# The maximum mass of neutron star

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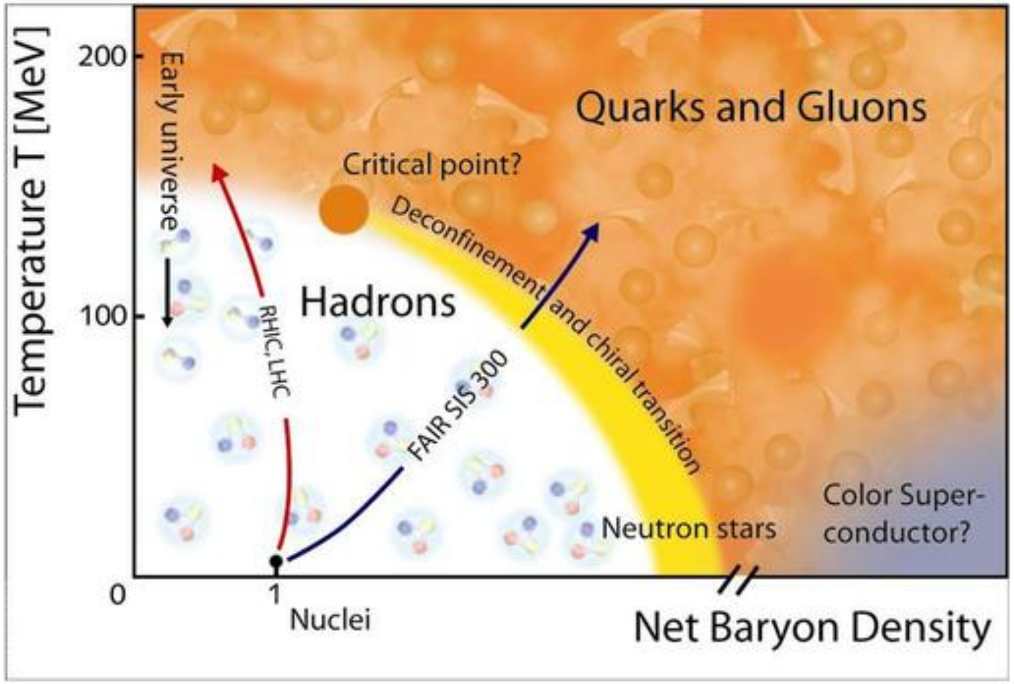
# Introduction

**The study of phase transition of matter at extreme condition (temperature/density) is important to understand the nature of strong interaction.**



# Introduction

Recent observation has emphasized the need to study compact astrophysical objects for better understanding of properties of matter under extreme condition.

The compact stars provide us with natural laboratory, where the matter is at extreme density.

Neutron stars, Quark stars and Magnetar.

X-ray satellites, radio satellites and gravitational wave detectors.

# Introduction

Neutron stars are dead stars formed after the Supernovae.

| | |
|---|---|
| Mass | 1 - 3 Solar mass |
| Radius | 10 - 20Km |

Two extreme properties

| | |
|---|---|
| High density | 3 - 10 $n_0$ |
| Surface magnetic field | $10^8$ - $10^{12}$ G |

Magnetars

| | |
|---|---|
| Extreme magnetic field | $10^{14}$ - $10^{15}$ G |

# Introduction

**Witten conjecture** : (1984, PRD 30, 272)
Strange quark matter (SQM) the true ground state of strongly interacting matter at high density and/or temperature.
SQM : Equal number of up (u), down (d) and strange (s) quark.

Constant effort to confirm the existence quark gluon plasma (QGP) and s quarks in ultra relativistic collisions.
SQM could naturally occur in the cores of compact stars.

Neutron star : high central density, gets converted to strange or hybrid star.

# Introduction

**Different scenarios of Phase transition**

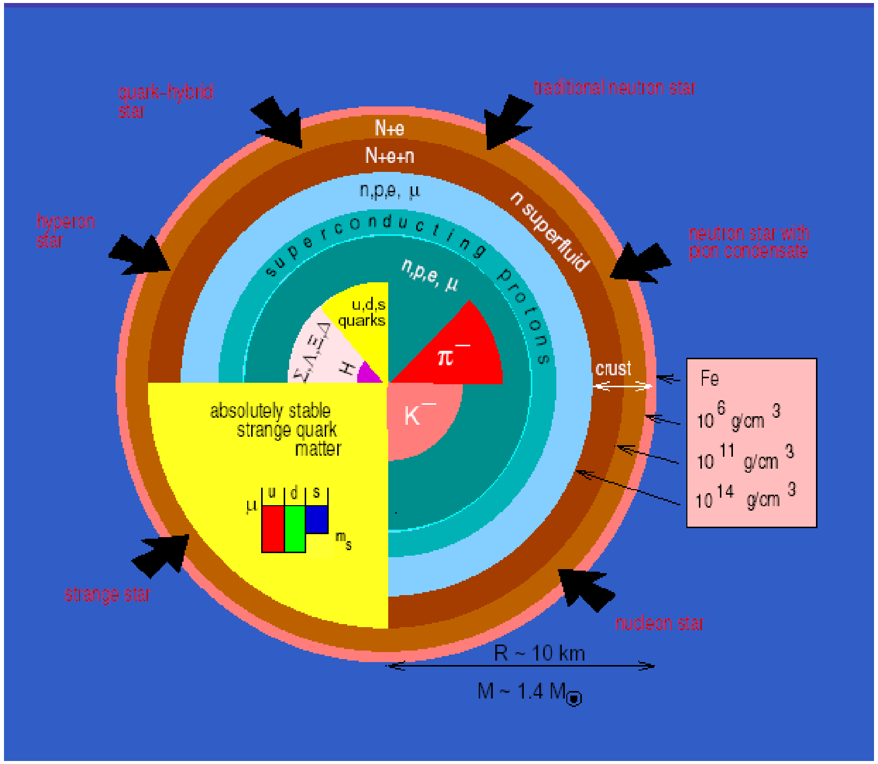
**Neutron star comes in contact with external seed.**
**Sudden density fluctuation at the centre of the star.**

**Shock wave travelling outwards from the center, converting hadronic matter to quark matter.**

**May travel throughout the star (SS), or may stop someway inside the star (HS).**

# Introduction

**Artistic model diagram of a neutron star**




**Different regions signifies different model predictions**

# Introduction

The star is defined by the EOS which describes the state of matter inside the star.

**Hadronic EOS**

Stiff EOS → Usually no Hyperons

Softer EOS → Hyperons present

**Quark EOS**

Stiff EOS → Colour superconducting models, strong coupling model

Softer EOS → MIT bag model

# Conversion Process

Conversion being a two step process ( PRC 74, 065804, 2006 ).

**First step** : Deconfinement of nuclear matter to 2-flavour (u & d) quark matter. Strong interaction time scale.

**Second step** : Generation of strange quark from excess down quark. Weak interaction process.

**Assumptions**

The neutron star is spherically symmetric.

Existence of a combustion front separating the two phases.

Front generates from the centre and propagates outwards.

# Conversion Process

**Conservation conditions**

**Energy density**
**Momentum density**
**Baryon number density**

$$w_n v_n^2 W_n^2 + p_n = w_s v_s^2 W_s^2 + p_s$$

$$w_n v_n W_n^2 = w_s v_s W_s^2$$

$$n_n v_n W_n = n_s v_s W_s$$

**Conservation conditions are simultaneously solved to obtain the velocity of matter in two phases**

$$v_s^2 = (p_s - p_n)(e_n + p_s)/(e_s - e_n)(e_s + p_n)$$

$$v_n^2 = (p_s - p_n)(e_s + p_n)/(e_s - e_n)(e_n + p_s)$$

# Conversion Process

**Front propgation**

**Evolution of the hydrodynamic combustion front with position and time.**

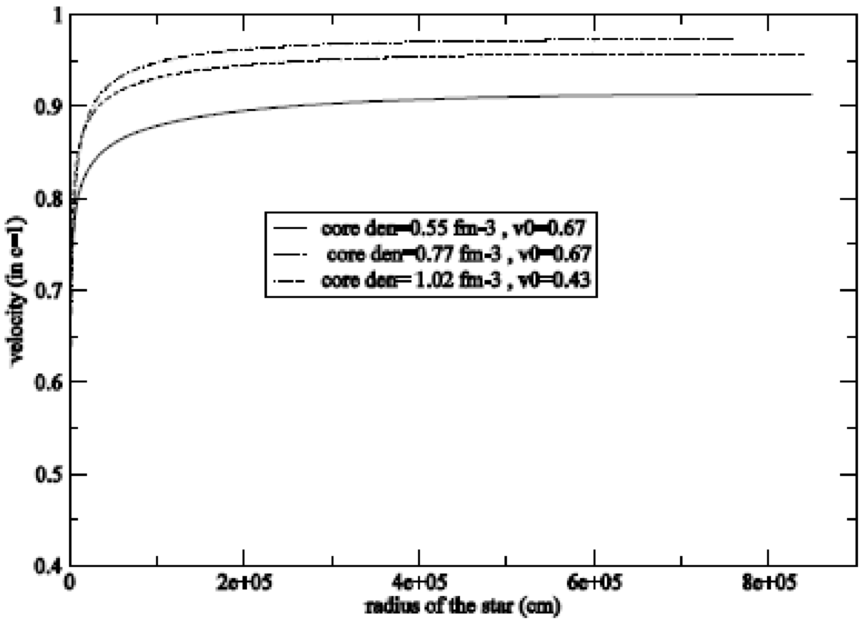
**Reference frame with nuclear matter at rest.**

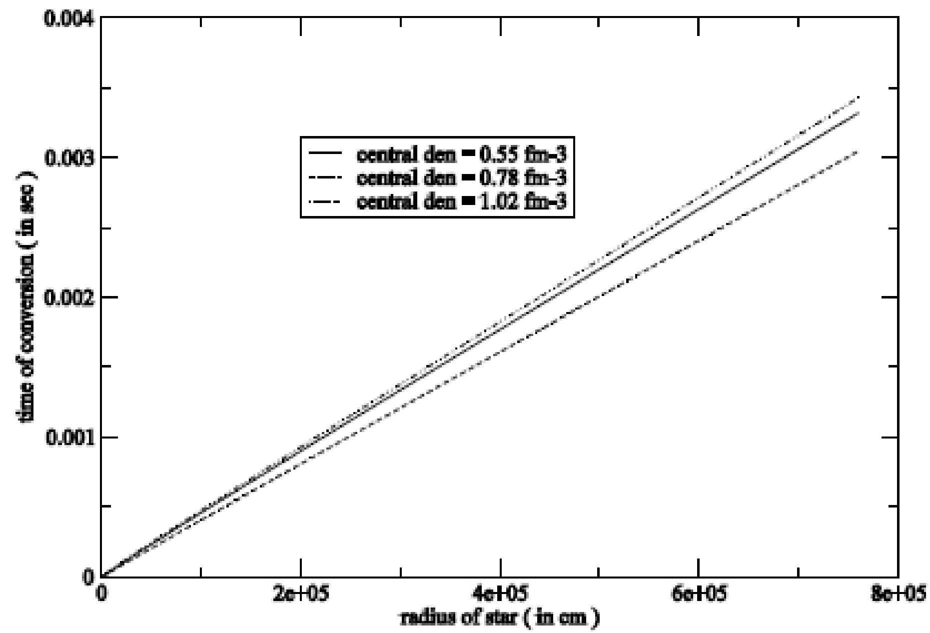
**The continuity and Euler's equation**

$$1/w.(\partial p/\partial r+v.\partial p/\partial t)+1/W^2(\partial v/\partial t+v.\partial v/\partial r)=0$$

$$1/w.(\partial e/\partial t+v.\partial e/\partial r)+1/W^2(\partial v/\partial r+v.\partial v/\partial t)+2v/r=0$$

# Conversion Process



# Conversion Process

**Results**

Velocity of the front shoots up near the centre and saturates at certain velocity at higher radius.

Time taken by the front to reach the surface is of the order of few ms.

# Conversion Process

**Metastable 2-flavour to stable 3-flavour**

Existence of a combusting front propagating outward

Conservation condition

**Assumptions**

Charge neutrality

Baryon number conservation

$$d \rightarrow u + e^{-} + \overline{\nu_{e-}}$$

$$s \rightarrow u + e^{-} + \overline{\nu_{e-}}$$

$$d + u \rightarrow s + u$$

**The Weak interaction**

Nonleptonic interaction is the governing rate equation.

Semileptonic equation for chemical equilibration.

# Conversion Process

**Define**

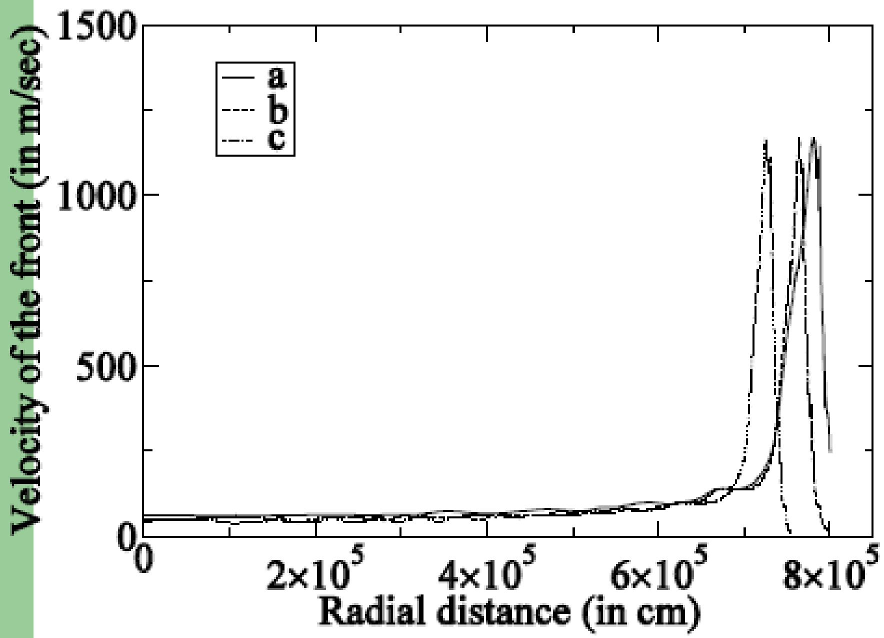
$$a(r) = [n_d - n_s]/2n_B$$

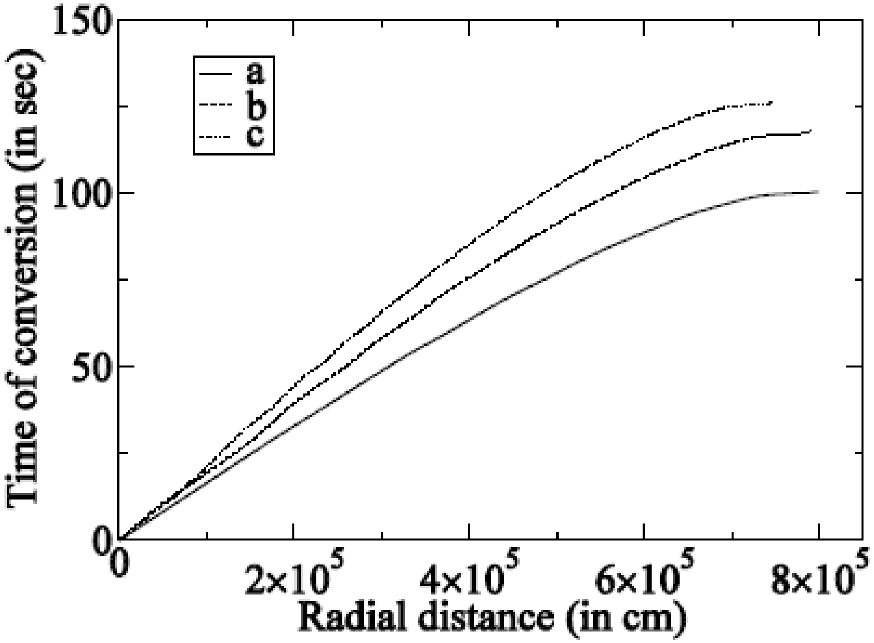
**a(0) is the number density of the strange quark at the centre for which the SQM is stable.**
**We can write a differential equation, which have decay and diffusion term**

$$D\ddot{a} - v\dot{a} - R(a) = 0$$

**The velocity increases as it reaches low density region and then drops to zero near the surface.**
**Time needed for the conversion is of the order of 100 s.**

# Conversion Process



# Conversion Process

**Actual calculation of the propagation of the front should involve GR, taking into account the curvature of the front.**
**( *PRC 76 ®, 052801, 2007* )**

**Metric of the star structure**

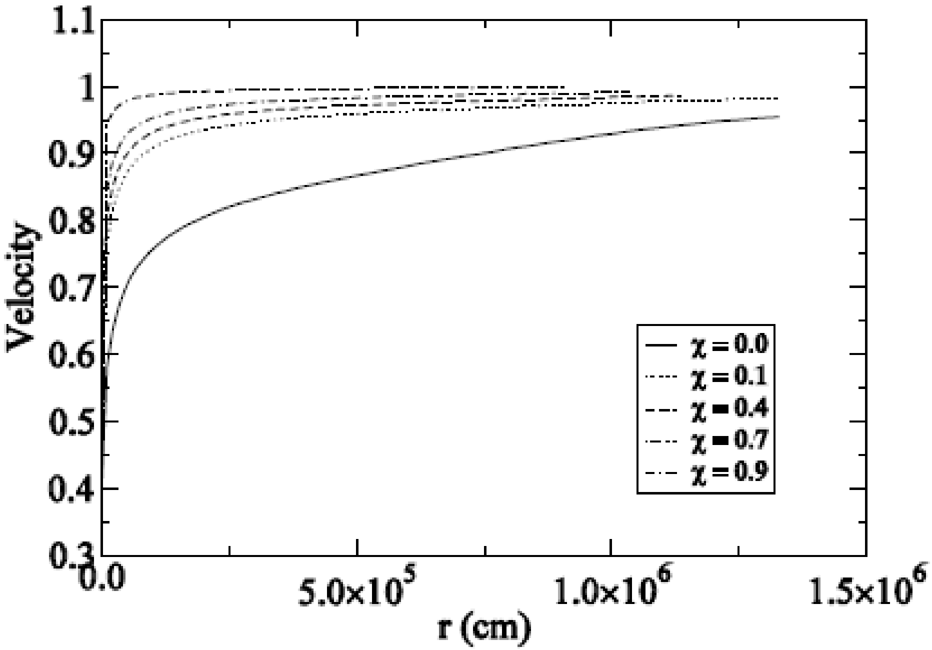
$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2)$$
$$+ e^{\gamma-\rho} r^2 \sin^2\theta (d\varphi - \omega dt)^2$$

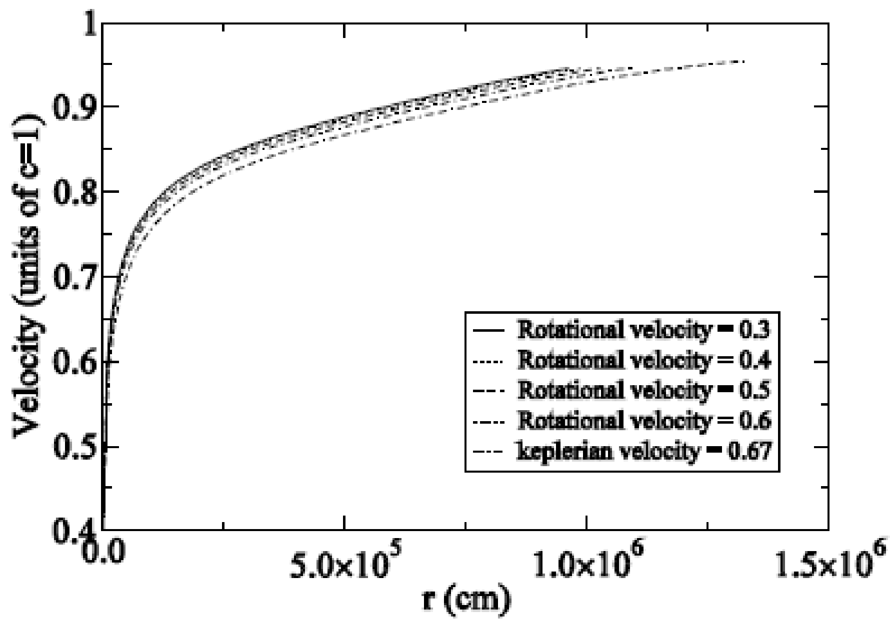
**The GR continuity and Euler's equation**

$$\frac{1}{\varpi}\left(\frac{\partial p}{\partial r} + v\frac{\partial p}{\partial \tau}\right) + \frac{1}{W^2}\left(\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial r}\right) = \frac{1}{2}\left(A\frac{\partial \gamma}{\partial r} + B\frac{\partial \rho}{\partial r} + C\frac{\partial \omega}{\partial r} + E\right)$$

$$\frac{1}{\varpi}\left(\frac{\partial \varepsilon}{\partial \tau} + v\frac{\partial \varepsilon}{\partial r}\right) + \frac{1}{W^2}\left(\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial \tau}\right) + \frac{2v}{r} + \frac{v}{r}\cos\theta = -v\left(\frac{\partial \gamma}{\partial r} + \frac{\partial \alpha}{\partial r}\right)$$

# Conversion Process



# Conversion Process

**Results**

The GR effect increases the velocity considerably. Most pronounced for static star.

The rotational effect suppress the GR effect.

The velocity is maximum along the pole and minimum along the equator.

At some distance from the centre, the front breaks up into several distinct front and propagates with different velocities along different directions.

# Remarks

The phase transition converts the NS to SS.

The effect from Magnetic field can stop the propagation front inside the star.
Resulting in a HS.
( PRC 84, 065805, 2011 and JPG 39, 095201, 2012)

# Recent Observations

Most of the pulsars discovered had mass 1 - 1.4 solar mass.

Recently a pulsar J1903+0327, mass 1.67 solar mass.

Most important recent discovery pulsar PSR J1614-2230 (Demorest et al. 2010, Nature 467, 1081).
Mass of compact star about 2 solar mass.

The quark matter EOS gets challenged. They cannot produce such high mass.

# Recent Observations

After the discovery of **PSR J1614-2230**, they predicted that the central density of the star can be as high as **5-6** times nuclear saturation density.

However, theoretical calculation had shown that the central density can be as high as **8-10** times.

The current quark matter EOS cannot give such masses.

Recent calculation, however gives hopes.
Mostly hybrid star with quark core.

# Construction of the mixed phase

**Hadronic EOS**

**Hyperons present**
**Softer : TM1L model**
**Stiffer : PL-Z model**

**Quark EOS**

**Simple model: MIT bag model.**
**New models NJL model.**
**MIT model  cannot give the mass predicted by**
**PSR J1614-2230**
**MIT bag model with varying bag constant**

# Construction of the mixed phase

**TM1L model**

**RMF model with hyperons.**
**Model parameters are fitted according to the bulk properties of hypernuclear matter.**
**(Schaffner & Mishustin, 1996, PRC 53, 1416).**

**Varying MIT bag model**

**The bag pressure varies with density of the star and as density increases towards the centre, the bag pressure reduces, reaching the lowest asymptotic value 130 MeV.**

$$B_{gn}(n_b) = B_\infty + (B_g - B_\infty)\exp\left[-\beta\left(\frac{n_b}{n_0}\right)^2\right]$$

**(Burgio et al., 2002, PRC 66, 025802)**

# The mixed phase

**The energy density and pressure of the hadronic EOS**

$$\varepsilon = \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_0^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2}$$
$$+\frac{1}{2}m_\phi^2\phi_0^2 + \frac{3}{4}d\omega_0^4 + U(\sigma) + \sum_b \varepsilon_b + \sum_l \varepsilon_l \,,$$

$$P = \sum_i \mu_i n_i - \varepsilon,$$

**Assuming charge neutrality and beta equilibrium in quark matter**

**The energy density and pressure of quark EOS**

$$\epsilon^Q = \sum_{i=u,d,s} \frac{g_i}{2\pi^2}\int_0^{k_F^i} dk k^2\sqrt{m_i^2+k^2} + B_G \,,$$

$$P^Q = \sum_{i=u,d,s} \frac{g_i}{6\pi^2}\int_0^{k_F^i} dk \frac{k^4}{\sqrt{m_i^2+k^2}} - B_G \,,$$

# Construction of the mixed phase

## Mixed Phase

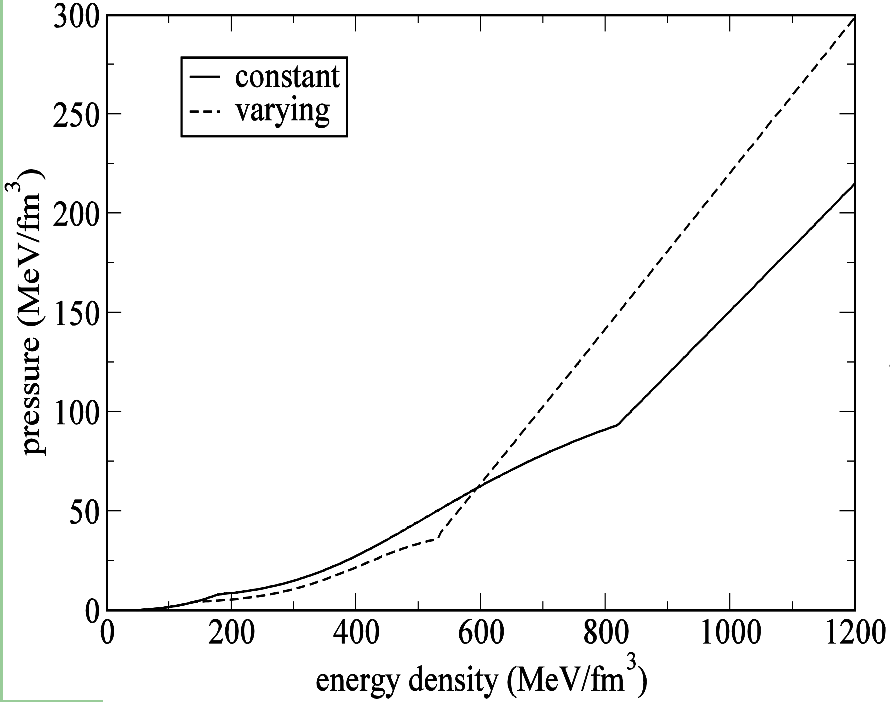
### Gibbs Construction

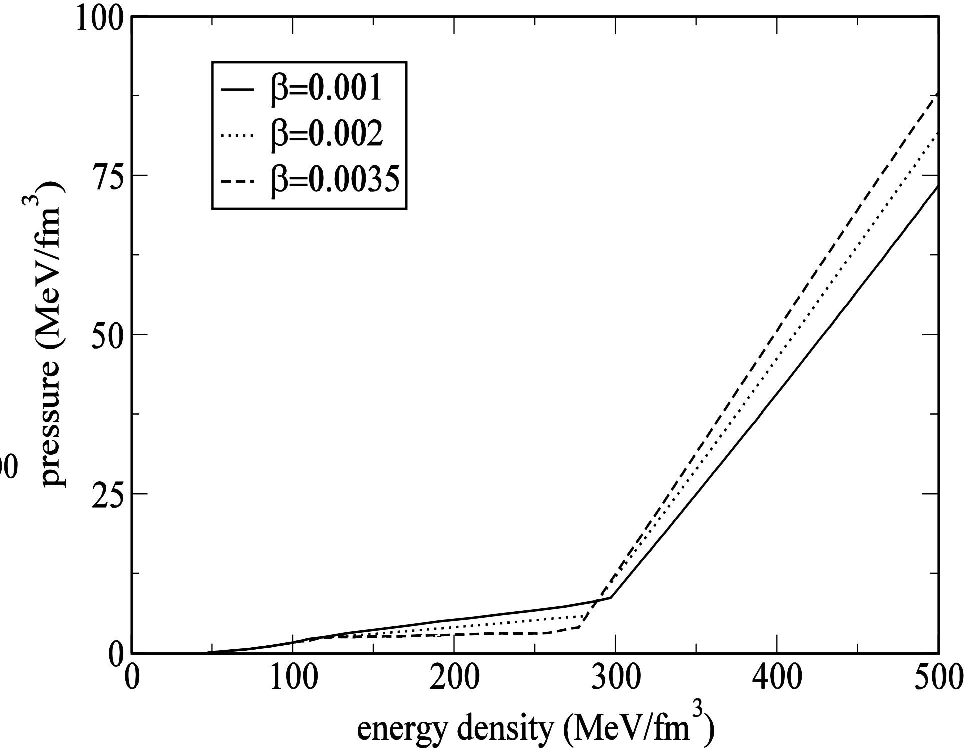
Smooth transition from nuclear to quark phase through a intermediate mixed phase (both hadrons and quarks present). Pressure varies smoothly.
Can be energetically expensive, and is expelled from the star.

### Maxwell Construction

Two pure phases are in direct contact.
Direct jump from quark to nuclear phase, no mixed phase.
Jump in the density for a constant pressure.

# The mixed phase



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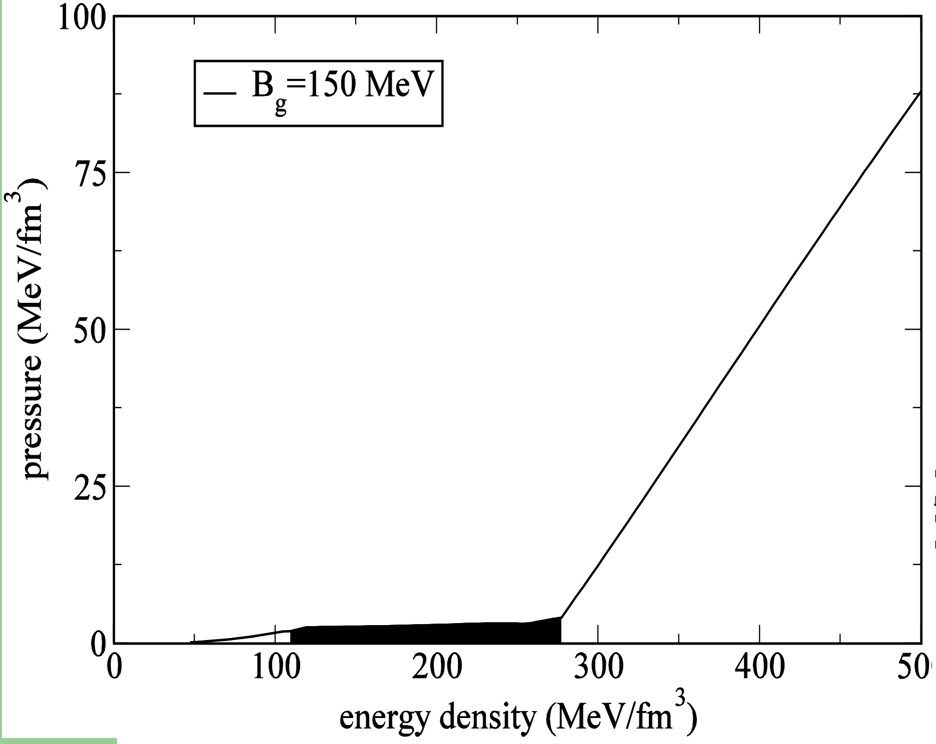


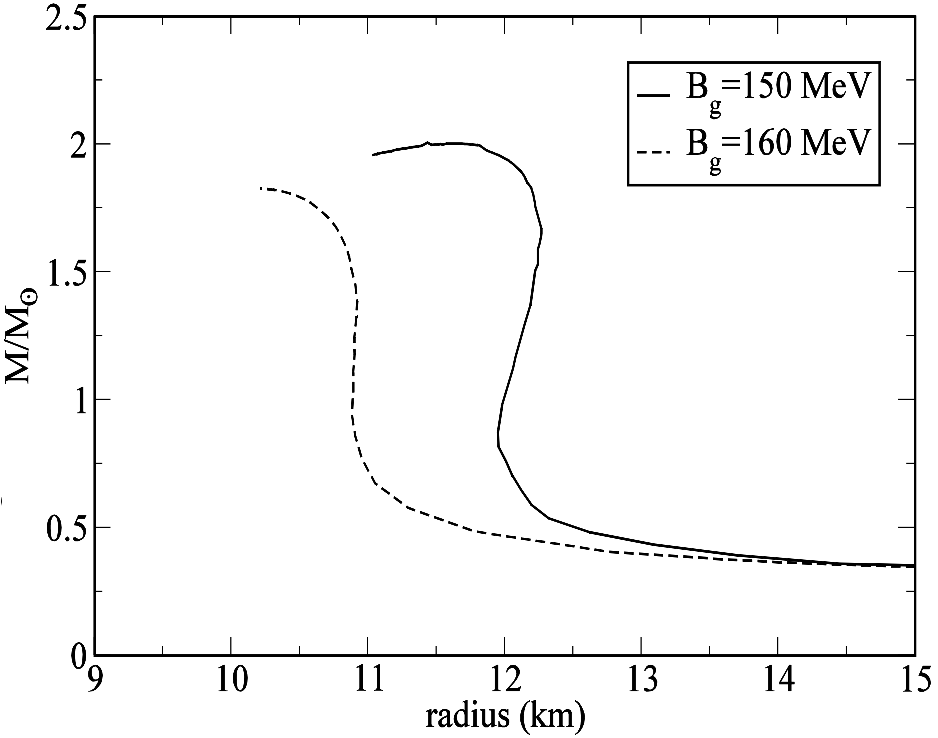

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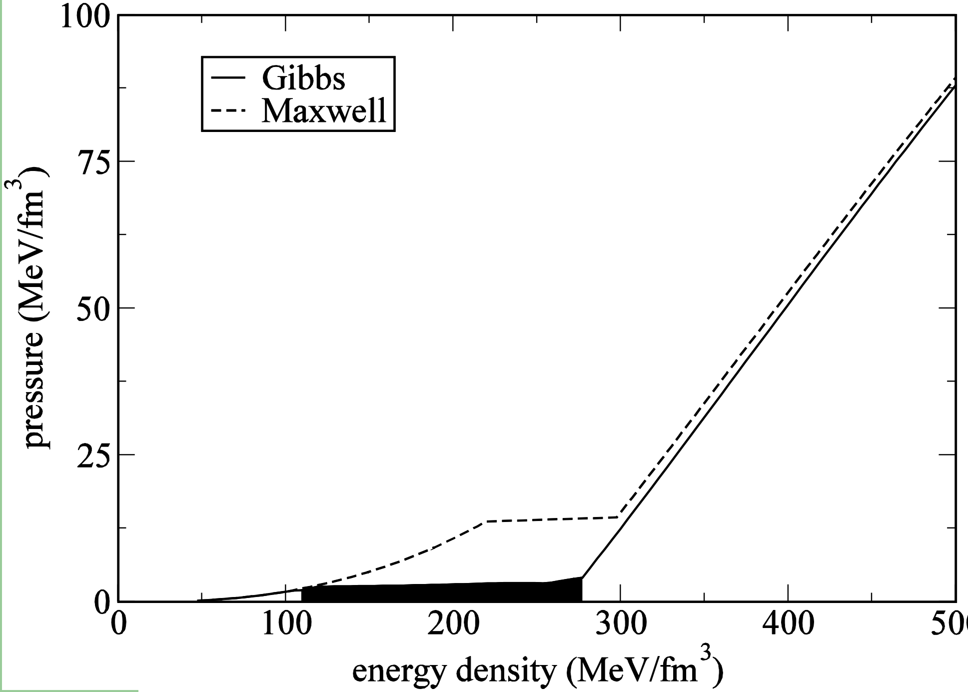
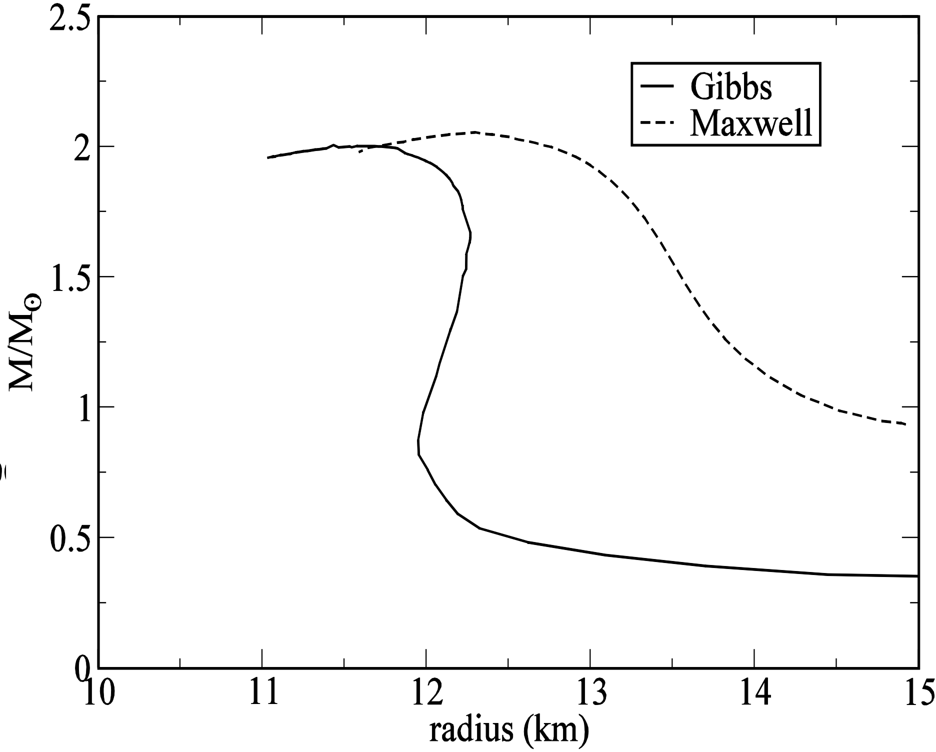
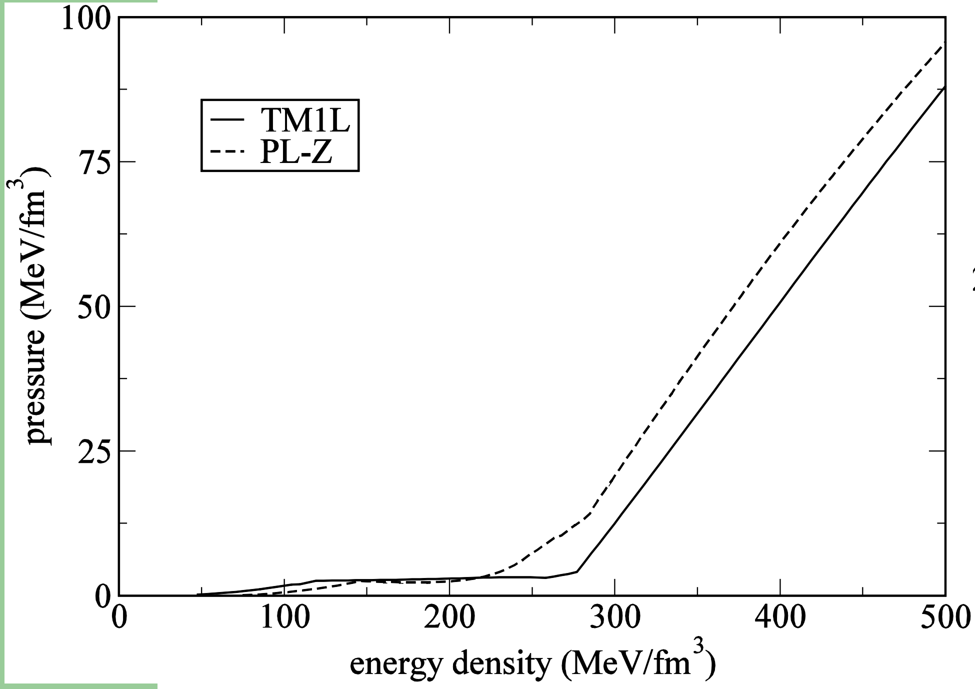
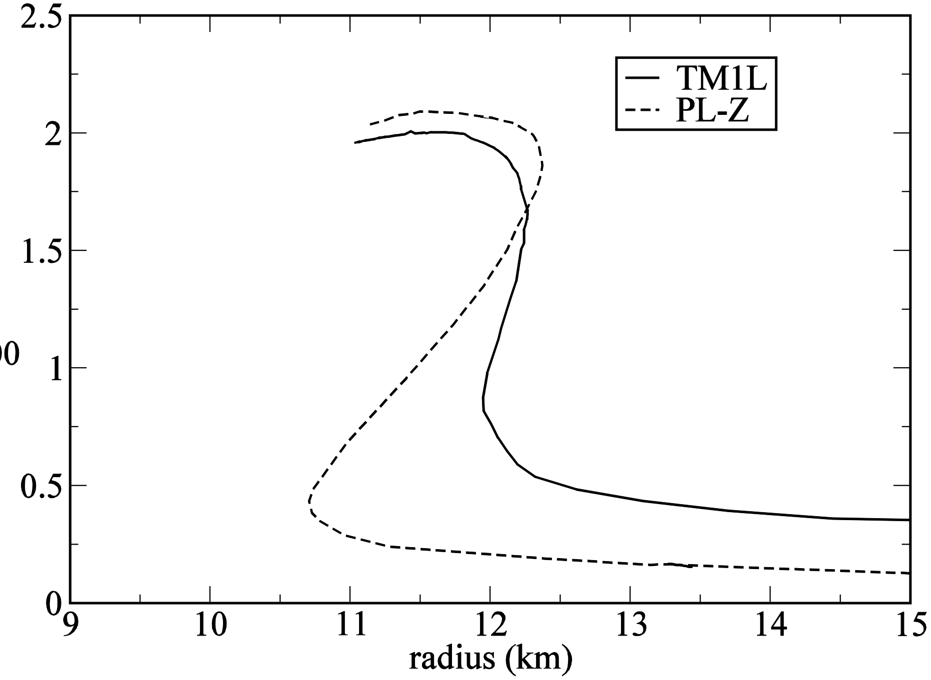


# The mixed phase



# The mixed phase

Higher bag pressure give very smooth mixed phase, but the Maximum mass is small.

Varying bag pressure with initial bag constant of 150 MeV, gives higher mass.

The beta defines how fast the bag pressure attains its lowest value. The higher the beta the faster the fall the stiffer the EOS.
(PRC 87, 025804, 2013)

# The mixed phase

**TM1L model**
Gibbs Construction
Maximum mass with initial bag pressure 150 MeV, beta equal to 0.0035. Mass is 2.01 solar mass.

Maxwell Construction
Maximum Mass 2.05 Solar mass.

**PL-Z model**
Gibbs Construction
Maximum mass 2.1 solar mass.

Maxwell Construction
Maximum mass 2.12 solar mass.

# Magnetic field

Other important property of neutron star is its high magnetic field.
Observed surface magnetic field in normal pulsars is $10^{8}$-$10^{12}$ G

Magnetars are compact stars with much higher magnetic field. Surface magnetic field is as high as $10^{14}$-$10^{15}$ G
Giant Flares activity ( SGR 0526-66, SGR 1900+14 and SGR 1806-20)

A simple calculation of the flux conservation of the progenitor star gives the central magnetic field of the order of $10^{18}$ G.

# Magnetic field

Magnetic field can effect NS and the phase transition.
The EOS describing the star can get modified.
The Field equations can also modify if we take the energy momentum tensor due to the magnetic field.

It may influence the structure and observational properties of neutron star. Therefore study of the compact star in presence of magnetic field is necessary.

# Magnetic field

**The magnetic field is assumed to be in the z direction. The motion of the charged particles are quantized in the perpendicular direction of the magnetic field ( Chakrabarty et al. PRL 78, 2898, 1997).**

$$E_i = \sqrt{{p_i}^2 + {m_i}^2 + |q_i|B(2n + s + 1)}.$$

**The number and energy density of the charged particles changes.**

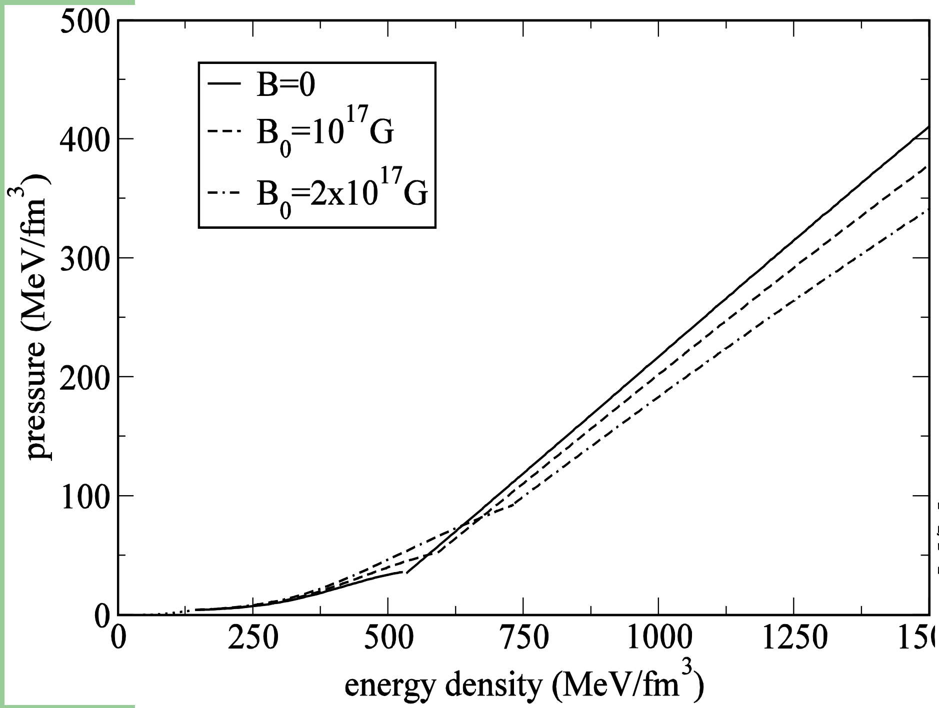
$$n_i = \frac{|q_i|B}{2\pi^2} \sum_{\nu} p^i_{f,\nu} ,$$

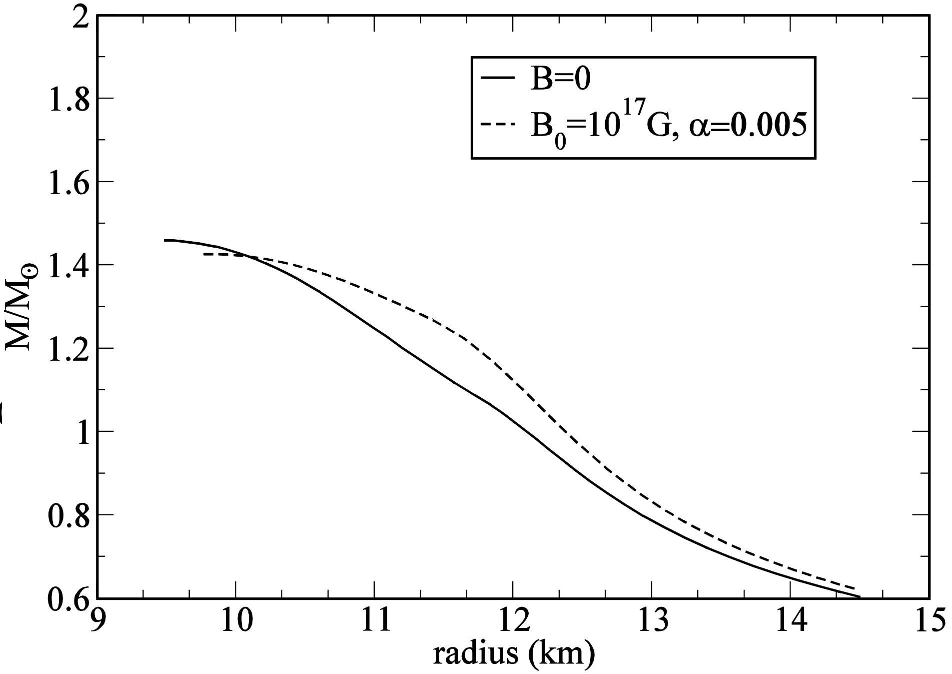
$$\varepsilon_i = \frac{|q_i|B}{4\pi^2} \sum_{\nu} \left[ E^i_f p^i_{f,\nu} + {\tilde{m}^i_\nu}^{\,2} \ln \left( \left| \frac{E^i_f + p^i_{f,\nu}}{\tilde{m}^i_\nu} \right| \right) \right] .$$

**The magnetic field variation is given by**

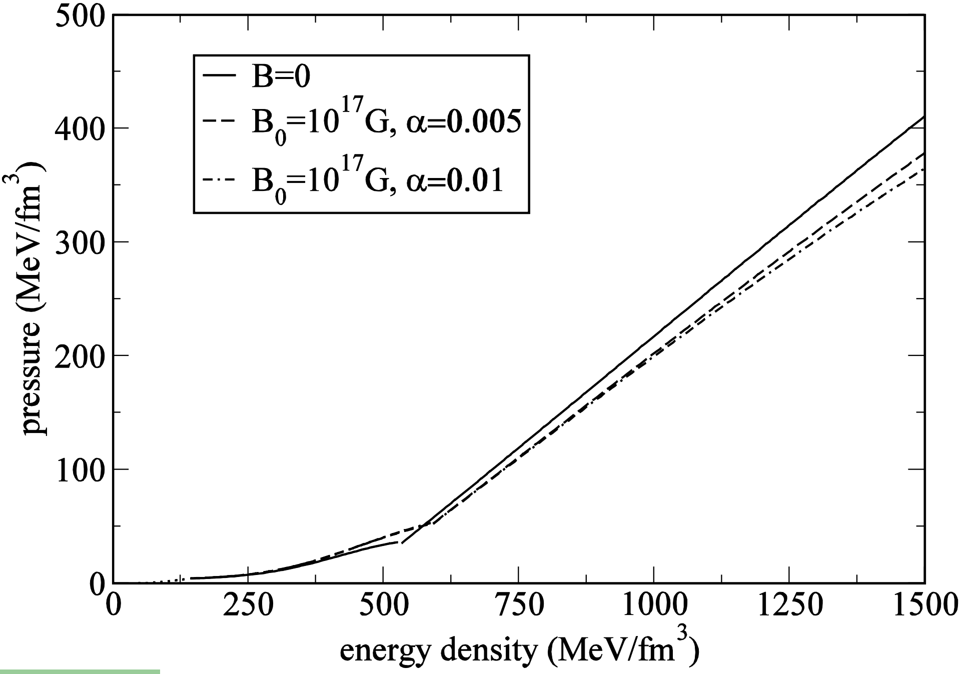
$$B(n_b) = B_s + B_0 \left\{ 1 - e^{-\alpha \left( \frac{n_b}{n_0} \right)^\gamma} \right\}$$

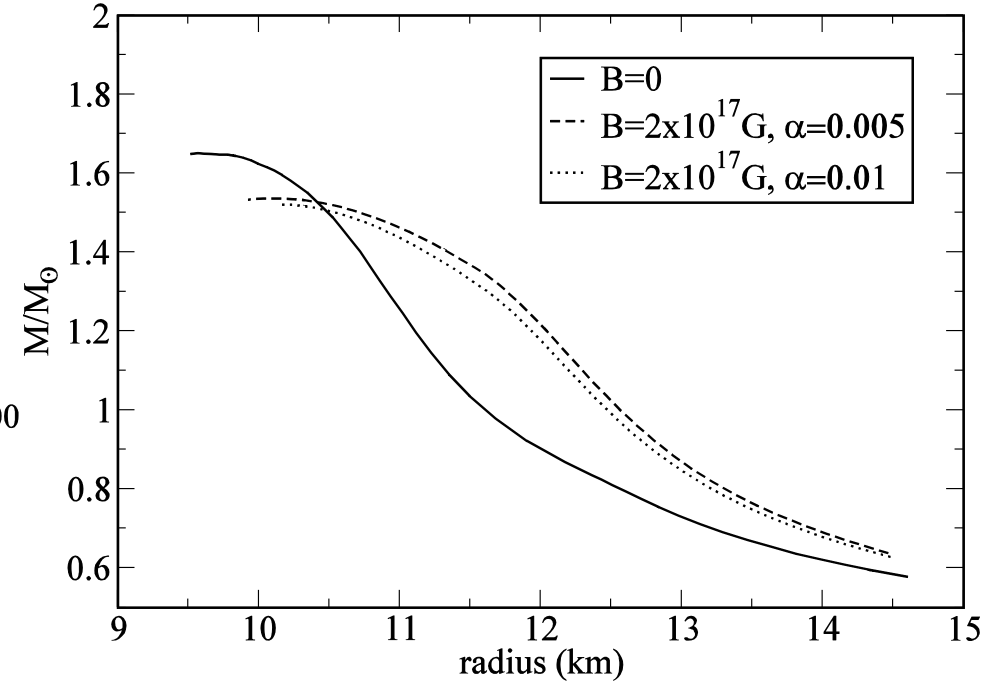
# Magnetic field



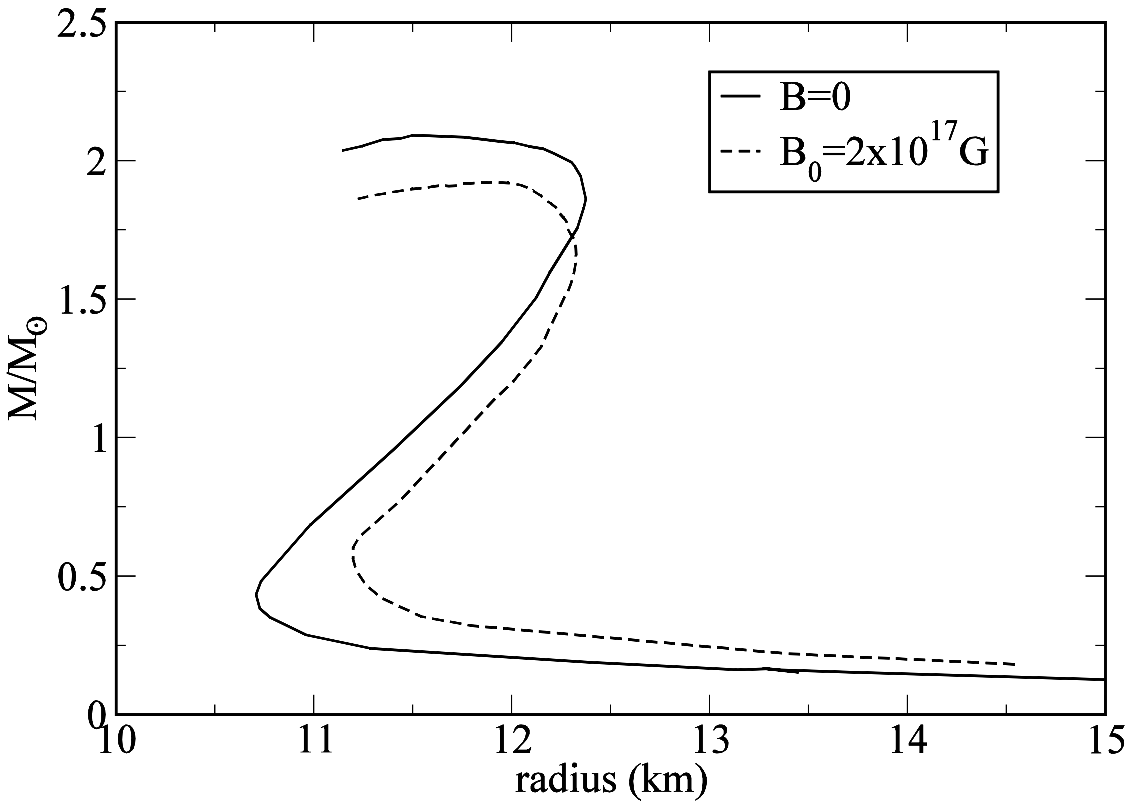



# Magnetic field



# Magnetic field



# Magnetic field

The maximum observed surface field is $10^{15}$G. At the centre it may increase to $10^{18}$G. We have assumed maximum central field of few $10^{17}$G.

Effect of magnetic field is insignificant for field strength lower than $10^{14}$G.
The magnetic field makes the EOS softer.
The mixed phase increases with magnetic field.

As expected, as the magnetic field makes the EOS curve soft, The mass-radius curve also becomes flat. The mass of magnetar is less than normal pulsar.
(Mallick & Sahu, submitted to MNRAS)

# Summary

Usually the hyperonic EOS are soft and cannot give massive stars. MIT bag model also cannot generate massive quark stars. New models with strong coupling and superconducting core (Alford et al., 2001, PRD 4, 074014)
coming up to explain the observation of PSR J1614-2230.

Our hybrid model with hyperonic EOS and MIT bag model EOS with varying bag constant. The varying bag makes the EOS stiff.

Gibbs Construction 2.1 solar mass HS
Maxwells Consruction 2.12 solar mass HS

# Summary

Huge magnetic field in magnetars are bound to effect the properties of the neutron star.

Our models deals only with the magnetic field affecting the EOS of the star matter. The charged particles are landau quantized perpendicular to the magnetic field.
The number density and the energy density changes.
The EOS becomes softer due to the magnetic field.

The maximum mass of the star reduces and the mass-radius curve becomes flat.

The heavier star are more likely to be pulsar than magnetar.

# Discussion

As stated earlier the discovery of PSR J1614-2230 with mass about 2 solar masses has posed a question mark over the exotic matter EOS.

New hadronic and quark model being prescribed to construct compact star with mass > 2 solar mass.

What happens due to the presence of magnetic field?
The single particle energy gets modified due to landau quantization, changing the EOS. The EOS gets softer making the star less heavier.

# Discussion

The field equation would get modified due to the magnetic field. It is likely to increase the mass of the star. Therefore the total effect of magnetic field is the combined effect of the metric and EOS modification.

# Discussion

Maximum mass related with EOS.

Along with the mass the more accurate measurement of radius could give a clear picture of the EOS of pulsars, and thereby establish them as NS and/or QS.
Is it possible ?

If not, the theorist have to come up with other models whose observable should be able to distinguish compact stars.

New detectors with more accurate observations, one expects to measure such details, to open new frontiers in astrophysics.

# Remarks

The recent work in this field

Color Superconducting matter.

At large densities the quark matter is expected to be in the state of color superconducting matter. The favored state is the CFL state. It can produce stiff EOS, and therefore high star mass.

Gravitational Wave detectors (LIGO)

Determines ripples in space-time, caused by massive axisymmetric NS. Can it determine the phase transition ?

**Thank you all for patiently following the talk**