Inferring the Nature of the Boson at 125 – 126 GeV

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Facts about the new resonance

ATLAS and CMS collaborations at CERN have observed a new resonance which has a mass of around 125-126 GeV.

- *We assume that exists only one resonance that decays to both $\gamma\gamma$ and ZZ. Denote by H.*
- *Since it decays to two-photons, it must be a Boson, but cannot have Spin 1 (forbidden by Landau-Yang Theorem).*
- *Is a charge conjugation $C = +$ state.*

Our Aim

If H is the Higgs boson of SM then $J^{PC} = 0^{++}$ and its couplings to other known particles follows the SM prediction exactly.

- *We propose a simple but efficient method of ascertaining the spin and parity of the particle and also determine information about its couplings to two Z bosons.*
- *Study the Golden Channel: $H \rightarrow ZZ \rightarrow 4$ leptons.*

The Approach

1. For each allowed spin possibility, write down the most general HZZ vertex factor assuming Lorentz invariance and Bose symmetry.
2. Identify the P-even and P-odd terms in the vertex factor.
3. Find out the most general angular distribution for $H \to ZZ^* \to (\ell_1^+ \ell_1^-)\,(\ell_2^+ \ell_2^-)$, where $\ell_1 \neq \ell_2$.
4. Express the angular distribution in terms of Helicity Fractions.
5. Outline a procedure to determine the spin, parity and couplings (to Z bosons) of H using experimentally measured distributions.

Kinematics

One of the Z's is on shell

$$q_1^2 = M_1^2 = M_Z^2$$

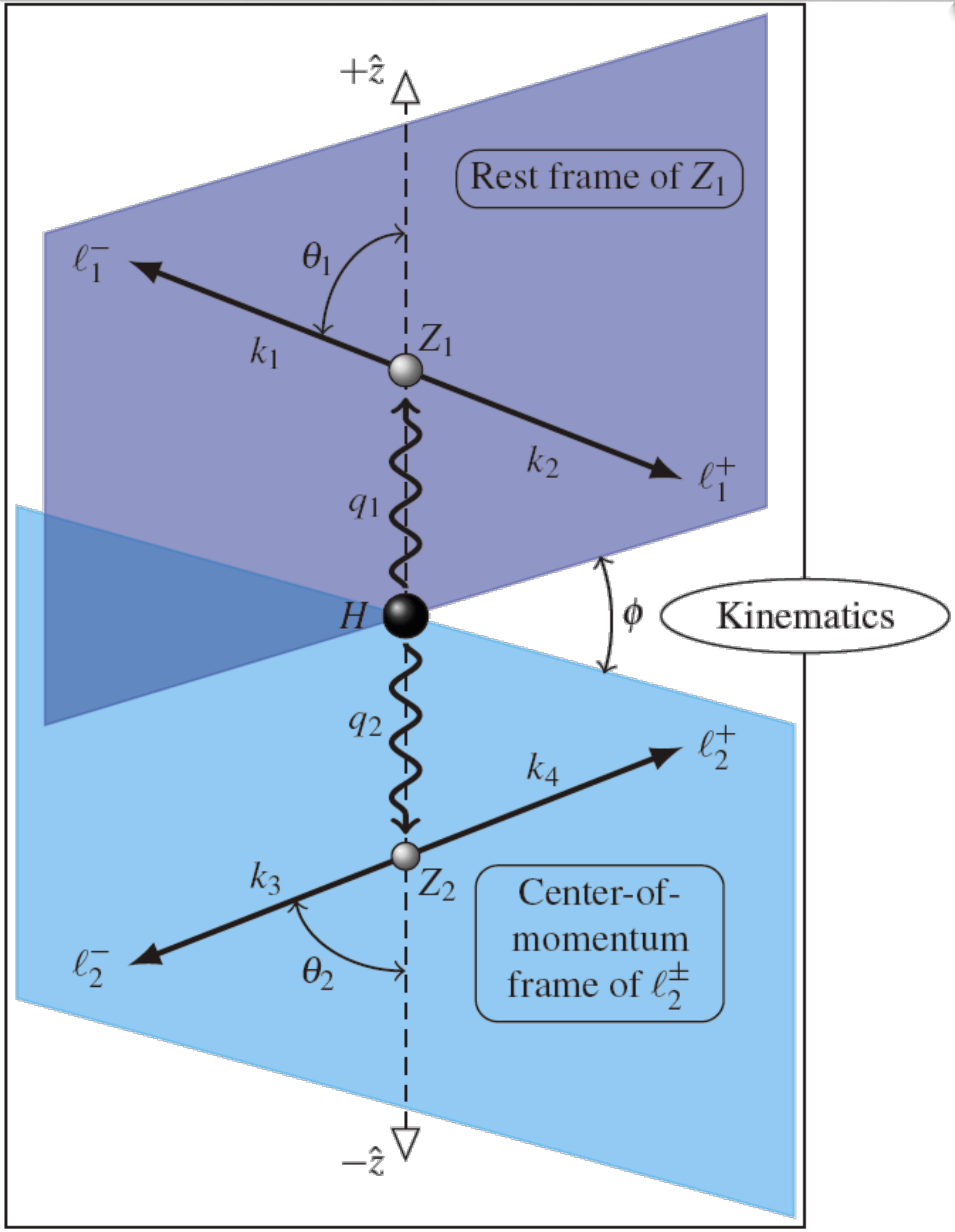
The other Z boson is off-shell

$$q_2^2 = M_2^2 = q^2$$

It is possible that both Z's are off-shell and we will consider that possibility as well.

Differential Decay Rate:

$$\frac{d^4\Gamma_f}{dq^2\, d\cos\theta_1\, d\cos\theta_2\, d\phi}$$



In the rest frame of H the momenta are defined as

$$P = \{M_H, 0, 0, 0\}$$

$$q_1 = \left\{\sqrt{M_1^2 + X^2}, 0, 0, X\right\} \qquad q_2 = \left\{\sqrt{M_2^2 + X^2}, 0, 0, -X\right\}$$

$$k_1 = \left\{\frac{1}{2}\left(\sqrt{M_1^2 + X^2} + X\cos\theta_1\right), \frac{1}{2}M_1\sin\theta_1, 0, \frac{1}{2}\left(\sqrt{M_1^2 + X^2}\cos\theta_1 + X\right)\right\}$$

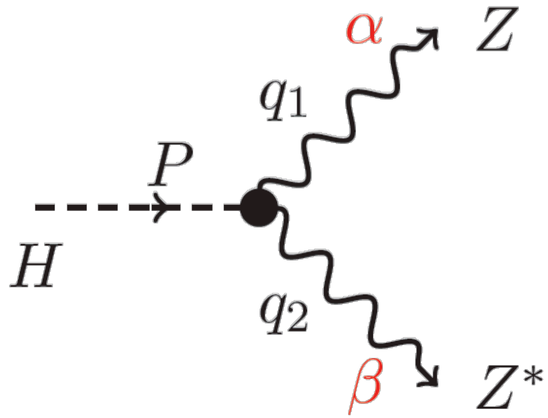
$$k_2 = \left\{\frac{1}{2}\left(\sqrt{M_1^2 + X^2} - X\cos\theta_1\right), -\frac{1}{2}M_1\sin\theta_1, 0, \frac{1}{2}\left(X - \sqrt{M_1^2 + X^2}\cos\theta_1\right)\right\}$$

$$k_3 = \left\{\frac{1}{2}\left(\sqrt{M_2^2 + X^2} - X\cos\theta_2\right), \frac{1}{2}M_2\sin\theta_2\cos\phi, \frac{1}{2}M_2\sin\theta_2\sin\phi, \frac{1}{2}\left(\sqrt{M_2^2 + X^2}\cos\theta_2 - X\right)\right\}$$

$$k_4 = \left\{\frac{1}{2}\left(\sqrt{M_2^2 + X^2} + X\cos\theta_2\right), -\frac{1}{2}M_2\sin\theta_2\cos\phi, -\frac{1}{2}M_2\sin\theta_2\sin\phi, \frac{1}{2}\left(-\sqrt{M_2^2 + X^2}\cos\theta_2 - X\right)\right\}$$

$$\lambda(x,y,z) = \left(x^2 + y^2 + z^2 - 2xy - 2xz - 2yz\right) \qquad X = \frac{\sqrt{\lambda(M_H^2, M_1^2, M_2^2)}}{2M_H}$$

Spin 0 Case



$$V_{HZZ}^{\alpha\beta} = \frac{igM_Z}{\cos\theta_W}\left[\, a\, g^{\alpha\beta} + b\, P^{\alpha} P^{\beta} + i\, c\, \epsilon^{\alpha\beta\rho\sigma} q_{1\rho} q_{2\sigma} \,\right]$$

P even   P odd

For J=0 we have 1 S-wave, 1 P-wave and 1 D-wave contributions and 3 helicity amplitudes. The amplitudes can be written in terms of 3 orthogonal helicity amplitudes in the transversity basis:

$$A_L = \frac{1}{2}(M_H^2 - M_1^2 - M_2^2)\, a + M_H^2 X^2\, b,$$

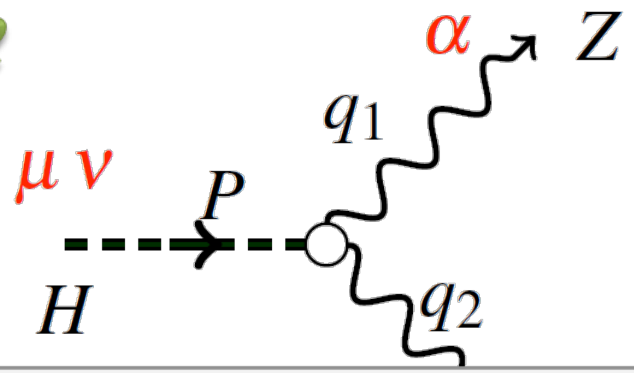
$$A_{\parallel} = \sqrt{2}\, M_1 M_2\, a$$

$$A_{\perp} = \sqrt{2} M_1 M_2 X\, M_H c$$

The vertex $V_{HZZ}^{\alpha\beta}$ is derived from an effective Lagrangian where higher dimensional operators contribute to the momentum dependence of the form factors.

Since the effective Lagrangian in the case of arbitrary new physics is not known, no momentum dependence of a, b and c can be assumed if the generality of the approach has to be retained.

Spin 2 Case



$$P = q_1 + q_2$$
$$Q = q_1 - q_2$$

$V^{\mu\nu;\alpha}_{HZZ}$ … even …)]

Is this the most general vertex for Spin 2?
We will show that other terms can be added
but the results remain the same…
It is hence the most general expression…

$$+2\,i\,E\left(g^{\beta\nu}\epsilon^{\alpha\mu\rho\sigma} - g^{\alpha\nu}\epsilon^{\beta\mu\rho\sigma} + g^{\beta\mu}\,\epsilon^{\alpha\nu\rho\sigma} - g^{\alpha\mu}\epsilon^{\beta\nu\rho\sigma}\right)q_{1\rho}q_{2\sigma}$$

$$+i\,F\begin{bmatrix}Q^{\beta}\,(Q^{\nu}\epsilon^{\alpha\mu\rho\sigma} + Q^{\mu}\,\epsilon^{\alpha\nu\rho\sigma}) \\ -\,Q^{\alpha}(Q^{\nu}\,\epsilon^{\beta\mu\rho\sigma} + Q^{\mu}\,\epsilon^{\beta\nu\rho\sigma})\end{bmatrix}q_{1\rho}\,q_{2\sigma}$$

P odd

$\hat{P}_{12}$ is the operator that that exchanges the two Z bosons: exchanges both their momenta and spins or polarizations

$$\hat{P}_{12}\left|2, S_z; L_{orbital}, L_{spin}\right\rangle = (-1)^{L_{orbital}+L_{spin}} \left|2, S_z; L_{orbital}, L_{spin}\right\rangle$$

L_{spin}	$L_{orbital}$	L_{total}	Partial wave	$L_{orbital} + L_{spin}$	Comments
0	2	2	$\mathcal{D}$-wave	2	Allowed
1	1	$\{2, 1, 0\}$	$\mathcal{P}$-wave	2	Allowed
1	2	$\{3, 2, 1\}$	$\mathcal{D}$-wave	3	Not allowed
1	3	$\{4, 3, 2\}$	$\mathcal{F}$-wave	4	Allowed
2	0	2	$\mathcal{S}$-wave	2	Allowed
2	1	$\{3, 2, 1\}$	$\mathcal{P}$-wave	3	Not allowed
2	2	$\{4, 3, 2, 1, 0\}$	$\mathcal{D}$-wave	4	Allowed
2	3	$\{5, 4, 3, 2, 1\}$	$\mathcal{F}$-wave	5	Not allowed
2	4	$\{6, 5, 4, 3, 2\}$	$\mathcal{G}$-wave	6	Allowed

For J=2 we have 1 S-wave, 1 P-wave, 2 D-wave, 1 F-wave and 1G-wave contributions and 6 helicity amplitudes.

$$A_L = \frac{4X}{3u_1}\left[E\left(u_2^4 - M_H^2 u_1^2\right) + F\left(4 u_1^2 M_H^2 X^2\right)\right]$$

$$A_M = \frac{8M_1 M_2 v X}{3\sqrt{3}u_1} E$$

$$A_1 = \frac{2\sqrt{2}}{3\sqrt{3}M_H^2}\left[A\left(M_H^4 - u_2^4\right) - B\left(8 M_H^4 X^2\right) + C\left(4M_H^2 X^2\right)\left(u_1^2 - M_H^2\right) - D\left(8 M_H^4 X^4\right)\right]$$

$$A_2 = \frac{8M_1 M_2}{3\sqrt{3}}\left(A + 4X^2 C\right)$$

$$A_3 = \frac{4}{3M_H u_1}\left[A\left(u_2^4 - M_H^2 u_1^2\right) - \ldots\right.$$

$$A_4 = \frac{8 M_1 M_2 w}{3M_H u_1} A$$

Where

$$u_1^2 = M_1^2 + M_2^2$$
$$u_2^2 = M_1^2 - M_2^2$$
$$v^2 = 4 M_H^2 u_1^2 + 3 u_2^4$$
$$w^2 = 2M_H^2 u_1^2 + u_2^4$$

It is possible to add extra terms to the vertex factor $V_{HZZ}^{\mu\nu;\alpha\beta}$, e.g.

$$i\,G\left[\epsilon^{\alpha\beta\nu\rho}\,P_\rho\,Q^\mu + \epsilon^{\alpha\beta\mu\rho}\,P_\rho\,Q^\nu\right]$$

Results in redefinition of A_L and A_M

$$A_L = \frac{4X}{3u_1}\left[(E-2G)(u_2^4 - M_H^2 u_1^2) + F(4\,u_1^2 M_H^2 X^2)\right]$$

$$A_M = \frac{8M_1M_2vX}{3\sqrt{3}u_1}(E-2G)$$

$$E \Rightarrow E-2G$$

It is possible to add only one more extra term to vertex factor $V_{HZZ}^{\mu\nu;\alpha\beta}$,

$i\,\epsilon^{\alpha\beta\rho\sigma}\,Q^\mu Q^\nu\,q_{1\rho}q_{2\sigma}$.

Schouten Identity:

$$g^{\lambda\mu}\,\epsilon^{\alpha\beta\rho\sigma} + g^{\lambda\alpha}\,\epsilon^{\beta\rho\sigma\mu} + g^{\lambda\beta}\,\epsilon^{\rho\sigma\mu\alpha} + g^{\lambda\rho}\,\epsilon^{\sigma\mu\alpha\beta} + g^{\lambda\sigma}\,\epsilon^{\mu\alpha\beta\rho} = 0$$

The identity holds in four dimensions simply because the left hand side is fully anti-symmetric in the five indices $\alpha, \beta, \rho, \sigma, \mu$.

$$\epsilon^{\alpha\beta\rho\sigma}Q^\mu Q^\nu q_{1\rho}q_{2\sigma} = \frac{1}{2}\begin{bmatrix} Q^\nu\left(\epsilon^{\alpha\mu\rho\sigma}Q^\beta - \epsilon^{\beta\mu\rho\sigma}Q^\alpha\right) \\ +Q^\mu\left(\epsilon^{\alpha\nu\rho\sigma}Q^\beta - \epsilon^{\beta\nu\rho\sigma}Q^\alpha\right)\end{bmatrix} q_{1\rho}q_{2\sigma}$$

$$+\frac{P.Q}{4}\left(\epsilon^{\alpha\beta\mu\sigma}Q^\nu + \epsilon^{\alpha\beta\nu\sigma}Q^\mu\right)Q_\sigma - \frac{Q^2}{4}\left(\epsilon^{\alpha\beta\mu\rho}Q^\nu + \epsilon^{\alpha\beta\nu\rho}Q^\mu\right)P_\rho$$

Only this term is new

P.Q is scalar and should be absorbed in form factor but it is odd under exchange of Z's. Such terms can't arise from effective Lagrangian where the two Z's are symmetric.

Helicity fractions defined as $F_i = \frac{A_i}{\sqrt{\sum_j |A_j|^2}}$ $\quad i,j \in \begin{cases}\{L, \parallel, \perp\} & J = 0 \\ \{L, M, 1, 2, 3, 4\} & J = 2\end{cases}$

Note $\quad \sum_i |F_i|^2 = 1$

Also $\quad \Gamma_f \equiv \frac{d\Gamma}{dq^2} = \mathcal{N}\sum_j |A_j|^2$ $\quad$ ***$\mathcal{N}$ is normalization that is different for $J = 0$ and $J = 2$***

Just to show you: $J^{PC} = 0^{++}$ *case*

$$
\begin{aligned}
\frac{8\pi}{\Gamma_f}\frac{d^4\Gamma}{dq_2^2\, d\cos\theta_1\, d\cos\theta_2\, d\phi} = 1 &+ \frac{|F_\parallel|^2 - |F_\perp|^2}{4}\cos 2\phi\big(1 - P_2(\cos\theta_1)\big)\big(1 - P_2(\cos\theta_2)\big)\\
&+ \frac{1}{2}\mathrm{Im}(F_\parallel F_\perp^*)\,\sin 2\phi\big(1 - P_2(\cos\theta_1)\big)\big(1 - P_2(\cos\theta_2)\big)\\
&+ \frac{1}{2}(1 - 3\,|F_L|^2)\,\big(P_2(\cos\theta_1) + P_2(\cos\theta_2)\big) + \frac{1}{4}(1 + 3\,|F_L|^2)\,P_2(\cos\theta_1)P_2(\cos\theta_2)\\
&+ \frac{9}{8\sqrt{2}}\Big[\mathrm{Re}(F_L F_\parallel^*)\,\cos\phi + \mathrm{Im}(F_L F_\perp^*)\,\sin\phi\Big]\,\sin 2\theta_1\,\sin 2\theta_2\\
&+ \eta\Big\{\frac{3}{2}\mathrm{Re}(F_\parallel F_\perp^*)\big[\cos\theta_2(2 + P_2(\cos\theta_1)) - \cos\theta_1(2 + P_2(\cos\theta_2))\big]\\
&\qquad + \frac{9}{2\sqrt{2}}\mathrm{Re}(F_L F_\perp^*)\big(\cos\theta_1 - \cos\theta_2\big)\cos\phi\sin\theta_1\sin\theta_2\\
&\qquad - \frac{9}{2\sqrt{2}}\mathrm{Im}(F_L F_\parallel^*)\big(\cos\theta_1 - \cos\theta_2\big)\sin\phi\sin\theta_1\sin\theta_2\Big\}\\
&- \frac{9}{4}\eta^2\Big\{(1 - |F_L|^2)\cos\theta_1\cos\theta_2 + \sqrt{2}\Big[\mathrm{Re}(F_L F_\parallel^*)\cos\phi + \mathrm{Im}(F_L F_\perp^*)\sin\phi\Big]\sin\theta_1\sin\theta_2\Big\},
\end{aligned}
$$

Uni-angular distribution in terms of helicity fractions for J=0 case

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_1} = \frac{1}{2} - \frac{3}{2}\eta\,\mathrm{Re}(F_\parallel F_\perp^*)\ \cos\theta_1 + \frac{1}{4}(1-3|F_L|^2)P_2(\cos\theta_1)$$

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_2} = \frac{1}{2} + \frac{3}{2}\eta\,\mathrm{Re}(F_\parallel F_\perp^*)\ \cos\theta_2 + \frac{1}{4}(1-3|F_L|^2)P_2(\cos\theta_2)$$

$$\frac{2\pi}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\phi} = 1 - \frac{9\pi^2}{32\sqrt{2}}\eta^2\,\mathrm{Re}(F_L F_\parallel^*)\ \cos\phi + \frac{1}{4}\left(|F_\parallel|^2 - |F_\perp|^2\right)\cos 2\phi - \frac{9\pi^2}{32\sqrt{2}}\eta^2\,\mathrm{Re}(F_L F_\perp^*)\ \sin\phi + \frac{1}{2}\mathrm{Im}(F_\parallel F_\perp^*)\ \sin 2\phi$$

$\eta = \frac{2\,v_\ell\,a_\ell}{v_\ell^2 + a_\ell^2}$ *where* $v_\ell = -1 + 4\sin^2\theta_W$ *and* $a_\ell = -1$

$P_2(x) = \frac{1}{2}(3\,x^2 - 1)$ *is 2nd degree Legendre Polynomial*

$\eta = 0.151$ *and* $\eta^2 = 0.0228$

Observables from uni-angular distributions $J = 0$ case.

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_1} = \frac{1}{2} + \frac{1}{4}\left(1 - 3|F_L|^2\right)P_2(\cos\theta_1)$$

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_2} = \frac{1}{2} + \frac{1}{4}\left(1 - 3|F_L|^2\right)P_2(\cos\theta_2)$$

$\frac{1}{4}\left(1 - 3|F_L|^2\right) \rightarrow T_1^{(0)}$

$$\frac{2\pi}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\phi} = \frac{1}{2} + \frac{1}{4}\left(|F_\parallel|^2 - |F_\perp|^2\right)\cos 2\phi + \frac{1}{2}\mathrm{Im}(\mathbf{F}_\parallel \mathbf{F}_\perp^*)\sin 2\phi$$

$\frac{1}{4}\left(|F_\parallel|^2 - |F_\perp|^2\right) \rightarrow T_2^{(0)}$

$\frac{1}{2}\mathrm{Im}(\mathbf{F}_\parallel \mathbf{F}_\perp^*) \rightarrow T_3^{(0)}$

F_L, $F_\parallel$ and $F_\perp$ can be solved in terms of $T_1^{(0)}, T_2^{(0)}$ and $T_3^{(0)}$

$$|F_L|^2 = \frac{1}{3}\left(1 - 4\,T_1^{(0)}\right),$$

$$\left|F_\parallel\right|^2 = \frac{1}{3}\left(1 + 2\,T_1^{(0)}\right) + 2T_2^{(0)}$$

$$|F_\perp|^2 = \frac{1}{3}\left(1 + 2\,T_1^{(0)}\right) - 2T_2^{(0)}$$

From the helicity fractions we can solve for a, b, c

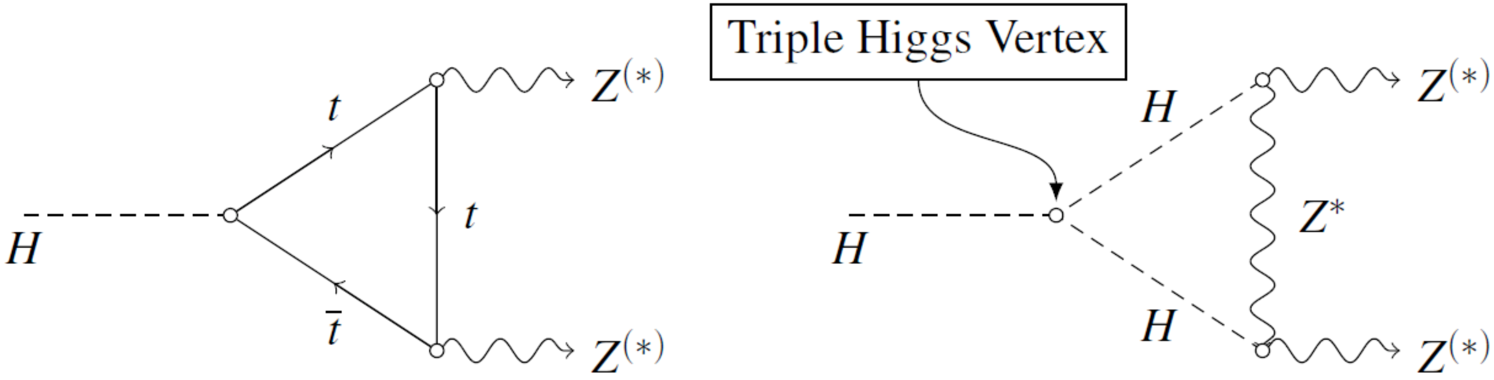
$$a = \frac{F_\parallel}{\sqrt{2} M_Z M_2}\sqrt{\frac{\Gamma_f}{\mathcal{N}}} \qquad b = \frac{1}{M_H^2 X^2}\sqrt{\frac{\Gamma_f}{\mathcal{N}}}\left[F_L - \frac{M_H^2 - M_Z^2 - M_2^2}{2\sqrt{2}\, M_Z M_2} F_\parallel\right]$$

$$c = \frac{F_\perp}{\sqrt{2} M_Z M_2 M_H\, X}\sqrt{\frac{\Gamma_f}{\mathcal{N}}}$$

If we find that $a = 1$, $b = c = 0$ *then and only then is H the SM Higgs*

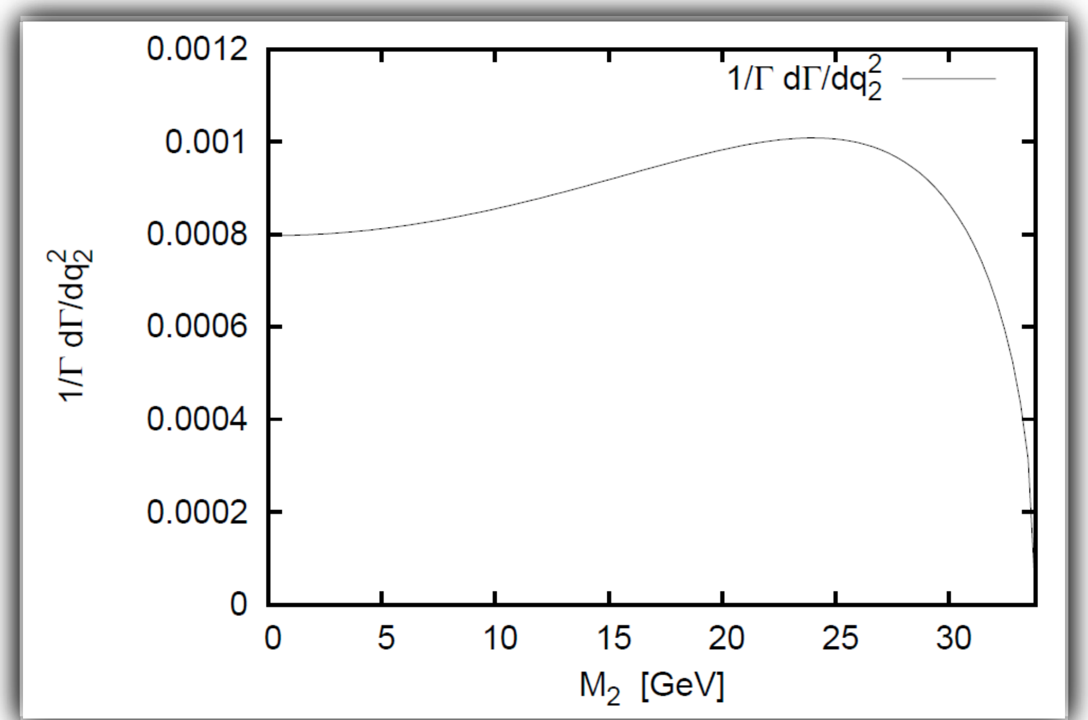
Testing Triple Higgs vertex?

The b term comes from higher derivative terms in the Lagrangian. Even in SM, the b term can manifest itself when we consider loop level corrections,



Measurement of b ⇒ first verification of the Higgs self-coupling.

SM tree-level expectations



In SM the values of $T_1^{(0)}$ and $T_2^{(0)}$ integrated over M_2 are -0.148 and 0.117 respectively.

$J^{PC} = 2^{++}$ *case*

$$
\begin{aligned}
&\frac{8\pi}{\Gamma_f}\frac{d^4\Gamma}{dq_2^2\, d\cos\theta_1\, d\cos\theta_2\, d\phi} \\
&= 1 + \left(\frac{1}{4}|F_2|^2 - \left[M_H^2\frac{u_1^2}{v^2}\right]|F_M|^2\right)\cos 2\phi\,(1 - P_2(\cos\theta_1))\,(1 - P_2(\cos\theta_2)) \\
&\quad + \left[M_H\frac{u_1}{v}\right]\mathrm{Im}(F_2F_M^*)\,\sin 2\phi\,(1 - P_2(\cos\theta_1))\,(1 - P_2(\cos\theta_2)) \\
&\quad + \frac{P_2(\cos\theta_1)}{2}\Bigg\{\left(-2|F_1|^2 + |F_2|^2\right) + \left(|F_3|^2 + |F_L|^2\right)\left[\frac{M_1^2 - 2M_2^2}{u_1^2}\right] \\
&\qquad\qquad + |F_M|^2\left[4M_H^2\frac{u_1^2}{v^2} + 3\frac{u_2^4}{u_1^2v^2}(M_2^2 - 2M_1^2)\right] + |F_4|^2\left[2M_H^2\frac{u_1^2}{w^2} + \frac{u_2^4}{u_1^2w^2}(M_2^2 - 2M_1^2)\right] \\
&\qquad\qquad + \left[6M_1M_2\frac{u_2^2}{u_1^2w}\right]\mathrm{Re}(F_3F_4^*) + \left[6\sqrt{3}M_1M_2\frac{u_2^2}{u_1^2v}\right]\mathrm{Re}(F_LF_M^*)\Bigg\} \\
&\quad + \frac{P_2(\cos\theta_2)}{2}\Bigg\{\left(-2|F_1|^2 + |F_2|^2\right) + \left(|F_3|^2 + |F_L|^2\right)\left[\frac{M_2^2 - 2M_1^2}{u_1^2}\right] \\
&\qquad\qquad + |F_M|^2\left[4M_H^2\frac{u_1^2}{v^2} + 3\frac{u_2^4}{u_1^2v^2}(M_1^2 - 2M_2^2)\right] + |F_4|^2\left[2M_H^2\frac{u_1^2}{w^2} + \frac{u_2^4}{u_1^2w^2}(M_1^2 - 2M_2^2)\right] \\
&\qquad\qquad - \left[6M_2M_1\frac{u_2^2}{u_1^2w}\right]\mathrm{Re}(F_3F_4^*) - \left[6\sqrt{3}M_2M_1\frac{u_2^2}{u_1^2v}\right]\mathrm{Re}(F_LF_M^*)\Bigg\} \\
&\quad + \frac{P_2(\cos\theta_1)P_2(\cos\theta_2)}{2}\left\{2|F_1|^2 + \frac{1}{2}|F_2|^2 - |F_3|^2 - |F_L|^2 - \left[\frac{u_2^4 - M_H^2u_1^2}{w^2}\right]|F_4|^2 + \left[\frac{2M_H^2u_1^2 - 3u_2^4}{v^2}\right]|F_M|^2\right\} \\
&\quad + \frac{9\sin 2\theta_1\sin 2\theta_2\cos\phi}{16}\Bigg\{\left(|F_3|^2 - |F_L|^2\right)\left[\frac{M_1M_2}{u_1^2}\right] + 3|F_M|^2\left[M_1M_2\frac{u_2^4}{u_1^2v^2}\right] - |F_4|^2\left[M_1M_2\frac{u_2^4}{u_1^2w^2}\right] \\
&\qquad\qquad - \left[\frac{u_2^4}{u_1^2w}\right]\mathrm{Re}(F_3F_4^*) + \left[\sqrt{3}\frac{u_2^4}{u_1^2v}\right]\mathrm{Re}(F_LF_M^*) - \sqrt{2}\,\mathrm{Re}(F_1F_2^*)\Bigg\} \\
&\quad + \frac{9\sin 2\theta_1\sin 2\theta_2\sin\phi}{16}\Bigg\{\left[2\frac{M_1M_2}{u_1^2}\right]\mathrm{Im}(F_3F_L^*) - \left[\sqrt{3}\frac{u_2^4}{u_1^2v}\right]\mathrm{Im}(F_3F_M^*) - \left[\frac{u_2^4}{u_1^2w}\right]\mathrm{Im}(F_4F_L^*) \\
&\qquad\qquad - \left[2\sqrt{3}M_1M_2\frac{u_2^4}{u_1^2vw}\right]\mathrm{Im}(F_4F_M^*) - \left[2\sqrt{2}M_H\frac{u_1}{v}\right]\mathrm{Im}(F_1F_M^*)\Bigg\}
\end{aligned}
$$

Uni-angular distributions in terms of helicity fractions $J = 2$ case.

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_1} = \frac{1}{2} + \frac{P_2(\cos\theta_1)}{4}\left\{-2|F_1|^2 + |F_2|^2 + \left(|F_3|^2 + |F_L|^2\right)\left(\frac{M_1^2 - 2M_2^2}{u_1^2}\right)\right.$$
$$+ |F_4|^2\left(2M_H^2\frac{u_1^2}{w^2} + \frac{u_2^4}{u_1^2 w^2}\left(M_2^2 - 2M_1^2\right)\right) + |F_M|^2\left(4M_H^2\frac{u_1^2}{v^2} + 3\frac{u_2^4}{u_1^2 v^2}\left(M_2^2 - 2M_1^2\right)\right)$$
$$\left. + 6M_1M_2\frac{u_2^2}{u_1^2 v\, w}\left(v\, \mathrm{Re}(F_3F_4^*) + \sqrt{3}w\,Re(F_LF_M^*)\right)\right\}$$

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_2} = \frac{1}{2} + \frac{P_2(\cos\theta_2)}{4}\left\{-2|F_1|^2 + |F_2|^2 + \left(|F_3|^2 + |F_L|^2\right)\left(\frac{M_2^2 - 2M_1^2}{u_1^2}\right)\right.$$
$$+ |F_4|^2\left(2M_H^2\frac{u_1^2}{w^2} + \frac{u_2^4}{u_1^2 w^2}\left(M_1^2 - 2M_2^2\right)\right) + |F_M|^2\left(4M_H^2\frac{u_1^2}{v^2} + 3\frac{u_2^4}{u_1^2 v^2}\left(M_1^2 - 2M_2^2\right)\right)$$
$$\left. - 6M_1M_2\frac{u_2^2}{u_1^2 v\, w}\left(v\, \mathrm{Re}(F_3F_4^*) + \sqrt{3}w\,Re(F_LF_M^*)\right)\right\}$$

$$\frac{2\pi}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\phi} = 1 + \left(\frac{1}{4}|F_2|^2 - \frac{M_H^2 u_1^2}{v^2}|F_M|^2\right)\cos 2\phi + M_H\frac{u_1}{v}\mathrm{Im}(F_2F_M^*)\ \sin 2\phi$$

... up to order η

The $\cos\theta_1$ and $\cos\theta_2$ uni-angular distributions differ for $J = 2$ case unlike for $J = 0$ case, unless $F_3 = F_4 = F_L = F_M = 0$ (called special case).

Uni-angular distribution for $J^{PC} = 2^{++}$ special case

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_1} = \frac{1}{2} + \frac{1}{4}\left(|F_2|^2 - 2|F_1|^2\right)P_2(\cos\theta_1)$$

$$\frac{1}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\cos\theta_2} = \frac{1}{2} + \frac{1}{4}\left(|F_2|^2 - 2|F_1|^2\right)P_2(\cos\theta_2)$$

$\rightarrow T_1^{(2)}$

$$\frac{2\pi}{\Gamma_f}\frac{d^2\Gamma}{dq^2 d\phi} = \frac{1}{2} + \frac{1}{4}|F_2|^2 \cos 2\phi$$

$\rightarrow T_2^{(2)}$

Cannot easily differentiate between 0^{++} and 2^{++} special case.

For $J^{PC} = 0^{++}$ $\quad T_2^{(0)} = \frac{1}{6}\left(1 + 2T_1^{(0)}\right)$

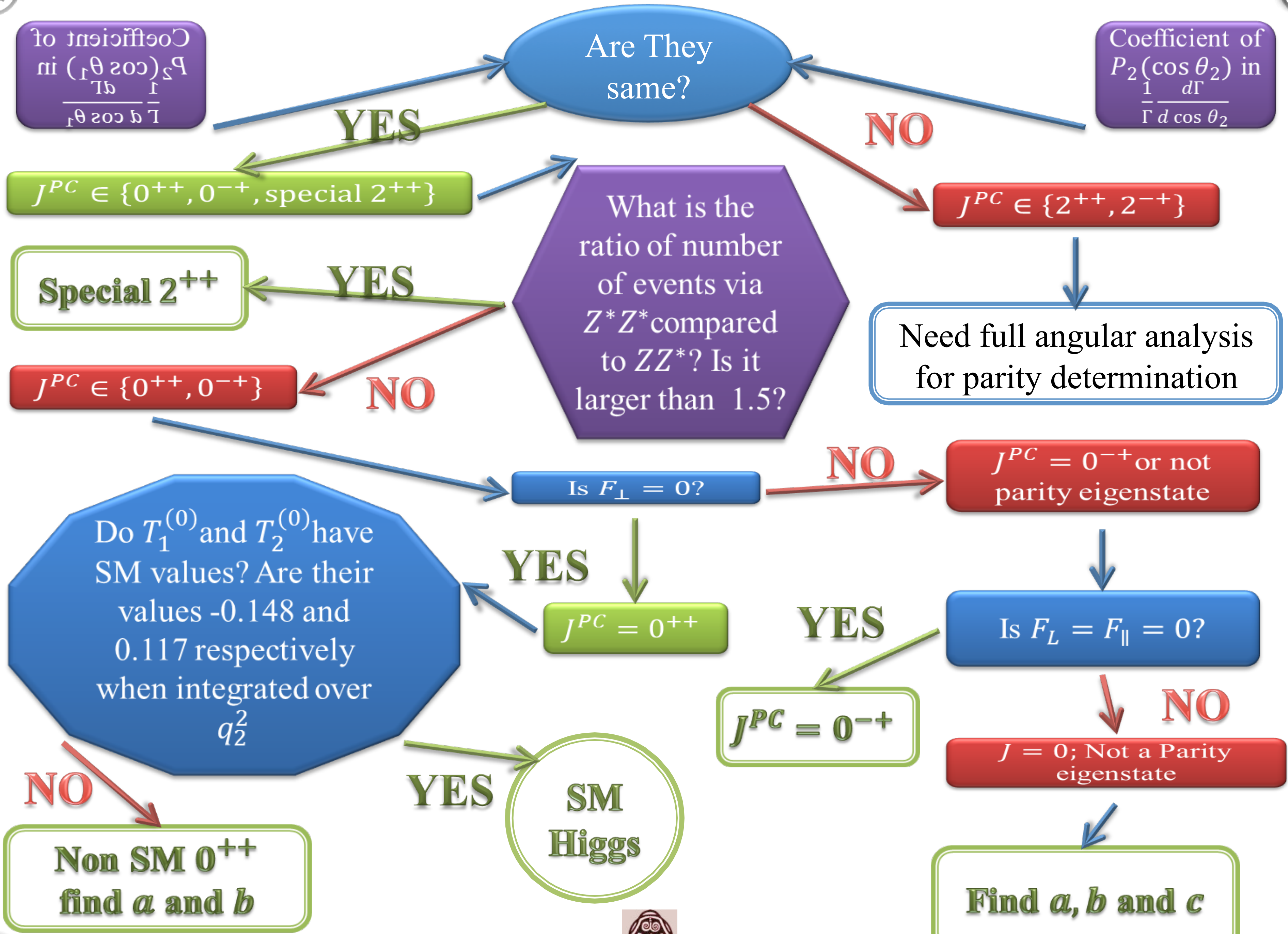
For $J^{PC} = 2^{++}$ special case $\quad T_2^{(2)} = \frac{1}{6}\left(1 + 2T_1^{(2)}\right)$

The extra X^2 dependence in the amplitude for the special 2^{++} case distinguishes it from the $J^{PC} = 0^{++}$ case.

$$\left.\begin{aligned} A_L &= \frac{1}{2}\left(M_H^2 - M_1^2 - M_2^2\right) a + M_H^2 X^2\, b\,, \\ A_\parallel &= \sqrt{2}\, M_1 M_2\, a\,, \end{aligned}\right\} \quad J^{PC} = 0^{++}$$

$$\left.\begin{aligned} A_1 &= \frac{16\sqrt{2}}{3\sqrt{3}} X^2 \left[\frac{1}{2}\left(M_H^2 - M_1^2 - M_2^2\right) C + M_H^2 X^2\, D\right], \\ A_2 &= \frac{32}{3\sqrt{3}} X^2 M_1 M_2\, C\,, \end{aligned}\right\} \quad J^{PC} = 2^{++}$$

The main difference between the $J^{PC} = 0^{++}$ and the special 2^{++} cases, is that they predict different ratios for the number of Z^*Z^* events to the number of ZZ^* events, due to the extra X^2 dependence.



Coefficient of $P_2(\cos\theta_1)$ in $\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_1}$
Are They same?
Coefficient of $P_2(\cos\theta_2)$ in $\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_2}$
YES
NO
$J^{PC} \in \{0^{++}, 0^{-+}, \text{special } 2^{++}\}$
$J^{PC} \in \{2^{++}, 2^{-+}\}$
What is the ratio of number of events via Z^*Z^*compared to ZZ^*? Is it larger than 1.5?
Special 2^{++}
YES
Need full angular analysis for parity determination
$J^{PC} \in \{0^{++}, 0^{-+}\}$
NO
Is $F_\perp = 0$?
NO
$J^{PC} = 0^{-+}$or not parity eigenstate
Do $T_1^{(0)}$and $T_2^{(0)}$have SM values? Are their values -0.148 and 0.117 respectively when integrated over q_2^2
YES
$J^{PC} = 0^{++}$
YES
Is $F_L = F_\parallel = 0$?
$J^{PC} = 0^{-+}$
NO
$J = 0$; Not a Parity eigenstate
NO
YES
SM Higgs
Non SM 0^{++} find a and b
Find a, b and c

Conclusions

By studying three uni-angular distributions and examining the number of Z^*Z^* to ZZ^* events one can unambiguously confirm whether the new boson is indeed the Higgs with $J^{PC} = 0^{++}$ and with couplings to Z bosons exactly as predicted in the Standard Model.