Non-zero θ_{13} and Beyond

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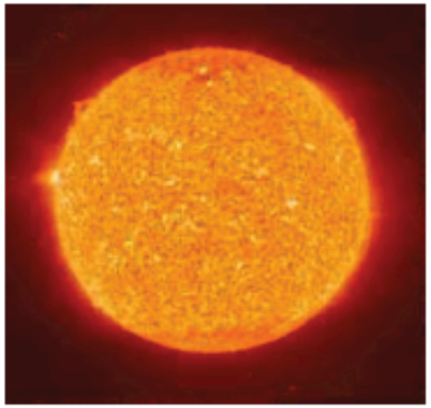
Golden Age of Neutrino Physics (1998 – 2012 & Beyond)

sun

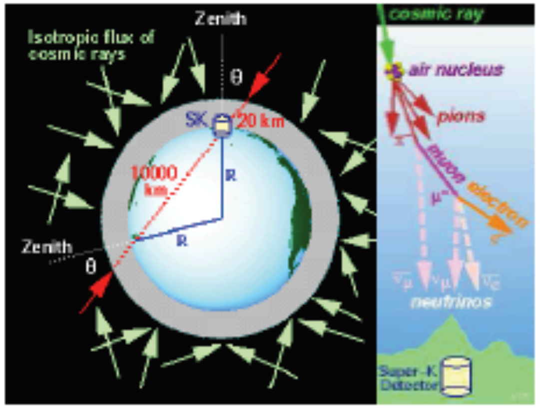


Homestake,SAGE,GALLEX
SuperK, SNO, Borexino

reactors

KamLAND, CHOOZ
Double Chooz, Daya Bay, RENO

atmosphere



SuperKamiokande
IceCube

accelerators

K2K, MINOS, T2K

Over the last fourteen years or so, marvellous data from world-class experiments

- Solar neutrinos (ν_e)
- Atmospheric neutrinos $(\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e)$
- Reactor anti-neutrinos $(\bar{\nu}_e)$
- Accelerator neutrinos $(\nu_\mu, \bar{\nu}_\mu)$

Data driven field – new data are coming

Data from various neutrino sources and vastly different energy and distance scales

We have just started our journey in the mysterious world of neutrinos!

Neutrino Flavor Oscillations

Neutrino oscillation experiments have revealed that neutrinos change flavor after propagating a finite distance. The rate of change depends on the **neutrino energy E_ν** and **the baseline L**

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments ["indisputable"];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments ["indisputable"];
- $\bar{\nu}_e \to \bar{\nu}_{\text{other}}$ — reactor neutrinos ["indisputable"];
- $\nu_\mu \to \nu_{\text{other}}$ from accelerator experiments ["indisputable"].

The simplest and **only satisfactory** explanation of **all** this data is that neutrinos have distinct masses, and they mix. A 3 flavor ν oscillation framework can accommodate all the data

Finite neutrino masses required by the experimental data provide the
first hint for physics beyond the Standard Model

!! An extension of the Standard Model is necessary !!

New parameters (masses, angles, phases) need to be measured and understood!

Neutrino Flavor Oscillations (continued..)

- Neutrino oscillations occur because the flavor (weak) eigenstates do not coincide with the mass eigenstates

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle \quad (\alpha = e, \mu, \tau) \qquad U_{PMNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

- The neutrinos interact as a flavor state, but propagate as a superposition of the three mass states. Over a distance *L*, changes in the relative phases of the mass states may induce neutrino flavor change
- Assume two neutrino flavors for simplicity:

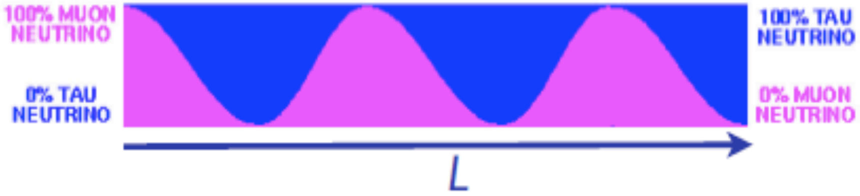
$$\begin{bmatrix} \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_2 \\ \nu_3 \end{bmatrix} \qquad |\nu_\mu(t)\rangle = \cos\theta e^{-iE_2 t}|\nu_2\rangle + \sin\theta e^{-iE_3 t}|\nu_3\rangle$$

$$E_i = \sqrt{p^2 + m_i^2}$$

- Probability that a ν_μ remains ν_μ after some time *t*:

$$\boxed{P(\nu_\mu \to \nu_\mu) = |\langle \nu_\mu|\nu_\mu(t)\rangle|^2 \simeq 1 - \sin^2(2\theta)\,\sin^2\left(1.27\,\Delta m^2_{32}\frac{L[\text{km}]}{E[\text{GeV}]}\right)}$$

- Neutrino oscillations depend on *L*, the neutrino energy *E*, and the mixing parameters

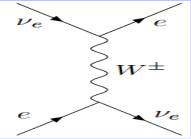


Courtesy to A. Sousa

Neutrino Oscillations in Matter

- ***Interactions in matter modify the oscillation probability significantly***
- ***Coherent forward elastic scattering of neutrinos with matter particles***
- ***Charged current interaction of ν_e with electrons creates a potential for ν_e***

$$A = \pm 2\sqrt{2} G_F \cdot E \cdot n_e$$



n_e = electron number density, + (-) for neutrinos (anti-neutrinos)

Creates an additional phase for ν_e and changes the oscillation probability

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq 0$$ → ***even if $\delta = 0$, causes fake CP asymmetry***

$$\Delta m^2 \simeq A \quad \Leftrightarrow \quad E_{\rm res}^{\rm Earth} = 6 - 8\,{\rm GeV}$$ → ***Resonant conversion – the MSW effect***

	ν	$\bar{\nu}$
$\Delta m^2 > 0$	MSW	-
$\Delta m^2 < 0$	-	MSW

→ ***Resonance occurs for neutrinos (anti-neutrinos) if Δm^2 is positive (negative)***

Three Flavor Mixing Hypothesis

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric sector

connection between
solar and atmospheric

$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

solar sector

U is a 3×3 unitary matrix containing θ_{23} , θ_{13} , θ_{12} and one CP violating (Dirac) phase δ

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

$\sin^2\theta_{23} = 0.52^{+0.06}_{-0.07}$ (θ_{23}= 46.14°)

$$\Delta m^2_{31} = \begin{array}{c} 2.50^{+0.09}_{-0.16} \\ -(2.40^{+0.08}_{-0.09}) \end{array} \times 10^{-3}\text{eV}^2$$

$\sin^2\theta_{12} = 0.312^{+0.017}_{-0.015}$ (θ_{12}= 33.95°)

$$\Delta m^2_{12} = (7.59^{+0.20}_{-0.18}) \times 10^{-5}\,\text{eV}^2$$

2011-2012: Important Breakthroughs in 1-3 mixing

- ☐ T2K (June 2011): $\sin^2 2\theta_{13} = 0.03 - 0.34$ @ 90% C.L

  T2K collaboration, arXiv:1106.2822 [hep-ex]

- ☐ MINOS (July 2011): $\sin^2 2\theta_{13} \neq 0$ @ 89% C.L.

  MINOS collaboration, arXiv:1108.0015 [hep-ex]

- ☐ Double CHOOZ (December 2011): $\sin^2 2\theta_{13} = 0.017 - 0.16$ @ 90% C.L

  Double CHOOZ collaboration, arXiv:1112.6353 [hep-ex]

- ☐ Daya Bay (March 2012): $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ @ 68% C.L.

  Daya Bay collaboration, arXiv:1203.1669 [hep-ex]

  $\sin^2 2\theta_{13} \neq 0$ @ 5.2σ

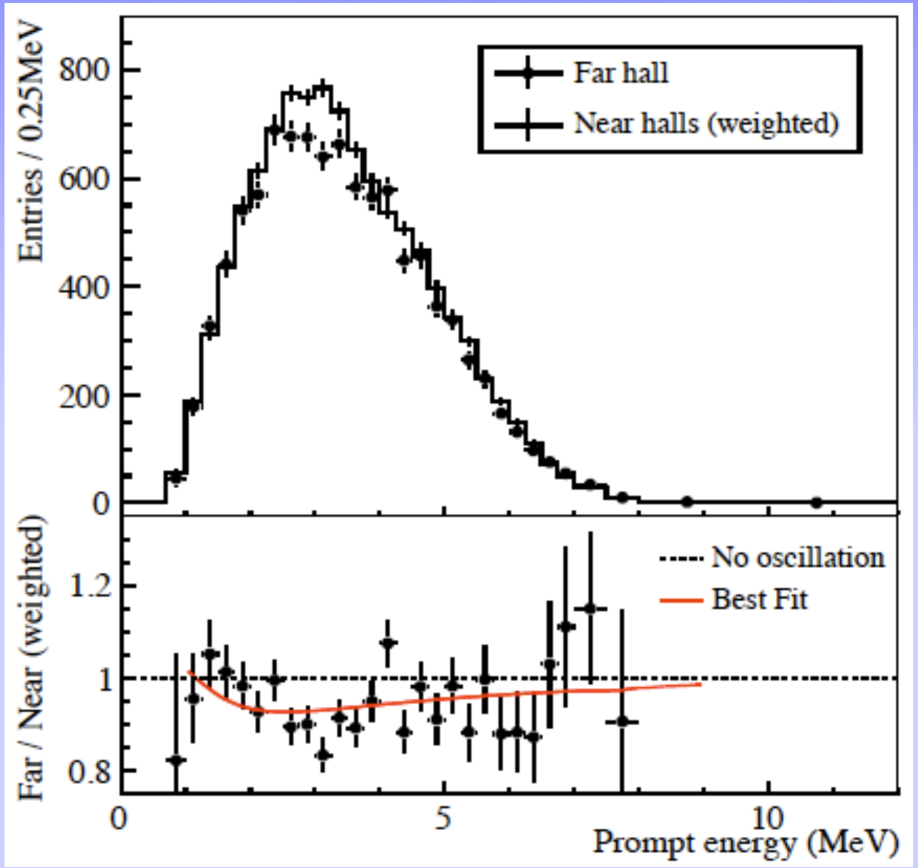
- ☐ RENO (April 2012): $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$ @ 68% C.L.

  RENO collaboration, arXiv:1204.0626v2 [hep-ex]

  $\sin^2 2\theta_{13} \neq 0$ @ 4.9σ

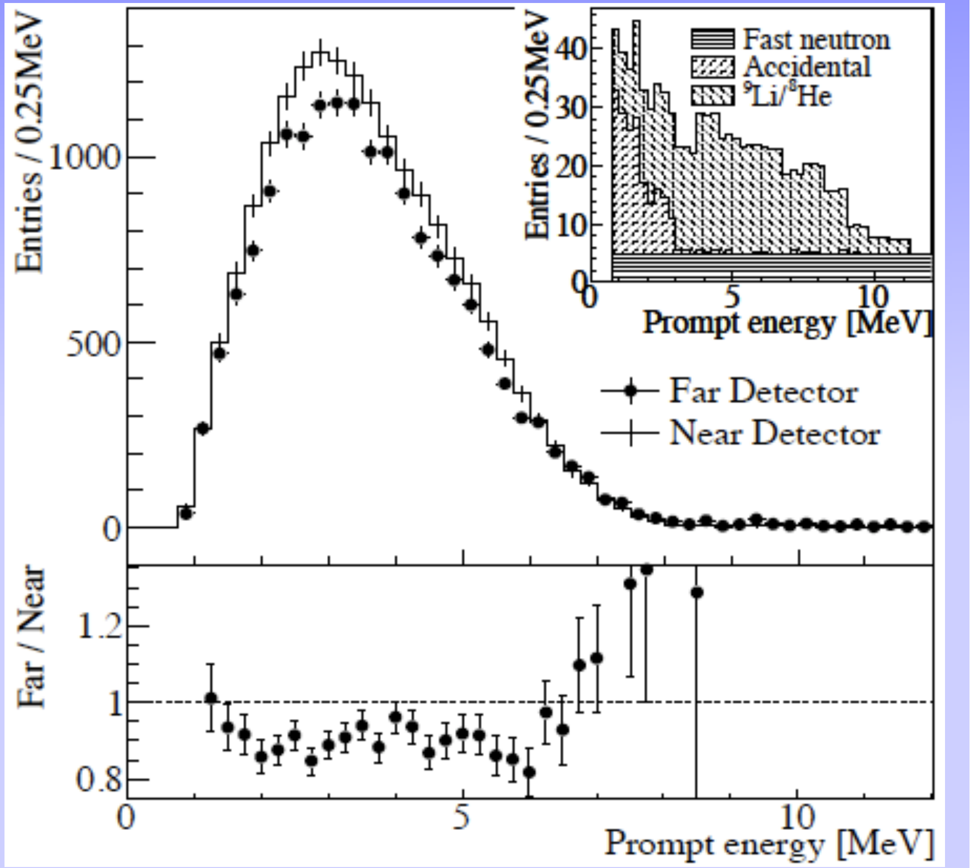
The θ_{13} Revolution

Daya Bay (5.2σ)



$$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$$

RENO (4.9σ)

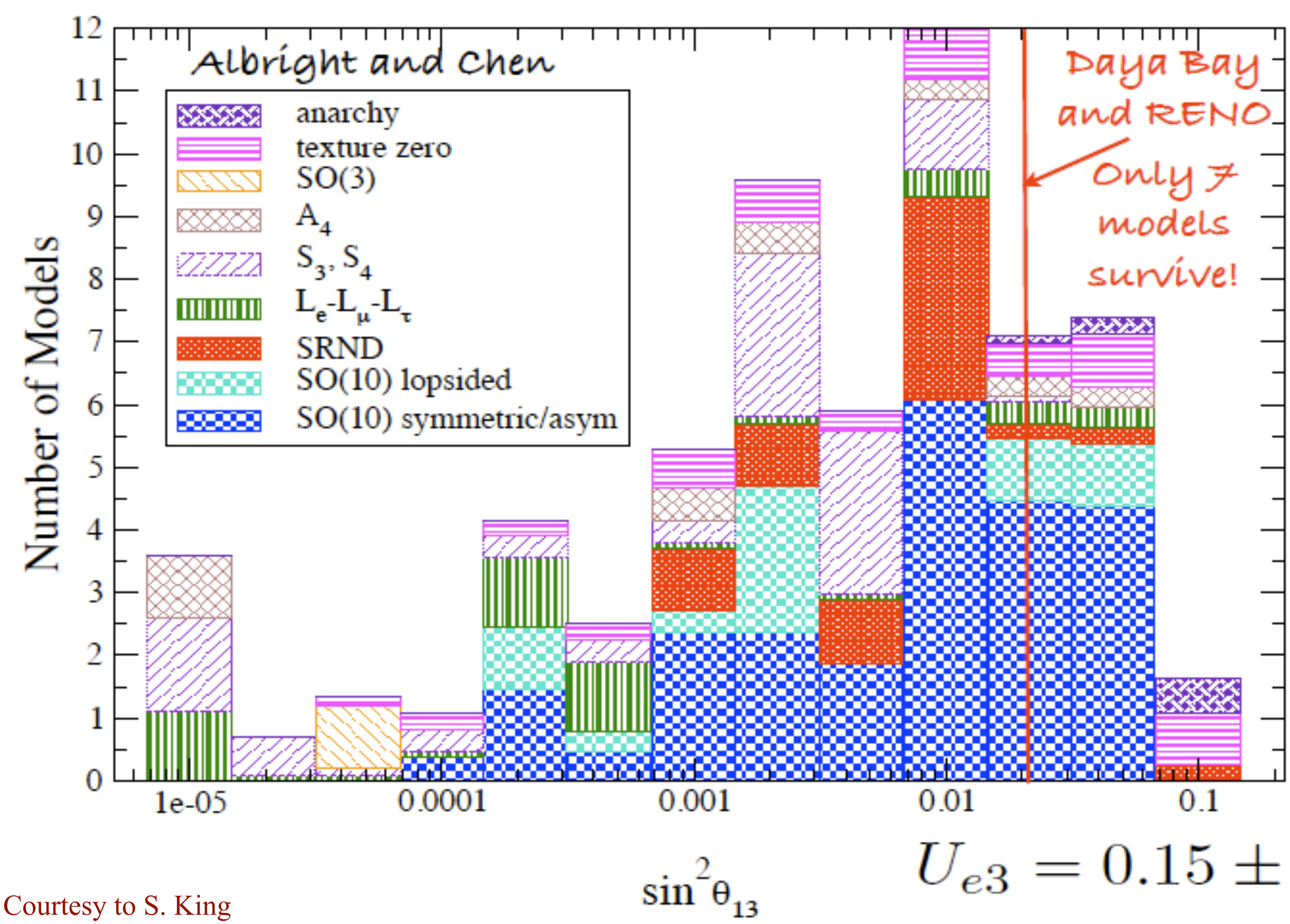


$$\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$$

Big News: We have discovered the 1-3 mixing angle!

By the end of 2012, this will be the most precisely known mixing angle in the PMNS matrix!

Latest results on θ_{13}: What happened to Mass models?



$$U_{e3} = 0.15 \pm 0.02$$

Courtesy to S. King

Is tri-bimaximal neutrino mixing pattern still allowed?

See, Brahmachari and Raychaudhuri, arXiv:1204.5619v1 [hep-ph]

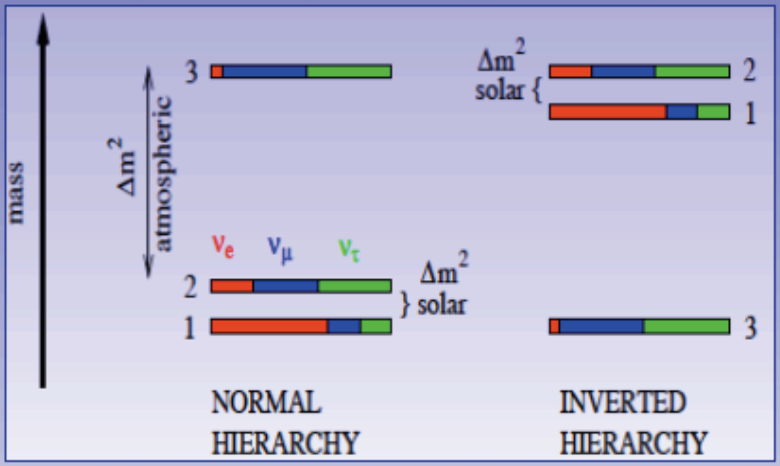
Unsolved Issues in Neutrino Oscillation

- The sign of Δm^2_{31} $(m^2_3 - m^2_1)$ is not known

It can be normal – $\Delta m^2_{31} > 0$

or

inverted hierarchical – $\Delta m^2_{31} < 0$



- Like in the quark sector, mixing can cause CP violation in leptonic sector

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq 0$$

(where $\alpha \neq \beta$)

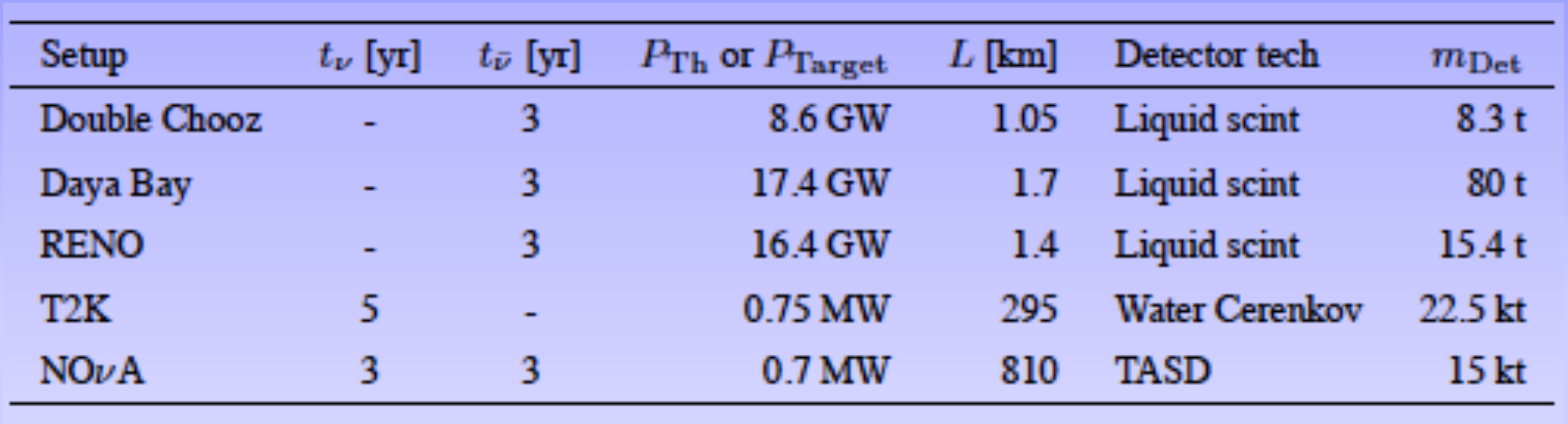
CP-odd asymmetries : ➔ $\boxed{\Delta P_{ab} \equiv P(\nu_a \to \nu_b; t) - P(\bar{\nu}_a \to \bar{\nu}_b; t)}$

$$\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e} = 4 s_{12}\, c_{12}\, s_{13}\, c^2_{13}\, s_{23}\, c_{23}\, \sin\delta \times \left[\sin\left(\frac{\Delta m^2_{21}}{2E}t\right) + \sin\left(\frac{\Delta m^2_{32}}{2E}t\right) + \sin\left(\frac{\Delta m^2_{13}}{2E}t\right)\right]$$

- δ is unknown, asymmetry vanishes if δ is 0° or 180° and maximum for 90° or 270°
- Need at least 3 generations to observe leptonic CP-violation, suppressed by θ_{13}

The Current Generation

Currently running & upcoming superbeam and reactor experiments



Setup	t_ν [yr]	$t_{\bar\nu}$ [yr]	P_{Th} or P_{Target}	L [km]	Detector tech	m_{Det}
Double Chooz	-	3	8.6 GW	1.05	Liquid scint	8.3 t
Daya Bay	-	3	17.4 GW	1.7	Liquid scint	80 t
RENO	-	3	16.4 GW	1.4	Liquid scint	15.4 t
T2K	5	-	0.75 MW	295	Water Cerenkov	22.5 kt
NOνA	3	3	0.7 MW	810	TASD	15 kt

P. Huber etal., JHEP 11 044 (2009)

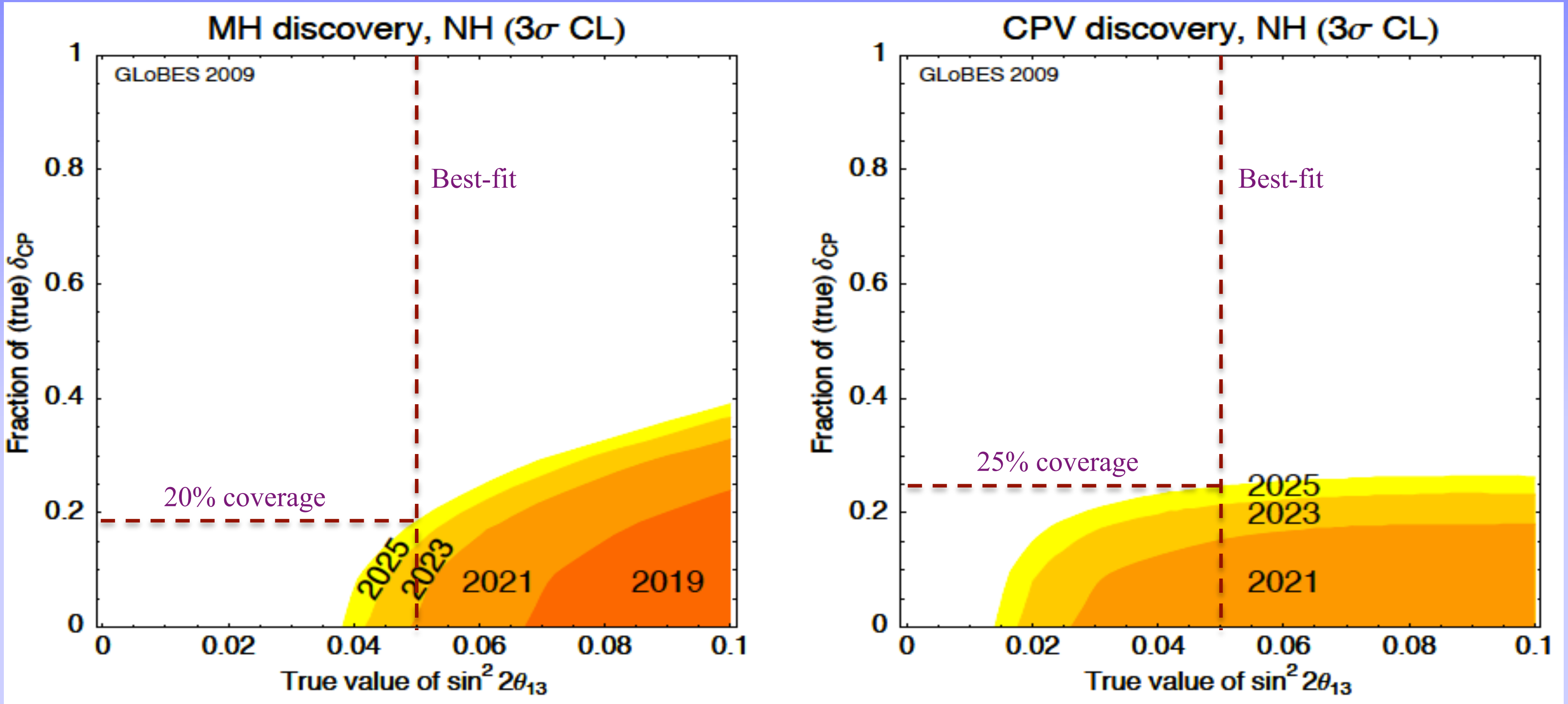
Double Chooz, Daya Bay, RENO: Reactor experiments

- Electron anti-neutrino disappearance at reactors with L ≈ 1 km
- "Clean" measurement of θ_{13} : $P \approx 1 - \sin^2 2\theta_{13} \sin^2(\Delta m^2_{31} L/4E)$

T2K, NOνA : ν_μ to ν_e transitions at Accelerator experiments

- Oscillation probability complicated : Depends on θ_{13}, δ_{CP}, and mass hierarchy

MH & CPV discovery without new experiments



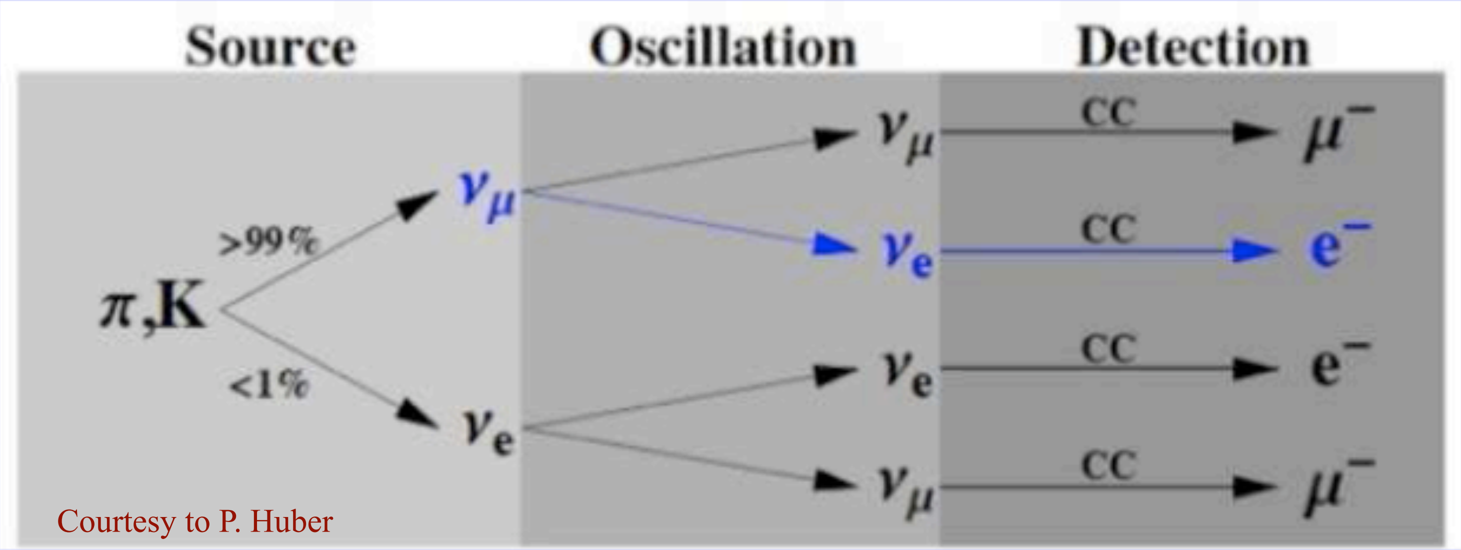
P. Huber etal., JHEP 11 044 (2009)

Expectation in 2025 without new facilities at 3σ C.L.

Combined results expected from: T2K + NOνA + Double Chooz + Daya Bay + RENO
(Including Project X and T2K operating at 1.66 MW)

More than 70% of parameter space are not accessible. New experiments needed

Superbeams



Courtesy to P. Huber

Neutrino beam from π-decay

They are called “super” : why?

- Beam power ∼ 1 MW
- Detector mass ∼ 100 kt
- Running time of the experiment ∼ 10 years
- Price

S. K. Agarwalla, Calcutta University, Kolkata, India, 26th April, 2012

Platinum Channel ($P_{\mu e}$)

The appearance probability $(\nu_\mu \to \nu_e)$ **in matter, upto second order in the small parameters** $\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31}$ **and** $\sin 2\theta_{13}$,

$$
\begin{aligned}
P_{\mu e} \quad \simeq \quad & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \Rightarrow \theta_{13}\text{ Driven} \\
- \quad & \alpha \sin 2\theta_{13}\, \xi \sin\delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \Rightarrow \text{CP odd} \\
+ \quad & \alpha \sin 2\theta_{13}\, \xi \cos\delta_{CP} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \Rightarrow \text{CP even} \\
+ \quad & \alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}; \Rightarrow \text{Solar Term}
\end{aligned}
$$

where $\Delta \equiv \Delta m^2_{31} L/(4E)$, $\xi \equiv \cos\theta_{13} \sin 2\theta_{21} \sin 2\theta_{23}$,
and $\hat{A} \equiv \pm(2\sqrt{2} G_F n_e E)/\Delta m^2_{31}$

Cervera etal., hep-ph/0002108
Freund etal., hep-ph/0105071

Eight-fold Degeneracy

- $(\theta_{13}, \delta_{CP})$ intrinsic degeneracy

  Burguet-Castell, Gavela, Gomez-Cadenas, Hernandez, Mena, hep-ph/0103258

- $(sgn(\Delta m^2_{31}), \delta_{CP})$ degeneracy

  Minakata, Nunokawa, hep-ph/0108085

- $(\theta_{23}, \pi/2 - \theta_{23})$ degeneracy

  Fogli, Lisi, hep-ph/9604415

Severely deteriorates the sensitivity

Future Facilities for Long Baseline Neutrinos

In USA

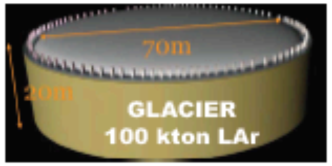
LBNE – a plan to build a new neutrino beam at Fermilab aimed at Homestake, where either a large water Cerenkov detector or a LAr tracking calorimeter would be built

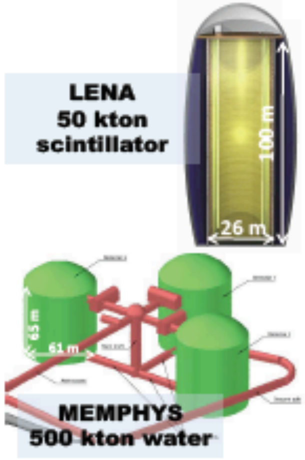
In Europe

LAGUNA

European design study for Large Apparatus for Grand Unification and Neutrino Astrophysics

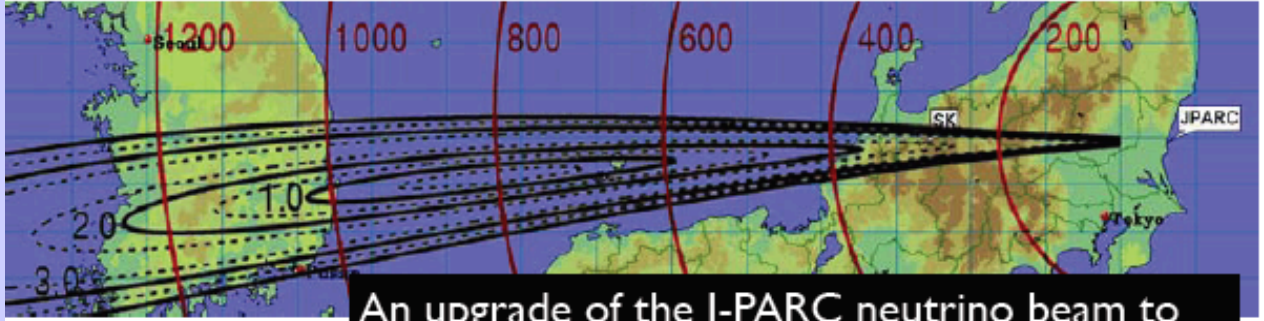
LAGUNA-LBNO





LAGUNA/LAGUNA-LBNO – study considering three detector options for astroparticle physics and new long baseline in Europe

In Japan

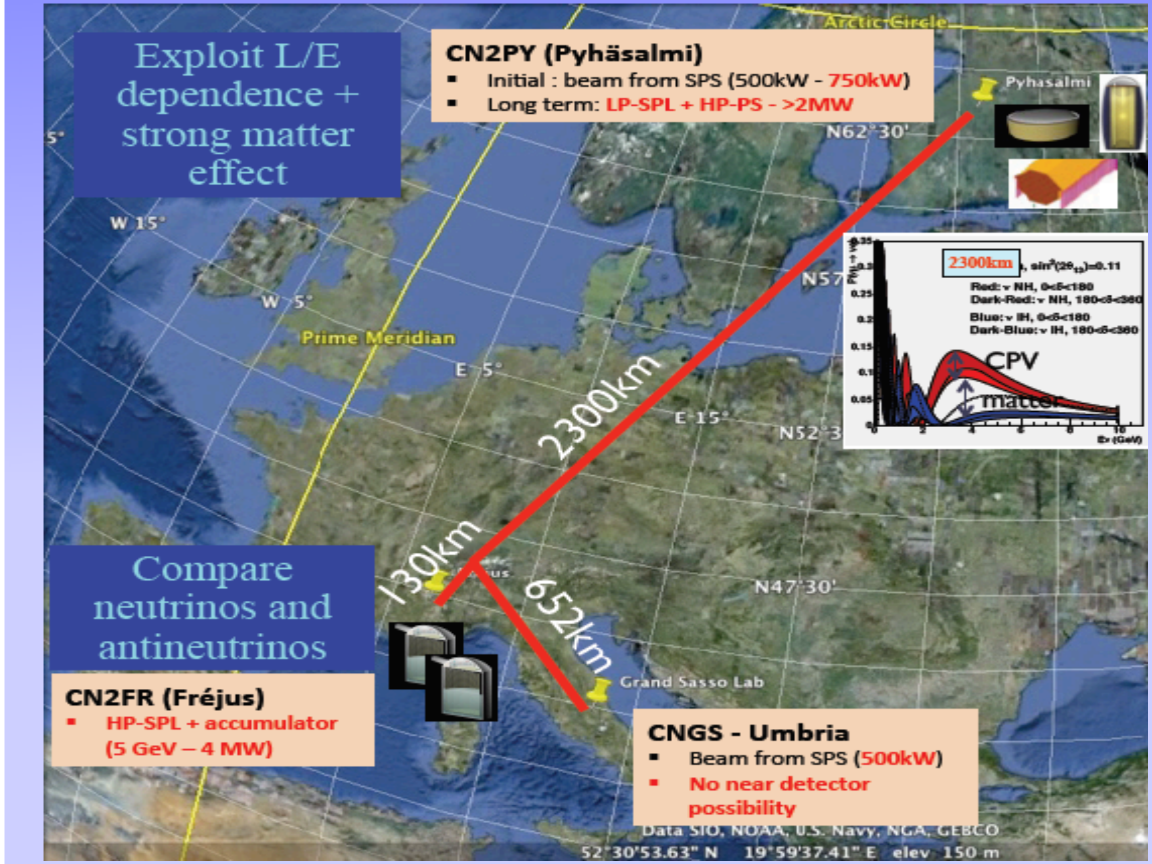


An upgrade of the J-PARC neutrino beam to reach 1.6 MW beam power and new far detector(s) at Kamioka, Okinoshima, or Korea

Each of the three community ≈ same size

Courtesy to A. Rubbia

European Policy: LAGUNA-LBNO



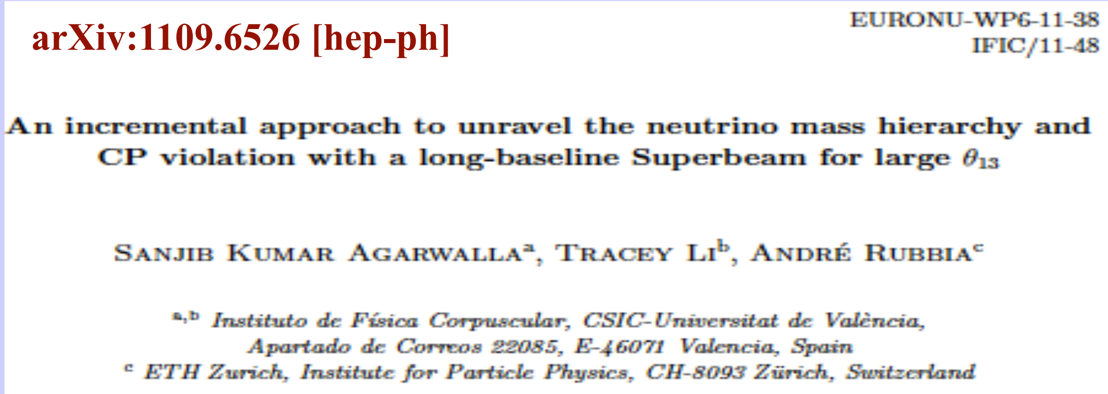
Exploit L/E dependence + strong matter effect

CN2PY (Pyhäsalmi)
- Initial : beam from SPS (500kW - 750kW)
- Long term: LP-SPL + HP-PS - >2MW

Compare neutrinos and antineutrinos

CN2FR (Fréjus)
- HP-SPL + accumulator (5 GeV – 4 MW)

CNGS - Umbria
- Beam from SPS (500kW)
- No near detector possibility



arXiv:1109.6526 [hep-ph]

EURONU-WP6-11-38
IFIC/11-48

An incremental approach to unravel the neutrino mass hierarchy and CP violation with a long-baseline Superbeam for large θ_{13}

Sanjib Kumar Agarwalla[a], Tracey Li[b], André Rubbia[c]

[a,b] *Instituto de Física Corpuscular, CSIC-Universitat de València, Apartado de Correos 22085, E-46071 Valencia, Spain*
[c] *ETH Zurich, Institute for Particle Physics, CH-8093 Zürich, Switzerland*

Large θ_{13} will have crucial impact on the optimization of the future long baseline Superbeam experiments

Large θ_{13} allows us to pursue a staged approach in terms of the size of the experiments

Progressive increase of the beam power & detector mass

Produce significant physics results at each phase !!

CERN-Pyhäsalmi Superbeam Experiment

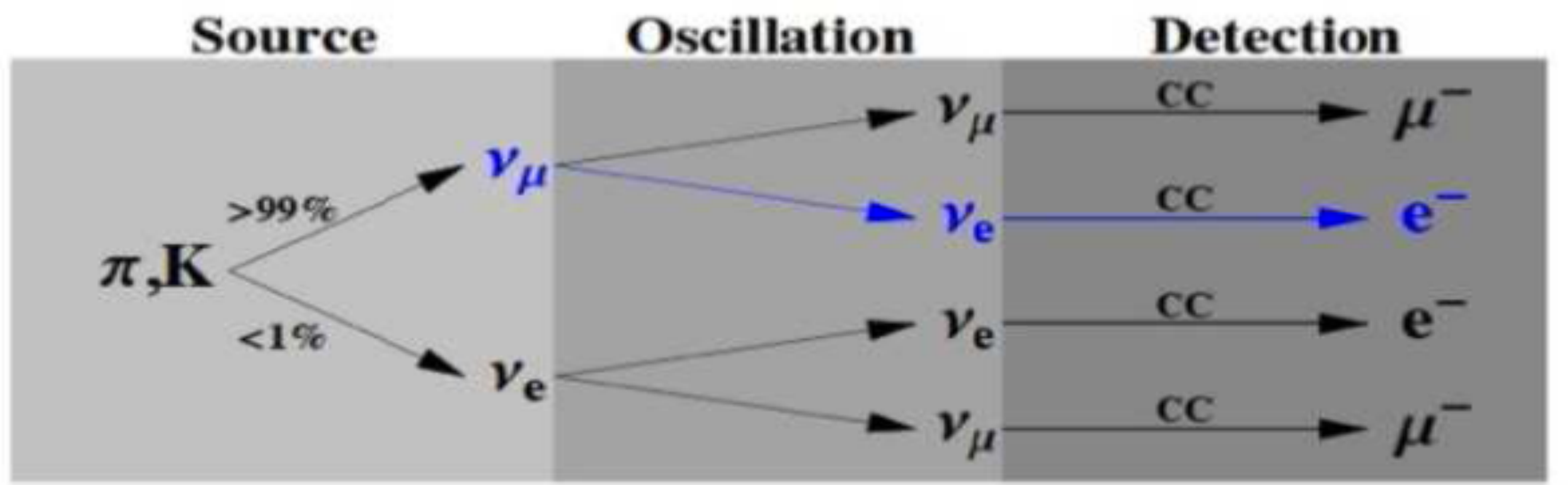
Longest Baseline in Europe → CERN-Pyhäsalm = 2290 km → Strong Matter Effect

New, upgraded, intense wide band superbeam from CERN

ν_μ oscillates into ν_e

2290 km

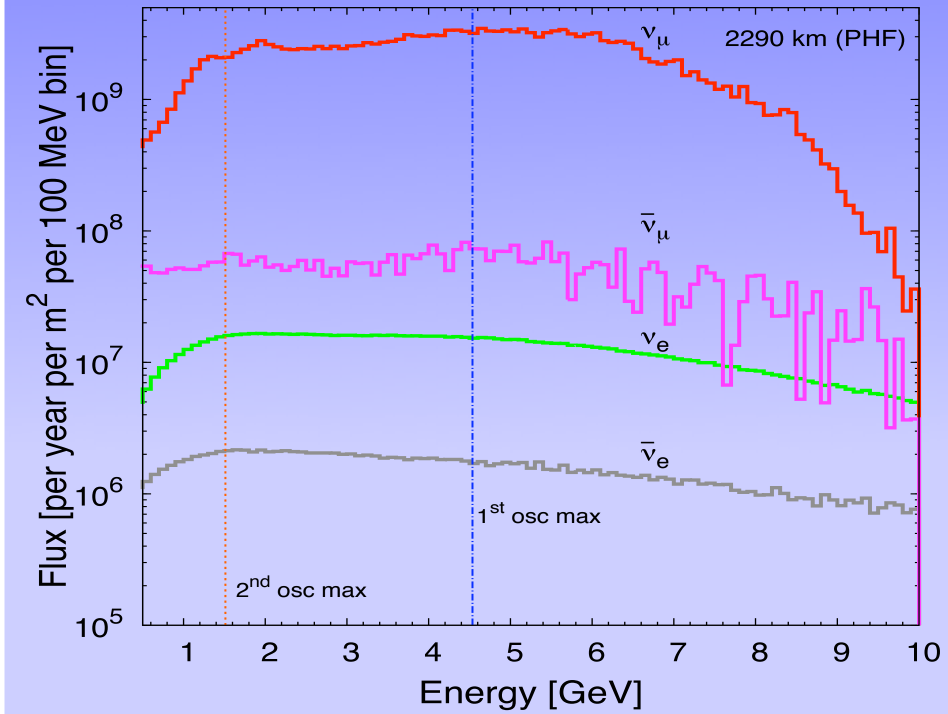
LAr Detector at Pyhäsalmi Pilot (10 kt) to Giant (100 kt)

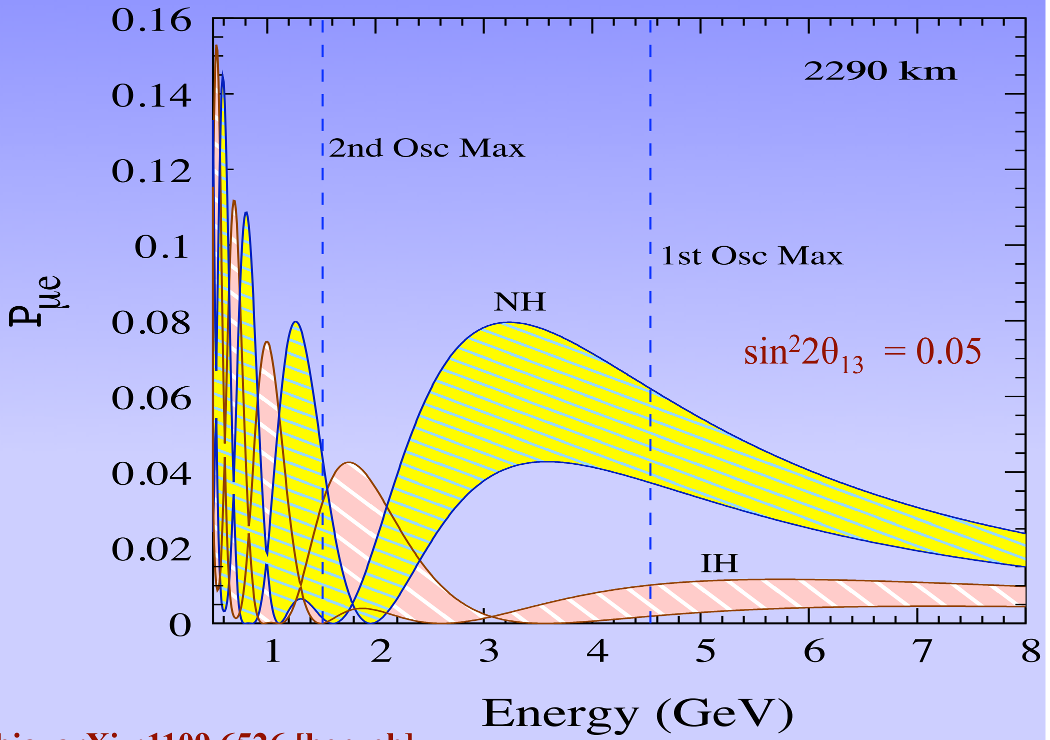


Wide Band Beam → Higher statistics → cover several L/E values → kill clone solutions

LAr Detector → Excellent Detection efficiency at 1st and 2nd Oscillation maxima

Superbeam Flux and Platinum Channel ($P_{\mu e}$)





Agarwalla, Li, Rubbia, arXiv:1109.6526 [hep-ph]

- *New high power accelerator (HP-PS2)*
- *50 GeV proton beam, power 1.6 MW*
- *3 × 10^{21} protons on target/yr (200 days/yr)*
- *@ flux level, 0.62% intrinsic ν_e contamination*

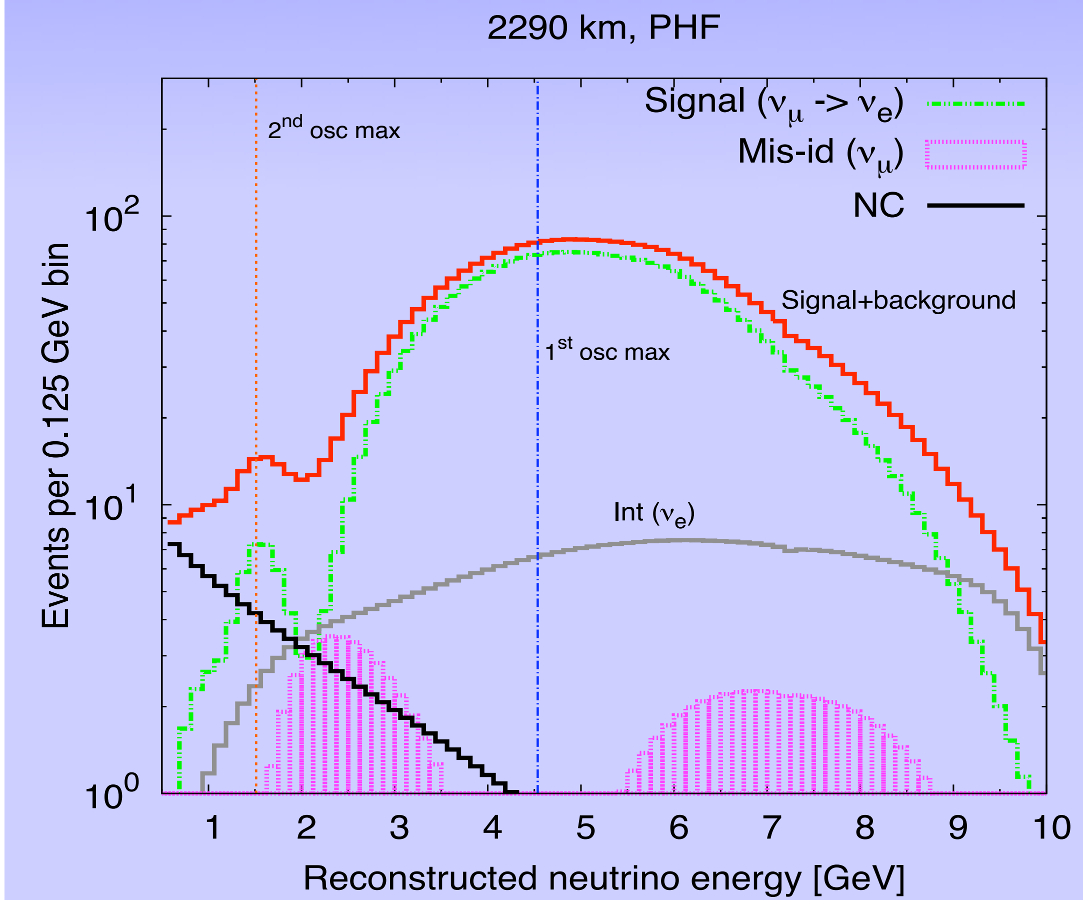
- *Both 1st and 2nd Osc. Maxima important*
- *High L, High E, High cross-section*
- *Less uncertainties in σ at high E*

Signal and Background

$$N_i = \frac{T\, n_n\, \epsilon}{4\pi L^2} \int_0^{E_{\rm max}} dE \int_{E_{A_i}^{\rm min}}^{E_{A_i}^{\rm max}} dE_A\, \phi(E)\, \sigma_{\nu_e}(E)\, R(E, E_A)\, P_{\mu e}(E)$$

exposure = (pot per year) × (fiducial mass of detector in kt) × (total runtime in years)

It has units of pot.kt



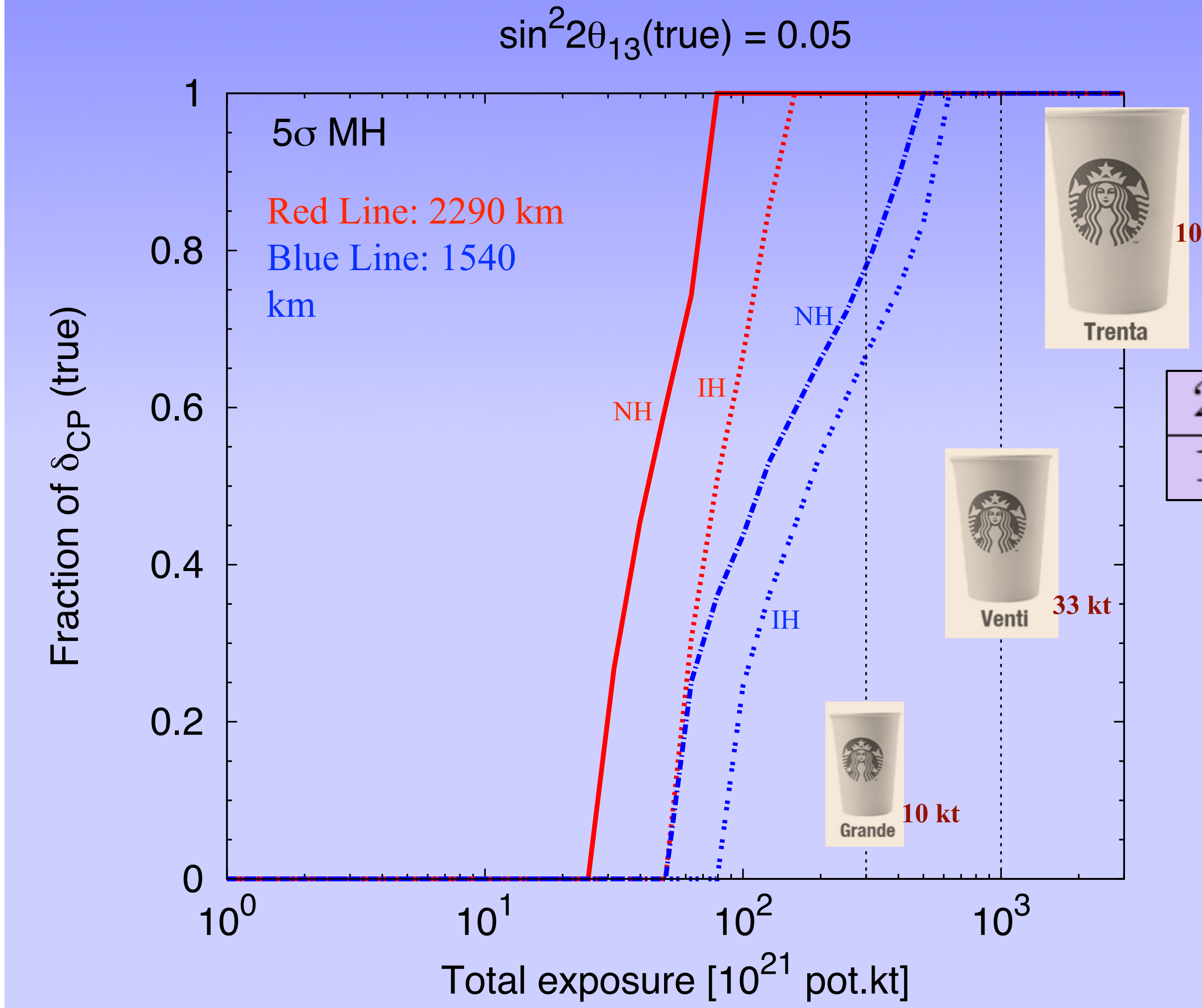
$\sin^2 2\theta_{13} = 0.05$, $\delta_{CP} = 0^o$, 1500×10^{21} pot.kt

Channel	CERN-Pyhäsalmi (2290 km)	
	Signal	Background
	CC	Int+Mis-id+NC = Total
$\nu_\mu \to \nu_e$ (NH)	**2364**	419+100+103=**622**
$\nu_\mu \to \nu_e$ (IH)	**485**	439+100+103=**642**
$\bar{\nu}_\mu \to \bar{\nu}_e$ (NH)	**304**	128+42+45=**215**
$\bar{\nu}_\mu \to \bar{\nu}_e$ (IH)	**1049**	122+43+45= **210**

Agarwalla, Li, Rubbia, arXiv:1109.6526 [hep-ph]

Intrinsic ν_e contamination causes highest background: Near Detector must

Neutrino Mass Hierarchy Discovery



1540 km: CERN-Slanic (Romania)
Similar to FNAL-Homestake (1290 km)

	Total exposure (× 10^{21} pot·kt)	
	NH true	IH true
2290 km	32 (80)	50 (158)
1540 km	158 (502)	250 (627)
	3σ 5σ	3σ 5σ

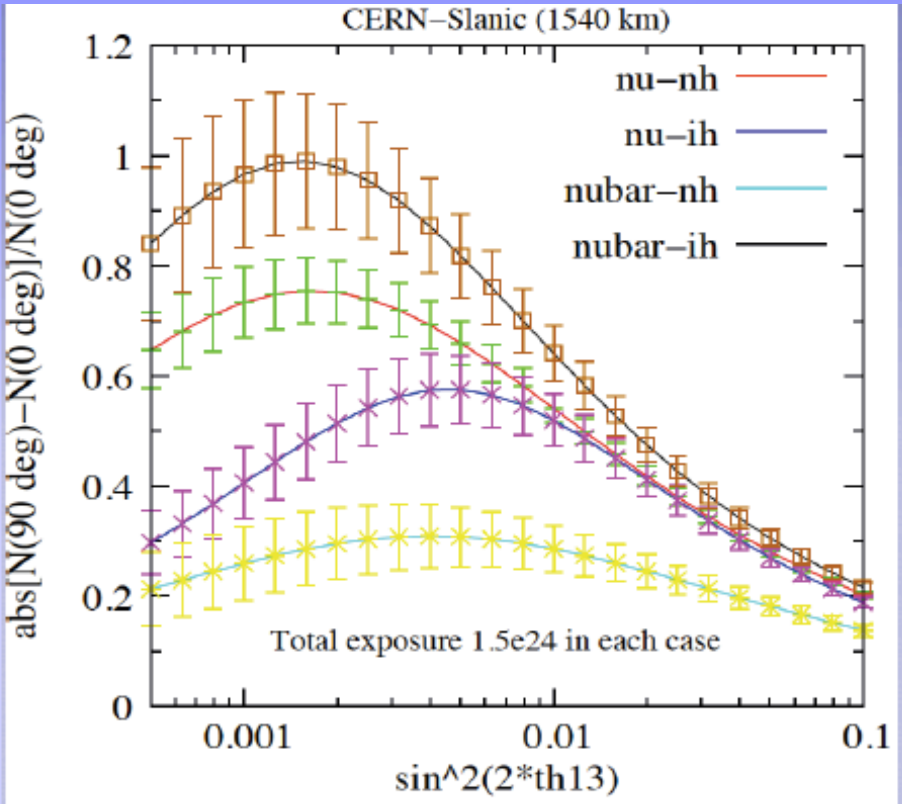
Total exposure needed to achieve MH discovery with 100% coverage in δ_{CP} (true)

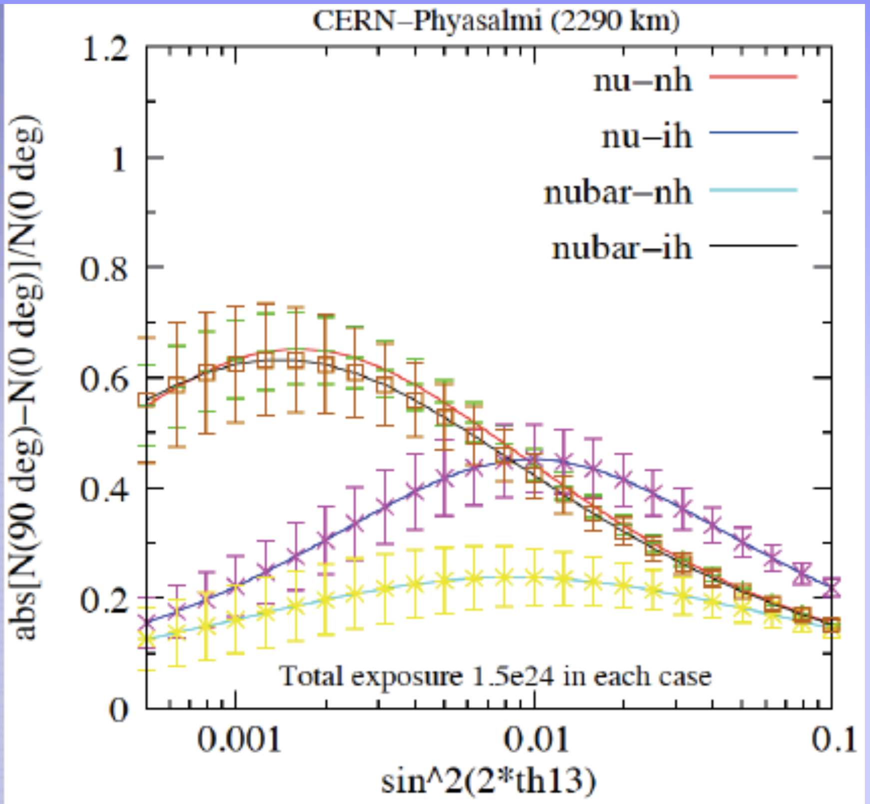
Agarwalla, Li, Rubbia, arXiv:1109.6526 [hep-ph]

Equal sharing of neutrino & anti-neutrino running. NH requires less exposure than IH

Neutrino Mass ordering will be discovered at 5σ with lowest exposure at 2290 km

CP violation discovery is never easy!

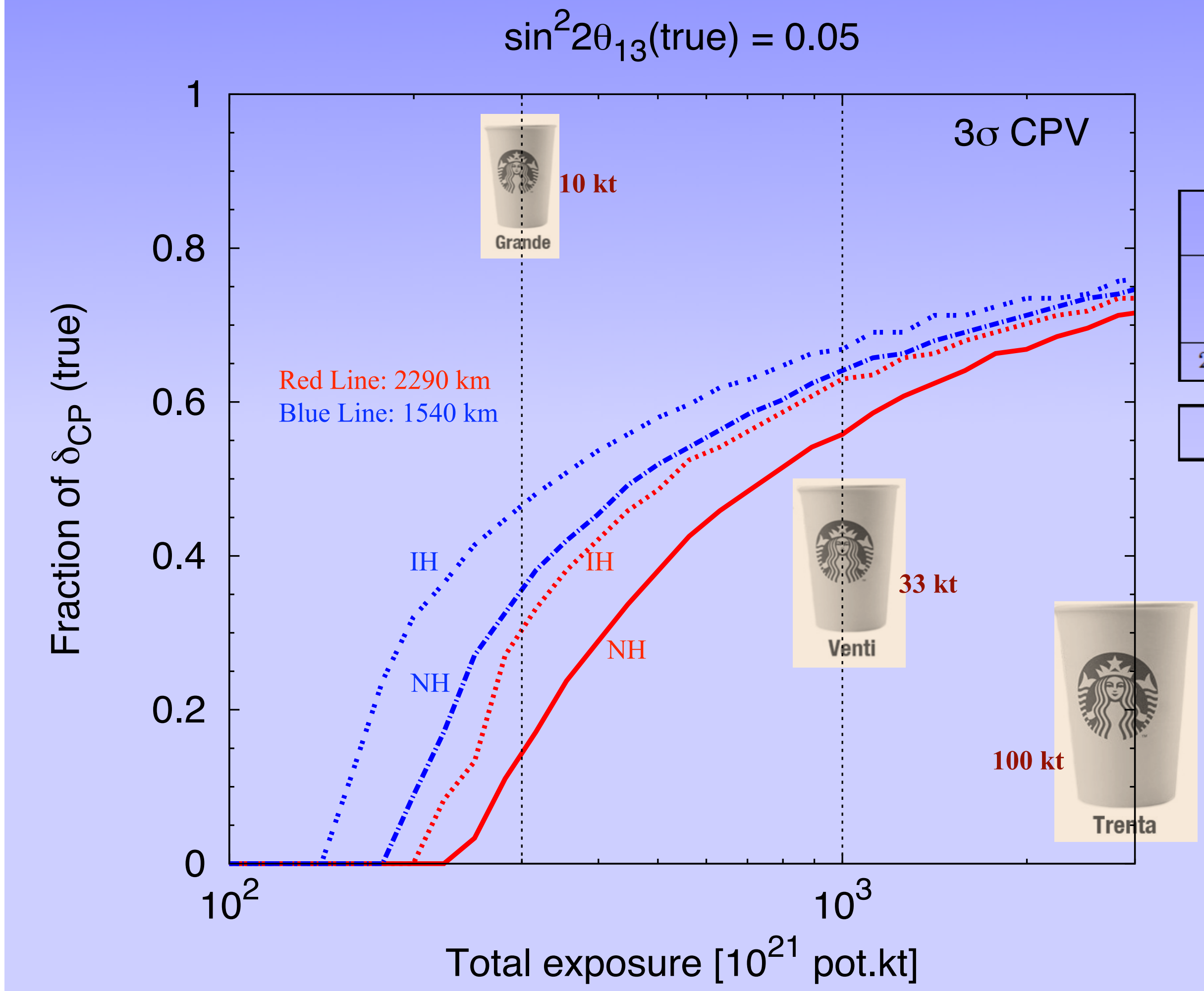




Agarwalla, work in progress

CP violation measurement is never easy even for the largest values of θ_{13}

Leptonic CP violation Discovery



Fraction of δ_{CP}(true) at 3 σ (%)

Exposure 300×10^{21} pot·kt		Exposure 1000×10^{21} pot·kt		Exposure 3000×10^{21} pot·kt	
2290 km	1540 km	2290 km	1540 km	2290 km	1540 km
14 (31)	36 (47)	56 (63)	65 (67)	72 (74)	75 (76)

NH (IH)

δ_{CP} fractions for which a discovery at 3σ is possible for CPV

Agarwalla, Li, Rubbia, arXiv:1109.6526 [hep-ph]

1540 & 2290 km are both optimal for CP measurement

Equal sharing of neutrino & anti-neutrino running. IH requires less exposure than NH

We can cover 30%, 50% & 70% of parameter space with 3 defining exposures

Is there any fastest way of determining the mass hierarchy?

We have one proposal!

Probing the Neutrino Mass Hierarchy with Super-Kamiokande

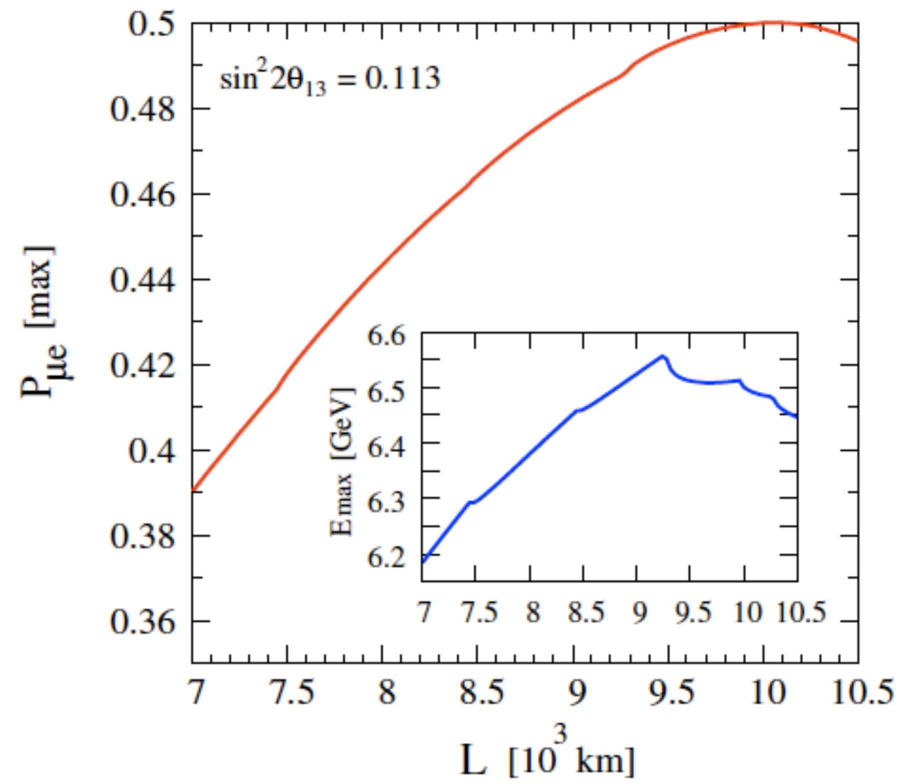
Agarwalla, Hernanadez, arXiv:1204.4217 [hep-ph]

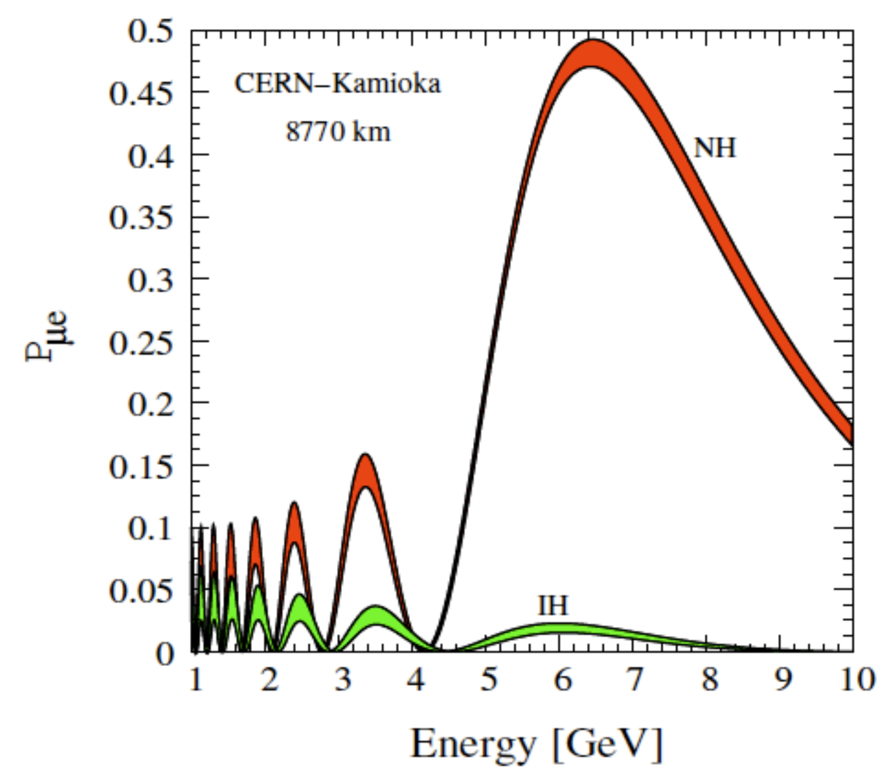
Send a superbeam (average energy of 5 GeV) from CERN towards existing and well-understood Super-Kamiokande (L = 8770 km)

This setup can reveal the neutrino MH at 5σ in less than two years irrespective of the true hierarchy and CP phase

The measurement relies on the near resonant matter effect in the ν_μ to ν_e oscillation channel, & can be done counting the total number of appearance events with just a neutrino beam

CERN-Kamioka (8770 km)





$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\tilde{\theta}_{13}\sin^2\left(\frac{\Delta\tilde{m}^2_{31}L}{4E}\right)$$

$$\sin^2 2\tilde{\theta}_{13} \equiv \sin^2 2\theta_{13}\left(\frac{\Delta m^2_{31}}{\Delta\tilde{m}^2_{31}}\right)^2$$

$$\Delta\tilde{m}^2_{31} \equiv \sqrt{(\Delta m^2_{31}\cos 2\theta_{13} - A)^2 + (\Delta m^2_{31}\sin 2\theta_{13})^2}$$

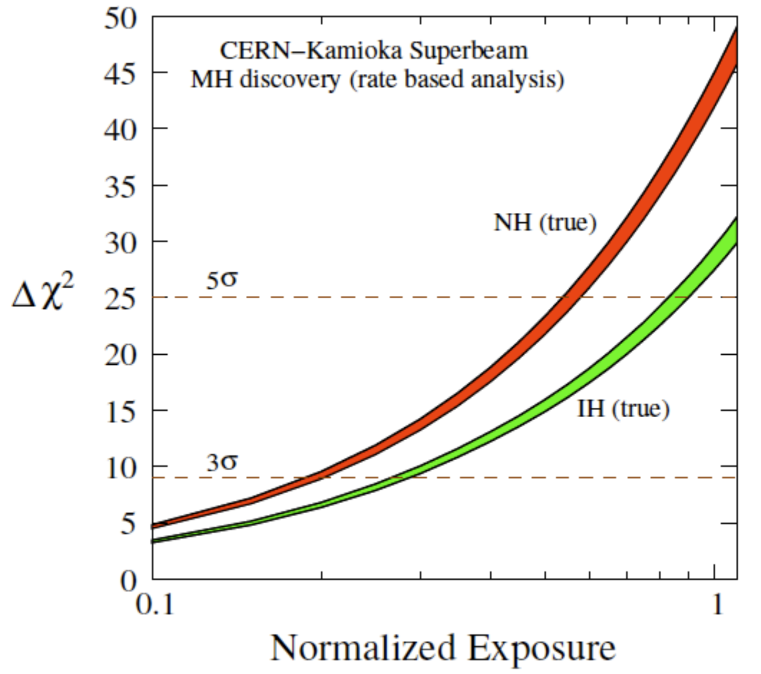
$$\sin^2 2\tilde{\theta}_{13}|_{E=E_{\rm res}} = 1, \quad E_{\rm res} \equiv \frac{\Delta m^2_{31}\cos 2\theta_{13}}{2\sqrt{2}G_F n_e}$$

$$n_e(L)L|_{L_{\max}} = \frac{\pi}{\sqrt{2}G_F\tan 2\theta_{13}}$$

Results

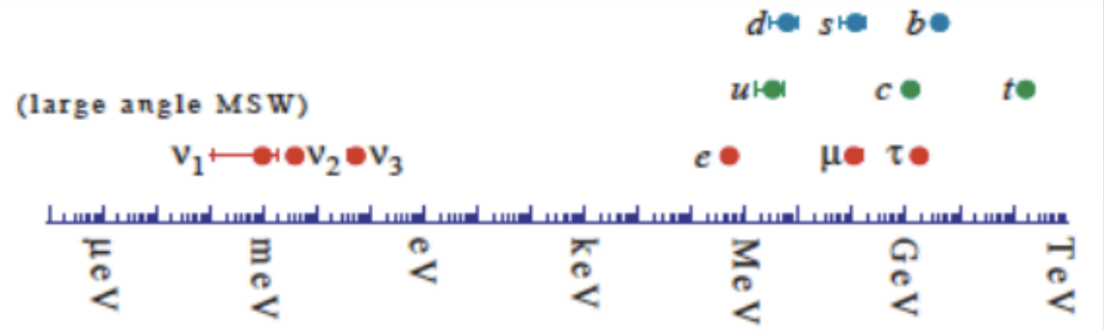
Central (true) Values	External 1σ error
$\sin^2 2\theta_{13}(\text{true}) = 0.113$	$\sigma(\sin^2 2\theta_{13}) = 0.023$
$\Delta m^2_{31}(\text{true}) = 2.45 \times 10^{-3}$ eV^2 (NH)	$\sigma(\Delta m^2_{31}) = 5\%$
$\Delta m^2_{31}(\text{true}) = -2.34 \times 10^{-3}$ eV^2 (IH)	$\sigma(\Delta m^2_{31}) = 5\%$
$\theta_{23}(\text{true}) = 45°$	$\sigma(\theta_{23}) = 10\%$
$\Delta m^2_{21}(\text{true}) = 7.59 \times 10^{-5}$ eV^2	$\sigma(\Delta m^2_{21}) = 3\%$
$\theta_{12}(\text{true}) = 33.96°$	$\sigma(\theta_{12}) = 3\%$
$\rho(\text{true}) = 1$	$\sigma(\rho) = 2\%$

Channel	CERN-Kamioka (8870 km)	
	Signal	Background
	CC-1 ring	Int+Mis-id+NC = Total
$\nu_\mu \to \nu_e$ (NH)	**44**	1+2+16=**19**
$\nu_\mu \to \nu_e$ (IH)	**2**	1+3+16=**20**
$\nu_\mu \to \nu_\mu$ (NH)	**83**	**2**
$\nu_\mu \to \nu_\mu$ (IH)	**91**	**2**



S. K. Agarwalla, Calcutta University, Kolkata, India, 26th April, 2012

Ultimately What Are We Trying To Understand?



Why are neutrino masses so small?

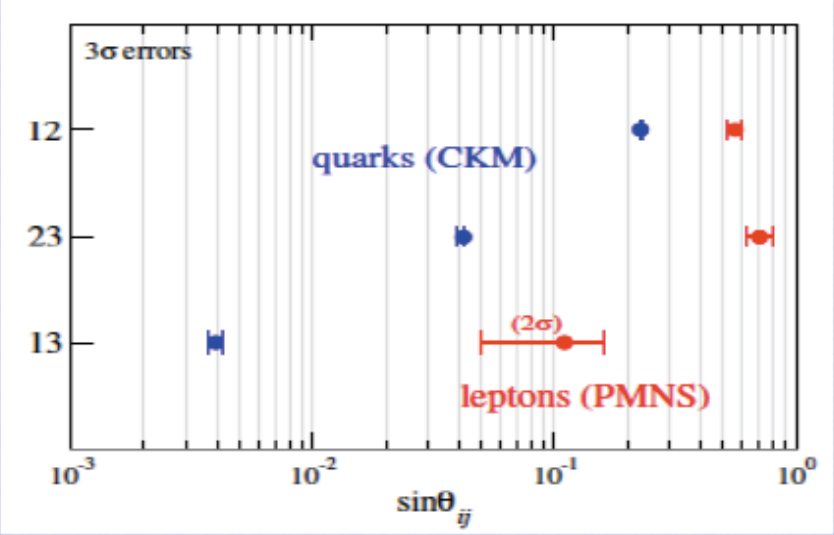
$$U_{CKM} = \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

Quark mixing

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \epsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Lepton mixing

Why are neutrino mixings so large?



Need to measure PMNS parameters as precisely as CKM parameters!

Our "Holly Grail"

Can shed light onto flavor puzzle!

New Experiments are must to achieve this!

Concluding Remarks

Neutrino oscillation is an exclusive example of experimental evidence for physics beyond the Standard Model

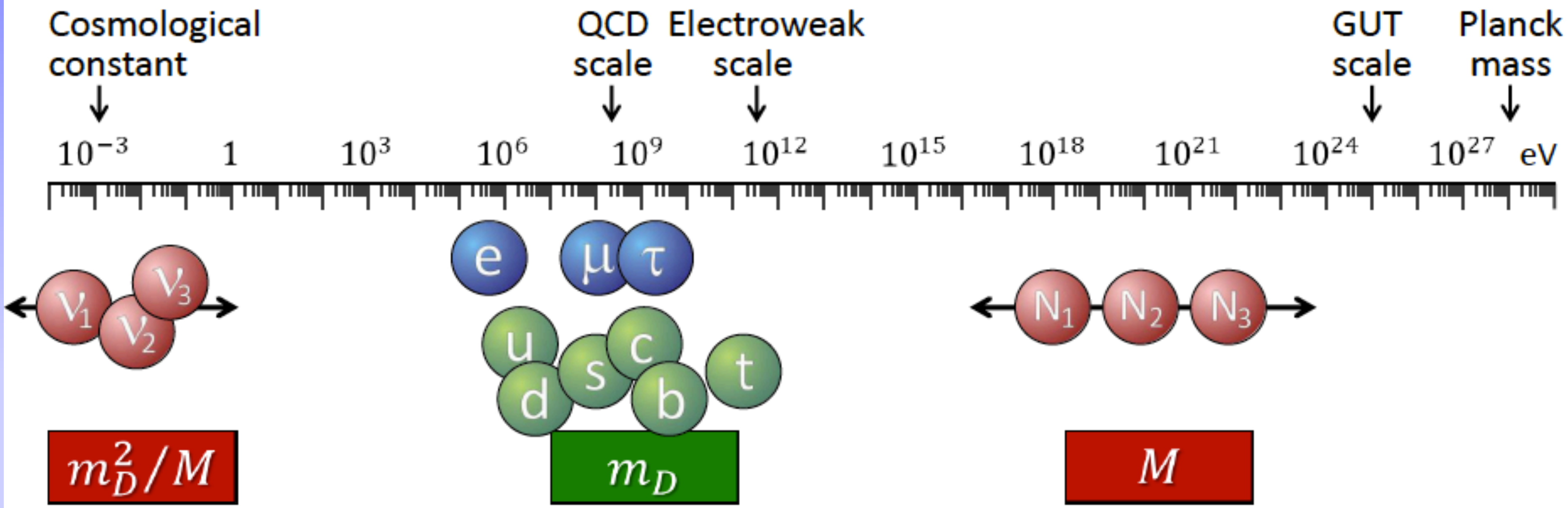
Recent results on 1-3 mixing angle is very exciting!

Following the recent discoveries, we need to re-optmize the future neutrino roadmap to explore the information on mass hierarchy and leptonic CP violation!

The work has been started just now……

Thank you!

Backup Slides: See-Saw & Neutrino Mass



Mass matrix for one family of ordinary and heavy r.h. neutrinos

$$(\bar{\nu}_L, \bar{N}_R)\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ N_R \end{pmatrix}$$

Diagonalization

$$(\bar{\nu}_L, \bar{N}_R)\begin{pmatrix} m_D^2/M & 0 \\ 0 & M \end{pmatrix}\begin{pmatrix} \nu_L \\ N_R \end{pmatrix}$$

One light and one heavy Majorana neutrino

Courtesy to G. Raffelt