\tilde{k}\,|y| + \ln\frac{2}{\omega_{pl}}\right]\left(-dt^2 + e^{2H_0 t}\delta_{ij}dx^i\,dx^j\right) + dy^2$$

Recast the induced metric of the four dimensional spacetime in the form :

$$ds_4^2 = -d\tilde{t}^2 + e^{2H(y)t}\delta_{ij}\,d\tilde{x}^i\,d\tilde{x}^j$$

Define following co-ordinate transformations :

$$d\tilde{t} = \omega_{pl}\sinh\left[-\tilde{k}\,|y| + \ln\frac{2}{\omega_{pl}}\right]dt$$

and

$$d\tilde{x}^i = \omega_{pl}\sinh\left[-\tilde{k}\,|y| + \ln\frac{2}{\omega_{pl}}\right]dx^i$$

On comparing the 5D metric and 4D effective metric

$$\omega_{pl}^2 \sinh^2\left[-\tilde{k}\,|y| + \ln\frac{2}{\omega_{pl}}\right] e^{2H_0 t}\,\delta_{ij}\,dx^i\,dx^j = e^{2\,H(y)\tilde{t}}\delta_{ij}\,d\tilde{x}^i\,d\tilde{x}^j$$

Therefore the effective 4D Hubble parameter on a 3-brane for a given value of $y$ is given by :

$$H(y) = \frac{H_0}{\omega_{pl}}\,cosech\left[-\tilde{k}\,|y| + \ln\frac{2}{\omega_{pl}}\right]$$

On the visible brane ($y = r\pi$) :

$$H_{vis} = \tilde{k}\,cosech\left[-\tilde{k}\pi + \ln\frac{2}{\omega_{pl}}\right] = \omega_{vis}\tilde{k}$$

Vanishing $H_{vis}$ implies a static as well as flat Universe, devoid of any matter.
Such a Universe is described by static RS model where both the branes are flat

possessing brane tensions only

# Energy-momentum tensor on 3-branes

The effective Einstein's equations on the visible brane are only related to energy momentum tensors of the two 3-branes and bulk curvature radius $l$:

$$^{(4)}G^{\mu}_{\nu} = \frac{\kappa^2}{l}\frac{1}{\Phi}\, T_2{}^{\mu}{}_{\nu} + \frac{\kappa^2}{l}\frac{(1+\Phi)^2}{\Phi}\, T_1{}^{\mu}{}_{\nu}$$

$\Phi = (e^{2\pi/l} - 1)$, $\kappa^2$ is related to the five dimensional gravitational constant. Combining previous equations

$$-\Omega_{vis} = \frac{\kappa^2}{4l(e^{2\pi r/l} - 1)}\left[e^{4\pi r/l}\, T_1{}^{\mu}{}_{\nu}\,\delta^{\nu}_{\mu} + T_2{}^{\mu}{}_{\nu}\,\delta^{\nu}_{\mu}\right]$$

n

1. So the existence of induced cosmological constant on the visible brane is related to the presence of matter density and pressure on both the 3-branes

   d

2. The effects of extra dimension on the induced brane cosmological constant however shows up through the multiplicative factor $\frac{\kappa^2}{4l(e^{2\pi r/l}-1)}$

3. It is interesting to find that even if the visible brane matter $T_2{}^{\mu}{}_{\nu} = 0$, there can be a net cosmological constant in the Universe solely due to the matter content of the hidden brane.

Visible brane: $T_{2\ \nu}^{\mu} = diag(-\rho_{vis}, p_{vis}, p_{vis}, p_{vis})$

Hidden brane: $T_{1\ \nu}^{\mu} = diag(-\rho_{pl}, p_{pl}, p_{pl}, p_{pl})$

Substituting the components of $T_i{}^{\mu}{}_{\nu}$ the visible brane induced cosmological constant is :

$$-\Omega_{vis} = \frac{\kappa^2}{4\, l(e^{2\pi r/l} - 1)} \left[-\left(e^{4\pi r/l}\rho_{pl} + \rho_{vis}\right) + 3\left(e^{4\pi r/l} p_{pl} + p_{vis}\right)\right]$$

In case of vacuum energy dominated Universe, the energy density and pressure are

$$\rho_{vis} = -p_{vis}$$

This yields:

$$\Omega_{vis} = \frac{\kappa^2}{l(e^{2\pi r/l} - 1)} \left[e^{4\pi r/l}\rho_{pl} + \rho_{vis}\right]$$

1. The non-zero value of brane matter as well as the bulk cosmological constant $(1/l)$ are essential to induce an effective cosmological constant on the visible brane when modulus field is independent of spacetime co-ordinates

2. Absence of matter in the visible brane i,e. $\rho_{vis} = 0$ does not necessarily imply a vanishing 4D cosmological constant as long as $\rho_{pl} \neq 0$
$\Omega_{vis} = \kappa^2 \frac{e^{4\pi r/l} \rho_{pl}}{l(e^{2\pi r/l} - 1)}$

3. The vacuum energy density of the Planck brane and five dimensional bulk cosmological constant $\Lambda_5$ may be possible origins of an effective cosmological constant on our Universe (i,e. the visible brane) which can result into an inflationary Universe.

# Radion driven acceleration without brane matter - time dependent radion field

Consider a time dependent radion field and look for a possible cosmological solution in the effective 4D theory

The metric :

$$ds^2 = e^{-2A(y)}\left[-dt^2 + v^2(t)\delta_{ij}\, dx^i dx^j\right] + e^{2\phi(t)}dy^2$$

The proper distance between the interval $y = 0$ to $y = r\pi$ is

$$d_0(t) = \int_0^{r\pi} e^{2\phi(t)}dy = r\pi e^{2\phi(t)}$$

In order to study the time evolution of the Universe which is located on the visible brane ($y = \pi$), we consider the effective 4D metric as :
$ds_4^2 = -dt^2 + v^2(t)\,\delta_{ij}\, dx^i dx^j$, where $v(t) = v(t, \pi)$.

Einstein's equations on the visible brane

$$\frac{\ddot{v}}{v} - \frac{\dot{v}^2}{v^2} = \frac{d}{dt}H(t) = \frac{\kappa^2}{2\,l(e^{2d_0/l}-1)}\left[({T_2}^t{}_t - {T_2}^i{}_i) + e^{4d_0/l}({T_1}^t{}_t - {T_1}^i{}_i)\right]$$
$$- \frac{e^{2d_0/l}}{l(e^{2d_0/l}-1)}\left(3\,\frac{\dot{v}}{v}\,\dot{d_0} + \ddot{d_0}\right) + \frac{e^{2d_0/l}}{2\,l^2\,(e^{2d_0/l}-1)}\,\dot{d_0}^{\,2}$$

$H(t) = \frac{\dot{v}(t)}{v(t)}$ is the Hubble parameter of the Universe located on the visible brane

Thus a time dependent modulus field can itself produce dynamical evolution of the Universe even in the absence of matter on the 3-branes i,e. when ${T_1}^\mu{}_\nu = {T_2}^\mu{}_\nu = 0$

For a slowly time varying radion field keeping term only upto leading order in $\dot{d_0}$ we obtain

$$\frac{\ddot{v}}{v} - \frac{\dot{v}^2}{v^2} = \frac{d}{dt}H(t) = \frac{3\, e^{2d_0/l}\, \dot{d_0}}{l(e^{2d_0/l} - 1)}\, H(t)$$

which gives

$$\frac{1}{H(t)}\frac{d}{dt}H(t) = \frac{3\, \dot{d_0}\, e^{2d_0/l}}{l(e^{2d_0/l} - 1)}$$

Finally we get,

$$H(t) = (e^{2d_0(t)/l} - 1)^{3/2}$$

Since $\dot{H(t)} > 0$, therefore Universe accelerates in the presence of a slowly

time-varying modulus field.

# Conclusion

In a generalized two-brane warped geometry model with visible 3-brane embedded in a five dimensional AdS ($\Lambda_5 < 0$) bulk we have considered different cases

1. The modulus field is constant:
2. An effective brane positive cosmological constant $\Omega$ is generated on our brane from the energy density in the hiddden/Planck brane. which leads to exponential solution of the scale factor $v(t) = e^{Ht}$ suggesting inflationary Universe
3. The generalised warp factor in the form of sine hyperbolic solution is determined which can resolve the gauge hierarchy problem for appropriate choice of the parameter
4. The effective brane cosmological constant $\Omega_{vis}$ depends on energy momentum tensors of both the 3-brane , the bulk curvature and the proper distance between the two branes
5. The value of visible brane Hubble parameter is of the order of visible brane cosmological constant

1. Time-varying modulus field
2. The dynamical evolution of the Universe is possible in the absence of any matter on the 3-branes i,e. when $T_1{}^{\mu}{}_{\nu} = T_2{}^{\mu}{}_{\nu} = 0$ with time dependent modulus field and bulk cosmological constant together leading to a non-zero Hubble parameter
3. In case of a slowly time varying radion field, the Hubble parameter on our Universe has been determined in terms of time varying proper length $d_0(t)$ modulated by bulk curvature $l$
4. $\dot{H}(t) > 0$ suggests an accelerating nature of the Universe driven solely by time dependent radion field.