# KINETICS OF CHIRAL TRANSITION IN HOT AND DENSE QUARK MATTER

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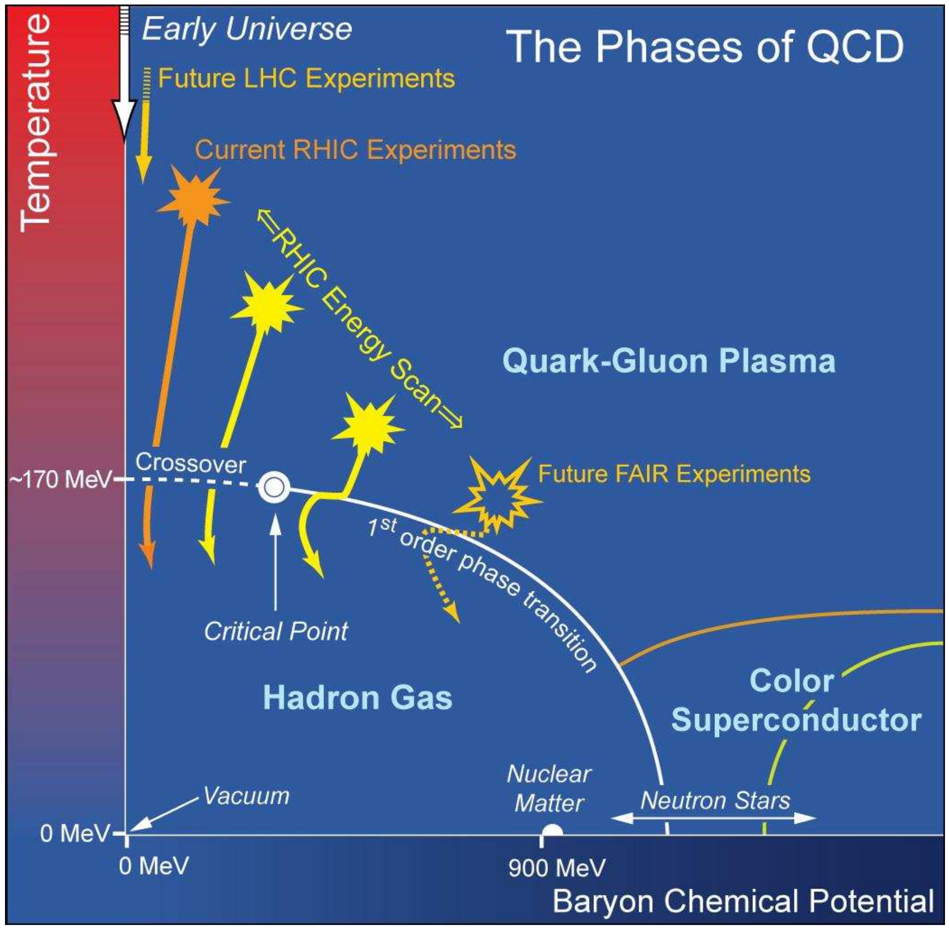
# OUTLINE

- **Introduction**
- **QCD phase digram**
- **Vacuum with quark condensates in NJL model and phase diagram**
- **Ginzburg-Landau expansion and TDGL equation**
- **Quench through second order transition and domain growth**
- **Quench through first order transition (bubble nucleation and spinodal decomposition)**
- **Summary and Outlook**

# QCD UNDER EXTEREME CONDITION

- Extreme conditions exist in the universe. (Compact astrophysical objects, Cosmology)
- Exploring QCD phase diagram is important to understand the phase we live in
- Fundamental properties of QCD

# QCD PHASE DIAGRAM (SCHEMATIC)



# INTRO. CONTD . . .

Mostly attention has been focussed on

- *Critical dynamics* (time dependent behaviour in the vicinity of critical point)
- *far from equllibrium dynamics*(dynamics subsequent to a quench from the disordered phase with vanishing quark condensate to the ordered phase)

We shall discuss the far from equllibrium dynamics and focus on the late stage of the phase separation kinetics of quark matter and the scaling properties of the correlation functions.

# CSB AND VAC. STRUCTURE IN NJL MODEL

*(HM and S.P. Misra, Phys Rev. D48,(1993)5376)*

$$\mathcal{L}_{NJL} = i\bar{\psi}\partial\!\!\!/\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2]$$

Two flavor, massless.

$$|vac\rangle = \exp(\int q^0(\mathbf{k})^\dagger \sigma \cdot \hat{\mathbf{k}} h(\mathbf{k}) \tilde{q}^0(-\mathbf{k}) d\mathbf{k} - h.c.)|0\rangle$$

$q^0|0\rangle = 0$

Determine the condensate function h(**k**) by minimising energy (T=0,$\mu$=0),/free energy ($T \neq 0$,$\mu = 0$)/, thermodynamic potential ($T \neq 0$, $\mu \neq 0$).

$$\tan 2h(\mathbf{k}) = \frac{M}{|\mathbf{k}|} = \frac{-2g\langle\bar{\psi}\psi\rangle}{|\mathbf{k}|}$$

$g = G(1 + \frac{1}{4N_c})$

# NJL MODEL CONTD. $\cdots$

Thermodynamic potential

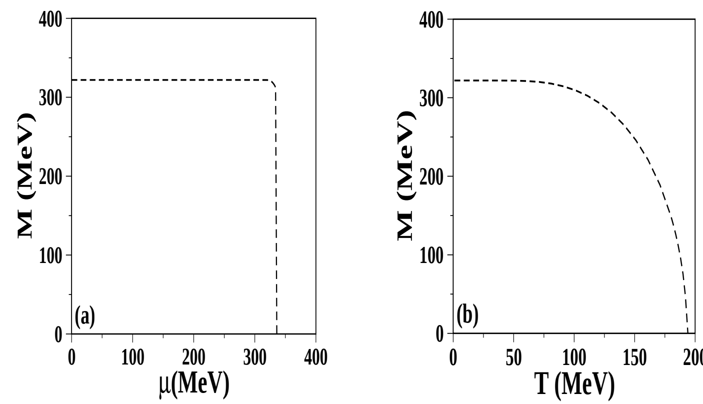
$$\begin{aligned}\Omega &= -\frac{12}{(2\pi)^3}\int(\sqrt{\mathbf{k}^2+M^2}-|\mathbf{k}|)d\mathbf{k}\\ &- \frac{12}{(2\pi)^3}\int[\log(1+\exp(-\beta\omega_-)+\log(1+\exp(-\beta\omega_+)]d\mathbf{k}\\ &+ \frac{M^2}{4g}\end{aligned} \quad (1)$$

$\omega_\mp = \sqrt{\mathbf{k}^2+M^2} \mp \nu$, $\nu = \mu - G\rho_v/N_c$.

Mass gap equation

$$M = 2g\frac{2N_cN_f}{(2\pi)^3}\int\frac{M}{\sqrt{\mathbf{k}^2+M^2}}[1-n_-(\mathbf{k},\beta,\mu)-n_+(\mathbf{k},\beta,\mu)]d\mathbf{k}$$

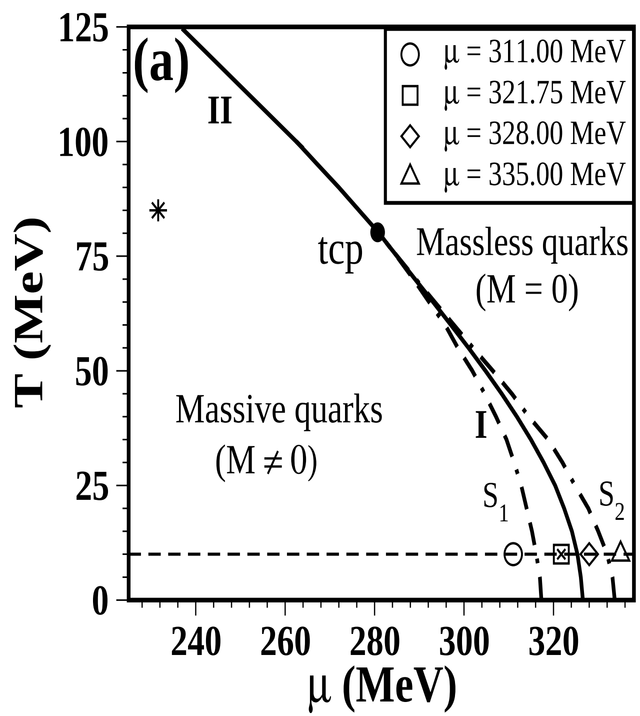
# PHASE DIAGRAM; NJL MODEL




a b

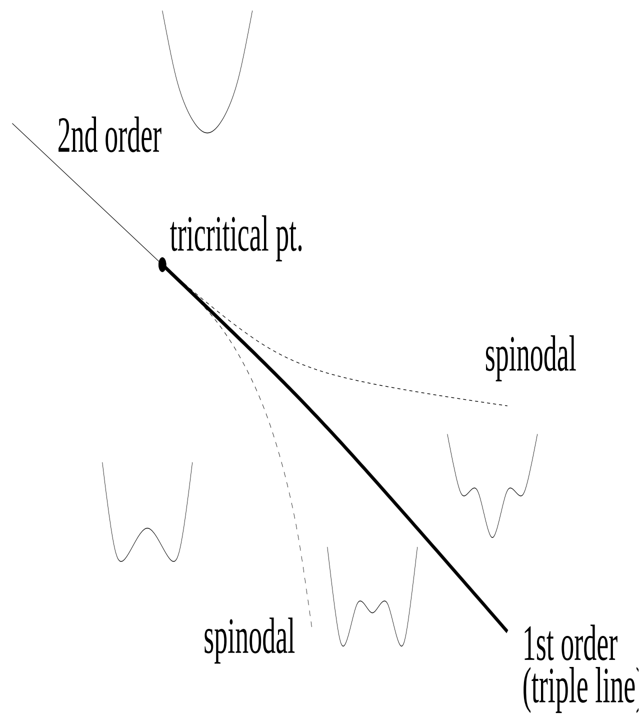
Mass$\sim G\langle\bar{\psi}\psi\rangle$ as a function of $\mu$ for T=0 (Fig a) and as a function of T for $\mu = 0$ (Fig b)

# PHASE DIAGRAM; NJL MODEL CONTD.$\cdots$




Phase diagram of the Nambu-Jona-Lasinio model in the $(\mu, T)$-plane for zero current quark mass. A line of first-order transitions (I) meets a line of second-order transitions (II) at the tricritical point (tcp). $(\mu_{\text{tcp}}, T_{\text{tcp}}) \simeq (282.58, 78)$ MeV. The dot-dashed lines $S_1$ and $S_2$ denote the spinodals or metastability limits for the first-order transitions.

# PHASE DIAGRAM; NJL MODEL CONTD. · · ·




$T_1 > T > T_c$, M>0 is a metastable state (superheated liq.)
$T_2 < T < T_c$, M=0 is metastable state (supercooled gas)

# GINZBURG LANDAU EXPANSION OF FREE ENERGY

In the mean field approx. close to the phase boundary, the thermodynamic potential may be expanded in power series of the order parameter $M$ upto logarithmic corrections:

*Sasaki,Friman,Redlich,PRD77, 034024 (2008); Iwasaki,PRD 70, 114031(2004)* $\cdots$

$$\tilde{\Omega}\left(M\right) = \tilde{\Omega}\left(0\right) + \frac{a}{2}M^2 + \frac{b}{4}M^4 + \frac{d}{6}M^6 + \cdots \equiv f\left(M\right).$$

$a, b, d$ —functions of ($\mu, T$)
Gap equation:

$$f'\left(M\right) = aM + bM^3 + dM^5 = 0.$$

Soln.s

$$\begin{cases} M_0 = 0, \\ M_{\pm}^2 = \dfrac{-b \pm \sqrt{b^2 - 4ad}}{2d}. \end{cases}$$

# G L FREE ENERGY–CONTD···

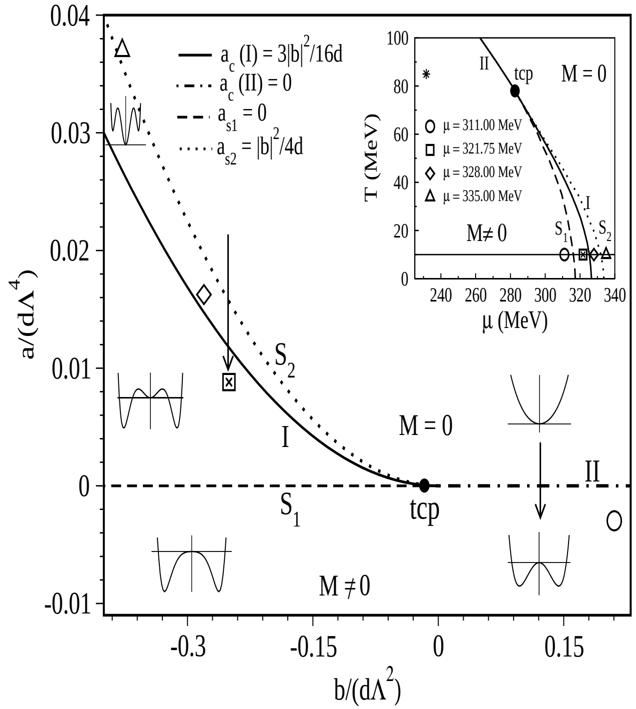
For $b > 0$ transition is second order.
Stationary pt.s are $M = 0$(for $a > 0$) OR M=0,$\pm M_+$(for $a < 0$)
For $b < 0$ phase transition is first order with the soln.s of gap eq.s

$$\begin{aligned} M &= 0, \quad a > b^2/4d, \\ M &= 0, \pm M_+, \pm M_-, \quad b^2/4d > a > 0, \\ M &= 0, \pm M_+, \quad a < 0. \end{aligned} \tag{2}$$

Condn. of degeneracy of two minima
($\Omega(M = 0)$=$\Omega(M = M_+)$ or $a_c = 3b^2/(16d)$) determines $T_c$.
$T_1$ ($T_2$) is determined by $a = b^2/4d$ ($a = 0$).

# GINZBURG LANDAU PHASE DIAGRAM

## in the (b,a) space




Phase diagram in ($b$, $a$)-space for the GL free energy. A line of first-order transitions (I) meets a line of second-order transitions (II) at the tricritical point (tcp), which is located at the origin. The equation for I is $a_c = 3|b|^2/(16d)$, and that for II is $a_c = 0$. The dashed lines denote the spinodals $S_1$ and $S_2$

# DYNAMICAL EQUATIONS (TDGL EQNS)

Consider a system which is rendered thermodynamically unstable by a rapid quench from the disordered (symmetric) phase to the ordered (broken-symmetric) phase.
The unstable homogeneous state (with $M \simeq 0$) evolves via the emergence and growth of domains rich in the preferred phase (with $M \neq 0$).
Such far-from-equilibrium evolution, is termed *phase ordering dynamics* or *domain growth* or *coarsening*. Most problems in this area historically arise from condensed matter systems.
Equally fascinating is the kinetics of chiral transition!

# TDGL CONTD. $\cdots$

Since coarsening system is inhomogeneous one includes a gradient term in the GL free energy

$$\Omega\left[M\right] = \int d\vec{r}\left[F\left(M\right) + \frac{K}{2}\left(\vec{\nabla}M\right)^2\right]$$

The evolution of the system is described by the Langevin equation with an inertial term:

$$\frac{\partial^2}{\partial t^2}M(\vec{r},t) + \bar{\gamma}\frac{\partial}{\partial t}M\left(\vec{r},t\right) = -\frac{\delta\Omega\left[M\right]}{\delta M} + \theta\left(\vec{r},t\right)$$

which models the relaxational dynamics of $M\left(\vec{r},t\right)$ to the minimum of $\Omega\left[M\right]$ (dissipative which damps the system towards the equillibrium configuration). $\gamma$: damping coefficient.

$\theta(\vec{r},t)$ represents the Langevin noise force assumed to be Gaussian and white satisfying the fluctuation-dissipation relation $\langle\theta\left(\mathbf{r},t\right)\rangle = 0$ and $\langle\theta(\mathbf{r}',t')\theta(\mathbf{r}'',t'')\rangle = 2\bar{\gamma}T\delta(\mathbf{r}'-\mathbf{r}'')\delta\left(t'-t''\right)$

# TDGL CONTD$\cdots$

Rescaling

$$\begin{aligned}
M &= M_0 M', \quad M_0 = \sqrt{|a|/|b|}, \\
\vec{r} &= \xi \vec{r'}, \quad \xi = \sqrt{K/|a|}, \\
t &= t_0 t', \quad t_0 = 1/\sqrt{|a|}, \\
\theta &= |a| M_0 \, \theta'.
\end{aligned} \tag{3}$$

Dropping primes, we obtain the dimensionless TDGL equation:

$$\frac{\partial^2}{\partial t^2} M(\vec{r}, t) + \gamma \frac{\partial}{\partial t} M(\vec{r}, t) = -\mathrm{sgn}\,(\mathrm{a}) M - \mathrm{sgn}\,(\mathrm{b}) M^3 - \lambda M^5 + \nabla^2 M + \theta(\vec{r}, t),$$

where $\lambda = |a|d/|b|^2 > 0$.

# Early time behavior

Consider the deterministic equation ($\theta = 0$) around an extremum pt. ($M(\vec{r}) = \bar{M} + \phi(\vec{r})$) in the Fourier space

$$\frac{\partial^2}{\partial t^2}\,\phi(\vec{k},t) + \gamma\frac{\partial}{\partial t}\,\phi(\vec{k},t) + (-\alpha + k^2)\phi(\vec{k},t) = 0,$$

$\alpha = -f''(M)$, ($\alpha > 0\bar{M}$- local Max; $\alpha < 0$– local Min.)
General soln.

$$\phi(\vec{k},t) = A_1 e^{\Lambda_+(\vec{k})t} + A_2 e^{\Lambda_-(\vec{k})t}$$

$$\Lambda_\pm(\vec{k}) = \frac{-\gamma \pm \sqrt{\gamma^2 + 4(\alpha - k^2)}}{2}.$$

For $\alpha > 0$ - instability for long wavelength ($k < \sqrt{\alpha}$)(exponential growth of fluctuations)
For $\alpha < 0$, no instability: fluctuations are exponentially damped. The damping is relaxational for $k^2 < (\gamma^2 - 4|\alpha|)/4$ and oscillatory for $k^2 > (\gamma^2 - 4|\alpha|)/4$

# Quenching through second order line $b > 0$

For $b > 0$, the chiral transition occurs when $a < 0$. The relevant evolution equation for order parameter is

$$\frac{\partial^2 M}{\partial t^2} + \gamma \frac{\partial M}{\partial t} = M - M^3 - \lambda M^5 + \nabla^2 M + \theta\left(\vec{r}, t\right).$$

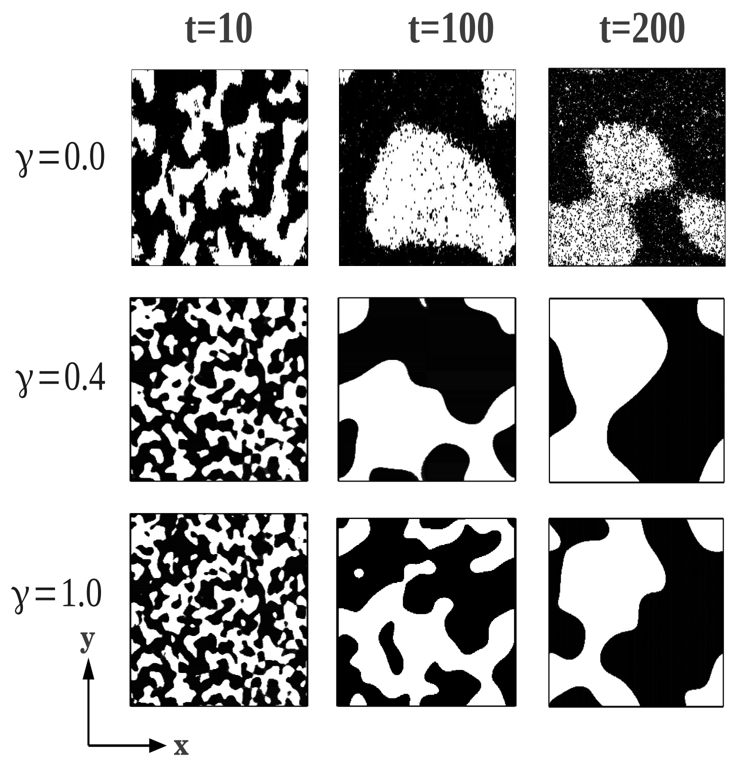
Numerically solve this equation using a simple Euler discretization scheme on a 3d lattice of size $256^3$ with periodic boundary condn.For numerical stabilty,

$$\Delta t < \frac{2\Delta x^2}{4d + \alpha_1 \Delta x^2} \quad \alpha_1 = 4 + (1 - \sqrt{1 + 4\lambda})/\lambda,$$

Mesh size $\Delta x = 1$ $\Delta t = 0.1$ obtained from a linear stability analysis. Euler discretized numerical scheme must respect the stabilty properties of the homogeneous solution.

Initial cond. :Small amplitude random fluctuation about $M = 0$. The system rapidly evolves with domains with nonzero value of the order parameter. Interface of these domains have $M = 0$. Dissipation coefficient controls the rapid growth of domains.

# DOMAIN GROWTH $(b > 0)$

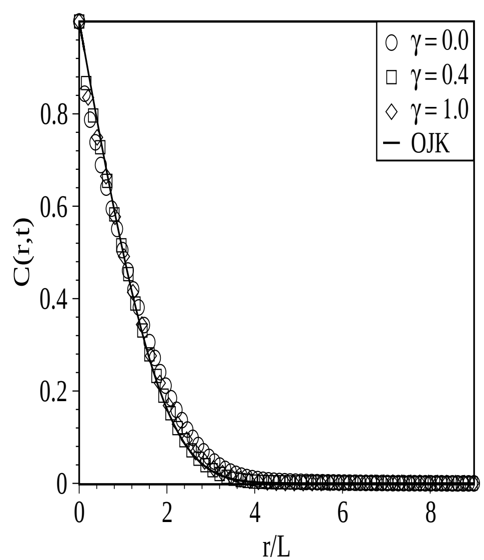



Domain evolution of the preferred massive phase: $M = M_+$ (marked black), after a deep temperature quench through the second-order line (II) . We show evolution pictures at $t = 10, 100, 200$ for three different values of $\gamma$. The frames are the cross-sections at $z = N/2$ of the 3-$d$ snapshots obtained by numerically solving the inertial TDGL Eq. with $\lambda = 0.14$. The noise strength is $\epsilon = 0.008$.

# CORRELATION FUNCTIONS

Domains have a characteristic length scale $L(t)$, which grows with time.

$$C(\vec{r},t) \equiv \frac{1}{V}\int d\vec{R}\left[\left\langle M(\vec{R},t)M(\vec{R}+\vec{r},t)\right\rangle - \left\langle M(\vec{R},t)\right\rangle\left\langle M(\vec{R}+\vec{r},t)\right\rangle\right],$$




Scaling of correlation function for $\lambda = 0.14$ for different values of dissipation parameters .

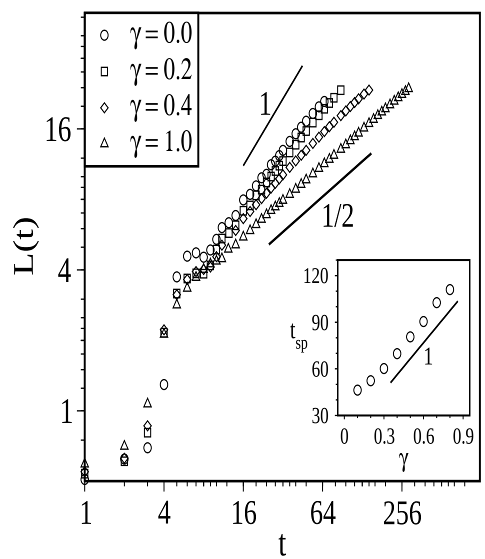
OJK function (as for usual $M^4$-free energy) has good agreement with simulation data.

# CORRELATION FUNCTIONS CONTD$\cdots$

The existence of characteristic scale results in the *dynamical scaling* of $C(\vec{r},t)$

$$C\left(\vec{r},t\right) = \; g\left(r/L\right) = \frac{2}{\pi}\sin^{-1}\left(e^{-r^2/L^2}\right).$$

Ohta-Jasnow-Kawasak (PRL49,1223 (1982)) scaling function.




Time-dependence of domain size, L(t) vs. t. The growth proceeds by the amplification of initial fluctuations, their saturation by the nonlinearity, and subsequent domain coarsening. There is a crossover from an early-time inertial growth [$L(t) \sim t(lnt)^{1/2}$] to a late-time Cahn-Allen (CA) growth [$L(t) \sim t^1/2$]

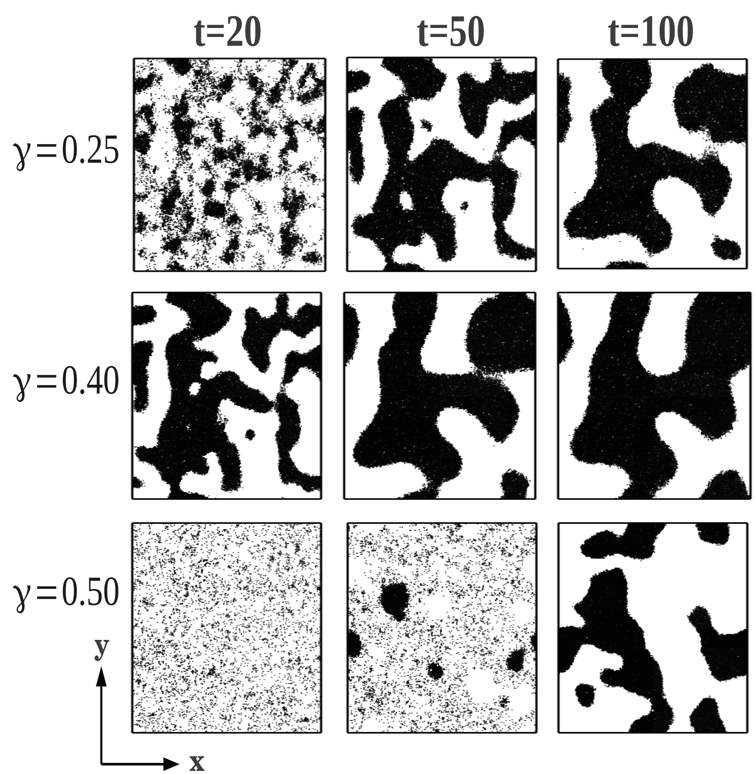
# QUENCH THROUGH FIRST ORDER LINE $(b < 0)$

First order transition occurs for $a < a_c = 3|b|^2/16d$ ($\lambda < \lambda_c = 3/16$)
For $a < 0$, double well structure for the free energy; the domain growth structure and ordering dynamics is similar to quenching through the 2nd order transition.
We confine our attention to $0 < a < a_c$ ($\lambda < \lambda_c$)

$$\frac{\partial^2 M}{\partial t^2} + \gamma \frac{\partial M}{\partial t} = -M + M^3 - \lambda M^5 + \nabla^2 M + \theta\,(\vec{r}, t)\,.$$

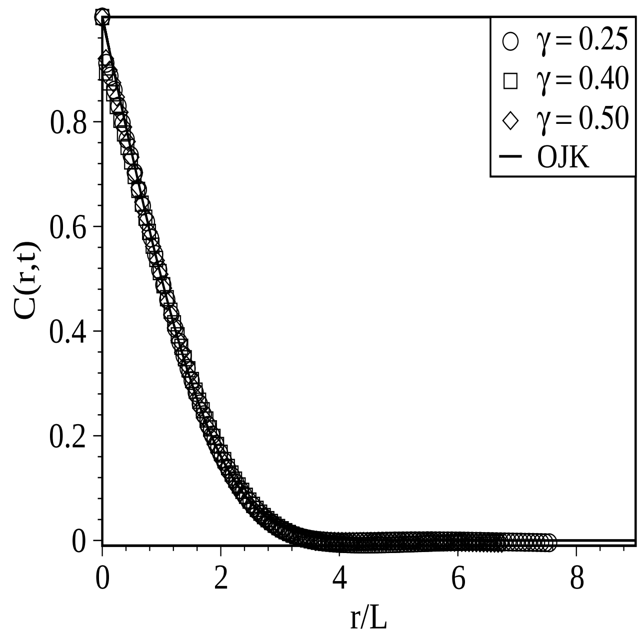
Evolve this equation with the initial state with $M = 0$ which is a metastable state. The chiral transition proceeds via the nucleation and growth of droplets of the preferred phase ($M = \pm M_+$). The thermal noise $\theta$ must be sufficiently large to enable the system to escape from the metastable state. Evolution begins with nucleation of droplets at the early stages. Droplets larger than a critical size $R_c$ grow while $R < R_c$ shrink. $R_c$ decided by the balance between free energy decrease due to bulk droplet and the free energy increase due to surface tension at the droplet boundary. Droplets grow with time and coalesce into domains.

# NUCLEATION AND SPINODAL DECOMPOSITION



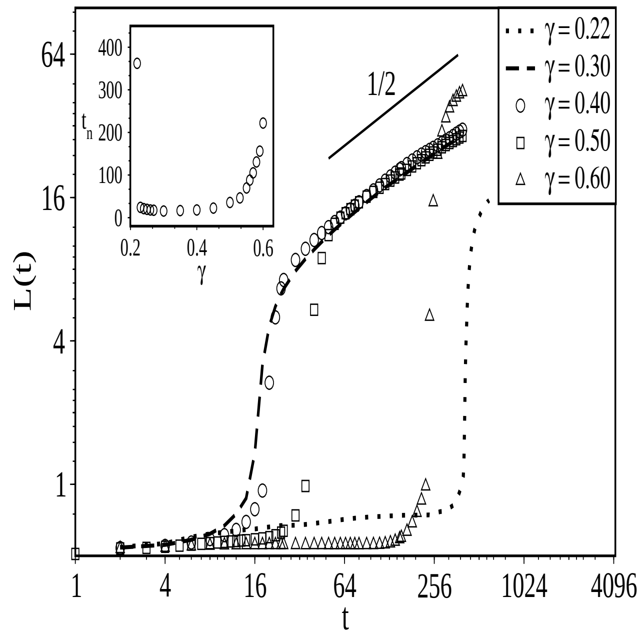

Domain growth after a shallow temperature quench through the first-order line (I) for $\gamma = 0.25, 0.4, 0.5$. The frames show the evolution of the preferred phase with $M = +M_+$ (marked black) at times $t = 20$, $50$ and $100$, respectively. Nucleation is fastest for moderate values of $\gamma$.

# Corrln. function and domain growth




Scaling of correlation function for $\lambda = 0.14$ for different values of dissipation parameters for the late stage dynamics subsequent to the nucleation regime. OJK function has good agreement with simulation data.

# Domain growth




Time-dependence of the domain size, $L(t)$ vs. $t$, for different $\gamma$-values. There is no growth in the early stages when droplets are being nucleated. The asymptotic growth is consistent with the CA growth law, $L(t) \sim t^{1/2}$. The inset shows the $\gamma$-dependence of the nucleation time $t_n$ for the onset of domain growth.

# SUMMARY

- We considered the equllibrium phase diagram in a two flavor NJL model.
- In the mean field approximation and near the chiral phase transition, the thermodynamic potential can be Ginzburg Landau effective theory.
- The kinetics of the transition is considered using the TDGL equations including the inertial terms.
- We studied the ordering dynamics resulting from a sudden quench of system parameters through both first order and second order transition lines. For quenches through the second order line the phase conversion is via spinodal decomposition. For quenches through the first order line, phase transition proceeds via nucleation and growth of droplets of the massive phase. Subsequent merger of these droplets results in late stage domain growth.
  Domain growth shows self similar dynamical scaling.
- Asymptotic growth law for domains is $L(t) \sim t^{1/2}$ The inertial terms gives a pre-asymptotic regime for a faster growth with $L(t) \sim t(lnt)^{1/2}$.

THANK YOU

# Droplet growth dynamics

$R_c$: Critical size of the droplet defined by the balance of *free energy reduction* due to the bulk of droplet and *free energy increase* due to the surface tension at the boundary. Droplet size $R > R_c$, grow while $R < R_c$ shrink. Solve the equation

$$\frac{\partial^2 M}{\partial t^2} + \gamma \frac{\partial M}{\partial t} = -M + M^3 - \lambda M^5 + \frac{M'}{r} + M"(r).$$

with an initial configuration of a 2 d bubble of radius $R_0 > R_c$ s.t,

$$M(r) = M_+(r < R) \quad M(r) = 0(r > R)$$

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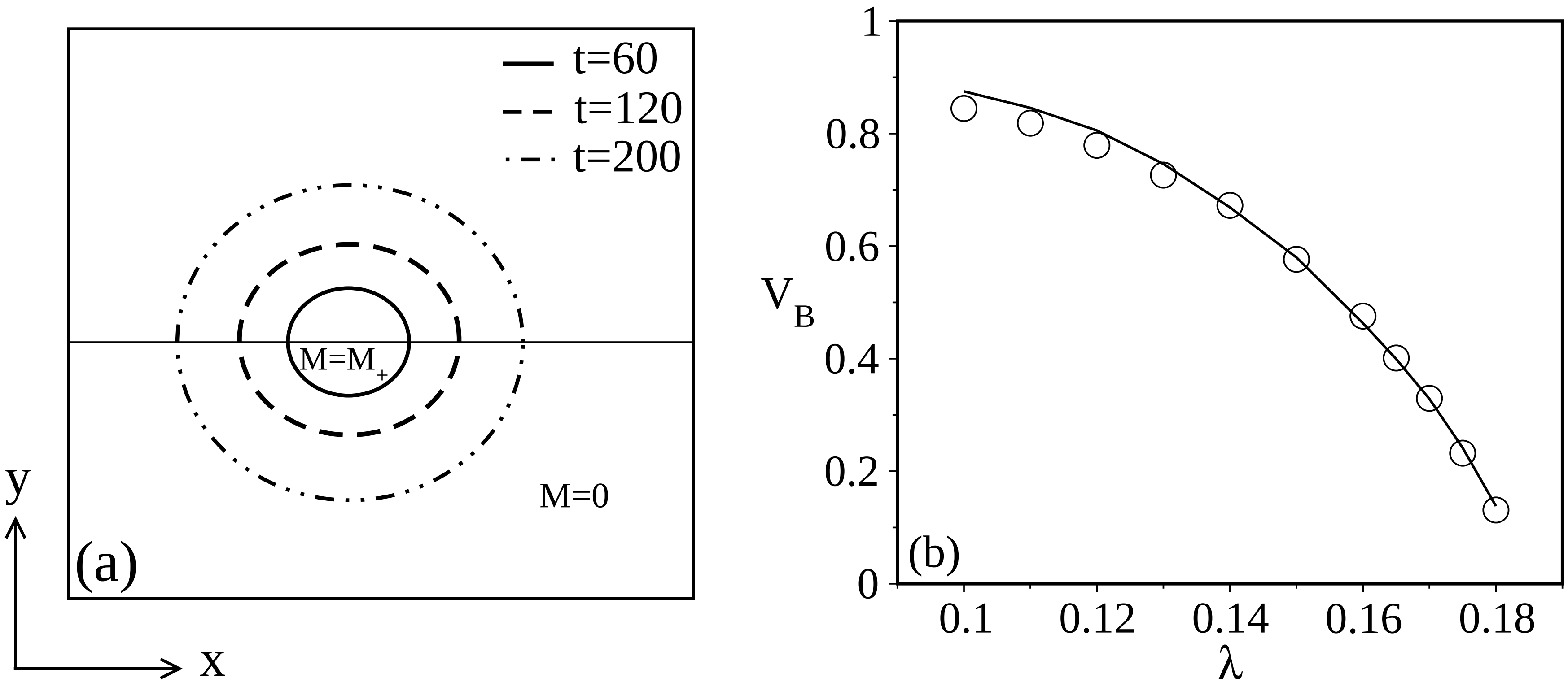
# Droplet growth dynamics contd· · ·




(a) Growth of a droplet of the preferred phase ($M = M_+$) in a background of the metastable phase ($M = 0$) for $\lambda = 0.14$. We show the boundary of the droplet at three different times, as specified. (b) Plot of the bubble growth velocity $v_B$ vs. $\lambda$.