Strong coupling from the τ hadronic width

Gauhar Abbas
Institute of Mathematical Sciences,
Chennai 600113 India

Institute of Physics, Bhubneswar
April 18, 2013

Work done with
B. Ananthanarayan, IISc Bangalore, I. Caprini, NIPNE Bucharest, and J. Fischer, IPAS Prague.
References: *Phys.Rev. D87 (2013) 014008* with BA, IC & JF, *Phys.Rev. D85 (2012) 094018* with BA and IC .

Outline

Outline

Introduction

- The inclusive hadronic decay width of the τ lepton provides a very clean way to determine α_s at low energies.
- The perturbative QCD contribution is known to $O(\alpha_s^4)$ and is very sensitive to α_s, allowing for an accurate determination of the strong coupling
- The nonperturbative corrections are predicted to be small and are suppressed by six powers of the τ mass.
- The main uncertainty originates from the treatment of higher-order corrections and improvement of the perturbative series through renormalization group method.
- In this talk we discuss the renormalization group improvement of the perturbative series using Renormalization-Group Summed expansion (RGS).
- We also study the large order behaviour of the perturbative series. We derive a new kind of expansions, called Renormalization-Group Summed Non-Power expansions (RGSNP).

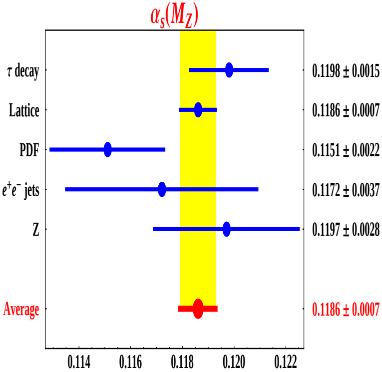


Figure: Summary of measurements of $\alpha_s(M_{Z^0})$. Taken from ' Review of α_s determinations ' Pich 2013

QCD description

- The inclusive character of the total τ hadronic width provides an accurate calculation of the ratio R

$$R_{\tau,V/A} \equiv \frac{\Gamma[\tau^- \to \text{hadrons}\, \nu_\tau]}{\Gamma[\tau^- \to e^- \overline{\nu}_e \nu_\tau]}. \tag{1}$$

- The theoretical analysis involves the two-point correlation functions for the vector $V_{ij}^{\mu} = \bar{\psi}_j \gamma^\mu \psi_i$ and axial-vector $A_{ij}^{\mu} = \bar{\psi}_j \gamma^\mu \gamma_5 \psi_i$ colour-singlet quark currents $(i, j = u, d, s)$:

$$\Pi_{ij,\mathcal{J}}^{\mu\nu}(q) \equiv i \int d^4x \, e^{iqx} \langle 0| T(\mathcal{J}_{ij}^{\mu}(x) \mathcal{J}_{ij}^{\nu}(0)^\dagger)|0\rangle, \tag{2}$$

- The Lorentz decompositions

$$\begin{aligned} \Pi_{ij,\mathcal{J}}^{\mu\nu}(q) &= \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi_{ij,\mathcal{J}}^{(1)}(q^2) \\ &\quad + q^\mu q^\nu \, \Pi_{ij,\mathcal{J}}^{(0)}(q^2)\,, \end{aligned} \tag{3}$$

where the superscript $(J = 0, 1)$ denotes the angular momentum in the hadronic rest frame.

QCD description

- The imaginary parts of $\Pi^{(J)}_{ij,\mathcal{J}}(q^2)$ are proportional to the spectral functions for hadrons with the corresponding quantum numbers.
- The hadronic decay rate of the τ can be written as an integral of these spectral functions over the invariant mass s of the final-state hadrons:

$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]. \quad (4)$$

- The appropriate combinations of correlators are

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2 \left(\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right). \quad (5)$$

The contributions coming from the first two terms correspond to $R_{\tau,V}$ and $R_{\tau,A}$ respectively, while $R_{\tau,S}$ contains the remaining Cabibbo-suppressed contributions.

QCD description

- R_τ can then be written as a contour integral in the complex s-plane running counter-clockwise around the circle $|s| = m_\tau^2$:

$$R_\tau = 6\pi i \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \Pi^{(0+1)}(s) - 2\frac{s}{m_\tau^2}\Pi^{(0)}(s)\right]. \quad (6)$$

Braaten-Narison-Pich 1991

- This expression requires the correlators only for complex s of order m_τ^2, which is significantly larger than the scale associated with non-perturbative effects.
- Using the Operator Product Expansion (OPE), $\Pi^{(J)}(s) = \sum_D C_D^{(J)}/(-s)^{D/2}$, to evaluate the contour integral, R_τ can be expressed as an expansion in powers of $1/m_\tau^2$.
- In the chiral limit ($m_{u,d,s} = 0$), the vector and axial-vector currents are conserved. This implies $s\,\Pi^{(0)}(s) = 0$. Therefore, only the correlator $\Pi^{(0+1)}(s)$ contributes to Eq. (6).

QCD description

- The Cabibbo-allowed combination $R_{\tau,V/A}$ can be written as

$$R_{\tau,V/A} = S_{\rm EW}|V_{ud}|^2\left(1+\delta^{(0)}+\delta'_{\rm EW}+\delta^{(2,m_q)}_{ud,V/A}+\sum_{D=4,6,\ldots}\delta^{(D)}_{ud,V/A}\right), \qquad (7)$$

  with the massless universal perturbative contribution $\delta^{(0)}$.
- The dimension $D=2$ perturbative contribution $\delta^{(2,m_q)}_{ud,V/A}$ from massive quarks is lower than 0.1% for u, d quarks.
- The term $\delta^{(D)}$ denotes the OPE contributions of mass dimension D

$$\delta^{(D)}_{ud,V/A} = \sum_{\dim\mathcal{O}=D} C_{ud,V/A}(s,\mu)\frac{\langle\mathcal{O}_D(\mu)\rangle_{V/A}}{s^{D/2}}\,, \qquad (8)$$

- Electroweak radiative corrections $S_{\rm EW}=1.0198\pm0.0006$
  Marciano and Sirlin 1988,
  and the residual non-logarithmic electroweak correction
  $\delta'_{\rm EW}=0.0010\pm0.0010$
  Braaten and Li 1990

QCD description

- Our main interest is in the perturbative corrections $\delta^{(0)}$ which can be written

$$\delta^{(0)} = \frac{1}{2\pi i} \oint_{|s|=M_\tau^2} \frac{ds}{s} \left(1 - \frac{s}{M_\tau^2}\right)^3 \left(1 + \frac{s}{M_\tau^2}\right) \hat{D}_{\rm pert}(s), \tag{9}$$

where the reduced function $\hat{D}_{\rm pert}(s) \equiv D^{(1+0)}(s) - 1$ is called the Adler function.

- The Adler function is defined as

$$D^{(1+0)}(s) \equiv -s \frac{d}{ds} \left[\Pi^{(1+0)}(s)\right], \tag{10}$$

Adler 1974

- The Adler function is expanded in power of $a \equiv a(\mu^2) \equiv \alpha_s(\mu^2)/\pi$.
- One has to use a renormalization group equation

$$\beta(a) \equiv \mu^2 \frac{\mathrm{d}a}{\mathrm{d}\mu^2} = -a^2 \sum_{k=0}^{\infty} \beta_k a^k. \tag{11}$$

- Finally, the integration (9) has to be performed, yielding $\delta^{(0)}$ as a function of $\alpha_s(m_\tau^2)$ with coefficients $c_{n,1}$ of the Adler function and β_i from the RGE as parameters.

QCD description

- A natural approach is to expand $\alpha_s(s)$ in a power series in $\alpha_s(m_\tau^2)$ and truncate it where the first unknown β_i coefficient appears and put $\mu^2 = M_\tau^2$. After performing the integration this gives $\delta^{(0)}$ as a power seires in $\alpha_s(m_\tau^2)$, where the coefficients of all terms included are exact and no higher ones are present. This is called '**Fixed-Order Perturbation Theory**' (FOPT).

$$\hat{D}_{FOPT}(s) = \sum_{n=1}^{\infty} a^n \sum_{k=1}^{n} k\, c_{n,k}\, L^{k-1} \,. \tag{12}$$

$L \equiv \ln \frac{-s}{\mu^2}$

- A different approach would be to keep the full solution of the RGE and perform a numerical integration and choose $\mu^2 = -s$. Now the results includes all the terms from FOPT and in addition some higher orders in $\alpha_s(m_\tau^2)$ which are generated by the running. This is called '**Contour Improved Perturbation Theory**'.
  Pivovarov 1991, Le Diberder and Pich 1992

$$\hat{D}_{\mathrm{CIPT}}(\alpha_s(-s)/\pi, 0) = \sum_{n=1}^{\infty} c_{n,1} \left(\frac{\alpha_s(-s)}{\pi}\right)^n \,. \tag{13}$$

QCD description

- In the expansion above, the leading known coefficients $c_{n,1}$ are

  $$c_{1,1} = 1,\ c_{2,1} = 1.640,\ c_{3,1} = 6.371,\ c_{4,1} = 49.076,\ c_{5,1} = 283\ (\textit{estimated}).$$

  Baikov, Chetyrkin and Kuhn 2008

- The β-function was calculated to four loops in the $\overline{\mathrm{MS}}$-renormalization scheme, the known coefficients are

  $$\beta_0 = 9/4,\ \beta_1 = 4,\ \beta_2 = 10.0599,\ \beta_3 = 47.228.$$

  Larin, Ritbergen and Vermaseren 1997 and Czakon 2005

QCD description

	$\delta^{(0)}_{\mathrm{FOPT}}$	$\delta^{(0)}_{\mathrm{CIPT}}$
$n = 1$	0.1082	0.1479
$n = 2$	0.1691	0.1776
$n = 3$	0.2025	0.1898
$n = 4$	0.2199	0.1984
$n = 5$	0.2287	0.2022

Table: Predictions of $\delta^{(0)}$ by the standard FOPT, CIPT for various truncation orders n, using $\alpha_s(m_\tau) = 0.34$.

To order $n = 4$, the difference between FOPT and CIPT is 0.0215.
Beneke and Jamin 2008

Outline

Renormalization Group Summed Expansion

- We use a method based on the explicit summation of all renormalization-group accessible logarithms.

$$\hat{D}_{RGS}(a, L) = a(c_{1,1} + 2c_{2,2}aL + 3c_{3,3}a^2L^2 + \cdots) + a^2(c_{2,1} + 2c_{3,2}aL + 3c_{4,3}a^2L^2 + \cdots) + a^3(c_{3,1} + 2c_{4,2}aL + 3c_{5,3}a^2L^2 + \cdots) + \cdots = \sum_{n=1}^{\infty} a^n D_n(aL). \quad (14)$$

Maxwell and A. Mirjalili 2000
Ahmady, Chishtie, Elias, Fariborz, Fattahi, McKeon, Sherry, Steele 2002, 03

$$D_n(u) \equiv \sum_{k=n}^{\infty}(k-n+1)c_{k,k-n+1}u^{k-n}. \quad (15)$$

$u = aL$

- The Adler function defined by (14) is scale independent

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\left\{\hat{D}_{\mathrm{RGS}}(a, L)\right\} = 0. \quad (16)$$

$$\beta(a)\frac{\partial \hat{D}_{\mathrm{RGS}}}{\partial a} - \frac{\partial \hat{D}_{\mathrm{RGS}}}{\partial L} = 0. \quad (17)$$

- We derive following RGE equation

$$0 = -\sum_{n=1}^{\infty}\sum_{k=2}^{n} k(k-1)c_{n,k}a^n L^{k-2}$$
$$-\left(\beta_0 a^2 + \beta_1 a^3 + \beta_2 a^4 + \ldots + \beta_l a^{l+2} + \ldots\right) \times \sum_{n=1}^{\infty}\sum_{k=1}^{n} nkc_{n,k}a^{n-1}L^{k-1}. \quad (18)$$

- By extracting the aggregate coefficient of $a^n L^{n-p}$ one obtains the recursion formula ($n \geq p$)

$$0 = (n-p+2)c_{n,n-p+2} + \sum_{\ell=0}^{p-2}(n-\ell-1)\beta_\ell c_{n-\ell-1,n-p+1}. \quad (19)$$

- Multiplying both sides of (19) by $(n-p+1)u^{n-p}$ and summing from $n = p$ to ∞, we obtain a set of first-order linear differential equation for the functions defined in (15), written as

$$\frac{\mathrm{d}D_n}{\mathrm{d}u} + \sum_{\ell=0}^{n-1}\beta_\ell\left(u\frac{\mathrm{d}}{\mathrm{d}u} + n - \ell\right)D_{n-\ell} = 0, \quad (20)$$

for $n \geq 1$, with the initial conditions $D_n(0) = c_{n,1}$ which follow from (15). The solution of the above Eq (20) can be found iteratively in an analytical closed form.

- The first two solutions are

$$D_1(u) = \frac{c_{1,1}}{y},\ D_2(u) = \frac{c_{2,1}}{y^2} - \frac{\beta_1 c_{1,1} \ln y}{\beta_0 w^2},\ y = 1 + \beta_0 u. \tag{21}$$

- The RGS expansion of the Adler function is

$$\hat{D}_{\mathrm{RGS}}(a, L) = \sum_{n=1}^{N} a^n D_n(aL), \tag{22}$$

$$\delta^{(0)}_{\mathrm{RGS}} = \sum_{n=1}^{\infty} a(M_\tau^2)^n d_n\,, \tag{23}$$

where

$$d_n = \frac{1}{2\pi i} \oint_{|s|=M_\tau^2} \frac{ds}{s} \left(1 - \frac{s}{M_\tau^2}\right)^3 \left(1 + \frac{s}{M_\tau^2}\right) D_n(a, L). \tag{24}$$

	$\delta^{(0)}_{\rm FOPT}$	$\delta^{(0)}_{\rm CIPT}$	$\delta^{(0)}_{\rm RGS}$
$n = 1$	0.1082	0.1479	0.1455
$n = 2$	0.1691	0.1776	0.1797
$n = 3$	0.2025	0.1898	0.1931
$n = 4$	0.2199	0.1984	0.2024
$n = 5$	0.2287	0.2022	0.2056

Table: Predictions of $\delta^{(0)}$ by the standard FOPT, CIPT and the RGS, for various truncation orders n using $\alpha_s = 0.34$.

For $n = 4$, the difference between the results of the RGS and the standard FOPT is 0.01754, and the difference from the RGS and CIPT is 0.0039, which confirms that the new expansion gives results close to those of the CIPT.

Adler function in the complex s-plane

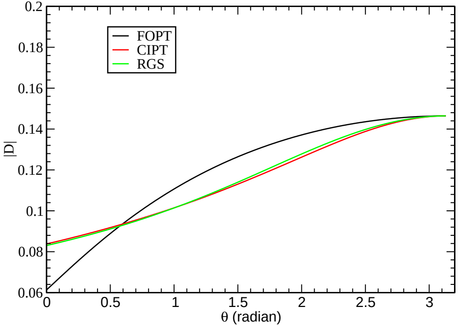


Figure: Adler function expansions, summed up to the order $N = 5$, along the circle $s = M_\tau^2 \exp(i\theta)$.

Outline

1. QCD description
2. Renormalization Group Summed Expansion
3. **Higher order behaviour of RGS expansion**
4. Determination of α_s from RGS expansion
5. RGS Non-Power Expansions
6. Higher order behaviour of RGSNP expansions
7. Determination of α_s from RGSNP expansions
8. Summary

Higher order behaviour of RGS expansion

- We consider the model where the Adler function is defined in terms of its Borel transform $B(u)$ by the principal value prescription

$$\widehat{D}(s) = \frac{1}{\beta_0} \mathrm{PV} \int_0^\infty e^{-\frac{u}{\beta_0 a(-s)}} B(u)\,\mathrm{d}u, \tag{25}$$

where the function $B(u)$ is expressed in terms of a few ultraviolet (UV) and infrared (IR) renormalons

$$B_{\mathrm{BJ}}(u) = B_1^{\mathrm{UV}}(u) + B_2^{\mathrm{IR}}(u) + B_3^{\mathrm{IR}}(u) + d_0^{\mathrm{PO}} + d_1^{\mathrm{PO}} u. \tag{26}$$

Beneke and Jamin 2008

- These terms were written as

$$B_p^{\mathrm{IR}}(u) = \frac{d_p^{\mathrm{IR}}}{(p-u)^{\gamma_p}} \Big[1 + \tilde{b}_1(p-u) + \dots \Big],$$

$$B_p^{\mathrm{UV}}(u) = \frac{d_p^{\mathrm{UV}}}{(p+u)^{\bar{\gamma}_p}} \Big[1 + \bar{b}_1(p+u) + \dots \Big],$$

- The parameters were obtained by imposing RG invariance at four loops. Finally, the free parameters of the model were fixed by the requirement of reproducing the perturbative coefficients $c_{n,1}$ for $n \leq 4$ and the estimate $c_{5,1} = 283$, and read:

$$d_1^{\mathrm{UV}} = -1.56\times 10^{-2}, \quad d_2^{\mathrm{IR}} = 3.16, \quad d_3^{\mathrm{IR}} = -13.5, d_0^{\mathrm{PO}} = 0.781, \quad d_1^{\mathrm{PO}} = 7.66\times 10^{-3}. \tag{27}$$

Higher order behaviour of RGS expansion

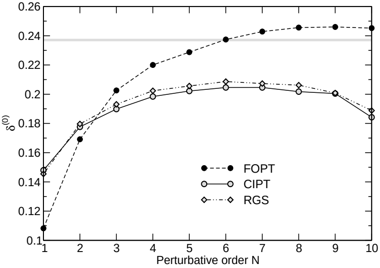


Figure: In the figure we show the dependence on the perturbative order of $\delta^{(0)}$ in FOPT, CIPT and RGS in the BJ model. The gray band is the true value obtained from Borel integral in this model.

Outline

1. QCD description
2. Renormalization Group Summed Expansion
3. Higher order behaviour of RGS expansion
4. Determination of α_s from RGS expansion
5. RGS Non-Power Expansions
6. Higher order behaviour of RGSNP expansions
7. Determination of α_s from RGSNP expansions
8. Summary

Determination of α_s from RGS expansion

- We use as input the recent phenomenological value of the pure perturbative correction to the hadronic τ width

$$\delta^{(0)}_{\rm phen} = 0.2037 \pm 0.0040_{exp} \pm 0.0037_{\rm PC}. \tag{28}$$

Beneke & Jamin 2011, Workshop on Precision Measurements of α_s 2011

- With this input we obtained from the above phenomenological value of $\delta^{(0)}$ the prediction

$$\alpha_s(M_\tau^2) = 0.3378 \pm 0.0046_{\rm exp} \pm 0.0042_{\rm PC}\ {}^{+0.0062}_{-0.0072}(c_{5,1})\ {}^{+0.0005}_{-0.0004}(\text{scale})^{+0.000085}_{-0.000082}(\beta_4). \tag{29}$$

- Combining errors in quadrature

$$\alpha_s(M_\tau^2) = 0.338 \pm 0.010. \tag{30}$$

$$\alpha_s(M_\tau^2) = 0.320^{+0.012}_{-0.007} \qquad \text{FOPT}$$
$$\alpha_s(M_\tau^2) = 0.342 \pm 0.012 \qquad \text{CIPT} \tag{31}$$

Outline

RGS Non-Power Expansions

- We improve the convergence of the RGS expansion by the analytical continuation in the Borel plane.
- We introduce the Borel transform of the RGS expansion of the Adler function

$$B_{\mathrm{RGS}}(u,y) = B(u) + \sum_{n=0}^{\infty} \frac{u^n}{\beta_0^n\, n!} \sum_{j=1}^{n} c_{j,1} d_{n+1,j}(y), \;\; B(u) = \sum_{n=0}^{\infty} c_{n+1,1} \frac{u^n}{\beta_0^n\, n!}. \quad (32)$$

- The function $\widehat{D}_{\mathrm{RGS}}(s)$ is recovered by the Laplace-Borel integral

$$\widehat{D}_{\mathrm{RGS}}(s) = \frac{1}{\beta_0}\,\mathrm{PV}\int_0^\infty \exp\left(\frac{-u}{\beta_0 \tilde{a}_s(-s)}\right) B_{\mathrm{RGS}}(u,y)\,\mathrm{d}u,$$

- The function $B(u)$ has singularities on the real axis in the u-plane, namely along the rays $u \geq 2$ and $u \leq -1$.
  Mueller 1985, Beneke 1999
- However the dominant singularities of $B_{\mathrm{RGS}}(u,y)$, *i.e.* the singularities closest to the origin $u = 0$, are those at $u = -1$ and $u = 2$ contained in $B(u)$.

RGS Non-Power Expansions

- Moreover, the dominant singularities are branch points, near which $B(u)$ behaves, respectively, as

$$B(u) \sim (1+u)^{-\gamma_1}, \qquad B(u) \sim (1-u/2)^{-\gamma_2},$$

where the exponents γ_1 and γ_2, calculated using renormalization-group invariance, have known positive values

$$\gamma_1 = 1.21, \qquad \gamma_2 = 2.58\,. \tag{33}$$

Mueller 1985, Beneke, Brown & Kivel 1997, Beneke & Jamin 2008

RGS Non-Power Expansions

- We consider the functions

$$\widetilde{w}_{lm}(u) = \frac{\sqrt{1+u/l} - \sqrt{1-u/m}}{\sqrt{1+u/l} + \sqrt{1-u/m}}, \quad l \geq 1, m \geq 2 \tag{34}$$

  where l, m are positive integers satisfying $l \geq 1$ and $m \geq 2$. The function $\widetilde{w}_{lm}(u)$ maps the u-plane cut along $u \leq -l$ and $u \geq m$ onto the disk $|w_{lm}| < 1$ in the plane $w_{lm} \equiv \widetilde{w}_{lm}(u)$.

- We define further the class of compensating factors of the simple form

$$S_{lm}(u) = \left(1 - \frac{\widetilde{w}_{lm}(u)}{\widetilde{w}_{lm}(-1)}\right)^{\gamma_1^{(l)}} \left(1 - \frac{\widetilde{w}_{lm}(u)}{\widetilde{w}_{lm}(2)}\right)^{\gamma_2^{(m)}}, \tag{35}$$

- where the exponents are

$$\gamma_1^{(l)} = \gamma_1(1+\delta_{l1}), \quad \gamma_2^{(m)} = \gamma_2(1+\delta_{m2}),$$

  are chosen such that $S_{lm}(u)$ cancel the dominant singularities on the real axis in the u-plane.

RGS Non-Power Expansions

- We further expand the product $S_{lm}(u)B_{\mathrm{RGS}}(u,y)$ in powers of the variable $\widetilde{w}_{lm}(u)$, as

$$S_{lm}(u)B_{\mathrm{RGS}}(u,y) = \sum_{n\geq 0} c_{n,\mathrm{RGS}}^{(lm)}(y)\,(\widetilde{w}_{lm}(u))^n. \tag{36}$$

- We are led to the class of RGSNP expansions

$$\widehat{D}_{\mathrm{RGSNP}}(s) = \sum_{n\geq 0} c_{n,\mathrm{RGS}}^{(lm)}(y)\,\mathcal{W}_{n,\mathrm{RGS}}^{(lm)}(s), \tag{37}$$

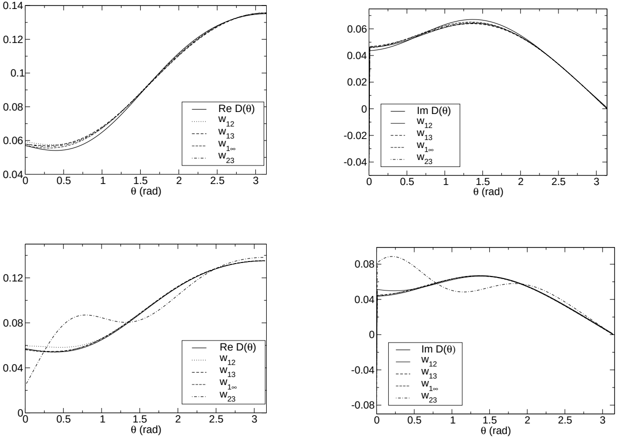
where

$$\mathcal{W}_{n,\mathrm{RGS}}^{(lm)}(s) = \frac{1}{\beta_0}\mathrm{PV}\int_0^\infty \exp\left(\frac{-u}{\beta_0\tilde{a}_s(-s)}\right)\frac{(\widetilde{w}_{lm}(u))^n}{S_{lm}(u)}\,\mathrm{d}u, \tag{38}$$

and the coefficients $c_{n,\mathrm{RGS}}^{(lm)}(y)$ are defined by the expansion (36).

- The coefficient, $\widetilde{a}_s(-s)$, entering in the Laplace-Borel integral is the one-loop solution of the RGE, a novel feature given by RGS.

Adler function in the complex plane



Clockwise from the left: The first figure is real part up to $N = 5$ terms. The same in the second with imaginary part. The third figure shows real part up to terms $N = 18$. The same in fourth figure with imaginary part.

- The RGSNP expansions provide a good description of the exact function along the whole circle, including the points near the timelike axis, which correspond to $\theta = 0$, and near the spacelike axis, where $\theta = \pi$.
- The worse approximation provided by the mapping w_{23} for $N = 18$ can be explained by the effect of the residual mild cut between $u = -1$ and $u = -2$, which limits the convergence radius of the expansion (36) in powers of w_{23} to $u < 1$.
- For other mappings, the divergence due to the residual cuts is manifest only for $u > 2$, and this region is more suppressed by the exponent in the Laplace-Borel integrals (38) defining the expansion functions .
  Caprini & Fischer, 2011

Outline

The convergence of RGSNP expansions

- The difference $\delta^{(0)} - \delta^{(0)}_{\rm exact}$ for the model $B_{\rm BJ}$ proposed in BJ model for $\alpha_s(M_\tau^2) = 0.34$ with the standard CI, FO and RGS expansions, and the new RGSNP expansions for various conformal mappings w_{lm}, truncated at order N. Exact value $\delta^{(0)}_{\rm exact} = 0.2371$

N	CI	FO	RGS	RGSNP w_{12}	RGSNP w_{13}	RGSNP $w_{1\infty}$	RGSNP w_{23}
2	-0.0595	-0.0679	-0.0574	-0.0347	-0.0239	-0.0417	-0.0177
3	-0.0473	-0.0345	-0.0440	-0.0333	-0.0301	-0.0349	-0.0303
4	-0.0388	-0.0171	-0.0347	-0.0089	-0.0142	-0.0067	-0.0132
5	-0.0349	-0.0083	-0.0315	-0.0070	-0.0086	-0.0058	-0.0070
6	-0.0325	-0.0043	-0.0284	-0.0073	-0.0071	-0.0064	-0.0072
7	-0.0325	-0.0029	-0.0298	-0.0059	-0.0057	-0.0056	-0.0044
8	-0.0354	-0.0018	-0.0309	-0.0041	-0.0035	-0.0041	-0.0011
9	-0.0367	-0.0004	-0.0363	-0.0023	-0.0019	-0.0028	-0.0010
10	-0.0529	0.0019	-0.0483	0.0014	-0.0012	-0.0020	0.0004
11	-0.0409	0.0031	-0.0458	0.0036	-0.0008	-0.0016	-0.0009
12	-0.1248	0.0065	-0.1335	0.0031	-0.0006	-0.0015	0.0005
13	0.0258	0.0037	0.0534	0.0026	-0.0004	-0.0015	-0.0005
14	-0.5286	0.0204	-0.7850	0.0018	-0.0003	-0.0015	-0.0011
15	0.8640	-0.0201	1.7734	0.0006	-0.0002	-0.0015	0.0044
16	-3.5991	0.1447	-7.7043	0.0001	$-7 \cdot 10^{-6}$	-0.0015	-0.0131
17	9.3560	-0.4252	24.8586	-0.0004	$4 \cdot 10^{-6}$	-0.0014	0.0238
18	-31.76	1.907	-94.26	-0.0013	-0.0001	-0.0013	-0.0310

Outline

Determination of α_s from RGSNP expansions

- We obtain with RGSNP expansions

$$\alpha_s(M_\tau^2) = 0.3189 \pm 0.0034_{\rm exp} \pm 0.0031_{\rm PC}\ {}^{+0.0138}_{-0.0105}({\rm c}_{5,1})\ \pm 0.0010_{\beta_4}, \tag{39}$$

after combining the errors in quadrature,

$$\alpha_s(M_\tau^2) = 0.3189\ {}^{+0.0145}_{-0.0115}. \tag{40}$$

- By evolving to the scale of M_Z our prediction reads

$$\alpha_s(M_Z^2) = 0.1184\ {}^{+0.0018}_{-0.0015}, \tag{41}$$

Outline

Summary

- This work is motivated by the well-known discrepancy between the predictions of $\alpha_s(M_\tau^2)$ from the standard fixed-order and RG-improved CIPT expansions.
- The main result is that the summation of leading logarithms provides a systematic expansion with good convergence properties in the complex plane.
- The results of the new RGS expansion is similar to those obtained by the CI expansion.
- The divergent character of the perturbative series is improved by analytic continuation in the Borel plane.
- The RGSNP exapansions lead to prediction for α_s which is similar to standard FOPT and CINP.