Dynamics of Fundamental Flavours
in
Holographic Duals of Large N Gauge Theories

Arnab Kundu
The University of Texas at Austin

Institute of Physics, Bhubaneswar
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Credits

Primarily based on:

ongoing enterprise in collaboration with
M. Sohaib Alam (Austin),
Matthias Ihl (Dublin),
Vadim Kaplunovsky (Austin),
Sandipan Kundu (Austin),
David Mateos (Barcelona)

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to appear, in progress, in conception etc ...

Outline

Introduction and Motivation

conventional wisdom

Gauge-gravity duality: specific realizations

Klebanov-Witten background

The dynamics of flavours and chiral symmetry breaking

non-susy D7/anti-D7 branes

External parameters and chiral symmetry breaking: phase structure

Electric and Magnetic field at finite temperature

in and beyond the probe limit

Conclusions and Outlook

General lessons etc.

Introduction & motivation

We want to learn about strongly coupled systems

e.g., Quark-Gluon Plasma at RHIC, strongly coupled condensed matter systems etc.

Gauge-gravity duality is a remarkable tool

soluble models

String theory provides concrete examples of this duality

specific brane constructions (top down)

Introduction & motivation

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QCD

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specific brane constructions (top down)

Gauge-gravity duality

General idea

QFT in d-dimensions is secretly a theory of Quantum gravity in (d+1)-dimensions

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Concrete example

AdS/CFT correspondence

classical gravity in (d+1)-dim anti de-Sitter space = strongly coupled conformal field theory in d-dimensions

string theory provides a large class of such examples including non-CFT

Controllable computations can be done for large N

Conventional wisdom

$3 = \infty$: an useful approximation

The duality works for large N gauge theories (super Yang-Mills)

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An example comparison:

	QCD	SYM	
$T = 0$	N=3, confinement, discrete spectrum, scattering, ...	large N, deconfined, conformal, supersymmetric, ...	
$T = T_c$	strongly coupled plasma of gluons and fundamental matter; deconfined, screening, finite correlation length, ...	strongly coupled plasma of gluons and adjoint + fundamental matter; deconfined, screening, finite correlation length, ...	⇨ Our focus
$T \gg T_c$	becomes weakly coupled	remains strongly coupled	

Elusive QCD features

Confinement

e.g. confining holographic duals: Klebanov-Strassler, global AdS

Confinement/deconfinement transition

e.g. Hawking-Page transition

Chiral symmetry breaking and chiral phase transition

e.g. Sakai-Sugimoto model

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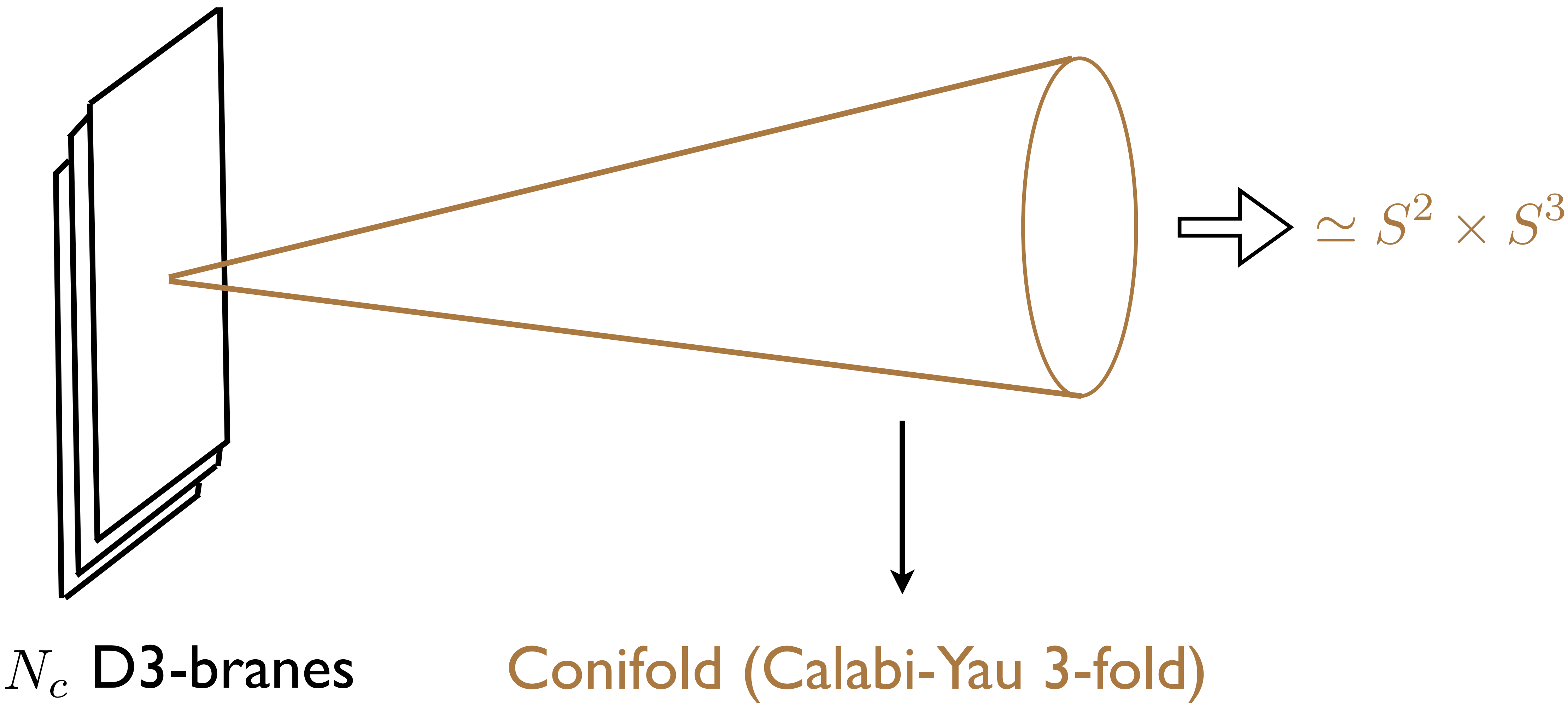
Kuperstein-Sonnenschein model

Specific example: the brane picture



Near-horizon geometry is: $AdS_5 \times T^{1,1}$ $(T^{1,1} \simeq S^2 \times S^3)$

Klebanov-Witten background (Romans' solution)

Specific example: the duality

String theory in $AdS_5 \times T^{1,1}$

isometry: $SO(4,2)$

$SU(2) \times SU(2) \times U(1)$

AdS-Schwarzschild geometry

$\mathcal{N} = 1$ quiver gauge theory

superconformal, global R-symmetry

$SU(N_c) \times SU(N_c)$ gauge group

two bi-fundamental chiral superfields

finite temperature
(broken susy)

The gravitational background

The metric: $$ds^2 = -\frac{r^2}{R^2} f(r) dt^2 + \frac{r^2}{R^2} d\vec{x}^2 + \frac{R^2}{r^2} \frac{dr^2}{f(r)} + R^2 ds^2_{T^{1,1}} \ ,$$

$$ds^2_{T^{1,1}} = \frac{1}{3}\left[\frac{1}{4}\left(f_1^2 + f_2^2\right) + \frac{1}{3} f_3^2 + \left(d\theta - \frac{1}{2} f_2\right)^2 + \left(\sin\theta d\phi - \frac{1}{2} f_1\right)^2\right] \ ,$$

$$R^4 = \frac{27}{4} \pi g_s N_c \alpha'^2 = \lambda \alpha'^2 \ . \qquad \{f_i\} \equiv S^3 \ , \quad \{\theta, \phi\} \equiv S^2 \ .$$

α' : string tension

λ : 't Hooft coupling

g_s : string coupling

$$f(r) = 1 - \left(\frac{r_H}{r}\right)^4 \ , \quad T = \frac{r_H}{\pi R^2} \ .$$

Euclideanize: $t \to i\tau$

Adding flavours

The background is obtained from near-horizon limit of a stack of N_c D-branes

We put N_f flavour-branes in the probe limit, i.e. $N_f \ll N_c$

The classical dynamics is determined by the probe action

$$S = -\mu_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det\left(\mathrm{P}[G+B] + 2\pi\alpha' F\right)} + S_{\mathrm{WZ}}$$

The DBI piece

The Wess-Zumino piece

Here we consider: $p = 7$

Adding flavours

	0	1	2	3	4	5	6	7	8	9
D3	–	–	–	–	•	•	•	•	•	•
D7, $\overline{\text{D7}}$	–	–	–	–	–	–	–	–	•	•

- stands for extended

• stands for point-like

3-3 strings: adjoint sector

3-7 strings: fundamental matter

7-7 strings: global symmetry

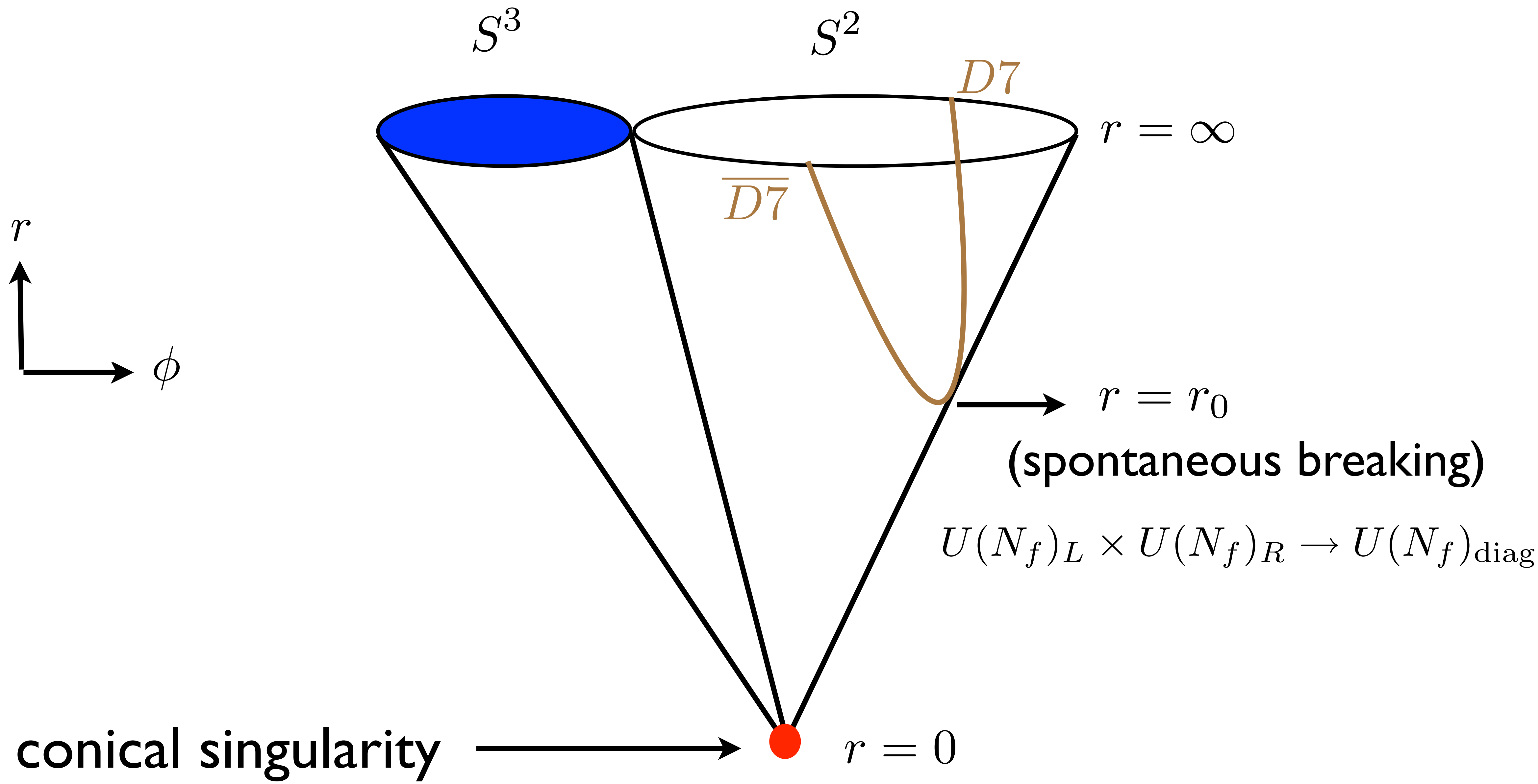
$$ds^2 = \frac{r^2}{R^2} f(r) d\tau^2 + \frac{r^2}{R^2} d\vec{x}^2 + \frac{R^2}{r^2} \frac{dr^2}{f(r)} + R^2 ds^2_{T^{1,1}} \ .$$

2-plane: $\{\theta(r), \phi(r)\}$

Equatorial embedding: $\theta = \frac{\pi}{2} \ , \quad \phi = \phi(r)$

Kuperstein & Sonnenschein '08

T=0 Physics



asymptotic angle separation: $\Delta\phi_\infty = \frac{\sqrt{6}}{4}\pi$.

Kuperstein & Sonnenschein '08

Key properties

The probe D7 and anti-D7 each break supersymmetry completely.

non-holomorphic embedding

Conformal theory.

$\Delta\phi_\infty$ *is* r_0 *independent.*

Spontaneous breaking of chiral symmetry.

$U(N_f)_L \times U(N_f)_R \to U(N_f)_{\rm diag}$

Spontaneous breaking of conformal symmetry.

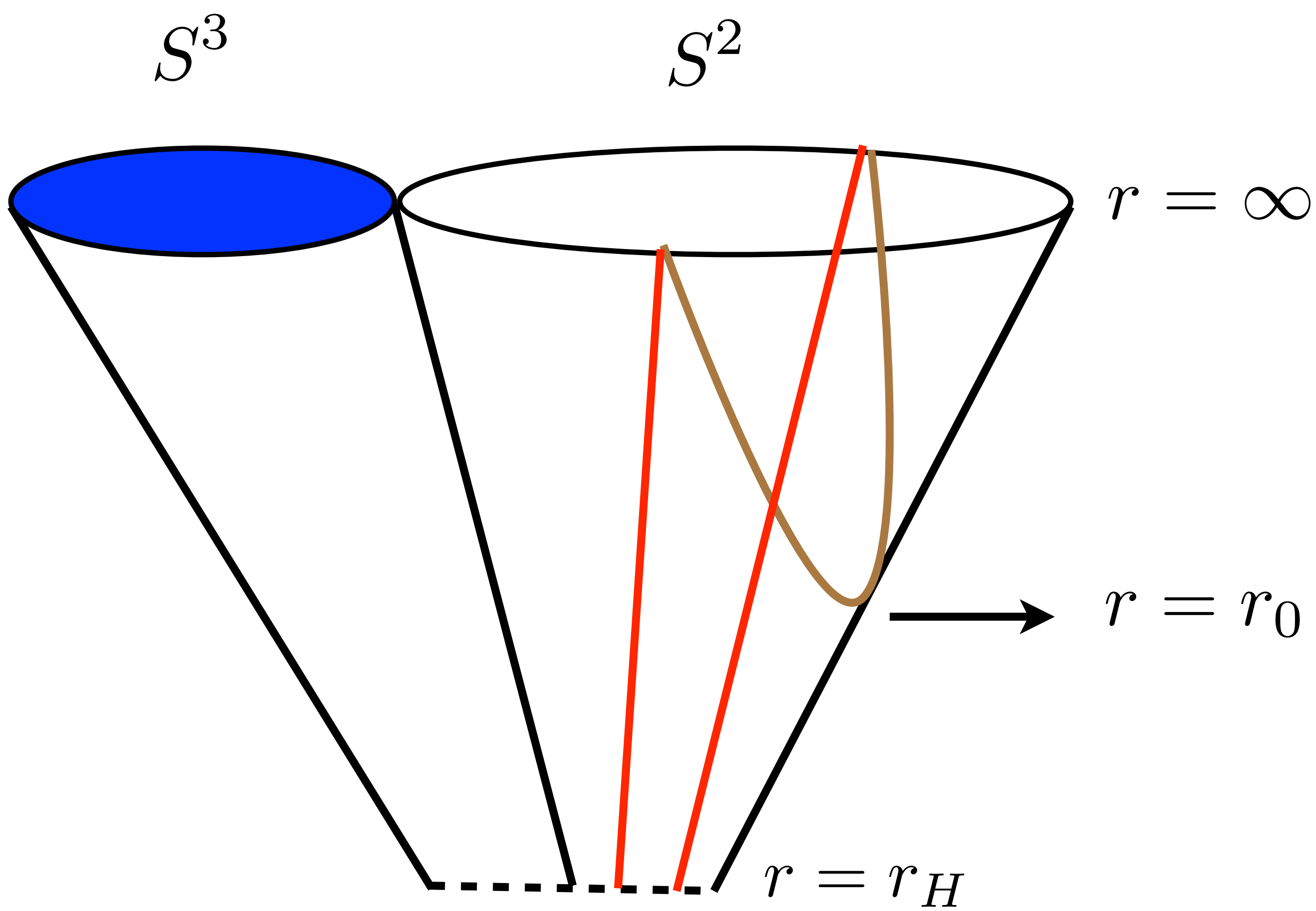
an IR scale (modulus) is generated

Perturbatively stable.

no pathology

Dymarsky, Melnikov & Sonnenschein '10

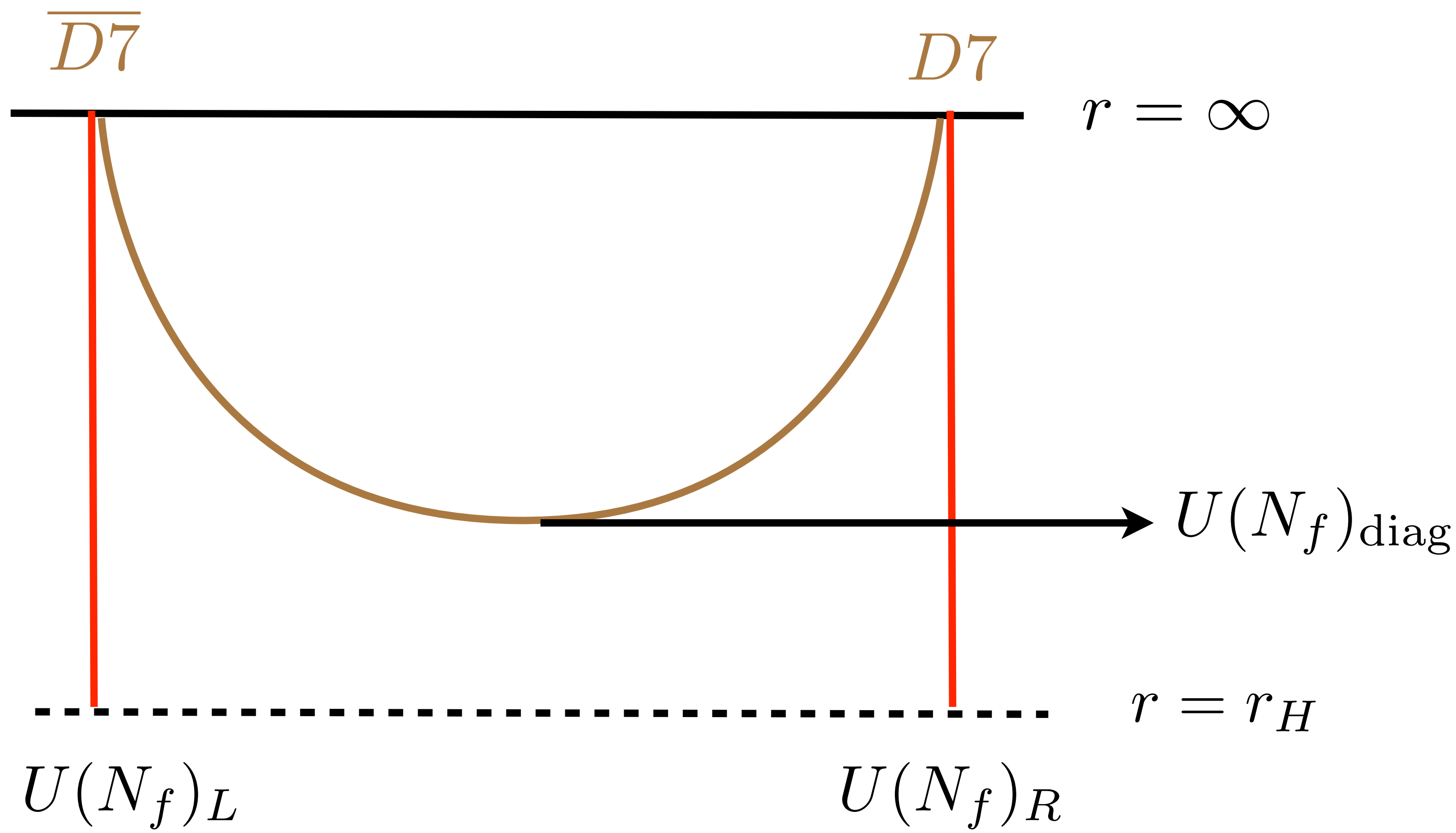
Introducing temperature



Two classes of embeddings: (i) probe branes reach black hole. *parallel-shaped*

(ii) probe branes join above the black hole. *U-shaped*

Introducing temperature



Favoured embedding determined by energetics.

$$S^{\text{on-shell}}_{\text{probe}} \leftrightarrow TF$$ ← Helmholtz free energy

Parallel shaped are always favoured.

Key features

Conformal symmetry explicitly broken by temperature.

Both chiral symmetry broken and restored phases are available as solutions.

No phase transition: symmetry restoration occurs at any temperature.

Need another scale to have a non-trivial phase structure.

New scale in the game

We can excite gauge fields on the world volume of the probe brane itself

Recall the DBI action contains: $\sqrt{-\det(\mathrm{P}[G+B]+2\pi\alpha' F)}$

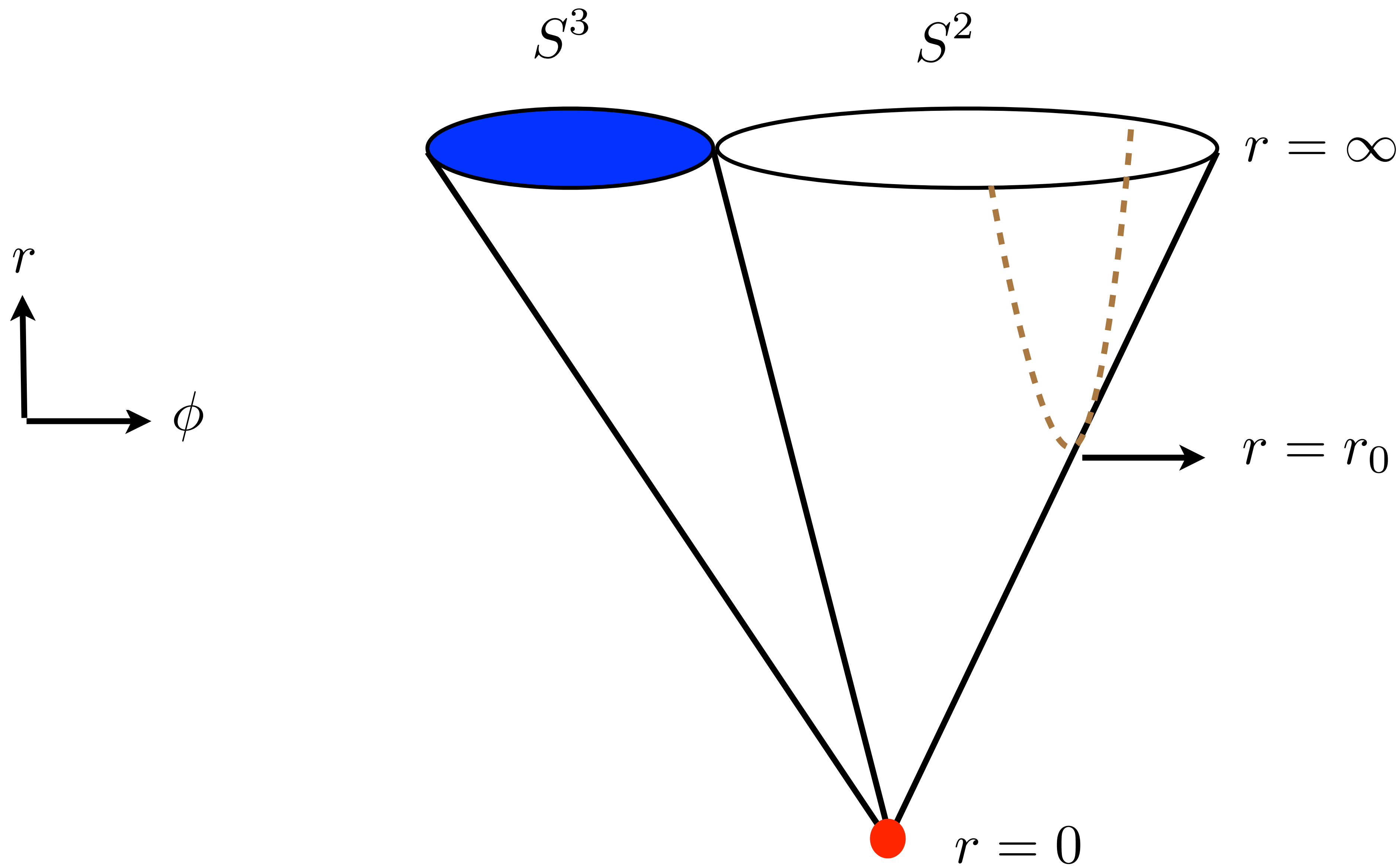
Still in the probe limit, background does not care

Have to satisfy the equations of motion

Simplest ansatz: $A_3 = Hx^2$

Introduces a magnetic field to which only the flavours couple: $F_{23} = H$

Physics at T=0



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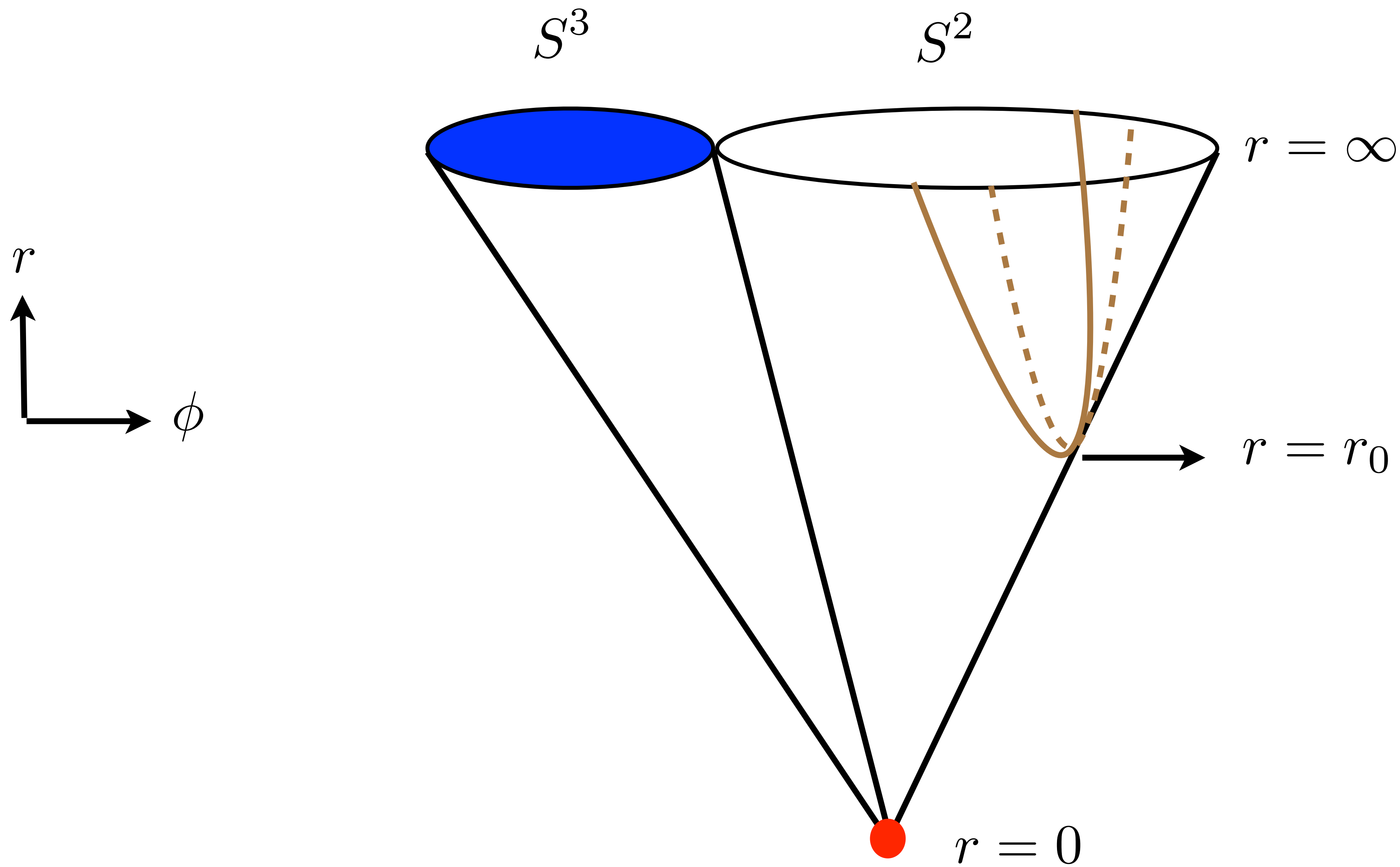


The magnetic field increases the (symmetry breaking) coupling:

$$\Delta\phi_\infty(H) > \Delta\phi_\infty(0)$$

Some features

Conformal symmetry explicitly broken by the magnetic field.

Magnetic field enhancing the symmetry breaking mechanism.

magnetic catalysis in chiral symmetry breaking

The coupling is not a constant anymore.

depends on the external parameters introduced.

Two competing scales

Finite temperature alone restores chiral symmetry.

Magnetic field alone enhances chiral symmetry breaking.

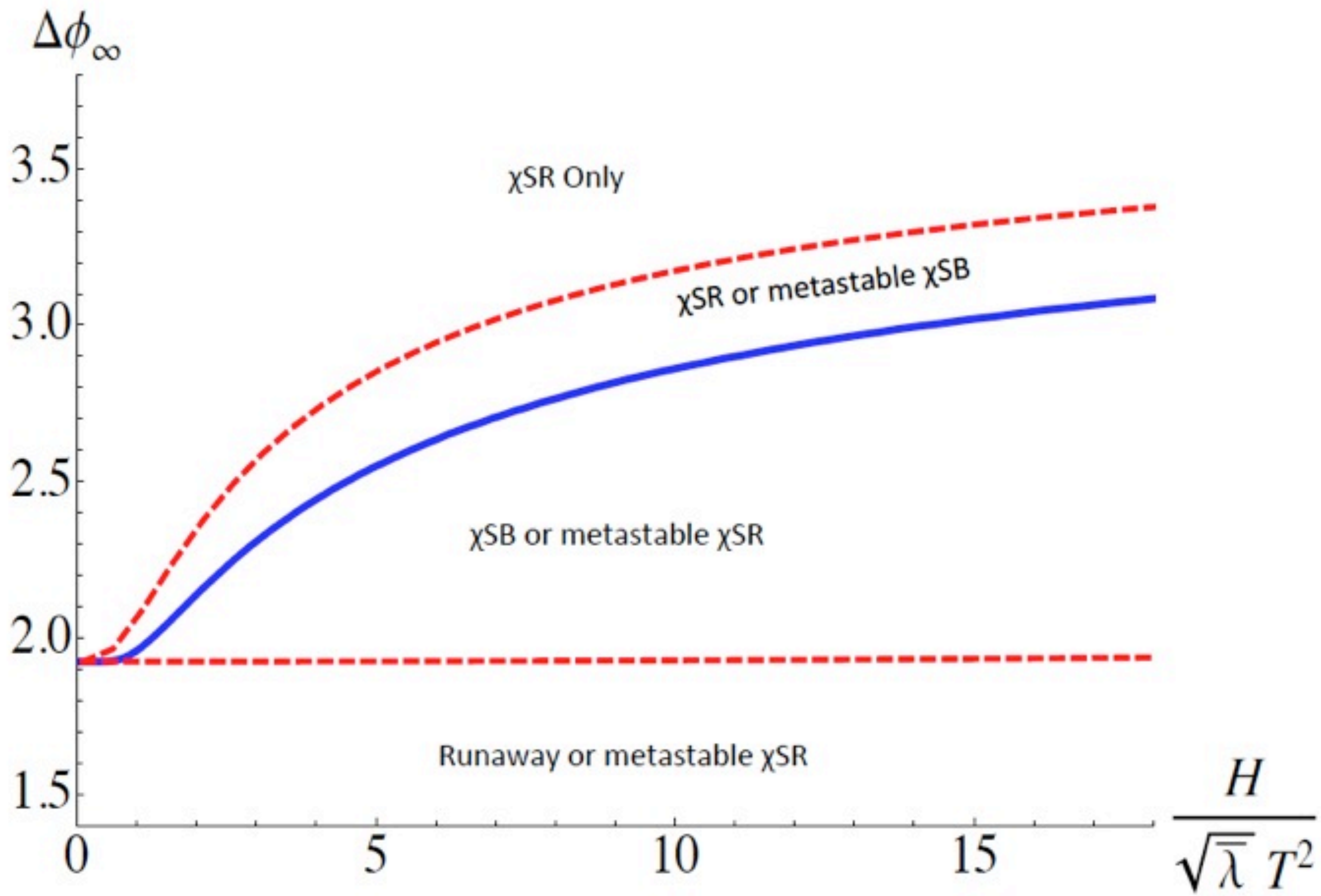
First order phase transition when both are present.

Phase boundary: obtained by looking at the energy difference of the two classes of embeddings.

Described by the curve: $\Delta\phi_\infty\left(H/T^2\right)$

monotonically increasing

The phase diagram



$$\bar{\lambda} = \frac{\pi^2}{4}\lambda$$

Magnetic Catalysis as expected

Introducing an electric field

Excite an appropriate gauge field on the probe: $A_x = -Et + A(r)$??

Gives a constant electric field: $F_{tx} = -E$

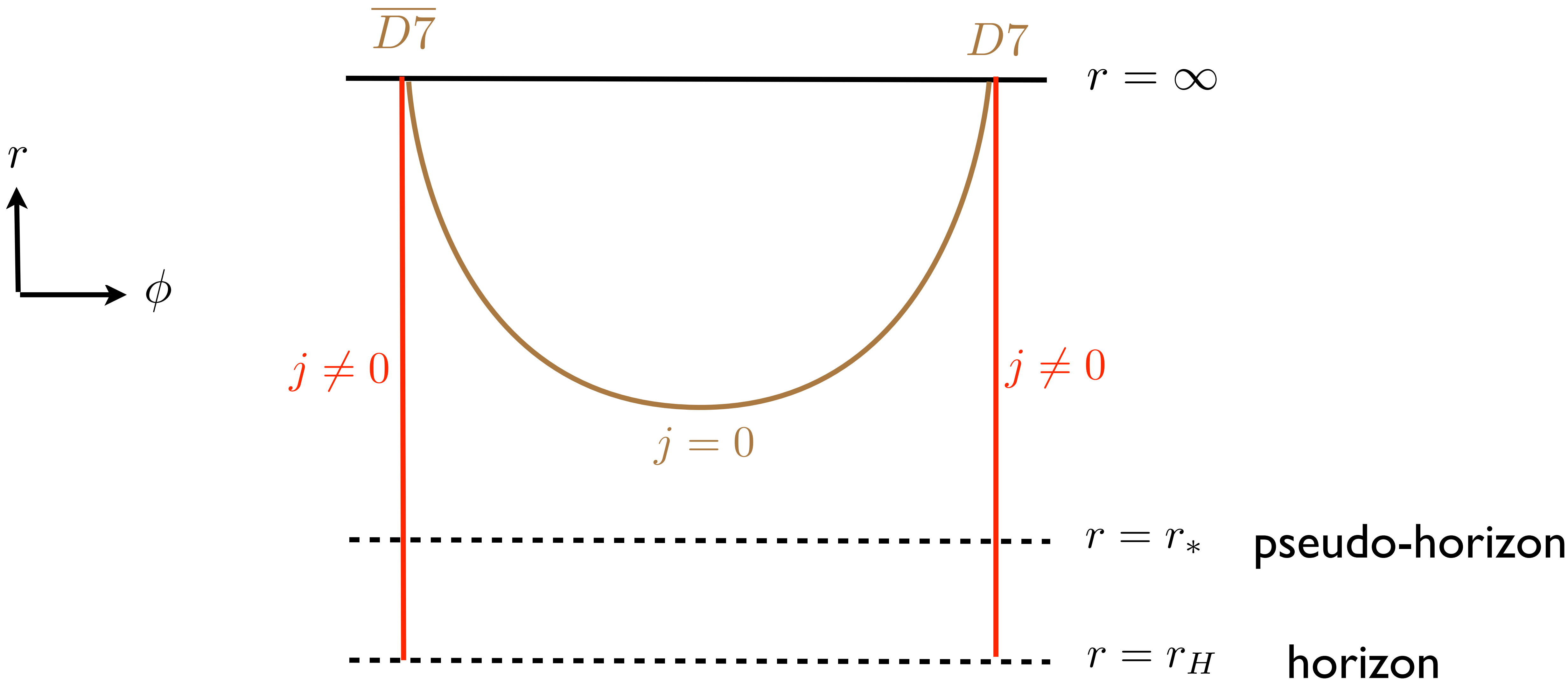
Assume: $A(r) = 0$, $\mathcal{L}_{\rm DBI} \sim \left(1 - \frac{E^2}{r^4 f(r)}\right)^{1/2}$, vanishes at: $r = r_* > r_H$

Cannot happen!

If: $A(r) \neq 0$, then E.O.M. $\Longrightarrow$ $\frac{\partial \mathcal{L}}{\partial A'} = j = \mathrm{const}$

AdS/CFT dictionary gives: $j \sim \langle J_x \rangle \equiv$ boundary current

Introducing an electric field



The condition of parallel branes reach the horizon $\Longrightarrow$ $j(E)$

Ohm's law

Introducing an electric field

Thermodynamic free energy ⟷ On-shell action

Parallel embeddings are subtle

There is a boundary term: $S_{\rm on-shell} \sim \int_{r_{\rm min}}^{\infty} \mathcal{L}(A', r) dr + jA|_{r_{\rm min}}$.

Usual identification: $r_{\rm min} = r_H$.

Introducing an electric field

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The gauge field blows up at the horizon!

Our proposal: $r_{\min} = r_* = \left(r_H^4 + R^4 E^2\right)^{1/4}$.

A brief summary

Ubiquitous effect of magnetic catalysis in chiral symmetry breaking persists at strong coupling

for field theory studies, see e. g. Miransky et al.

Electric field restores the symmetry, drives a flavour current

non-linear conductivity.

Emergence of pseudo-horizon: a natural way to define thermodynamics in a steady-state system

an intriguing feature: effective temperature

Beyond the probe limit

Consider the probes without any external parameters

Need to work with the action: $S = S_{\rm sugra} + S_{\rm probe}$

$$S_{\rm sugra} \sim N_c^2 \ , \quad S_{\rm probe} \sim \lambda N_f N_c$$

Geometry receives perturbative corrections as powers in: $\epsilon \sim \dfrac{N_f}{N_c}$

Analytical solution can be obtained at the leading order in ϵ

Key features

Analytical solution

we're lucky!

Conformal symmetry explicitly broken by the back-reaction

existence of a Landau pole, UV-incomplete

The dual field theory is now a deformed CFT

irrelevant deformations by dim 8 & dim 6 operators

The Landau pole provides a scale

finite T phase transition is possible now

Introducing external parameters

The qualitative physics is not modified

perhaps expected, since the back-reaction is only a small parameter

But there is a phase transition at finite temperature

caused by the presence of the Landau pole, which breaks conformal symmetry

The back-reaction does not change the order of the phase transitions

may be a limitation of the model

Conclusions and Outlook

Interesting phenomenological consequences of a magnetic field

universal magnetic catalysis at strong coupling + more

Our proposal of thermodynamic free energy in an electric field

intriguing possibility for non-equilibrium systems, fluctuation-dissipation relation implies an effective temperature

Back-reaction with more relevant parameters are being explored

e.g. chemical potential, possibility of colour superconductivity(?)

e.g. magnetic field, possibility of anisotropic background

etc...

More general lessons of strong coupling physics

perhaps applicable elsewhere

Pictures speak a thousand dualities

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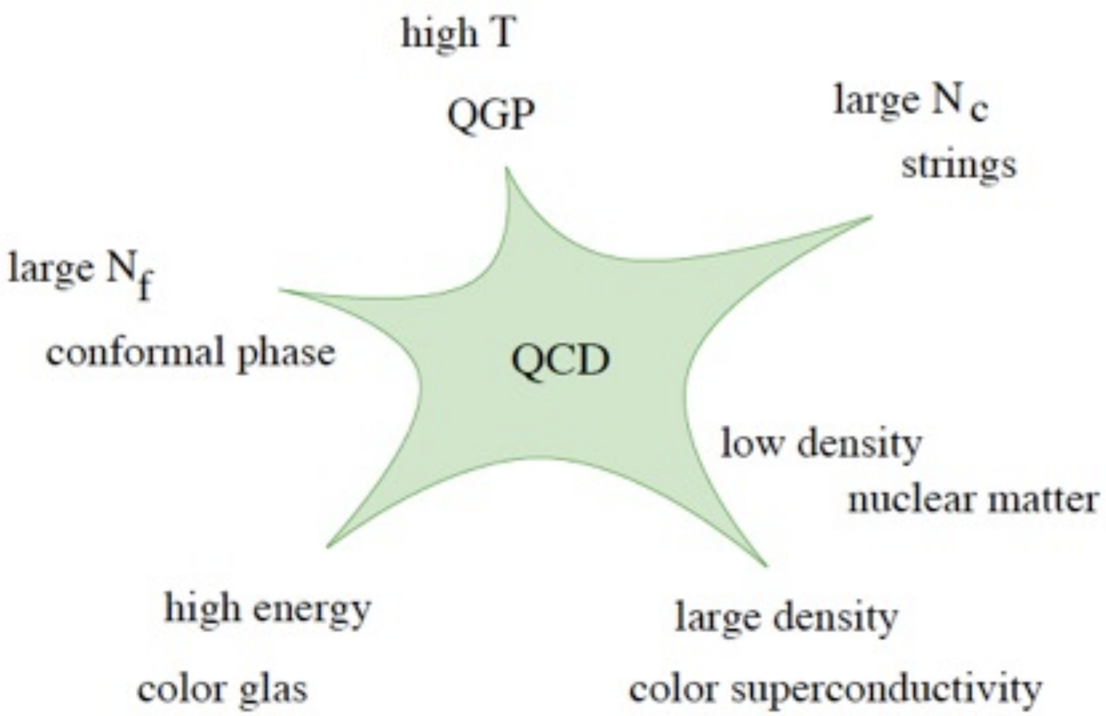
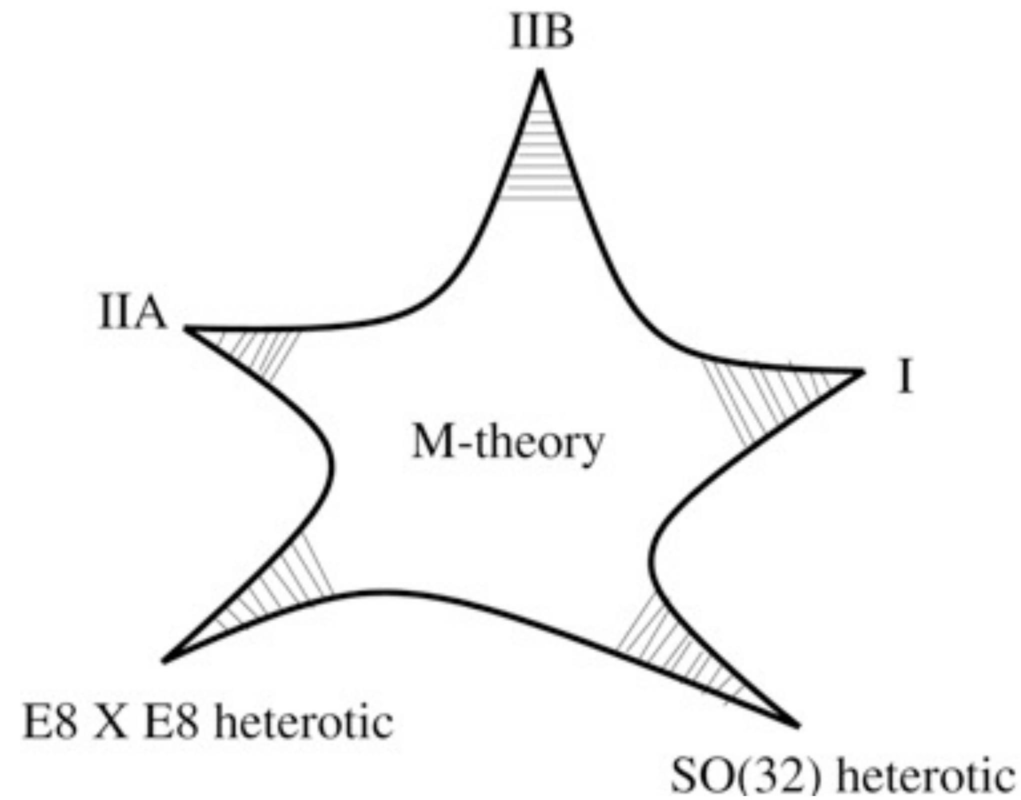
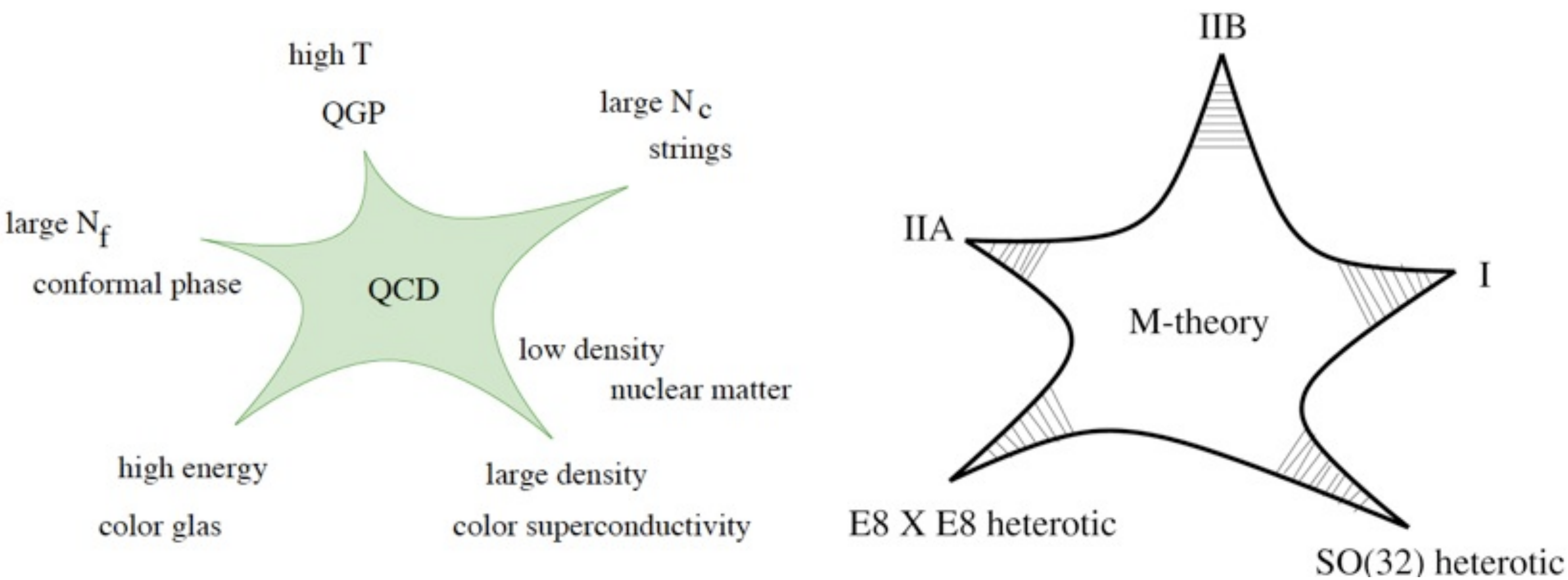




Picture courtesy: Phases of QCD by T. Schafer and the web

Don't they look alike?

Pictures speak a thousand dualities



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Thank You

Other holographic models

D7-probes in $AdS_5 \times S^5$ $\longleftrightarrow$ $\mathcal{N}=4$ SYM $+$ $\mathcal{N}=2$ hypers

transverse 2-plane $\longleftrightarrow$ $SO(2) \simeq U(1)$

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transverse 2-plane $\longleftrightarrow$ $SO(2) \simeq U(1)$

T=0 physics is supersymmetric, no chiral symmetry breaking

T=0, non-zero magnetic field induces a chiral condensate

non-trivial phase diagram when both T and H are present

Other holographic models

N_c D4-branes wrapped on a circle ⟺ (4+1)-dim Yang-Mills with flavours

add D8 and anti-D8 (probe) branes ⟺ $U(N_f)_L \times U(N_f)_R$

(Sakai-Sugimoto model)

spontaneous breaking of chiral symmetry

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spontaneous breaking of chiral symmetry

chiral symmetry restoration transition at finite temperature

once again, magnetic catalysis in chiral symmetry breaking

not an honest (3+1)-dim gauge theory

running dilaton, lacks UV completion