TESTABLE CONSTRAINT ON NEAR-TRIBIMAXIMAL NEUTRINO MIXING

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Plan

1. Neutrino mixing puzzle
2. TBM and additive perturbation
3. Lowest order results
4. Discussion
5. Bottom line

Neutrino mixing puzzle

Quark and charged lepton masses hierarchical.

Neutrinos ? Quark mixing hierarchical:1-2 » 2-3 » 1-3.

(1) $$|V_{CKM}| \sim \begin{pmatrix} 0.9 & 0.2 & 0.004 \\ 0.2 & 0.9 & 0.01 \\ 0.008 & 0.04 & 0.9 \end{pmatrix}$$

Neutrino mixing pattern is different

(2) $$|U_{PMNS}| \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

Either neutrinos are quasi-degenerate or the neutrino mass hierarchy is mild and does not affect neutrino mixing significantly. A non-hierarchical mixing principle seems operative in the neutrino sector.

For fermion of type t (=u,d,l,ν)

(3) $$U^{t\dagger} M_{tf}^{\dagger} M_{tf} U^{t} = M_t^{\dagger} M_t = diag.(|m_{t1}^2|, |m_{t2}|^2, |m_{t3}|^2)$$

Take neutrinos to be Majorana particles

(4) $$M_\nu = diag.(|m_{\nu 1}|, |m_{\nu 2}|e^{-i\alpha_{21}}, |m_{\nu 3}|e^{-i\alpha_{32}})$$

Majorana phase matrix is,

(5) $$K = diag.(1, e^{i\alpha_{21}/2}, e^{i\alpha_{32}/2})$$

(6) $$U_{PMNS} = U^{l\dagger} U^{\nu} K$$

Neutrino mixing puzzle

(7) $$U_{PMNS} =$$

(8) $$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} K.$$

Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

Global fits (3σ)

$$31^\circ < \theta_{12} < 36^\circ$$
$$36^\circ < \theta_{23} < 55^\circ$$
$$7.2^\circ < \theta_{13} < 10^\circ$$

Neutrino mixing puzzle

$$7.00 < \delta^2_{21}(10^5\ eV^2) < 8.09$$

$$2.195 < \delta^2_{32}(10^3\ eV^2) < 2.625 \quad \text{Direct}$$

$$-2.649 < \delta^2_{32}(10^3\ eV^2) < -2.242 \quad \text{Inverted}$$

We repeat our operative principle. Either neutrino masses are quasi degenerate or at least a non-hierarchical principle seems to be involved in neutrino mixing.

TBM and additive perturbations

Mass basis and flavor basis of neutrinos (ν) and charged leptons (l)

(9) $$\underbrace{\psi^f_{\nu l}}_{flavor\ basis} = U_{\nu l} \underbrace{\psi_{\nu l}}_{mass\ basis}$$

Majorana mass matrix is complex symmetric $M_{\nu f}$ in flavor basis.

(10) $$diag\ (m_1, m_2, m_3) = M_\nu = U^T_\nu M_f U_\nu$$

(11) $$diag\ (|m_1|^2, |m_2|^2, |m_3|^2) = M^\dagger_\nu M_\nu = U^\dagger_\nu M^\dagger_{\nu f} M_{\nu f} U_\nu$$

(12) $$U^\dagger_l U_\nu K = U_{PMNS}$$

We will now examine $M_{\nu f}$ and U_ν in the limit of tribimaximal symmetry(superscript zero). Theoretically tribimaximal mixing can be related to A_4, S_3 Δ_{27} symmetries.

(13)
$$M^0_{\nu f} = \begin{pmatrix} X & Y & -Y \\ Y & X+Z & -Y+Z \\ -Y & -Y+Z & X+Z \end{pmatrix},$$

Here X, Y, Z are arbitrary complex mass dimensional parameters.

(14)
$$(M^0_{\nu f})_{12} = -(M^0_{\nu f})_{13},$$

(15)
$$(M^0_{\nu f})_{22} = (M^0_{\nu f})_{33},$$

(16)
$$(M^0_{\nu f})_{11} - (M^0_{\nu f})_{13} = (M^0_{\nu f})_{22} - (M^0_{\nu f})_{23}$$

(17)
$$\underbrace{\theta^0_{12} = \sin^{-1}\frac{1}{\sqrt{3}} \sim 35.3^\circ, \theta^0_{23} = 45^\circ,}_{allowed\ \ 3\sigma\ \ level} \underbrace{\theta^0_{13} = 0}_{violated}.$$

Chosen weak basis is where

(18) $$M_\ell^0 = diag.\,(m_\tau^0, m_\mu^0, m_\tau^0)$$

In the TBM limit

(19) $$U_\nu^{0\dagger}\; M_{\nu f}^{0}{}^{\dagger} M_{\nu f}^0\; U_\nu^0 = \mathrm{diag}\,(|m_{\nu 1}^0|^2, |m_{\nu 2}^0|^2, |m_{\nu 3}^0|^2),$$

(20) $$U_\nu^0 = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix}.$$

(21) $$U_\ell^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

TBM and additive perturbations

(22) $$M_\ell = \underbrace{M_\ell^0}_{TBM\ \ conserving} + \underbrace{M'_\ell}_{TBM\ \ violating}$$

(23) $$M_{\nu f} = \underbrace{M^0_{\nu f}}_{TBM\ \ conserving} + \underbrace{M'_{\nu f}}_{TBM\ \ violating}$$

M' elements are of $O(\epsilon)$.

$|\epsilon^\nu| \sim s_{13} \sim 0.16$, $|\epsilon^\ell|$ **expected smaller than** $|\epsilon^\nu|$ **because they are related to inverse powers of charged lepton mass squares**.

Neglecting $O(\epsilon^2)$ terms, we can write,

(24) $$M^\dagger M \sim {M^0}^\dagger M^0 + {M^0}^\dagger M' + {M'}^\dagger M^0 + O(\epsilon^2)$$

Lowest order results

Flavor Basis:

(25)
$$|\psi_i^{\nu,\ell}\rangle_f = |\psi_i^{0\nu,\ell}\rangle_f + \sum_{k\neq i}\epsilon_{ik}^{\nu,\ell}|\psi_k^{0\nu,\ell}\rangle_f + O(\epsilon^2).$$

In mass basis:

(26)
$$|\psi_i^{\nu,\ell}\rangle = |\psi_i^{0\nu,\ell}\rangle + \sum_{k\neq i}\epsilon_{ik}^{\nu,\ell}|\psi_k^{0\nu,\ell}\rangle + O(\epsilon^2).$$

(27)
$$\epsilon_{ik}^{\nu,\ell} = -\epsilon^{\nu,\ell *}{}_{ki} = (|m_{\nu,\ell i}^0|^2 - |m_{\nu,\ell k}^0|^2)^{-1} p_{ki}^{\nu,\ell},$$

(28)
$$p_{ik}^{\nu,\ell} = \langle\psi_i^{0\nu,\ell}|{M_{\nu,\ell}^0}^\dagger M'_{\nu,\ell} + {M'_{\nu,\ell}}^\dagger M_{\nu,\ell}^0|\psi_k^{0\nu,\ell}\rangle.$$

One can explicitly check that $\epsilon_{ik}^{\nu,\ell}$ and $p_{ik}^{\nu,l}$ are identical in either basis.

Arranging first order flavor eigenstates column by column we get mixing matrices for charged leptons and neutrinos.

(29)
$$U_\ell^\dagger = \begin{pmatrix} 1 & \epsilon_{12}^{\ell *} & \epsilon_{13}^{\ell *} \\ -\epsilon_{12}^{\ell *} & 1 & \epsilon_{23}^{\ell *} \\ -\epsilon_{13}^{\ell} & -\epsilon_{23}^{\ell} & 1 \end{pmatrix}$$

$U_\nu =$

(30)
$$\begin{pmatrix} \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{3}}\epsilon_{12}^{\nu} & \sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}}\epsilon^{\nu *}{}_{12} & -\sqrt{\frac{2}{3}}\epsilon^{\nu *}{}_{13} - \sqrt{\frac{1}{3}}\epsilon^{\nu *}{}_{23} \\ -\sqrt{\frac{1}{6}} + \sqrt{\frac{1}{3}}\epsilon_{12}^{\nu} + \sqrt{\frac{1}{2}}\epsilon_{13}^{\nu} & \sqrt{\frac{1}{3}} + \sqrt{\frac{1}{6}}\epsilon^{\nu *}{}_{12} + \sqrt{\frac{1}{2}}\epsilon_{23}^{\nu} & \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{6}}\epsilon_{13}^{\nu *} - \sqrt{\frac{1}{3}}\epsilon^{\nu *}{}_{23} \\ \sqrt{\frac{1}{6}} - \sqrt{\frac{1}{3}}\epsilon_{12}^{\nu} + \sqrt{\frac{1}{2}}\epsilon_{13}^{\nu} & -\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{6}}\epsilon^{\nu *}{}_{12} + \sqrt{\frac{1}{2}}\epsilon_{23}^{\nu} & \sqrt{\frac{1}{2}} - \sqrt{\frac{1}{6}}\epsilon^{\nu *}{}_{13} + \sqrt{\frac{1}{3}}\epsilon^{\nu *}{}_{23} \end{pmatrix}$$

(31)
$$U_{PMNS} = U_\ell^\dagger U_\nu K$$

U_{PMNS} given in PDG convention in Eqn. (7). In identification the matrix K cancels out.

Relations among $\epsilon_{ij}^{\nu\ell}, c_{ij}, s_{ij}$ and δ_{CP} emerge.

c_{ij} and s_{ij} are real and $e^{i\delta_{CP}}$ is complex.

Conditions emerge on $\mathrm{Im}\ \epsilon_{ij}^{\nu,\ell}$

$$\mathrm{Im}\ \epsilon_{12}^{\nu} = O(\epsilon^2), \tag{32}$$

$$\mathrm{Im}\ (\epsilon_{13}^{\nu} - \sqrt{2}\epsilon_{23}^{\nu}) = O(\epsilon^2), \tag{33}$$

$$\mathrm{Im}\ \epsilon_{23}^{l} = O(\epsilon^2), \tag{34}$$

$$\mathrm{Im}\ (\epsilon_{12}^{l} - \epsilon_{13}^{l}) = O(\epsilon^2). \tag{35}$$

Three measurable deviants from tribimaximal mixing:

$$c_{12} - \sqrt{2/3} = \sqrt{1/2}\left(\sqrt{1/3} - s_{12}\right) + O(\epsilon^2) = \sqrt{1/3}\,\epsilon^{\nu}_{12} - \sqrt{1/6}\left(\epsilon^{l}_{12} - \epsilon^{l}_{13}\right) + O(\epsilon^2),$$

(36)

$$c_{23} - s_{23} = -\sqrt{2/3}\left(\epsilon^{\nu}_{13} - \sqrt{2}\,\epsilon^{\nu}_{23}\right) - \sqrt{2}\,\epsilon^{l}_{23} + O(\epsilon^2),$$

(37)

$$s_{13}\,e^{i\delta_{CP}} = -\sqrt{1/3}\,\left(\sqrt{2}\,\epsilon^{\nu}_{13} + \epsilon^{\nu}_{23}\right) + \sqrt{1/2}\left(\epsilon^{l}_{12} + \epsilon^{l}_{13}\right) + O(\epsilon^2).$$

(38)

The basis independent Jarlskog invariant $J = \mathrm{Im}[(U^{\dagger}_{\ell}U_{\nu})_{e1}(U^{\dagger}_{\ell}U_{\nu})_{\mu 2}(U^{\dagger}_{\ell}U_{\nu})^{*}_{e2}(U^{\dagger}_{\ell}U_{\nu})^{*}_{\mu 1}]$ now turns out to be

(39) $$J = -\frac{1}{\sqrt{6}}\,\mathrm{Im}\,[\epsilon^{\nu}_{23} - \frac{1}{\sqrt{6}}(\epsilon^{l}_{12} + \epsilon^{l}_{13})] + O(\epsilon^2).$$

Let us now take the perturbing mass matrices for neutrinos and charged leptons, with respective complex mass dimensional parameters $\mu_{ij} = \mu_{ji}$ and λ_{ij}, as

(40) $$M'_{\nu f} = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{pmatrix}, \quad M'_{\ell f} = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} = M'_{\ell} + O(\epsilon^2)$$

Off diagonal entries of neutrino perturbation in mass basis:

$$(M'_\nu)_{12} = \frac{1}{3\sqrt{2}}(2\mu_{11} + \mu_{12} - \mu_{13} - \mu_{22} + 2\mu_{23} - \mu_{33})$$

$$(M'_\nu)_{13} = \frac{1}{2\sqrt{3}}(2\mu_{12} + 2\mu_{13} - \mu_{22} + \mu_{33})$$

$$(M'_\nu)_{23} = \frac{1}{\sqrt{6}}(\mu_{12} + \mu_{13} + \mu_{22} + \mu_{33})$$

Explicit substitution in Eqn. (25) yields

(41) $$\epsilon^l_{12} = ({m^0_e}^2 - {m^0_\mu}^2)^{-1}\,(m^0_\mu \lambda_{21} + m^0_e \lambda^*_{12}),$$

(42) $$\epsilon^l_{23} = ({m^0_\mu}^2 - {m^0_\tau}^2)^{-1}\,(m^0_\tau \lambda_{32} + m^0_\mu \lambda^*_{23}),$$

(43) $$\epsilon^l_{13} = ({m^0_e}^2 - {m^0_\tau}^2)^{-1}\,(m^0_\tau\, \lambda_{31} + m^0_e \lambda^*_{13}).$$

Because of the hierarchical nature of charged lepton masses, (34) and (35) can only be satisfied, without unnatural cancellations, by $\lambda_{12}, \lambda_{21}, \lambda_{13}, \lambda_{31}, \lambda_{32}$ and λ_{23} all being real to order ϵ. That immediately implies

(44) $$\mathrm{Im}\ \epsilon^l_{12} = O(\epsilon^2) = \mathrm{Im}\ \epsilon^l_{13}.$$

Using (44)

(45) $$\tan \delta_{CP} = \frac{3\ \mathrm{Im}\ \epsilon^{\nu}_{23} + O(\epsilon^2)}{\mathrm{Re}\ [\sqrt{2}\ \epsilon^{\nu}_{13} + \epsilon^{\nu}_{23}] - \sqrt{3/2}\ \mathrm{Re}\ [\epsilon^{l}_{12} + \epsilon^{l}_{13}] + O(\epsilon^2)}.$$

Also,

(46) $$J = -\frac{1}{\sqrt{6}}\ \mathrm{Im}\ \epsilon^{\nu}_{23} + O(\epsilon^2).$$

Lowest order results

(47) $$\Delta^0_{ij} \equiv |m^0_{\nu i}|^2 - |m^0_{\nu j}|^2,$$

(48) $$a^{\mp}_{ij} \equiv m^0_{\nu i} \mp m^0_{\nu j}.$$

$$6\sqrt{2}\,\Delta^0_{12} \begin{pmatrix} \mathrm{i}\ \mathrm{Im}\ \epsilon^\nu_{12} \\ \mathrm{Re}\ \epsilon^\nu_{12} \end{pmatrix} = a^{\mp}{}^*_{21}\,(2\mu_{11} + \mu_{12} - \mu_{13} - \mu_{22} + 2\mu_{23} - \mu_{33}) \mp c.c.,$$

(49)

$$2\sqrt{6}\,\Delta^0_{23} \begin{pmatrix} \mathrm{i}\ \mathrm{Im}\ \epsilon^\nu_{23} \\ \mathrm{Re}\ \epsilon^\nu_{23} \end{pmatrix} = a^{\mp}{}^*_{32}(\mu_{12} + \mu_{13} + \mu_{22} - \mu_{33}) \mp c.c.,$$

(50)

$$2\sqrt{3}\,\Delta^0_{13} \begin{pmatrix} \mathrm{i}\ \mathrm{Im}\ \epsilon^\nu_{13} \\ \mathrm{Re}\ \epsilon^\nu_{13} \end{pmatrix} = a^{\mp}{}^*_{31}(\mu_{12} + \mu_{13} - \frac{1}{2}\mu_{22} + \frac{1}{2}\mu_{33}) \mp c.c.,$$

(51)

In the absence of unnatural cancellations, (32) and (49) would require $2\mu_{11} + \mu_{12} - \mu_{13} - \mu_{22} + 2\mu_{23} - \mu_{33}$ and $m^0_{\nu 2}$ to be real; the latter constrains the Majorana phase α^0_{21} to equal 0 or π in the TBM limit. These statements are valid neglecting $O(\epsilon^2)$ terms. Furthermore, (33), (50) and (51) would require the equality.

$$\begin{aligned} &\mathrm{Im}[(m^{0*}_{\nu 3} - m^{0*}_{\nu 2})(\mu_{12} + \mu_{13} + \mu_{22} - \mu_{33})] \\ &= \mathrm{Im}[(m^{0*}_{\nu 2} - m^{0*}_{\nu 1})(\mu_{12} + \mu_{13} + \frac{1}{2}\mu_{22} - \frac{1}{2}\mu_{33})] \end{aligned} \tag{52}$$

Either, one must have **condition 1:** $m^0_{\nu 1} = m^0_{\nu 2}$, meaning $m^0_{\nu 1} = |m^0_{\nu 2}|$ plus $\alpha^0_{21} = 0$, and $\mu_{22} = \mu_{33}$ in which case, $\sqrt{2}\,\mathrm{Re}\ \epsilon^\nu_{23} = \mathrm{Re}\ \epsilon^\nu_{13} + O(\epsilon^2)$ and then, from (37), $c_{23} - s_{23} = -\sqrt{2}\,\epsilon^l_{23} + O(\epsilon^2)$,i.e, $s_{23} = (1/\sqrt{2})(1 + \epsilon^l_{23}) + O(\epsilon^2)$; the latter implies via (37) and (42) that any deviation from maximality in the atmospheric neutrino mixing angle θ_{23} must come solely from the 2-3 off-diagonal element in the charged lepton mass perturbation M'_l and is expected to be small since ϵ^l_{23} is scaled by $(m_\tau)^{-1}$, cf. (42) .

Or, what becomes necessary is **condition 2:** $m^0_{\nu 3} - m^0_{\nu 1}$, $m^0_{\nu 3} - m^0_{\nu 2}$ as well as $\mu_{12} + \mu_{13}$ and $\mu_{22} - \mu_{33}$ have to be real; this means that the Majorana phase α^0_{21} and α^0_{32} in the TBM limit are 0 or π and ϵ^{ν}_{23} is real, in which case, by virtue of (45) as well as (46), $s_{13}\ \sin\ \delta_{CP} = O(\epsilon^2)$ and $J = O(\epsilon^2)$ so that, any observable CP-violation in neutrino oscillation experiments would vanish to the lowest order of TBM violating perturbations.

Forthcoming experiments

Let us finally remark on the relevance of our results to planned neutrino experiments at the proton beam intensity frontier. A combination of data from the ongoing and upcoming runs of the T2K and NOvA experiments will sensitively probe the existence of any possible deviation from $\sqrt{\frac{1}{2}}$ in s_{23} from converison probability $P(\nu_\mu \longrightarrow \nu_e)$. Suppose such a deviation is discovered at a significant level, say comparable in percentage terms to (100 s_{13})$\%$ of the maximal value, then within our framework we would expect a non-observation of CP-violation in neutrino oscillations in the above data

Contrariwise, the failure to measure any such deviation outside error bars would bolster the hope of detecting CP-nonconservation for oscillating neutrinos which would be allowed to order ϵ. The latter would be good news not only for a combined analysis of data from forthcoming runs of T2K and NOvA, but also for future experiments with superbeams, such as LBNE, LBNO or a neutrino factory at 10 GeV, aiming to observe CP-violation from the difference in oscillation probabilities, $P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

Bottom line

In summary, linear expressions have been analytically derived, valid with any possible neutrino mass hierarchy, for the three measurable TBM-variants in terms of lowest order perturbation parameters. Constrained interrelations among those parameters have been obtained from the requirement that the perturbed charged lepton and neutrino flavor eigenstate vectors have to constitute the columns of the respective unitary matrices U_ℓ and U_ν.

Bottom line

With the plausible assumptions of the mixing caused by charged leptons being significantly smaller than that due to neutrinos and no unnatural cancellations, we have obtained a result, forcing one of two possibilities, which should be testable in the foreseeable future. Our bottom line is that if both a deviation from maximality in θ_{23} and a non-zero amount of CP-violation are discovered in forthcoming neutrino oscillation experiments, perturbed tribimaximality will cease to be an attractive theoretical option.

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THANK YOU